Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.3-c+d-x-^m-trig^n-trig^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [397]. This is test number [137].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (397)	% 0. (0)
Mathematica	% 100. (397)	% 0. (0)
Maple	% 90.43 (359)	% 9.57 (38)
Maxima	% 73.3 (291)	% 26.7 (106)
Fricas	% 91.94 (365)	% 8.06 (32)
Sympy	% 25.69 (102)	% 74.31 (295)
Giac	% 55.92 (222)	% 44.08 (175)

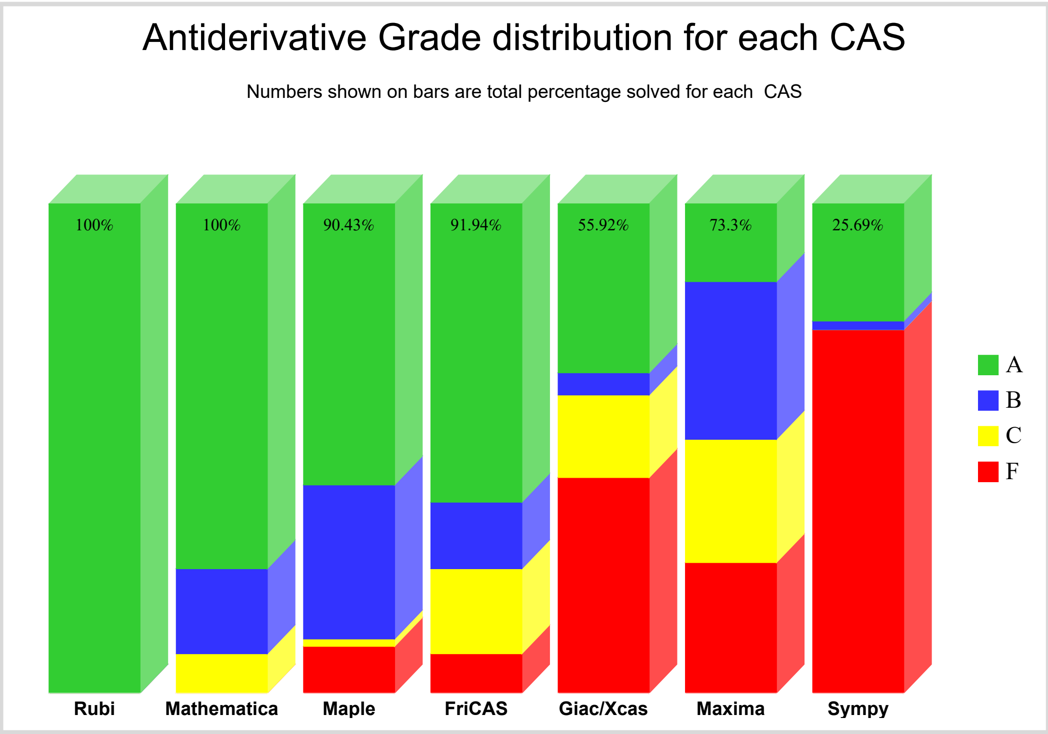
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

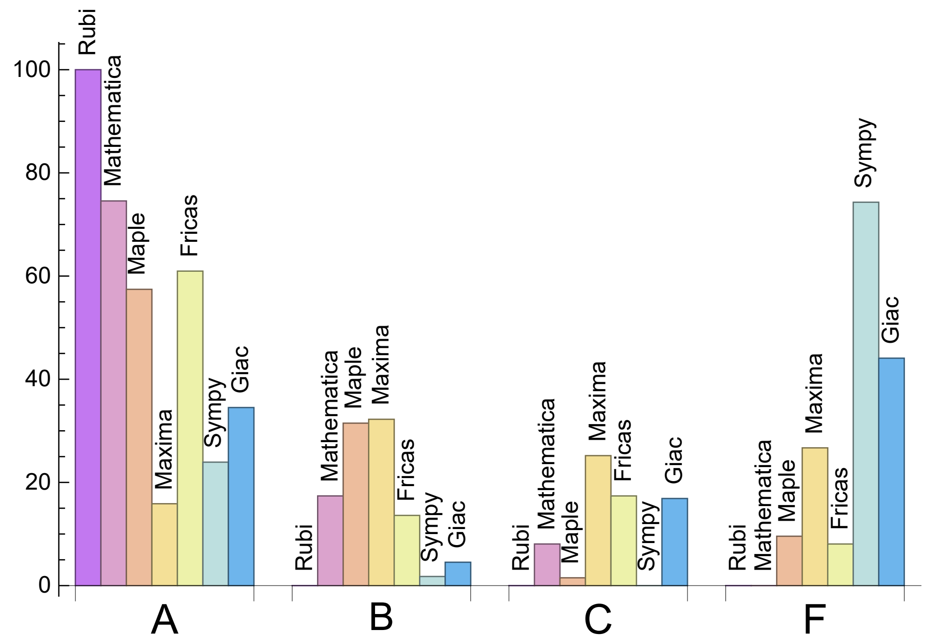
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	74.56	17.38	8.06	0.
Maple	57.43	31.49	1.51	9.57
Maxima	15.87	32.24	25.19	26.7
Fricas	60.96	13.6	17.38	8.06
Sympy	23.93	1.76	0.	74.31
Giac	34.51	4.53	16.88	44.08

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	149.21	0.77	115.	1.
Mathematica	4.79	277.11	1.3	132.	0.92
Maple	0.36	344.87	1.81	242.	1.41
Maxima	2.2	1482.18	7.27	706.	4.6
Fricas	0.48	1006.59	5.1	599.	3.08
Sympy	10.68	208.78	1.5	0.	0.
Giac	0.86	1058.52	7.85	155.	1.53

1.4 list of integrals that has no closed form antiderivative

{31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {32, 33, 34, 35, 39, 41, 46, 47, 58, 63, 99, 105, 106, 107, 130, 131, 134, 135, 164, 165, 166, 171, 173, 178, 179, 180, 181, 190, 195, 222, 223, 236, 237, 241, 242, 249, 254, 255, 261, 267, 268, 280, 285, 291, 292, 300, 304, 305, 306, 310, 311, 312, 318, 319, 331, 376, 382, 383, 384, 385, 389, 390}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

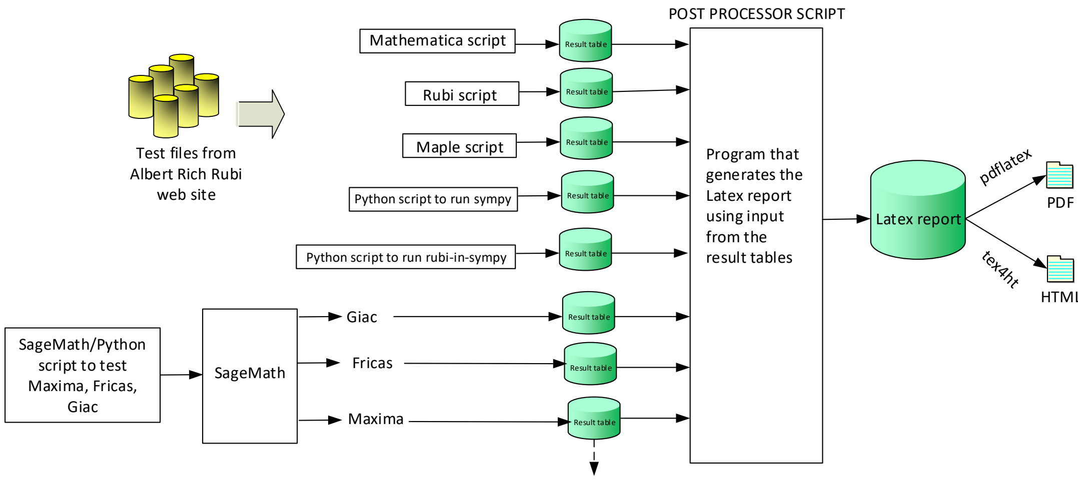
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 43, 44, 45, 49, 50, 51, 52, 53, 54, 55, 56, 57, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 109, 110, 111, 113, 116, 117, 124, 125, 126, 127, 128, 129, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 219, 220, 221, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 279, 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 296, 297, 298, 301, 302, 303, 307, 308, 309, 313, 314, 315, 316, 317, 320, 321, 322, 323, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 343, 344, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 32, 33, 34, 35, 40, 41, 42, 46, 47, 98, 105, 106, 107, 112, 114, 115, 164, 165, 166, 171, 172, 173, 178, 179, 180, 181, 216, 218, 222, 223, 228, 235, 236, 241, 242, 250, 254, 255, 260, 261, 273, 274, 280, 281, 286, 287, 291, 292, 299, 300, 304, 305, 306, 310, 311, 312, 318, 324, 325, 331, 340, 342, 382, 383, 384, 385, 389, 390, 397 }

C grade: { 48, 58, 59, 60, 61, 62, 63, 108, 118, 119, 120, 121, 122, 123, 130, 131, 132, 133, 134, 135, 190, 191, 192, 193, 194, 195, 237, 319, 345, 347, 356, 358 }

F grade: { }

2.1.3 Maple

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 102, 103, 104, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 159, 160, 161, 162, 163, 168, 169, 170, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 213, 214, 215, 219, 220, 221, 224, 225, 226, 227, 232, 233, 234, 237, 238, 239, 240, 244, 245, 246, 250, 251, 252, 253, 256, 257, 258, 259, 263, 264, 265, 269, 270, 271, 272, 276, 277, 278, 281, 282, 283, 284, 287, 288, 289, 290, 293, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 362, 363, 364, 365, 366, 367, 371, 372, 373, 374, 375, 379, 380, 381, 385, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 46, 47, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 158, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 204, 209, 210, 211, 212, 216, 217, 218, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 273, 274, 275, 279, 280, 286, 291, 292, 298, 299, 300, 304, 305, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 361, 368, 369, 370, 376, 377, 378, 382, 383, 384, 389, 390, 397 }

C grade: { 174, 262, 331, 340, 347, 356 }

F grade: { 1, 13, 22, 70, 79, 88, 136, 145, 154, 285, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359 }

2.1.4 Maxima

A grade: { 5, 17, 26, 31, 36, 37, 38, 45, 74, 92, 97, 102, 104, 111, 116, 117, 140, 149, 158, 163, 168, 170, 177, 208, 213, 214, 215, 221, 224, 227, 232, 233, 234, 240, 246, 253, 259, 265, 272, 278, 284, 290, 297, 303, 307, 308, 309, 314, 316, 322, 360, 361, 362, 363, 364, 368, 369, 370, 371, 379, 384, 386, 387

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 164, 165, 166, 167, 172, 173, 174, 178, 179, 180, 181, 205, 206, 207, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 250, 254, 255, 256, 260, 262, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 304, 305, 306, 310, 311, 312, 313, 317, 318, 323, 324, 325, 376, 377, 382, 383, 391, 396 }

C grade: { 6, 7, 8, 9, 10, 11, 12, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 365, 366, 367, 372, 373, 374, 375 }

F grade: { 1, 13, 22, 43, 44, 50, 51, 70, 79, 88, 103, 109, 110, 136, 145, 154, 169, 171, 175, 176, 182, 183, 202, 203, 204, 218, 219, 220, 225, 226, 237, 238, 239, 244, 245, 249, 251, 252, 257, 258, 261, 263, 264, 270, 271, 276, 277, 282, 283, 288, 289, 295, 296, 300, 301, 302, 315, 319, 320, 321, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 378, 380, 381, 385, 388, 389, 390, 392, 393, 394, 395, 397 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 31, 36, 37, 38, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 97, 102, 103, 104, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 154, 157, 158, 159, 160, 161, 163, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 293, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 396 }

B grade: { 8, 9, 21, 30, 35, 41, 42, 47, 78, 80, 81, 82, 86, 87, 96, 101, 107, 108, 115, 144, 153, 155, 156, 162, 167, 173, 181, 203, 207, 212, 218, 224, 231, 237, 243, 249, 250, 255, 261, 269, 274, 275, 281, 287, 292, 300, 313, 319, 325, 378, 385, 390, 395, 397 }

C grade: { 32, 33, 34, 39, 40, 46, 98, 99, 100, 105, 106, 112, 113, 114, 164, 165, 166, 171, 172, 178, 179, 180, 202, 205, 206, 209, 210, 211, 216, 217, 222, 223, 228, 229, 230, 235, 236, 241, 242, 247, 248, 254, 260, 266, 267, 268, 273, 279, 280, 285, 286, 291, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 376, 377, 382, 383, 384, 389 }

F grade: { 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359 }

2.1.6 Sympy

A grade: { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 36, 37, 43, 44, 50, 51, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 102, 103, 104, 108, 109, 110, 111, 116, 117, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 168, 169, 175, 176, 182, 183, 213, 214, 219, 220, 232, 233, 238, 239, 244, 245, 246, 251, 252, 253, 256, 257, 258, 263, 264, 270, 271, 276, 277, 282, 295, 296, 301, 302, 303, 307, 308, 314, 315, 360, 364 }

B grade: { 53, 54, 55, 56, 362, 363, 396 }

C grade: { }

F grade: { 1, 6, 7, 8, 9, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 216, 217, 218, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 254, 255, 259, 260, 261, 262, 265, 266, 267, 268, 269, 272, 273, 274, 275, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 298, 299, 300, 304, 305, 306, 309, 310, 311, 312, 313, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 361, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 31, 36, 37, 38, 43, 44, 45, 50, 51, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 97, 102, 103, 104, 109, 110, 111, 116, 117, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 321, 322, 326, 327, 360, 361, 362, 363, 364, 365, 366, 379, 380, 381, 386, 387, 388, 392, 393, 394 }

B grade: { 42, 48, 49, 108, 174, 204, 250, 256, 262, 293, 294, 367, 368, 369, 370, 371, 391, 396 }

C grade: { 6, 8, 9, 18, 27, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 84, 86, 87, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 159, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 372, 374 }

F grade: { 1, 7, 13, 19, 20, 21, 22, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 70, 76, 77, 78, 79, 85, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 136, 142, 143, 144, 145, 150, 151, 152, 153, 154, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 282, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 320, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 373, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	138	0	0	246	0	0
normalized size	1	1.	1.01	0.	0.	1.8	0.	0.
time (sec)	N/A	0.156	0.085	0.209	0.	0.516	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	86	853	791	518	502	244
normalized size	1	1.	0.55	5.47	5.07	3.32	3.22	1.56
time (sec)	N/A	0.107	0.513	0.034	1.221	0.492	7.636	1.093

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	71	466	462	348	342	163
normalized size	1	1.	0.59	3.88	3.85	2.9	2.85	1.36
time (sec)	N/A	0.083	0.3	0.015	1.132	0.483	3.107	1.145

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	50	215	231	203	175	99
normalized size	1	1.	0.56	2.42	2.6	2.28	1.97	1.11
time (sec)	N/A	0.054	0.233	0.015	1.156	0.474	1.374	1.122

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	34	74	88	108	80	51
normalized size	1	1.	0.68	1.48	1.76	2.16	1.6	1.02
time (sec)	N/A	0.026	0.101	0.014	1.121	0.464	0.599	1.142

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	84	190	213	0	768
normalized size	1	1.	0.92	1.29	2.92	3.28	0.	11.82
time (sec)	N/A	0.139	0.131	0.026	1.38	0.47	0.	1.208

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	124	221	333	0	0
normalized size	1	1.	0.94	1.46	2.6	3.92	0.	0.
time (sec)	N/A	0.149	0.316	0.02	1.451	0.493	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	102	162	269	525	0	7287
normalized size	1	1.	0.89	1.42	2.36	4.61	0.	63.92
time (sec)	N/A	0.175	1.091	0.02	1.741	0.513	0.	1.605

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	164	200	336	703	0	10249
normalized size	1	1.	1.14	1.39	2.33	4.88	0.	71.17
time (sec)	N/A	0.198	0.661	0.021	2.113	0.533	0.	1.809

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	18	31	5	8
normalized size	1	1.	1.	0.88	2.25	3.88	0.62	1.
time (sec)	N/A	0.029	0.006	0.026	1.219	0.452	1.075	1.117

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	20	95	22	26
normalized size	1	1.	1.	0.94	1.25	5.94	1.38	1.62
time (sec)	N/A	0.046	0.006	0.026	1.208	0.459	2.16	1.121

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	20	96	24	35
normalized size	1	1.	1.	0.9	0.69	3.31	0.83	1.21
time (sec)	N/A	0.059	0.008	0.029	1.265	0.469	1.596	1.134

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	237	0	0	471	0	0
normalized size	1	1.	0.86	0.	0.	1.71	0.	0.
time (sec)	N/A	0.33	0.723	0.335	0.	0.554	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	385	835	1188	759	646	473
normalized size	1	1.	1.88	4.07	5.8	3.7	3.15	2.31
time (sec)	N/A	0.2	1.514	0.058	1.358	0.521	11.338	1.129

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	121	447	674	491	391	312
normalized size	1	1.	0.8	2.96	4.46	3.25	2.59	2.07
time (sec)	N/A	0.135	0.992	0.019	1.118	0.502	5.268	1.166

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	204	324	296	216	185
normalized size	1	1.	0.9	1.98	3.15	2.87	2.1	1.8
time (sec)	N/A	0.077	0.614	0.016	1.11	0.48	2.62	1.141

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	71	115	149	85	93
normalized size	1	1.	0.86	1.39	2.25	2.92	1.67	1.82
time (sec)	N/A	0.033	0.182	0.018	1.132	0.472	1.178	1.102

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	370	404	0	8180
normalized size	1	1.	0.84	1.37	3.06	3.34	0.	67.6
time (sec)	N/A	0.27	0.335	0.017	1.478	0.483	0.	1.737

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	139	242	408	593	0	0
normalized size	1	1.	0.83	1.44	2.43	3.53	0.	0.
time (sec)	N/A	0.301	1.435	0.023	1.771	0.541	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	183	311	455	914	0	0
normalized size	1	1.	0.83	1.41	2.06	4.14	0.	0.
time (sec)	N/A	0.358	2.238	0.021	2.273	0.575	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	298	384	522	1243	0	0
normalized size	1	1.	1.1	1.42	1.93	4.6	0.	0.
time (sec)	N/A	0.42	1.752	0.024	2.808	0.637	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	246	0	0	486	0	0
normalized size	1	1.	0.91	0.	0.	1.79	0.	0.
time (sec)	N/A	0.331	0.3	0.265	0.	0.551	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	158	1143	1305	936	976	487
normalized size	1	1.	0.61	4.4	5.02	3.6	3.75	1.87
time (sec)	N/A	0.241	1.81	0.063	1.313	0.54	18.541	1.085

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	135	594	741	621	634	325
normalized size	1	1.	0.69	3.03	3.78	3.17	3.23	1.66
time (sec)	N/A	0.165	0.97	0.019	1.213	0.518	10.49	1.133

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	91	260	355	359	350	196
normalized size	1	1.	0.68	1.94	2.65	2.68	2.61	1.46
time (sec)	N/A	0.092	0.524	0.02	1.186	0.486	5.116	1.139

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	75	85	124	192	160	101
normalized size	1	1.	1.04	1.18	1.72	2.67	2.22	1.4
time (sec)	N/A	0.045	0.116	0.021	1.093	0.474	2.371	1.114

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	370	421	0	8162
normalized size	1	1.	0.85	1.38	2.87	3.26	0.	63.27
time (sec)	N/A	0.232	0.416	0.021	1.437	0.48	0.	1.72

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	406	621	0	0
normalized size	1	1.	0.84	1.43	2.27	3.47	0.	0.
time (sec)	N/A	0.28	1.296	0.023	1.778	0.554	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	199	329	454	976	0	0
normalized size	1	1.	0.87	1.44	1.98	4.26	0.	0.
time (sec)	N/A	0.344	2.826	0.024	2.171	0.617	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	521	1293	0	0
normalized size	1	1.	1.1	1.41	1.82	4.51	0.	0.
time (sec)	N/A	0.389	2.406	0.024	3.091	0.685	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	2.539	0.234	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	799	1150	1704	2938	0	0
normalized size	1	1.	5.29	7.62	11.28	19.46	0.	0.
time (sec)	N/A	0.22	5.769	0.329	1.978	0.694	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	560	783	1008	2056	0	0
normalized size	1	1.	4.41	6.17	7.94	16.19	0.	0.
time (sec)	N/A	0.192	2.76	0.296	1.701	0.627	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	356	468	545	1330	0	0
normalized size	1	1.	3.83	5.03	5.86	14.3	0.	0.
time (sec)	N/A	0.166	1.425	0.27	1.526	0.567	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	65	65	188	215	255	721	0	0
normalized size	1	1.	2.89	3.31	3.92	11.09	0.	0.
time (sec)	N/A	0.096	5.18	0.248	1.532	0.547	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	3.585	0.244	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	7.065	0.279	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	1.242	0.155	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	308	716	3974	2569	0	0
normalized size	1	1.	1.48	3.44	19.11	12.35	0.	0.
time (sec)	N/A	0.172	1.375	0.28	2.5	0.7	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	311	433	2390	1740	0	0
normalized size	1	1.	2.13	2.97	16.37	11.92	0.	0.
time (sec)	N/A	0.116	1.187	0.256	1.817	0.605	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	90	90	234	212	751	1006	0	0
normalized size	1	1.	2.6	2.36	8.34	11.18	0.	0.
time (sec)	N/A	0.062	2.051	0.205	1.634	0.559	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	131	52	350	181	0	1081
normalized size	1	1.	4.37	1.73	11.67	6.03	0.	36.03
time (sec)	N/A	0.02	0.058	0.024	1.092	0.488	0.	1.527

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	16.76	0.295	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	20.204	0.411	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	3.292	0.162	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	504	716	6129	2562	0	0
normalized size	1	1.	3.68	5.23	44.74	18.7	0.	0.
time (sec)	N/A	0.256	6.602	0.188	2.477	0.695	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	277	409	1411	1445	0	0
normalized size	1	1.	2.41	3.56	12.27	12.57	0.	0.
time (sec)	N/A	0.174	6.412	0.164	2.279	0.61	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	94	95	1526	231	0	4701
normalized size	1	1.	1.74	1.76	28.26	4.28	0.	87.06
time (sec)	N/A	0.065	0.918	0.029	1.346	0.504	0.	2.577

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	387	103	0	710
normalized size	1	1.	1.37	1.74	11.06	2.94	0.	20.29
time (sec)	N/A	0.031	0.074	0.029	1.166	0.457	0.	1.209

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.137	11.553	0.445	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	11.151	0.595	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	179	234	910	540	0	1327
normalized size	1	1.	0.91	1.19	4.64	2.76	0.	6.77
time (sec)	N/A	0.448	2.37	0.037	1.956	0.551	0.	1.318

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	157	187	869	413	665	732
normalized size	1	1.	0.93	1.11	5.17	2.46	3.96	4.36
time (sec)	N/A	0.295	0.886	0.025	1.888	0.532	130.886	1.23

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	134	142	788	308	389	316
normalized size	1	1.	0.94	1.	5.55	2.17	2.74	2.23
time (sec)	N/A	0.231	0.269	0.023	2.06	0.521	5.903	1.169

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	134	142	788	308	389	316
normalized size	1	1.	0.94	1.	5.55	2.17	2.74	2.23
time (sec)	N/A	0.221	0.026	0.023	1.978	0.52	5.846	1.158

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	157	187	869	413	665	732
normalized size	1	1.	0.93	1.11	5.17	2.46	3.96	4.36
time (sec)	N/A	0.275	0.042	0.026	1.941	0.531	132.766	1.261

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	179	234	910	540	0	1327
normalized size	1	1.	0.91	1.19	4.64	2.76	0.	6.77
time (sec)	N/A	0.334	0.081	0.024	2.034	0.546	0.	1.315

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	1171	474	1866	918	0	2722
normalized size	1	1.	2.88	1.17	4.6	2.26	0.	6.7
time (sec)	N/A	1.137	15.523	0.04	2.503	0.643	0.	1.589

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	677	386	1787	756	0	1517
normalized size	1	1.	1.92	1.09	5.06	2.14	0.	4.3
time (sec)	N/A	0.684	9.307	0.036	2.536	0.612	0.	1.37

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	280	294	1648	639	0	662
normalized size	1	1.	0.92	0.97	5.42	2.1	0.	2.18
time (sec)	N/A	0.47	6.566	0.036	2.312	0.583	0.	1.244

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	280	294	1648	639	0	662
normalized size	1	1.	0.92	0.97	5.42	2.1	0.	2.18
time (sec)	N/A	0.468	6.489	0.035	2.431	0.586	0.	1.28

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	677	386	1787	756	0	1517
normalized size	1	1.	1.92	1.09	5.06	2.14	0.	4.3
time (sec)	N/A	0.571	9.306	0.04	2.375	0.607	0.	1.374

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	1171	474	1866	918	0	2722
normalized size	1	1.	2.88	1.17	4.6	2.26	0.	6.7
time (sec)	N/A	0.668	14.663	0.036	2.456	0.638	0.	1.562

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	1874	1011	0	2681
normalized size	1	1.	1.35	1.15	4.6	2.48	0.	6.59
time (sec)	N/A	1.051	15.219	0.036	2.361	0.693	0.	1.861

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	1804	802	0	1485
normalized size	1	1.	1.12	1.07	5.14	2.28	0.	4.23
time (sec)	N/A	0.674	3.344	0.039	2.318	0.647	0.	1.558

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	1655	630	0	643
normalized size	1	1.	0.88	0.96	5.54	2.11	0.	2.15
time (sec)	N/A	0.499	0.874	0.034	2.308	0.607	0.	1.337

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	1655	630	0	643
normalized size	1	1.	0.88	0.96	5.54	2.11	0.	2.15
time (sec)	N/A	0.46	0.326	0.035	2.447	0.608	0.	1.364

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	1804	802	0	1485
normalized size	1	1.	1.12	1.07	5.14	2.28	0.	4.23
time (sec)	N/A	0.566	3.129	0.033	2.471	0.646	0.	1.539

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	1874	1011	0	2681
normalized size	1	1.	1.35	1.15	4.6	2.48	0.	6.59
time (sec)	N/A	0.697	11.788	0.033	2.415	0.687	0.	1.799

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	250	0	0	460	0	0
normalized size	1	1.	0.94	0.	0.	1.72	0.	0.
time (sec)	N/A	0.283	0.476	0.239	0.	0.547	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	150	835	1200	636	646	473
normalized size	1	1.	0.73	4.07	5.85	3.1	3.15	2.31
time (sec)	N/A	0.203	1.623	0.043	1.395	0.526	10.47	1.205

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	127	447	682	404	391	312
normalized size	1	1.	0.84	2.96	4.52	2.68	2.59	2.07
time (sec)	N/A	0.133	0.961	0.018	1.175	0.497	5.385	1.203

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	86	204	328	236	216	185
normalized size	1	1.	0.83	1.98	3.18	2.29	2.1	1.8
time (sec)	N/A	0.079	0.51	0.018	1.208	0.483	2.627	1.153

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	71	71	116	112	85	93
normalized size	1	1.	1.39	1.39	2.27	2.2	1.67	1.82
time (sec)	N/A	0.034	0.153	0.018	1.117	0.474	1.135	1.204

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	100	167	369	404	0	8477
normalized size	1	1.	0.83	1.38	3.05	3.34	0.	70.06
time (sec)	N/A	0.224	0.304	0.02	1.383	0.482	0.	1.771

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	139	240	405	586	0	0
normalized size	1	1.	0.83	1.43	2.41	3.49	0.	0.
time (sec)	N/A	0.266	1.135	0.022	1.725	0.538	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	181	313	452	907	0	0
normalized size	1	1.	0.82	1.42	2.05	4.1	0.	0.
time (sec)	N/A	0.324	2.615	0.02	2.171	0.59	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	300	381	520	1231	0	0
normalized size	1	1.	1.11	1.41	1.93	4.56	0.	0.
time (sec)	N/A	0.377	1.933	0.023	2.787	0.626	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	213	0	0	342	0	0
normalized size	1	1.	1.31	0.	0.	2.11	0.	0.
time (sec)	N/A	0.205	1.104	0.209	0.	0.527	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	132	1915	992	986	1209	302
normalized size	1	1.	1.01	14.62	7.57	7.53	9.23	2.31
time (sec)	N/A	0.164	1.367	0.076	1.371	0.531	17.601	1.16

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	106	1074	597	647	813	207
normalized size	1	1.	1.01	10.23	5.69	6.16	7.74	1.97
time (sec)	N/A	0.13	0.685	0.022	1.304	0.514	9.976	1.14

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	77	519	313	398	484	127
normalized size	1	1.	0.97	6.57	3.96	5.04	6.13	1.61
time (sec)	N/A	0.123	0.438	0.02	1.228	0.501	5.12	1.084

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	194	130	204	231	65
normalized size	1	1.	1.02	3.66	2.45	3.85	4.36	1.23
time (sec)	N/A	0.054	0.298	0.02	1.256	0.482	2.355	1.087

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	105	216	239	0	903
normalized size	1	1.	0.83	1.35	2.77	3.06	0.	11.58
time (sec)	N/A	0.14	0.165	0.026	1.456	0.487	0.	1.197

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	81	156	231	346	0	0
normalized size	1	1.	0.78	1.5	2.22	3.33	0.	0.
time (sec)	N/A	0.169	0.463	0.025	1.469	0.55	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	105	193	278	582	0	7560
normalized size	1	1.	0.83	1.52	2.19	4.58	0.	59.53
time (sec)	N/A	0.198	0.883	0.024	1.834	0.564	0.	1.641

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	123	230	346	880	0	11486
normalized size	1	1.	0.78	1.46	2.19	5.57	0.	72.7
time (sec)	N/A	0.228	1.754	0.026	2.338	0.601	0.	1.87

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	376	0	0	709	0	0
normalized size	1	1.	0.92	0.	0.	1.74	0.	0.
time (sec)	N/A	0.403	0.67	0.334	0.	0.57	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	238	1812	1808	1121	1098	717
normalized size	1	1.	0.72	5.49	5.48	3.4	3.33	2.17
time (sec)	N/A	0.391	3.416	0.087	1.428	0.611	79.215	1.131

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	369	992	1034	706	690	474
normalized size	1	1.	1.42	3.83	3.99	2.73	2.66	1.83
time (sec)	N/A	0.279	1.599	0.024	1.385	0.538	23.666	1.112

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	127	466	506	406	382	282
normalized size	1	1.	0.69	2.53	2.75	2.21	2.08	1.53
time (sec)	N/A	0.197	0.923	0.026	1.251	0.506	20.576	1.129

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	163	188	194	163	143
normalized size	1	1.	0.86	1.5	1.72	1.78	1.5	1.31
time (sec)	N/A	0.097	0.324	0.023	1.235	0.49	9.738	1.204

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	154	253	549	612	0	0
normalized size	1	1.	0.83	1.37	2.97	3.31	0.	0.
time (sec)	N/A	0.339	0.523	0.024	1.727	0.509	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	213	365	591	883	0	0
normalized size	1	1.	0.83	1.42	2.3	3.44	0.	0.
time (sec)	N/A	0.416	1.562	0.027	2.122	0.64	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	279	475	639	1358	0	0
normalized size	1	1.	0.83	1.41	1.89	4.02	0.	0.
time (sec)	N/A	0.505	4.217	0.027	3.01	0.76	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	457	580	706	1841	0	0
normalized size	1	1.	1.11	1.4	1.71	4.46	0.	0.
time (sec)	N/A	0.59	3.174	0.028	4.008	0.815	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	143	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	6.433	0.297	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	837	1295	2076	3281	0	0
normalized size	1	1.	2.51	3.89	6.23	9.85	0.	0.
time (sec)	N/A	0.284	1.371	0.354	2.329	0.849	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	330	847	1241	2280	0	0
normalized size	1	1.	1.3	3.33	4.89	8.98	0.	0.
time (sec)	N/A	0.198	0.983	0.218	1.991	0.742	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	221	479	684	1462	0	0
normalized size	1	1.	1.29	2.8	4.	8.55	0.	0.
time (sec)	N/A	0.138	0.874	0.184	1.47	0.638	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	176	203	269	787	0	0
normalized size	1	1.	1.87	2.16	2.86	8.37	0.	0.
time (sec)	N/A	0.062	0.175	0.164	1.654	0.607	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	8.493	0.302	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	4.125	0.63	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	1.173	0.16	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	795	913	4359	2034	0	0
normalized size	1	1.	5.13	5.89	28.12	13.12	0.	0.
time (sec)	N/A	0.229	6.727	0.213	3.246	0.619	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	374	573	2626	1439	0	0
normalized size	1	1.	2.94	4.51	20.68	11.33	0.	0.
time (sec)	N/A	0.197	6.445	0.194	2.369	0.565	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	198	297	872	950	0	0
normalized size	1	1.	2.04	3.06	8.99	9.79	0.	0.
time (sec)	N/A	0.13	6.318	0.152	2.282	0.543	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	82	49	394	242	100	1856
normalized size	1	1.	2.	1.2	9.61	5.9	2.44	45.27
time (sec)	N/A	0.026	0.451	0.041	1.683	0.49	1.291	1.607

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	4.647	0.332	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	2.418	0.447	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	10.297	0.18	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	966	1673	9385	6342	0	0
normalized size	1	1.	2.32	4.02	22.56	15.25	0.	0.
time (sec)	N/A	0.502	8.439	0.381	18.885	1.049	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	528	1056	5227	4091	0	0
normalized size	1	1.	1.71	3.43	16.97	13.28	0.	0.
time (sec)	N/A	0.344	5.051	0.342	6.897	0.809	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	471	546	2607	2379	0	0
normalized size	1	1.	2.63	3.05	14.56	13.29	0.	0.
time (sec)	N/A	0.223	7.595	0.281	2.404	0.688	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	260	246	1037	1191	0	0
normalized size	1	1.	2.41	2.28	9.6	11.03	0.	0.
time (sec)	N/A	0.107	1.836	0.139	1.909	0.576	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	36.712	2.23	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	42.607	3.468	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	1168	476	1874	856	0	2724
normalized size	1	1.	2.88	1.17	4.62	2.11	0.	6.71
time (sec)	N/A	0.667	16.39	0.033	2.533	0.653	0.	1.521

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	676	384	1793	716	0	1513
normalized size	1	1.	1.92	1.09	5.08	2.03	0.	4.29
time (sec)	N/A	0.526	9.261	0.031	2.521	0.624	0.	1.354

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	278	296	1656	613	0	659
normalized size	1	1.	0.91	0.97	5.45	2.02	0.	2.17
time (sec)	N/A	0.416	6.64	0.03	2.401	0.591	0.	1.227

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	278	296	1656	613	0	659
normalized size	1	1.	0.91	0.97	5.45	2.02	0.	2.17
time (sec)	N/A	0.419	6.643	0.03	2.215	0.592	0.	1.257

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	676	384	1793	716	0	1513
normalized size	1	1.	1.92	1.09	5.08	2.03	0.	4.29
time (sec)	N/A	0.528	9.215	0.029	2.497	0.611	0.	1.39

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	406	1168	476	1874	856	0	2724
normalized size	1	1.	2.88	1.17	4.62	2.11	0.	6.71
time (sec)	N/A	0.627	16.242	0.029	2.42	0.672	0.	1.532

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	206	251	922	840	0	1467
normalized size	1	1.	0.9	1.1	4.04	3.68	0.	6.43
time (sec)	N/A	0.398	3.657	0.037	2.088	0.652	0.	1.686

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	187	206	886	621	0	805
normalized size	1	1.	0.94	1.03	4.43	3.1	0.	4.02
time (sec)	N/A	0.329	3.155	0.036	1.947	0.597	0.	1.458

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	161	159	817	447	0	347
normalized size	1	1.	0.93	0.91	4.7	2.57	0.	1.99
time (sec)	N/A	0.27	0.885	0.036	1.936	0.569	0.	1.29

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	161	159	817	447	0	347
normalized size	1	1.	0.93	0.91	4.7	2.57	0.	1.99
time (sec)	N/A	0.249	0.137	0.036	2.013	0.572	0.	1.3

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	187	206	886	621	0	805
normalized size	1	1.	0.94	1.03	4.43	3.1	0.	4.02
time (sec)	N/A	0.318	1.694	0.036	2.16	0.606	0.	1.439

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	206	251	922	840	0	1467
normalized size	1	1.	0.9	1.1	4.04	3.68	0.	6.43
time (sec)	N/A	0.378	2.734	0.034	2.138	0.649	0.	1.639

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	615	615	3348	719	2943	1337	0	4084
normalized size	1	1.	5.44	1.17	4.79	2.17	0.	6.64
time (sec)	N/A	1.153	25.825	0.043	3.005	0.843	0.	2.235

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	534	534	1041	580	2790	1118	0	2269
normalized size	1	1.	1.95	1.09	5.22	2.09	0.	4.25
time (sec)	N/A	0.879	12.284	0.042	2.901	0.78	0.	1.807

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	432	447	2531	952	0	988
normalized size	1	1.	0.94	0.97	5.51	2.07	0.	2.15
time (sec)	N/A	0.671	7.401	0.041	2.849	0.712	0.	1.425

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	432	447	2531	952	0	988
normalized size	1	1.	0.94	0.97	5.51	2.07	0.	2.15
time (sec)	N/A	0.658	7.346	0.042	2.759	0.721	0.	1.455

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	534	534	1041	580	2790	1118	0	2269
normalized size	1	1.	1.95	1.09	5.22	2.09	0.	4.25
time (sec)	N/A	0.801	12.183	0.038	2.936	0.767	0.	1.815

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	615	615	3348	719	2943	1337	0	4084
normalized size	1	1.	5.44	1.17	4.79	2.17	0.	6.64
time (sec)	N/A	0.953	25.314	0.043	2.95	0.832	0.	2.218

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	245	0	0	487	0	0
normalized size	1	1.	0.9	0.	0.	1.78	0.	0.
time (sec)	N/A	0.288	0.248	0.266	0.	0.544	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	158	1150	1305	802	976	487
normalized size	1	1.	0.61	4.42	5.02	3.08	3.75	1.87
time (sec)	N/A	0.234	1.883	0.049	1.394	0.536	21.918	1.11

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	135	594	741	518	634	325
normalized size	1	1.	0.69	3.03	3.78	2.64	3.23	1.66
time (sec)	N/A	0.161	1.012	0.021	1.268	0.506	11.69	1.145

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	89	260	355	294	350	196
normalized size	1	1.	0.66	1.94	2.65	2.19	2.61	1.46
time (sec)	N/A	0.087	0.486	0.019	1.155	0.493	13.536	1.105

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	75	85	124	147	160	101
normalized size	1	1.	1.04	1.18	1.72	2.04	2.22	1.4
time (sec)	N/A	0.047	0.152	0.02	1.112	0.488	2.812	1.115

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	370	421	0	8162
normalized size	1	1.	0.85	1.38	2.87	3.26	0.	63.27
time (sec)	N/A	0.212	0.344	0.021	1.487	0.481	0.	1.755

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	406	597	0	0
normalized size	1	1.	0.84	1.43	2.27	3.34	0.	0.
time (sec)	N/A	0.268	1.673	0.023	1.741	0.558	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	197	329	454	918	0	0
normalized size	1	1.	0.85	1.42	1.97	3.97	0.	0.
time (sec)	N/A	0.329	3.792	0.023	2.254	0.625	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	521	1256	0	0
normalized size	1	1.	1.1	1.41	1.82	4.38	0.	0.
time (sec)	N/A	0.451	2.55	0.024	2.91	0.683	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	409	0	0	726	0	0
normalized size	1	1.	0.98	0.	0.	1.73	0.	0.
time (sec)	N/A	0.435	0.606	0.331	0.	0.577	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	563	1842	1808	1245	1098	717
normalized size	1	1.	1.71	5.58	5.48	3.77	3.33	2.17
time (sec)	N/A	0.368	3.757	0.052	1.489	0.568	48.01	1.147

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	195	1016	1034	798	690	474
normalized size	1	1.	0.75	3.92	3.99	3.08	2.66	1.83
time (sec)	N/A	0.273	2.217	0.021	1.268	0.54	53.576	1.118

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	252	484	506	470	382	282
normalized size	1	1.	1.37	2.63	2.75	2.55	2.08	1.53
time (sec)	N/A	0.19	1.011	0.02	1.177	0.5	11.542	1.114

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	110	175	188	235	163	143
normalized size	1	1.	1.01	1.61	1.72	2.16	1.5	1.31
time (sec)	N/A	0.094	0.365	0.023	1.053	0.488	5.467	1.116

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	154	252	551	612	0	0
normalized size	1	1.	0.83	1.36	2.98	3.31	0.	0.
time (sec)	N/A	0.28	0.521	0.023	1.627	0.497	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	212	367	593	867	0	0
normalized size	1	1.	0.82	1.43	2.31	3.37	0.	0.
time (sec)	N/A	0.345	2.139	0.026	1.968	0.632	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	338	283	473	640	1312	0	0
normalized size	1	1.	0.84	1.4	1.89	3.88	0.	0.
time (sec)	N/A	0.438	3.296	0.026	2.603	0.724	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	451	583	707	1814	0	0
normalized size	1	1.	1.09	1.41	1.71	4.39	0.	0.
time (sec)	N/A	0.538	3.48	0.024	3.975	0.845	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	255	0	0	487	0	0
normalized size	1	1.	0.89	0.	0.	1.71	0.	0.
time (sec)	N/A	0.317	3.412	0.264	0.	0.557	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	153	2061	1395	1153	1334	485
normalized size	1	1.	0.66	8.85	5.99	4.95	5.73	2.08
time (sec)	N/A	0.266	1.553	0.074	1.448	0.562	46.242	1.131

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	132	1100	813	747	867	325
normalized size	1	1.	0.73	6.08	4.49	4.13	4.79	1.8
time (sec)	N/A	0.219	2.39	0.027	1.334	0.533	60.503	1.132

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	91	498	409	447	471	196
normalized size	1	1.	0.71	3.86	3.17	3.47	3.65	1.52
time (sec)	N/A	0.144	0.545	0.024	1.238	0.504	22.629	1.145

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	176	161	221	201	101
normalized size	1	1.	0.82	2.29	2.09	2.87	2.61	1.31
time (sec)	N/A	0.074	0.223	0.022	1.18	0.486	11.178	1.122

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	370	421	0	8162
normalized size	1	1.	0.85	1.38	2.87	3.26	0.	63.27
time (sec)	N/A	0.246	0.309	0.024	1.535	0.491	0.	1.783

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	189	256	406	632	0	0
normalized size	1	1.	1.06	1.43	2.27	3.53	0.	0.
time (sec)	N/A	0.297	0.981	0.028	1.761	0.597	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	239	329	454	1010	0	0
normalized size	1	1.	1.02	1.4	1.93	4.3	0.	0.
time (sec)	N/A	0.353	1.091	0.028	2.301	0.706	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	554	404	521	1419	0	0
normalized size	1	1.	1.93	1.41	1.82	4.94	0.	0.
time (sec)	N/A	0.419	5.131	0.028	3.221	0.762	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	151	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.172	7.752	0.192	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	2828	1492	2207	3460	0	0
normalized size	1	1.	9.21	4.86	7.19	11.27	0.	0.
time (sec)	N/A	0.34	6.522	0.49	2.635	0.922	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	246	246	1918	1001	1305	2437	0	0
normalized size	1	1.	7.8	4.07	5.3	9.91	0.	0.
time (sec)	N/A	0.278	6.402	0.438	2.02	0.825	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	564	590	705	1538	0	0
normalized size	1	1.	3.12	3.26	3.9	8.5	0.	0.
time (sec)	N/A	0.227	2.988	0.47	1.678	0.689	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	131	271	300	844	0	0
normalized size	1	1.	1.15	2.38	2.63	7.4	0.	0.
time (sec)	N/A	0.128	0.353	0.383	1.536	0.602	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	0.794	0.355	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.17	2.499	0.63	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	153	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.239	3.96	0.164	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	798	1056	0	3009	0	0
normalized size	1	1.	2.67	3.53	0.	10.06	0.	0.
time (sec)	N/A	0.293	1.722	0.209	0.	0.887	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	539	649	14876	2018	0	0
normalized size	1	1.	2.5	3.	68.87	9.34	0.	0.
time (sec)	N/A	0.222	1.464	0.189	6.757	0.726	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	310	332	4435	1172	0	0
normalized size	1	1.	2.23	2.39	31.91	8.43	0.	0.
time (sec)	N/A	0.149	3.956	0.16	5.39	0.611	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	104	124	2849	269	0	2655
normalized size	1	1.	1.79	2.14	49.12	4.64	0.	45.78
time (sec)	N/A	0.063	0.707	0.119	1.184	0.521	0.	2.221

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	74	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	3.705	0.346	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.256	4.008	0.492	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	5.536	0.178	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	1534	1868	9600	3915	0	0
normalized size	1	1.	5.08	6.19	31.79	12.96	0.	0.
time (sec)	N/A	0.462	7.107	0.403	19.409	0.707	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	256	256	994	1194	5335	2637	0	0
normalized size	1	1.	3.88	4.66	20.84	10.3	0.	0.
time (sec)	N/A	0.368	6.881	0.363	6.142	0.621	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	168	168	540	635	2654	1581	0	0
normalized size	1	1.	3.21	3.78	15.8	9.41	0.	0.
time (sec)	N/A	0.266	6.672	0.313	2.509	0.564	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	109	109	240	281	1133	859	0	0
normalized size	1	1.	2.2	2.58	10.39	7.88	0.	0.
time (sec)	N/A	0.128	6.144	0.161	1.696	0.537	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	8.051	2.391	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	9.165	3.544	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	1879	944	0	2680
normalized size	1	1.	1.35	1.15	4.62	2.32	0.	6.58
time (sec)	N/A	0.797	13.652	0.035	2.311	0.698	0.	1.775

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	1806	749	0	1485
normalized size	1	1.	1.12	1.07	5.15	2.13	0.	4.23
time (sec)	N/A	0.556	3.297	0.035	2.256	0.675	0.	1.533

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	1661	599	0	643
normalized size	1	1.	0.88	0.96	5.56	2.	0.	2.15
time (sec)	N/A	0.447	0.835	0.032	2.184	0.61	0.	1.338

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	1661	599	0	643
normalized size	1	1.	0.88	0.96	5.56	2.	0.	2.15
time (sec)	N/A	0.455	0.391	0.033	2.36	0.614	0.	1.324

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	1806	749	0	1485
normalized size	1	1.	1.12	1.07	5.15	2.13	0.	4.23
time (sec)	N/A	0.572	1.619	0.033	2.276	0.64	0.	1.537

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	1879	944	0	2680
normalized size	1	1.	1.35	1.15	4.62	2.32	0.	6.58
time (sec)	N/A	0.672	10.917	0.034	2.242	0.694	0.	1.741

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	615	615	1795	716	2940	1397	0	4077
normalized size	1	1.	2.92	1.16	4.78	2.27	0.	6.63
time (sec)	N/A	1.145	23.492	0.051	2.648	0.839	0.	2.167

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	1043	583	2790	1162	0	2271
normalized size	1	1.	1.95	1.09	5.22	2.18	0.	4.25
time (sec)	N/A	0.838	12.585	0.051	2.617	0.766	0.	1.78

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	435	444	2531	973	0	988
normalized size	1	1.	0.95	0.97	5.51	2.12	0.	2.15
time (sec)	N/A	0.693	7.043	0.049	2.535	0.716	0.	1.442

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	435	444	2531	973	0	988
normalized size	1	1.	0.95	0.97	5.51	2.12	0.	2.15
time (sec)	N/A	0.672	7.044	0.05	2.515	0.719	0.	1.421

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	534	534	1043	583	2790	1162	0	2271
normalized size	1	1.	1.95	1.09	5.22	2.18	0.	4.25
time (sec)	N/A	0.858	12.406	0.052	2.625	0.769	0.	1.812

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	615	615	1795	716	2940	1397	0	4077
normalized size	1	1.	2.92	1.16	4.78	2.27	0.	6.63
time (sec)	N/A	1.015	23.409	0.051	2.696	0.83	0.	2.129

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	477	1916	1089	0	2677
normalized size	1	1.	1.35	1.17	4.71	2.68	0.	6.58
time (sec)	N/A	0.897	5.347	0.043	2.248	0.767	0.	2.7

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	391	383	1829	821	0	1481
normalized size	1	1.	1.11	1.09	5.21	2.34	0.	4.22
time (sec)	N/A	0.628	3.086	0.041	2.221	0.706	0.	2.076

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	293	1681	632	0	643
normalized size	1	1.	0.88	0.98	5.62	2.11	0.	2.15
time (sec)	N/A	0.458	1.385	0.041	2.194	0.646	0.	1.555

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	264	293	1681	632	0	643
normalized size	1	1.	0.88	0.98	5.62	2.11	0.	2.15
time (sec)	N/A	0.448	0.613	0.037	2.169	0.652	0.	1.561

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	391	383	1829	821	0	1481
normalized size	1	1.	1.11	1.09	5.21	2.34	0.	4.22
time (sec)	N/A	0.559	0.216	0.039	2.218	0.72	0.	2.078

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	550	477	1916	1089	0	2677
normalized size	1	1.	1.35	1.17	4.71	2.68	0.	6.58
time (sec)	N/A	0.669	3.002	0.04	2.202	0.765	0.	2.743

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	104	150	0	851	0	0
normalized size	1	1.	0.93	1.34	0.	7.6	0.	0.
time (sec)	N/A	0.185	0.176	0.106	0.	0.595	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	112	0	551	0	0
normalized size	1	1.	0.87	1.35	0.	6.64	0.	0.
time (sec)	N/A	0.17	0.1	0.106	0.	0.558	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	76	0	138	0	278
normalized size	1	1.	1.	2.3	0.	4.18	0.	8.42
time (sec)	N/A	0.055	0.025	0.093	0.	0.505	0.	1.18

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	159	240	5022	1566	0	0
normalized size	1	1.	0.88	1.33	27.9	8.7	0.	0.
time (sec)	N/A	0.401	0.416	0.157	3.474	0.644	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	108	170	3856	1204	0	0
normalized size	1	1.	1.02	1.6	36.38	11.36	0.	0.
time (sec)	N/A	0.278	0.324	0.224	2.631	0.613	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	62	109	2349	640	0	0
normalized size	1	1.	0.85	1.49	32.18	8.77	0.	0.
time (sec)	N/A	0.164	0.108	0.126	1.881	0.582	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	2.517	0.2	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	157	616	1069	3355	0	0
normalized size	1	1.	0.99	3.9	6.77	21.23	0.	0.
time (sec)	N/A	0.21	0.069	0.315	2.058	0.742	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	126	423	662	2367	0	0
normalized size	1	1.	0.95	3.2	5.02	17.93	0.	0.
time (sec)	N/A	0.183	0.087	0.289	1.857	0.664	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	100	257	378	1534	0	0
normalized size	1	1.	1.04	2.68	3.94	15.98	0.	0.
time (sec)	N/A	0.153	0.041	0.277	1.851	0.597	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	123	154	837	0	0
normalized size	1	1.	1.06	1.86	2.33	12.68	0.	0.
time (sec)	N/A	0.093	0.015	0.252	1.726	0.572	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	3.57	0.222	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.02	5.364	0.235	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.136	6.586	0.323	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	557	901	1247	2591	0	0
normalized size	1	1.	2.03	3.28	4.53	9.42	0.	0.
time (sec)	N/A	0.215	1.546	0.418	2.243	0.776	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	315	512	689	1666	0	0
normalized size	1	1.	1.69	2.75	3.7	8.96	0.	0.
time (sec)	N/A	0.146	0.883	0.361	1.989	0.677	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	213	221	0	905	0	0
normalized size	1	1.	2.07	2.15	0.	8.79	0.	0.
time (sec)	N/A	0.068	0.424	0.227	0.	0.595	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	6.078	0.34	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	7.24	0.607	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	151	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	7.772	0.274	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	251	251	1720	641	925	2753	0	0
normalized size	1	1.	6.85	2.55	3.69	10.97	0.	0.
time (sec)	N/A	0.298	7.147	0.285	2.097	0.843	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	518	379	512	1746	0	0
normalized size	1	1.	2.82	2.06	2.78	9.49	0.	0.
time (sec)	N/A	0.228	6.459	0.353	1.835	0.706	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	134	179	196	965	0	0
normalized size	1	1.	1.17	1.56	1.7	8.39	0.	0.
time (sec)	N/A	0.128	0.304	0.355	1.77	0.614	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	81	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.151	0.779	0.488	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	101	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	2.503	0.812	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	5.942	0.141	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	578	1242	2402	6280	0	0
normalized size	1	1.	2.34	5.03	9.72	25.43	0.	0.
time (sec)	N/A	0.229	1.379	0.329	2.506	1.04	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	350	816	1435	4410	0	0
normalized size	1	1.	1.78	4.14	7.28	22.39	0.	0.
time (sec)	N/A	0.166	1.016	0.313	2.201	0.865	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	213	469	797	2850	0	0
normalized size	1	1.	1.68	3.69	6.28	22.44	0.	0.
time (sec)	N/A	0.117	0.628	0.297	1.975	0.738	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	141	208	360	1544	0	0
normalized size	1	1.	1.99	2.93	5.07	21.75	0.	0.
time (sec)	N/A	0.056	0.119	0.234	1.95	0.631	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	4.53	0.2	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	5.883	0.197	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	9.154	0.163	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	739	1158	4374	4446	0	0
normalized size	1	1.	2.11	3.31	12.5	12.7	0.	0.
time (sec)	N/A	0.64	6.417	0.679	4.389	0.951	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	593	556	2202	2844	0	0
normalized size	1	1.	2.62	2.46	9.74	12.58	0.	0.
time (sec)	N/A	0.384	6.291	0.449	2.355	0.764	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	517	235	0	1215	0	0
normalized size	1	1.	3.95	1.79	0.	9.27	0.	0.
time (sec)	N/A	0.135	2.947	0.299	0.	0.607	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	11.7	2.356	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	11.908	2.649	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.237	11.017	0.178	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	1477	1223	6939	8092	0	0
normalized size	1	1.	4.54	3.76	21.35	24.9	0.	0.
time (sec)	N/A	0.823	6.993	0.421	7.781	1.261	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	872	632	3405	4868	0	0
normalized size	1	1.	4.34	3.14	16.94	24.22	0.	0.
time (sec)	N/A	0.443	6.757	0.369	2.945	0.915	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	210	270	1397	2457	0	0
normalized size	1	1.	1.49	1.91	9.91	17.43	0.	0.
time (sec)	N/A	0.139	0.958	0.173	2.219	0.722	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.147	14.386	3.076	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	16.889	4.781	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.93	0.162	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	428	767	3974	2930	0	0
normalized size	1	1.	1.89	3.38	17.51	12.91	0.	0.
time (sec)	N/A	0.186	1.24	0.315	3.043	0.785	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	256	463	2395	1993	0	0
normalized size	1	1.	1.61	2.91	15.06	12.53	0.	0.
time (sec)	N/A	0.126	0.855	0.274	2.261	0.669	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	174	227	0	1166	0	0
normalized size	1	1.	1.79	2.34	0.	12.02	0.	0.
time (sec)	N/A	0.068	1.713	0.135	0.	0.596	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	93	49	350	163	0	2075
normalized size	1	1.	3.21	1.69	12.07	5.62	0.	71.55
time (sec)	N/A	0.019	0.047	0.022	1.5	0.493	0.	1.685

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	11.262	0.268	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	19.338	0.32	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	3.14	0.166	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	128	128	424	348	1840	913	0	0
normalized size	1	1.	3.31	2.72	14.38	7.13	0.	0.
time (sec)	N/A	0.21	6.564	0.197	1.93	0.503	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	276	191	563	522	0	0
normalized size	1	1.	2.88	1.99	5.86	5.44	0.	0.
time (sec)	N/A	0.141	6.381	0.162	1.845	0.49	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	76	47	320	131	65	301
normalized size	1	1.	1.9	1.18	8.	3.28	1.62	7.52
time (sec)	N/A	0.028	0.265	0.044	1.467	0.472	0.373	1.403

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	3.957	0.264	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	5.266	0.324	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	149	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	5.097	0.17	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	532	677	14923	2237	0	0
normalized size	1	1.	2.33	2.97	65.45	9.81	0.	0.
time (sec)	N/A	0.211	1.61	0.296	6.936	0.745	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	362	345	0	1316	0	0
normalized size	1	1.	2.5	2.38	0.	9.08	0.	0.
time (sec)	N/A	0.131	3.118	0.158	0.	0.637	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	107	123	2866	251	0	3729
normalized size	1	1.	1.91	2.2	51.18	4.48	0.	66.59
time (sec)	N/A	0.054	0.316	0.129	1.763	0.512	0.	2.529

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	3.818	0.299	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.181	4.183	0.378	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	8.947	0.16	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	694	1866	7688	6263	0	0
normalized size	1	1.	1.48	3.98	16.39	13.35	0.	0.
time (sec)	N/A	0.795	3.63	0.758	9.458	1.358	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	473	1152	4327	4352	0	0
normalized size	1	1.	1.38	3.36	12.62	12.69	0.	0.
time (sec)	N/A	0.573	1.439	0.651	4.02	1.005	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	317	568	2157	2773	0	0
normalized size	1	1.	1.45	2.59	9.85	12.66	0.	0.
time (sec)	N/A	0.377	2.588	0.448	2.265	0.776	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	122	212	160	1085	1046	0	0
normalized size	1	1.08	1.88	1.42	9.6	9.26	0.	0.
time (sec)	N/A	0.13	0.504	0.29	2.046	0.581	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	9.847	2.332	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.18	10.095	2.663	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.19	2.785	0.173	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	285	687	3179	4263	0	0
normalized size	1	1.	2.42	5.82	26.94	36.13	0.	0.
time (sec)	N/A	0.28	2.231	0.336	2.355	0.898	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	277	351	1049	2573	0	0
normalized size	1	1.	3.15	3.99	11.92	29.24	0.	0.
time (sec)	N/A	0.191	1.759	0.332	2.129	0.746	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	182	416	197	0	0
normalized size	1	1.	0.91	5.2	11.89	5.63	0.	0.
time (sec)	N/A	0.059	0.204	0.074	1.501	0.5	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	7.335	0.518	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	7.586	0.811	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	11.762	0.21	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	601	601	907	1613	10858	7588	0	0
normalized size	1	1.	1.51	2.68	18.07	12.63	0.	0.
time (sec)	N/A	2.313	8.316	0.727	18.615	1.347	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	889	802	5157	4454	0	0
normalized size	1	1.	2.91	2.63	16.91	14.6	0.	0.
time (sec)	N/A	0.865	7.923	0.508	5.042	0.958	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	174	520	267	2029	1652	0	0
normalized size	1	1.13	3.38	1.73	13.18	10.73	0.	0.
time (sec)	N/A	0.191	5.329	0.352	2.806	0.634	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	23.06	2.755	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	31.86	4.368	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.924	11.334	0.102	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	387	387	672	0	5385	4625	0	0
normalized size	1	1.	1.74	0.	13.91	11.95	0.	0.
time (sec)	N/A	0.96	7.193	1.401	3.74	0.911	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	613	429	2996	3302	0	0
normalized size	1	1.	2.61	1.83	12.75	14.05	0.	0.
time (sec)	N/A	0.537	6.663	0.402	2.521	0.77	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	282	182	1598	1459	0	0
normalized size	1	1.	2.24	1.44	12.68	11.58	0.	0.
time (sec)	N/A	0.168	2.563	0.299	2.344	0.608	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.5	49.516	0.784	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.513	20.98	0.884	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.152	2.416	0.17	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	418	489	4641	2211	0	0
normalized size	1	1.	3.01	3.52	33.39	15.91	0.	0.
time (sec)	N/A	0.258	6.585	0.193	2.318	0.777	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	286	301	900	1374	0	0
normalized size	1	1.	2.49	2.62	7.83	11.95	0.	0.
time (sec)	N/A	0.174	6.395	0.158	2.241	0.691	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	66	95	1334	208	0	6040
normalized size	1	1.	1.2	1.73	24.25	3.78	0.	109.82
time (sec)	N/A	0.062	0.524	0.029	1.575	0.508	0.	2.833

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	382	95	0	771
normalized size	1	1.	1.37	1.74	10.91	2.71	0.	22.03
time (sec)	N/A	0.032	0.062	0.027	0.977	0.465	0.	1.237

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	7.371	0.367	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	10.436	0.382	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	8.989	0.174	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	530	1127	5168	3214	0	0
normalized size	1	1.	1.57	3.34	15.34	9.54	0.	0.
time (sec)	N/A	0.408	3.635	0.355	6.806	0.945	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	526	584	2556	2033	0	0
normalized size	1	1.	2.73	3.03	13.24	10.53	0.	0.
time (sec)	N/A	0.271	7.225	0.31	2.898	0.747	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	555	267	0	1169	0	0
normalized size	1	1.	4.74	2.28	0.	9.99	0.	0.
time (sec)	N/A	0.129	6.522	0.162	0.	0.624	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	26.7	1.455	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	29.522	2.21	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	5.543	0.197	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	259	259	803	720	3247	1443	0	0
normalized size	1	1.	3.1	2.78	12.54	5.57	0.	0.
time (sec)	N/A	0.356	6.923	0.368	4.481	0.537	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	454	400	1655	883	0	0
normalized size	1	1.	2.69	2.37	9.79	5.22	0.	0.
time (sec)	N/A	0.222	6.64	0.321	2.442	0.518	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	240	183	701	447	0	0
normalized size	1	1.	2.22	1.69	6.49	4.14	0.	0.
time (sec)	N/A	0.117	6.154	0.159	2.042	0.493	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	6.457	1.853	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	7.034	2.576	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.269	9.561	0.185	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	2090	1729	11840	8008	0	0
normalized size	1	1.	5.24	4.33	29.67	20.07	0.	0.
time (sec)	N/A	0.971	7.508	0.483	23.457	1.968	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	1486	1115	6639	5605	0	0
normalized size	1	1.	4.57	3.43	20.43	17.25	0.	0.
time (sec)	N/A	0.64	6.926	0.4	7.39	1.418	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	875	614	3297	3629	0	0
normalized size	1	1.	4.35	3.05	16.4	18.05	0.	0.
time (sec)	N/A	0.414	6.742	0.339	2.849	0.963	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	141	212	270	1397	2072	0	0
normalized size	1	1.01	1.53	1.94	10.05	14.91	0.	0.
time (sec)	N/A	0.135	0.565	0.158	2.224	0.752	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	9.068	3.113	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	6.709	4.859	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	11.962	0.215	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	819	1629	10843	5646	0	0
normalized size	1	1.	1.69	3.35	22.31	11.62	0.	0.
time (sec)	N/A	1.206	7.913	0.738	20.687	1.453	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	341	341	889	770	5154	3596	0	0
normalized size	1	1.	2.61	2.26	15.11	10.55	0.	0.
time (sec)	N/A	0.648	7.562	0.499	5.443	0.973	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	182	669	344	0	1644	0	0
normalized size	1	1.12	4.13	2.12	0.	10.15	0.	0.
time (sec)	N/A	0.196	6.577	0.355	0.	0.696	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.187	21.19	2.754	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	26.433	4.312	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.265	10.558	0.187	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	582	1329	7574	9802	0	0
normalized size	1	1.	1.83	4.18	23.82	30.82	0.	0.
time (sec)	N/A	0.321	9.182	0.295	11.693	1.741	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	381	716	3676	5801	0	0
normalized size	1	1.	2.01	3.77	19.35	30.53	0.	0.
time (sec)	N/A	0.213	8.167	0.239	3.668	1.117	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	236	325	1455	3087	0	0
normalized size	1	1.	2.15	2.95	13.23	28.06	0.	0.
time (sec)	N/A	0.106	2.211	0.209	2.468	0.797	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	29.331	2.741	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	33.482	4.84	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	0.335	0.095	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.39	0.087	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	52	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.152	0.092	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	33	33	181	310	0	0	0	0
normalized size	1	1.	5.48	9.39	0.	0.	0.	0.
time (sec)	N/A	0.026	1.771	0.212	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.165	0.089	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	0	0	0	0	0
normalized size	1	1.	0.9	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.198	0.087	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	0.24	0.09	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	0.276	0.09	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	65	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.287	0.101	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	61	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	0.243	0.095	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	54	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.195	0.096	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.141	0.101	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	132	310	0	0	0	0
normalized size	1	1.	2.49	5.85	0.	0.	0.	0.
time (sec)	N/A	0.037	2.211	0.184	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	63	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.248	0.097	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	212	0	0	0	0	0
normalized size	1	1.	2.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	8.098	0.097	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	89	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.339	0.099	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	67	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.546	0.104	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	108	0	0	0	0	0
normalized size	1	1.	1.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.926	0.092	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.189	0.095	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	86	308	0	0	0	0
normalized size	1	1.	2.26	8.11	0.	0.	0.	0.
time (sec)	N/A	0.022	1.186	0.204	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.023	0.181	0.098	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	56	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.184	0.096	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	57	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.233	0.092	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	73	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.299	0.096	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	73	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.272	0.116	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.298	0.11	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	56	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	0.184	0.104	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	46	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.148	0.109	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	106	308	0	0	0	0
normalized size	1	1.	1.83	5.31	0.	0.	0.	0.
time (sec)	N/A	0.032	0.73	0.212	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.245	0.102	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	114	0	0	0	0	0
normalized size	1	1.	1.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.998	0.103	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	93	0	0	0	0	0
normalized size	1	1.	0.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.42	0.099	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	26	24	54	37	24
normalized size	1	1.	0.71	0.84	0.77	1.74	1.19	0.77
time (sec)	N/A	0.041	0.015	0.048	0.99	0.518	6.234	1.143

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	154	260	197	419	0	225
normalized size	1	1.	1.18	1.98	1.5	3.2	0.	1.72
time (sec)	N/A	0.188	0.219	0.066	1.049	0.515	0.	1.136

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	109	179	136	278	289	151
normalized size	1	1.	0.95	1.56	1.18	2.42	2.51	1.31
time (sec)	N/A	0.141	0.157	0.051	1.028	0.496	146.651	1.144

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	107	81	159	155	86
normalized size	1	1.	0.82	1.47	1.11	2.18	2.12	1.18
time (sec)	N/A	0.103	0.11	0.054	0.995	0.513	21.014	1.128

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	34	52	36	78	56	36
normalized size	1	1.	0.83	1.27	0.88	1.9	1.37	0.88
time (sec)	N/A	0.056	0.02	0.048	1.022	0.492	14.303	1.099

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	58	128	182	0	69
normalized size	1	1.	0.86	1.02	2.25	3.19	0.	1.21
time (sec)	N/A	0.252	0.063	0.051	1.234	0.505	0.	1.121

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	82	437	251	0	150
normalized size	1	1.	0.78	1.05	5.6	3.22	0.	1.92
time (sec)	N/A	0.238	0.138	0.054	1.225	0.514	0.	1.12

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	77	104	489	392	0	271
normalized size	1	1.	0.78	1.05	4.94	3.96	0.	2.74
time (sec)	N/A	0.329	0.234	0.053	1.372	0.53	0.	1.114

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	128	1000	329	579	0	6323
normalized size	1	1.	0.65	5.05	1.66	2.92	0.	31.93
time (sec)	N/A	0.252	0.693	0.049	1.2	0.565	0.	1.743

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	105	580	234	389	0	4238
normalized size	1	1.	0.61	3.39	1.37	2.27	0.	24.78
time (sec)	N/A	0.184	0.415	0.043	1.129	0.505	0.	1.464

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	73	294	146	240	0	2538
normalized size	1	1.	0.65	2.62	1.3	2.14	0.	22.66
time (sec)	N/A	0.136	0.394	0.041	1.073	0.498	0.	1.309

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	46	119	74	132	0	1242
normalized size	1	1.	0.7	1.8	1.12	2.	0.	18.82
time (sec)	N/A	0.068	0.135	0.037	1.064	0.484	0.	1.206

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	63	116	158	228	0	1509
normalized size	1	1.	0.89	1.63	2.23	3.21	0.	21.25
time (sec)	N/A	0.281	0.153	0.037	1.286	0.489	0.	1.298

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	81	169	159	321	0	0
normalized size	1	1.	0.79	1.66	1.56	3.15	0.	0.
time (sec)	N/A	0.276	0.545	0.041	1.329	0.519	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	104	207	176	516	0	12712
normalized size	1	1.	0.76	1.52	1.29	3.79	0.	93.47
time (sec)	N/A	0.374	0.999	0.038	1.45	0.531	0.	2.116

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	125	243	190	748	0	0
normalized size	1	1.	0.61	1.19	0.93	3.65	0.	0.
time (sec)	N/A	0.38	1.127	0.039	1.511	0.556	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	459	849	813	2314	0	0
normalized size	1	1.	1.8	3.33	3.19	9.07	0.	0.
time (sec)	N/A	0.347	1.572	0.196	1.809	0.756	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	223	481	552	1480	0	0
normalized size	1	1.	1.3	2.8	3.21	8.6	0.	0.
time (sec)	N/A	0.229	1.156	0.259	1.56	0.638	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	171	205	0	814	0	0
normalized size	1	1.	1.8	2.16	0.	8.57	0.	0.
time (sec)	N/A	0.109	0.341	0.249	0.	0.584	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	71	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	6.232	0.349	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.283	6.787	0.7	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.328	7.249	0.71	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	2482	956	819	3866	0	0
normalized size	1	1.	8.3	3.2	2.74	12.93	0.	0.
time (sec)	N/A	0.503	6.742	0.255	1.679	1.002	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	1719	639	597	2703	0	0
normalized size	1	1.	7.1	2.64	2.47	11.17	0.	0.
time (sec)	N/A	0.446	6.608	0.214	1.5	0.866	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	516	377	406	1728	0	0
normalized size	1	1.	2.98	2.18	2.35	9.99	0.	0.
time (sec)	N/A	0.328	6.541	0.243	1.399	0.719	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	254	177	0	937	0	0
normalized size	1	1.	2.37	1.65	0.	8.76	0.	0.
time (sec)	N/A	0.18	5.824	0.324	0.	0.622	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	79	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.299	3.157	0.371	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.342	4.256	0.663	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	128	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.398	5.894	0.504	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	547	677	0	2240	0	0
normalized size	1	1.	2.38	2.94	0.	9.74	0.	0.
time (sec)	N/A	0.33	2.335	0.302	0.	0.755	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	364	345	0	1320	0	0
normalized size	1	1.	2.48	2.35	0.	8.98	0.	0.
time (sec)	N/A	0.212	3.737	0.308	0.	0.638	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	105	87	4496	252	0	3883
normalized size	1	1.	1.84	1.53	78.88	4.42	0.	68.12
time (sec)	N/A	0.091	0.454	0.046	1.853	0.514	0.	2.289

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	77	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.266	13.558	0.388	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	97	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.344	16.554	0.5	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	120	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.436	18.928	0.996	0.	0.	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	77	81	0	405	0	0
normalized size	1	1.	1.35	1.42	0.	7.11	0.	0.
time (sec)	N/A	0.067	0.031	0.192	0.	0.529	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	150	73	144	159
normalized size	1	1.	1.	1.07	10.71	5.21	10.29	11.36
time (sec)	N/A	0.033	0.023	0.042	1.568	0.486	53.066	1.161

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	146	102	0	500	0	0
normalized size	1	1.	2.18	1.52	0.	7.46	0.	0.
time (sec)	N/A	0.137	0.274	0.223	0.	0.539	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [205] had the largest ratio of [1.25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.	20	0.2
2	A	5	4	1.	20	0.2
3	A	5	5	1.	20	0.25
4	A	3	2	1.	20	0.1
5	A	3	3	1.	18	0.167
6	A	5	5	1.	20	0.25
7	A	6	6	1.	20	0.3
8	A	7	6	1.	20	0.3
9	A	8	6	1.	20	0.3
10	A	3	3	1.	8	0.375
11	A	4	4	1.	8	0.5
12	A	5	4	1.	8	0.5
13	A	8	3	1.	22	0.136
14	A	9	5	1.	22	0.227
15	A	7	5	1.	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	4	4	1.	22	0.182
17	A	3	2	1.	20	0.1
18	A	8	4	1.	22	0.182
19	A	10	5	1.	22	0.227
20	A	12	5	1.	22	0.227
21	A	14	5	1.	22	0.227
22	A	8	3	1.	22	0.136
23	A	9	4	1.	22	0.182
24	A	9	5	1.	22	0.227
25	A	4	2	1.	22	0.091
26	A	4	3	1.	20	0.15
27	A	8	4	1.	22	0.182
28	A	10	5	1.	22	0.227
29	A	12	5	1.	22	0.227
30	A	14	5	1.	22	0.227
31	A	0	0	0.	0	0.
32	A	7	6	1.	14	0.429
33	A	6	6	1.	14	0.429
34	A	5	5	1.	14	0.357
35	A	4	4	1.	12	0.333
36	A	0	0	0.	0	0.
37	A	0	0	0.	0	0.
38	A	0	0	0.	0	0.
39	A	10	6	1.	20	0.3
40	A	8	5	1.	20	0.25
41	A	6	4	1.	20	0.2
42	A	2	2	1.	18	0.111
43	A	0	0	0.	0	0.
44	A	0	0	0.	0	0.
45	A	0	0	0.	0	0.
46	A	7	7	1.	22	0.318
47	A	6	6	1.	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	3	3	1.	22	0.136
49	A	3	3	1.	20	0.15
50	A	0	0	0.	0	0.
51	A	0	0	0.	0	0.
52	A	10	8	1.	22	0.364
53	A	9	8	1.	22	0.364
54	A	8	8	1.	22	0.364
55	A	8	8	1.	22	0.364
56	A	9	8	1.	22	0.364
57	A	10	8	1.	22	0.364
58	A	18	7	1.	24	0.292
59	A	16	7	1.	24	0.292
60	A	14	7	1.	24	0.292
61	A	14	7	1.	24	0.292
62	A	16	7	1.	24	0.292
63	A	18	7	1.	24	0.292
64	A	18	7	1.	24	0.292
65	A	16	7	1.	24	0.292
66	A	14	7	1.	24	0.292
67	A	14	7	1.	24	0.292
68	A	16	7	1.	24	0.292
69	A	18	7	1.	24	0.292
70	A	8	3	1.	22	0.136
71	A	9	5	1.	22	0.227
72	A	7	5	1.	22	0.227
73	A	4	4	1.	22	0.182
74	A	3	2	1.	20	0.1
75	A	8	4	1.	22	0.182
76	A	10	5	1.	22	0.227
77	A	12	5	1.	22	0.227
78	A	14	5	1.	22	0.227
79	A	5	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	7	3	1.	24	0.125
81	A	6	3	1.	24	0.125
82	A	5	3	1.	24	0.125
83	A	4	3	1.	22	0.136
84	A	5	4	1.	24	0.167
85	A	6	5	1.	24	0.208
86	A	7	5	1.	24	0.208
87	A	8	5	1.	24	0.208
88	A	11	3	1.	24	0.125
89	A	17	3	1.	24	0.125
90	A	14	3	1.	24	0.125
91	A	11	3	1.	24	0.125
92	A	8	3	1.	22	0.136
93	A	11	4	1.	24	0.167
94	A	14	5	1.	24	0.208
95	A	17	5	1.	24	0.208
96	A	20	5	1.	24	0.208
97	A	0	0	0.	0	0.
98	A	17	8	1.	20	0.4
99	A	14	8	1.	20	0.4
100	A	11	7	1.	20	0.35
101	A	8	6	1.	18	0.333
102	A	0	0	0.	0	0.
103	A	0	0	0.	0	0.
104	A	0	0	0.	0	0.
105	A	8	8	1.	16	0.5
106	A	7	7	1.	16	0.438
107	A	6	6	1.	16	0.375
108	A	3	2	1.	14	0.143
109	A	0	0	0.	0	0.
110	A	0	0	0.	0	0.
111	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	31	7	1.	22	0.318
113	A	25	9	1.	22	0.409
114	A	17	7	1.	22	0.318
115	A	12	5	1.	20	0.25
116	A	0	0	0.	0	0.
117	A	0	0	0.	0	0.
118	A	18	7	1.	24	0.292
119	A	16	7	1.	24	0.292
120	A	14	7	1.	24	0.292
121	A	14	7	1.	24	0.292
122	A	16	7	1.	24	0.292
123	A	18	7	1.	24	0.292
124	A	10	7	1.	26	0.269
125	A	9	7	1.	26	0.269
126	A	8	7	1.	26	0.269
127	A	8	7	1.	26	0.269
128	A	9	7	1.	26	0.269
129	A	10	7	1.	26	0.269
130	A	26	7	1.	26	0.269
131	A	23	7	1.	26	0.269
132	A	20	7	1.	26	0.269
133	A	20	7	1.	26	0.269
134	A	23	7	1.	26	0.269
135	A	26	7	1.	26	0.269
136	A	8	3	1.	22	0.136
137	A	9	4	1.	22	0.182
138	A	9	5	1.	22	0.227
139	A	4	2	1.	22	0.091
140	A	4	3	1.	20	0.15
141	A	8	4	1.	22	0.182
142	A	10	5	1.	22	0.227
143	A	12	5	1.	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	14	5	1.	22	0.227
145	A	11	3	1.	24	0.125
146	A	17	3	1.	24	0.125
147	A	14	3	1.	24	0.125
148	A	11	3	1.	24	0.125
149	A	8	3	1.	22	0.136
150	A	11	4	1.	24	0.167
151	A	14	5	1.	24	0.208
152	A	17	5	1.	24	0.208
153	A	20	5	1.	24	0.208
154	A	8	3	1.	24	0.125
155	A	12	3	1.	24	0.125
156	A	10	3	1.	24	0.125
157	A	8	3	1.	24	0.125
158	A	6	3	1.	22	0.136
159	A	8	4	1.	24	0.167
160	A	10	5	1.	24	0.208
161	A	12	5	1.	24	0.208
162	A	14	5	1.	24	0.208
163	A	0	0	0.	0	0.
164	A	13	11	1.	22	0.5
165	A	12	12	1.	22	0.546
166	A	9	8	1.	22	0.364
167	A	8	8	1.	20	0.4
168	A	0	0	0.	0	0.
169	A	0	0	0.	0	0.
170	A	0	0	0.	0	0.
171	A	16	9	1.	22	0.409
172	A	13	8	1.	22	0.364
173	A	10	7	1.	22	0.318
174	A	5	5	1.	20	0.25
175	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	0	0	0.	0	0.
177	A	0	0	0.	0	0.
178	A	15	8	1.	16	0.5
179	A	13	10	1.	16	0.625
180	A	9	7	1.	16	0.438
181	A	7	7	1.	14	0.5
182	A	0	0	0.	0	0.
183	A	0	0	0.	0	0.
184	A	18	7	1.	24	0.292
185	A	16	7	1.	24	0.292
186	A	14	7	1.	24	0.292
187	A	14	7	1.	24	0.292
188	A	16	7	1.	24	0.292
189	A	18	7	1.	24	0.292
190	A	26	7	1.	26	0.269
191	A	23	7	1.	26	0.269
192	A	20	7	1.	26	0.269
193	A	20	7	1.	26	0.269
194	A	23	7	1.	26	0.269
195	A	26	7	1.	26	0.269
196	A	18	7	1.	26	0.269
197	A	16	7	1.	26	0.269
198	A	14	7	1.	26	0.269
199	A	14	7	1.	26	0.269
200	A	16	7	1.	26	0.269
201	A	18	7	1.	26	0.269
202	A	12	10	1.	12	0.833
203	A	11	10	1.	12	0.833
204	A	6	5	1.	10	0.5
205	A	26	15	1.	12	1.25
206	A	19	11	1.	12	0.917
207	A	16	10	1.	10	1.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
208	A	0	0	0.	0	0.
209	A	7	6	1.	14	0.429
210	A	6	6	1.	14	0.429
211	A	5	5	1.	14	0.357
212	A	4	4	1.	12	0.333
213	A	0	0	0.	0	0.
214	A	0	0	0.	0	0.
215	A	0	0	0.	0	0.
216	A	14	8	1.	20	0.4
217	A	11	7	1.	20	0.35
218	A	8	6	1.	18	0.333
219	A	0	0	0.	0	0.
220	A	0	0	0.	0	0.
221	A	0	0	0.	0	0.
222	A	12	12	1.	22	0.546
223	A	9	8	1.	22	0.364
224	A	8	8	1.	20	0.4
225	A	0	0	0.	0	0.
226	A	0	0	0.	0	0.
227	A	0	0	0.	0	0.
228	A	12	6	1.	20	0.3
229	A	10	6	1.	20	0.3
230	A	8	5	1.	20	0.25
231	A	6	4	1.	18	0.222
232	A	0	0	0.	0	0.
233	A	0	0	0.	0	0.
234	A	0	0	0.	0	0.
235	A	23	14	1.	22	0.636
236	A	19	15	1.	22	0.682
237	A	10	10	1.	20	0.5
238	A	0	0	0.	0	0.
239	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
240	A	0	0	0.	0	0.
241	A	22	18	1.	22	0.818
242	A	17	13	1.	22	0.591
243	A	11	10	1.	20	0.5
244	A	0	0	0.	0	0.
245	A	0	0	0.	0	0.
246	A	0	0	0.	0	0.
247	A	10	6	1.	20	0.3
248	A	8	5	1.	20	0.25
249	A	6	4	1.	20	0.2
250	A	2	2	1.	18	0.111
251	A	0	0	0.	0	0.
252	A	0	0	0.	0	0.
253	A	0	0	0.	0	0.
254	A	7	7	1.	16	0.438
255	A	6	6	1.	16	0.375
256	A	3	2	1.	14	0.143
257	A	0	0	0.	0	0.
258	A	0	0	0.	0	0.
259	A	0	0	0.	0	0.
260	A	13	8	1.	22	0.364
261	A	10	7	1.	22	0.318
262	A	5	5	1.	20	0.25
263	A	0	0	0.	0	0.
264	A	0	0	0.	0	0.
265	A	0	0	0.	0	0.
266	A	27	14	1.	22	0.636
267	A	23	14	1.	22	0.636
268	A	19	15	1.	22	0.682
269	A	10	10	1.08	20	0.5
270	A	0	0	0.	0	0.
271	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	A	0	0	0.	0	0.
273	A	7	7	1.	24	0.292
274	A	6	6	1.	24	0.25
275	A	3	3	1.	22	0.136
276	A	0	0	0.	0	0.
277	A	0	0	0.	0	0.
278	A	0	0	0.	0	0.
279	A	64	24	1.	24	1.
280	A	36	22	1.	24	0.917
281	A	13	12	1.13	22	0.546
282	A	0	0	0.	0	0.
283	A	0	0	0.	0	0.
284	A	0	0	0.	0	0.
285	A	40	18	1.	20	0.9
286	A	29	19	1.	20	0.95
287	A	13	12	1.	18	0.667
288	A	0	0	0.	0	0.
289	A	0	0	0.	0	0.
290	A	0	0	0.	0	0.
291	A	7	7	1.	22	0.318
292	A	6	6	1.	22	0.273
293	A	3	3	1.	22	0.136
294	A	3	3	1.	20	0.15
295	A	0	0	0.	0	0.
296	A	0	0	0.	0	0.
297	A	0	0	0.	0	0.
298	A	25	9	1.	22	0.409
299	A	17	7	1.	22	0.318
300	A	12	5	1.	20	0.25
301	A	0	0	0.	0	0.
302	A	0	0	0.	0	0.
303	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
304	A	13	10	1.	16	0.625
305	A	9	7	1.	16	0.438
306	A	7	7	1.	14	0.5
307	A	0	0	0.	0	0.
308	A	0	0	0.	0	0.
309	A	0	0	0.	0	0.
310	A	25	16	1.	22	0.727
311	A	22	18	1.	22	0.818
312	A	17	13	1.	22	0.591
313	A	11	10	1.01	20	0.5
314	A	0	0	0.	0	0.
315	A	0	0	0.	0	0.
316	A	0	0	0.	0	0.
317	A	44	19	1.	24	0.792
318	A	31	19	1.	24	0.792
319	A	13	12	1.12	22	0.546
320	A	0	0	0.	0	0.
321	A	0	0	0.	0	0.
322	A	0	0	0.	0	0.
323	A	16	9	1.	24	0.375
324	A	10	7	1.	24	0.292
325	A	7	5	1.	22	0.227
326	A	0	0	0.	0	0.
327	A	0	0	0.	0	0.
328	A	4	3	1.	18	0.167
329	A	3	3	1.	18	0.167
330	A	3	3	1.	18	0.167
331	A	2	2	1.	18	0.111
332	A	2	2	1.	18	0.111
333	A	3	3	1.	18	0.167
334	A	3	3	1.	18	0.167
335	A	4	3	1.	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
336	A	5	4	1.	18	0.222
337	A	4	4	1.	18	0.222
338	A	4	4	1.	18	0.222
339	A	3	3	1.	18	0.167
340	A	3	3	1.	18	0.167
341	A	4	4	1.	18	0.222
342	A	4	4	1.	18	0.222
343	A	5	4	1.	18	0.222
344	A	4	3	1.	18	0.167
345	A	3	3	1.	18	0.167
346	A	3	3	1.	18	0.167
347	A	2	2	1.	18	0.111
348	A	2	2	1.	18	0.111
349	A	3	3	1.	18	0.167
350	A	3	3	1.	18	0.167
351	A	4	3	1.	18	0.167
352	A	5	4	1.	18	0.222
353	A	4	4	1.	18	0.222
354	A	4	4	1.	18	0.222
355	A	3	3	1.	18	0.167
356	A	3	3	1.	18	0.167
357	A	4	4	1.	18	0.222
358	A	4	4	1.	18	0.222
359	A	5	4	1.	18	0.222
360	A	6	3	1.	8	0.375
361	A	14	5	1.	14	0.357
362	A	10	4	1.	14	0.286
363	A	10	5	1.	14	0.357
364	A	6	2	1.	12	0.167
365	A	12	5	1.	14	0.357
366	A	12	6	1.	14	0.429
367	A	16	7	1.	14	0.5

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
368	A	14	5	1.	23	0.217
369	A	10	4	1.	23	0.174
370	A	10	5	1.	23	0.217
371	A	6	2	1.	21	0.095
372	A	12	5	1.	23	0.217
373	A	12	6	1.	23	0.261
374	A	16	7	1.	23	0.304
375	A	16	8	1.	23	0.348
376	A	20	9	1.	25	0.36
377	A	16	8	1.	25	0.32
378	A	12	7	1.	23	0.304
379	A	0	0	0.	0	0.
380	A	0	0	0.	0	0.
381	A	0	0	0.	0	0.
382	A	20	12	1.	23	0.522
383	A	19	13	1.	23	0.565
384	A	14	9	1.	23	0.391
385	A	13	9	1.	21	0.429
386	A	0	0	0.	0	0.
387	A	0	0	0.	0	0.
388	A	0	0	0.	0	0.
389	A	19	9	1.	25	0.36
390	A	15	8	1.	25	0.32
391	A	9	6	1.	23	0.261
392	A	0	0	0.	0	0.
393	A	0	0	0.	0	0.
394	A	0	0	0.	0	0.
395	A	12	7	1.	8	0.875
396	A	5	4	1.	10	0.4
397	A	19	6	1.	10	0.6

Chapter 3

Listing of integrals

3.1 $\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=137

$$-\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{b}$$

```
[Out] -((2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-3 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m)
```

Rubi [A] time = 0.155546, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4406, 12, 3308, 2181}

$$-\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-3 - m)*(c + d*x)^m*Gamma[
```

```
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx &= \int \frac{1}{2}(c+dx)^m \sin(2a+2bx)\,dx \\
&= \frac{1}{2}\int (c+dx)^m \sin(2a+2bx)\,dx \\
&= \frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx - \frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx \\
&= -\frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c
\end{aligned}$$

Mathematica [A] time = 0.0851013, size = 138, normalized size = 1.01

$$-\frac{2^{-m-3}e^{-\frac{2i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{4ia}\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)+e^{\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(m+1,\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((2^(-3 - m)*(c + d*x)^m*(E^((4*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, (
(-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1
 + m, ((2*I)*b*(c + d*x))/d]))/(b*E^(((2*I)*(b*c + a*d))/d)*((b^2*(c + d*x)
^2)/d^2)^m))
```

Maple [F] time = 0.209, size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

Fricas [A] time = 0.515529, size = 246, normalized size = 1.8

$$-\frac{e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,\frac{2ibdx+2ibc}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(m+1,\frac{-2ibdx-2ibc}{d}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m + 1, (2*I*b*d*x
 + 2*I*b*c)/d) + e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1
, (-2*I*b*d*x - 2*I*b*c)/d))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

3.2 $\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=156

$$-\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}-\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{d(c+dx)^3\sin(a+bx)\cos(a+bx)}{b^2}+\frac{3d^4\sin^2(a+b}{4b^5}$$

```
[Out] (3*c*d^3*x)/(2*b^3) + (3*d^4*x^2)/(4*b^3) - (c + d*x)^4/(4*b) - (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 + (3*d^4*Sin[a + b*x]^2)/(4*b^5) - (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) + ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.106998, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4404, 3311, 32, 3310}

$$-\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}-\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{d(c+dx)^3\sin(a+bx)\cos(a+bx)}{b^2}+\frac{3d^4\sin^2(a+b}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```


```
[Out] (3*c*d^3*x)/(2*b^3) + (3*d^4*x^2)/(4*b^3) - (c + d*x)^4/(4*b) - (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 + (3*d^4*Sin[a + b*x]^2)/(4*b^5) - (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) + ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```


Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx &= \frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{(2d)\int (c+dx)^3\sin^2(a+bx)\,dx}{b}\\ &= \frac{d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2}-\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}+\frac{(c+dx)^4\sin^2(\\ &= -\frac{(c+dx)^4}{4b}-\frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4}+\frac{d(c+dx)^3\cos(a+bx)\sin(a\\ &= \frac{3cd^3x}{2b^3}+\frac{3d^4x^2}{4b^3}-\frac{(c+dx)^4}{4b}-\frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4}+\frac{d(c+dx)^3}{}\end{aligned}$$

Mathematica [A] time = 0.512725, size = 86, normalized size = 0.55

$$\frac{4bd(c+dx)\sin(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)-2\cos(2(a+bx))\left(-6b^2d^2(c+dx)^2+2b^4(c+dx)^4+3d^4\right)}{16b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
4*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^5)
```

Maple [B] time = 0.034, size = 853, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/2*(b*x+a)^4*cos(b*x+a)^2+2*(b*x+a)^3*(1/2*cos(b*x+a)*sin
(b*x+a)+1/2*b*x+1/2*a)+3/2*(b*x+a)^2*cos(b*x+a)^2-3*(b*x+a)*(1/2*cos(b*x+a)
*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2+3/4*sin(b*x+a)^2-3/4*(b*x+a)^4)-4/
b^4*a*d^4*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*
x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)-3/8*
b*x-3/8*a-1/2*(b*x+a)^3)+4/b^3*c*d^3*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+
a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8
*cos(b*x+a)*sin(b*x+a)-3/8*b*x-3/8*a-1/2*(b*x+a)^3)+6/b^4*a^2*d^4*(-1/2*(b*
x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(
b*x+a)^2-1/4*sin(b*x+a)^2)-12/b^3*a*c*d^3*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x
+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^
2)+6/b^2*c^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b
*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-4/b^4*a^3*d^4*(-1/2*(b
*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+12/b^3*a^2*c*d^
3*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-12/b^
2*a*c^2*d^2*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/
4*a)+4/b*c^3*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x
+1/4*a)-1/2/b^4*a^4*d^4*cos(b*x+a)^2+2/b^3*a^3*c*d^3*cos(b*x+a)^2-3/b^2*a^2
*c^2*d^2*cos(b*x+a)^2+2/b*a*c^3*d*cos(b*x+a)^2-1/2*c^4*cos(b*x+a)^2)
```

Maxima [B] time = 1.22132, size = 791, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^4*cos(b*x + a)^2 - 16*a*c^3*d*cos(b*x + a)^2/b + 24*a^2*c^2*d^2*c
os(b*x + a)^2/b^2 - 16*a^3*c*d^3*cos(b*x + a)^2/b^3 + 4*a^4*d^4*cos(b*x + a
)^2/b^4 + 4*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c^3*d/b - 12*
(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 12*(2*(b*
x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^2*c*d^3/b^3 - 4*(2*(b*x + a)*
cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^3*d^4/b^4 + 6*((2*(b*x + a)^2 - 1)*c
```

```
os(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 12*((2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 6
*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a^2*
d^4/b^4 + 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(2*(b*x +
 a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)
*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((2
*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b
*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

Fricas [A] time = 0.49188, size = 518, normalized size = 3.32

$$\frac{b^4d^4x^4 + 4\,b^4cd^3x^3 + 3\left(2\,b^4c^2d^2 - b^2d^4\right)x^2 - \left(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 2\,b^4c^4 - 6\,b^2c^2d^2 + 3\,d^4 + 6\left(2\,b^4c^2d^2 - b^2d^4\right)x^2 + 4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b
^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4
*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 +
 2*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^
2*d^2 - b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)
*x)/b^5
```

Sympy [A] time = 7.63633, size = 502, normalized size = 3.22

$$\begin{cases} \frac{c^4\sin^2{(a+bx)}}{2b} + \frac{c^3dx\sin^2{(a+bx)}}{b} - \frac{c^3dx\cos^2{(a+bx)}}{b} + \frac{3c^2d^2x^2\sin^2{(a+bx)}}{2b} - \frac{3c^2d^2x^2\cos^2{(a+bx)}}{2b} + \frac{cd^3x^3\sin^2{(a+bx)}}{b} - \frac{cd^3x^3\cos^2{(a+bx)}}{b} + \frac{d^4x^4 s}{} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin{(a)}\cos{(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**2/(2*b) + c**3*d*x*sin(a + b*x)**2/b - c**3*d
*x*cos(a + b*x)**2/b + 3*c**2*d**2*x**2*sin(a + b*x)**2/(2*b) - 3*c**2*d**2
*x**2*cos(a + b*x)**2/(2*b) + c*d**3*x**3*sin(a + b*x)**2/b - c*d**3*x**3*c
os(a + b*x)**2/b + d**4*x**4*sin(a + b*x)**2/(4*b) - d**4*x**4*cos(a + b*x)
```

```
**2/(4*b) + c**3*d*sin(a + b*x)*cos(a + b*x)/b**2 + 3*c**2*d**2*x*sin(a + b
*x)*cos(a + b*x)/b**2 + 3*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)/b**2 + d**4
*x**3*sin(a + b*x)*cos(a + b*x)/b**2 - 3*c**2*d**2*sin(a + b*x)**2/(2*b**3)
 - 3*c*d**3*x*sin(a + b*x)**2/(2*b**3) + 3*c*d**3*x*cos(a + b*x)**2/(2*b**3
) - 3*d**4*x**2*sin(a + b*x)**2/(4*b**3) + 3*d**4*x**2*cos(a + b*x)**2/(4*b
**3) - 3*c*d**3*sin(a + b*x)*cos(a + b*x)/(2*b**4) - 3*d**4*x*sin(a + b*x)*
cos(a + b*x)/(2*b**4) + 3*d**4*sin(a + b*x)**2/(4*b**5), Ne(b, 0)), ((c**4*
x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*co
s(a), True))
```

Giac [A] time = 1.09295, size = 244, normalized size = 1.56

$$-\frac{\left(2\,b^4d^4x^4+8\,b^4cd^3x^3+12\,b^4c^2d^2x^2+8\,b^4c^3dx+2\,b^4c^4-6\,b^2d^4x^2-12\,b^2cd^3x-6\,b^2c^2d^2+3\,d^4\right)\cos\left(2\,bx+2\,a\right)}{8\,b^5}+\frac{\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x
+ 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2
*b*x + 2*a)/b^5 + 1/4*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

3.3 $\int (c+dx)^3 \cos(a+bx) \sin(a+bx)\,dx$

Optimal. Leaf size=120

$$-\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}+\frac{(c+dx)^3\sin^2(a+bx)}{2b}$$

```
[Out] (3*d^3*x)/(8*b^3) - (c + d*x)^3/(4*b) - (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(
8*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) - (3*d^2*(c +
d*x)*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^3*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0834477, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4404, 3311, 32, 2635, 8}

$$-\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}+\frac{(c+dx)^3\sin^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (3*d^3*x)/(8*b^3) - (c + d*x)^3/(4*b) - (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(
8*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) - (3*d^2*(c +
d*x)*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^3*Sin[a + b*x]^2)/(2*b)
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2\sin^2(a+bx)\,dx}{2b}$$
$$= \frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3} + \frac{(c+dx)^3\sin}{2b}$$
$$= -\frac{(c+dx)^3}{4b} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx}{4b^2}$$
$$= \frac{3d^3x}{8b^3} - \frac{(c+dx)^3}{4b} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos(a+bx)\sin(}{4b^2}$$

Mathematica [A] time = 0.300051, size = 71, normalized size = 0.59

$$\frac{3d\sin(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)-2b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 3*d*(-d^2 +
 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^4)
```

Maple [B] time = 0.015, size = 466, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*s
in(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)
-3/8*b*x-3/8*a-1/2*(b*x+a)^3)-3/b^3*a*d^3*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x
+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^
2)+3/b^2*c*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+3/b^3*a^2*d^3*(-1/2*(b*x
+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-6/b^2*a*c*d^2*(-1
/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+3/b*c^2*d*
(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+1/2/b^3
*a^3*d^3*cos(b*x+a)^2-3/2/b^2*a^2*c*d^2*cos(b*x+a)^2+3/2/b*a*c^2*d*cos(b*x+
a)^2-1/2*c^3*cos(b*x+a)^2)
```

Maxima [B] time = 1.13191, size = 462, normalized size = 3.85

$$-\frac{8\,c^3\cos\left(bx+a\right)^2 - \frac{24\,ac^2d\cos(bx+a)^2}{b} + \frac{24\,a^2cd^2\cos(bx+a)^2}{b^2} - \frac{8\,a^3d^3\cos(bx+a)^2}{b^3} + \frac{6\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))c^2d}{b} - \frac{12\,(2\,(bx+a)\,\mathrm{co}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*(8*c^3*cos(b*x + a)^2 - 24*a*c^2*d*cos(b*x + a)^2/b + 24*a^2*c*d^2*co
s(b*x + a)^2/b^2 - 8*a^3*d^3*cos(b*x + a)^2/b^3 + 6*(2*(b*x + a)*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)*cos(2*b*x + 2*a) - sin
(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x +
2*a))*a^2*d^3/b^3 + 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*s
in(2*b*x + 2*a))*c*d^2/b^2 - 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b
*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(
2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

Fricas [A] time = 0.483148, size = 348, normalized size = 2.9

$$\frac{2\,b^3d^3x^3+6\,b^3cd^2x^2-2\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+2\,b^3c^3-3\,bcd^2+3\left(2\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^2+3\left(2\,b^2d^3x^2+4\,b^2\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 +
 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b
^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) +
 3*(2*b^3*c^2*d - b*d^3)*x)/b^4
```

Sympy [A] time = 3.10669, size = 342, normalized size = 2.85

$$\begin{cases}\frac{c^3\sin^2\left(a+bx\right)}{2b}+\frac{3c^2dx\sin^2\left(a+bx\right)}{4b}-\frac{3c^2dx\cos^2\left(a+bx\right)}{4b}+\frac{3cd^2x^2\sin^2\left(a+bx\right)}{4b}-\frac{3cd^2x^2\cos^2\left(a+bx\right)}{4b}+\frac{d^3x^3\sin^2\left(a+bx\right)}{4b}-\frac{d^3x^3\cos^2\left(a+bx\right)}{4b}+\frac{3c^2d\,\mathrm{s}}{} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin\left(a\right)\cos\left(a\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**2/(2*b) + 3*c**2*d*x*sin(a + b*x)**2/(4*b) -
3*c**2*d*x*cos(a + b*x)**2/(4*b) + 3*c*d**2*x**2*sin(a + b*x)**2/(4*b) - 3*
c*d**2*x**2*cos(a + b*x)**2/(4*b) + d**3*x**3*sin(a + b*x)**2/(4*b) - d**3*
x**3*cos(a + b*x)**2/(4*b) + 3*c**2*d*sin(a + b*x)*cos(a + b*x)/(4*b**2) +
3*c*d**2*x*sin(a + b*x)*cos(a + b*x)/(2*b**2) + 3*d**3*x**2*sin(a + b*x)*co
s(a + b*x)/(4*b**2) - 3*c*d**2*sin(a + b*x)**2/(4*b**3) - 3*d**3*x*sin(a +
b*x)**2/(8*b**3) + 3*d**3*x*cos(a + b*x)**2/(8*b**3) - 3*d**3*sin(a + b*x)*
cos(a + b*x)/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
+ d**3*x**4/4)*sin(a)*cos(a), True))
```

Giac [A] time = 1.14494, size = 163, normalized size = 1.36

$$-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(2\,bx+2\,a\right)}{8\,b^4}+\frac{3\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\mathrm{s}}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3
*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^4 + 3/16*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x
+ 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a)/b^4
```

3.4 $\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{cdx}{2b}-\frac{d^2x^2}{4b}$$

```
[Out] -(c*d*x)/(2*b) - (d^2*x^2)/(4*b) + (d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/
(2*b^2) - (d^2*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^2*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0541044, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4404, 3310}

$$\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{cdx}{2b}-\frac{d^2x^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -(c*d*x)/(2*b) - (d^2*x^2)/(4*b) + (d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/
(2*b^2) - (d^2*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^2*Sin[a + b*x]^2)/(2*b)
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)^2\sin^2(a+bx)}{2b} - \frac{d\int (c+dx)\sin^2(a+bx)\,dx}{b}$$
$$= \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} - \frac{d^2\sin^2(a+bx)}{4b^3} + \frac{(c+dx)^2\sin^2(a+bx)}{2b} - \cdots$$
$$= -\frac{cdx}{2b} - \frac{d^2x^2}{4b} + \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} - \frac{d^2\sin^2(a+bx)}{4b^3} + \frac{(c+dx)^2}{\cdots}$$

Mathematica [A] time = 0.233082, size = 50, normalized size = 0.56

$$\frac{\cos(2(a+bx))\left(d^2-2b^2(c+dx)^2\right)+2bd(c+dx)\sin(2(a+bx))}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] ((d^2 - 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 2*b*d*(c + d*x)*Sin[2*(a + b*
x)])/(8*b^3)
```

Maple [B] time = 0.015, size = 215, normalized size = 2.4

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(-\frac{(bx+a)^2(\cos(bx+a))^2}{2}+(bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{(\sin(bx+a))^2}{4}\right)-2\frac{ad^2}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-2/b^2*a*d^2*(-1/2*(b*x+a
)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+2/b*c*d*(-1/2*(b*x+
a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-1/2/b^2*a^2*d^2*co
s(b*x+a)^2+1/b*a*c*d*cos(b*x+a)^2-1/2*c^2*cos(b*x+a)^2)
```

Maxima [B] time = 1.1559, size = 231, normalized size = 2.6

$$-\frac{4c^2\cos(bx+a)^2-\frac{8acd\cos(bx+a)^2}{b}+\frac{4a^2d^2\cos(bx+a)^2}{b^2}+\frac{2(2(bx+a)\cos(2bx+2a)-\sin(2bx+2a))cd}{b}-\frac{2(2(bx+a)\cos(2bx+2a)-\sin(2bx+2a))a\cdots}{b^2}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^2*cos(b*x + a)^2 - 8*a*c*d*cos(b*x + a)^2/b + 4*a^2*d^2*cos(b*x +
 a)^2/b^2 + 2*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c*d/b - 2*(
2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*d^2/b^2 + ((2*(b*x + a)^
2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

Fricas [A] time = 0.474177, size = 203, normalized size = 2.28

$$\frac{b^2d^2x^2 + 2\,b^2cdx - \left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(bx + a\right)^2 + 2\left(bd^2x + bcd\right)\cos\left(bx + a\right)\sin\left(bx + a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 -
 d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/b^3
```

Sympy [A] time = 1.37364, size = 175, normalized size = 1.97

$$\begin{cases} \frac{c^2\sin^2\left(a+bx\right)}{2b} + \frac{cdx\sin^2\left(a+bx\right)}{2b} - \frac{cdx\cos^2\left(a+bx\right)}{2b} + \frac{d^2x^2\sin^2\left(a+bx\right)}{4b} - \frac{d^2x^2\cos^2\left(a+bx\right)}{4b} + \frac{cd\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b^2} + \frac{d^2x\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b^2} - \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin\left(a\right)\cos\left(a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**2/(2*b) + c*d*x*sin(a + b*x)**2/(2*b) - c*d*x
*cos(a + b*x)**2/(2*b) + d**2*x**2*sin(a + b*x)**2/(4*b) - d**2*x**2*cos(a
+ b*x)**2/(4*b) + c*d*sin(a + b*x)*cos(a + b*x)/(2*b**2) + d**2*x*sin(a + b
*x)*cos(a + b*x)/(2*b**2) - d**2*sin(a + b*x)**2/(4*b**3), Ne(b, 0)), ((c**
2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a), True))
```

Giac [A] time = 1.12213, size = 99, normalized size = 1.11

$$-\frac{(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2)\cos\left(2\,bx + 2\,a\right)}{8\,b^3} + \frac{(bd^2x + bcd)\sin\left(2\,bx + 2\,a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^3 +
 1/4*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a)/b^3
```

3.5 $\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=50

$$\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{dx}{4b}$$

```
[Out] -(d*x)/(4*b) + (d*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) + ((c + d*x)*Sin[a + b
*x]^2)/(2*b)
```

Rubi [A] time = 0.0259538, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4404, 2635, 8}

$$\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{dx}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```


```
[Out] -(d*x)/(4*b) + (d*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) + ((c + d*x)*Sin[a + b
*x]^2)/(2*b)
```


Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\int \sin^2(a+bx)\,dx}{2b}$$
$$= \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\int 1\,dx}{4b}$$
$$= -\frac{dx}{4b} + \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{(c+dx)\sin^2(a+bx)}{2b}$$

Mathematica [A] time = 0.100934, size = 34, normalized size = 0.68

$$\frac{d\sin(2(a+bx)) - 2b(c+dx)\cos(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)])/(8*b^2)
```

Maple [A] time = 0.014, size = 74, normalized size = 1.5

$$\frac{1}{b}\left(\frac{d}{b}\left(-\frac{(bx+a)(\cos(bx+a))^2}{2} + \frac{\cos(bx+a)\sin(bx+a)}{4} + \frac{bx}{4} + \frac{a}{4}\right) + \frac{ad(\cos(bx+a))^2}{2b} - \frac{c(\cos(bx+a))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(d/b*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a
)+1/2/b*a*d*cos(b*x+a)^2-1/2*c*cos(b*x+a)^2)
```

Maxima [A] time = 1.1213, size = 88, normalized size = 1.76

$$-\frac{4c\cos(bx+a)^2 - \frac{4ad\cos(bx+a)^2}{b} + \frac{(2(bx+a)\cos(2bx+2a) - \sin(2bx+2a))d}{b}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c*cos(b*x + a)^2 - 4*a*d*cos(b*x + a)^2/b + (2*(b*x + a)*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a))*d/b)/b
```

Fricas [A] time = 0.464452, size = 108, normalized size = 2.16

$$\frac{bdx - 2\,(bdx + bc)\cos\left(bx + a\right)^2 + d\cos\left(bx + a\right)\sin\left(bx + a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a))/
b^2
```

Sympy [A] time = 0.599258, size = 80, normalized size = 1.6

$$\begin{cases} \frac{c\sin^2\left(a+bx\right)}{2b} + \frac{dx\sin^2\left(a+bx\right)}{4b} - \frac{dx\cos^2\left(a+bx\right)}{4b} + \frac{d\sin\left(a+bx\right)\cos\left(a+bx\right)}{4b^2} & \text{for}\: b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin\left(a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((c*sin(a + b*x)**2/(2*b) + d*x*sin(a + b*x)**2/(4*b) - d*x*cos(a
+ b*x)**2/(4*b) + d*sin(a + b*x)*cos(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x +
d*x**2/2)*sin(a)*cos(a), True))
```

Giac [A] time = 1.14172, size = 51, normalized size = 1.02

$$-\frac{(bdx + bc)\cos\left(2\,bx + 2\,a\right)}{4\,b^2} + \frac{d\sin\left(2\,bx + 2\,a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 + 1/8*d*sin(2*b*x + 2*a)/b^2
```

3.6 $\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=65

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) + (Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rubi [A] time = 0.139341, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4406, 12, 3303, 3299, 3302}

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```


```
[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) + (Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx\\
&= \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\\
&= \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\\
&= \frac{\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.130577, size = 60, normalized size = 0.92

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)+\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d] + Cos[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Maple [A] time = 0.026, size = 84, normalized size = 1.3

$$\frac{1}{2d}\mathrm{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\cos\left(2\frac{-ad+bc}{d}\right)-\frac{1}{2d}\mathrm{Ci}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\sin\left(2\frac{-ad+bc}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] 1/2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-1/2*Ci(2*b*x+2*a+2*(
-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d
```

Maxima [C] time = 1.38025, size = 190, normalized size = 2.92

$$-\frac{b\left(iE_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-iE_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_1\left(-\frac{2i\,bc+2i\,(bx+a)d}{d}\right.\right.}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I*e
xp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a
*d)/d) + b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + ex
p_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*
d)/d))/(b*d)
```

Fricas [A] time = 0.470119, size = 213, normalized size = 3.28

$$\frac{\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*s
in(-2*(b*c - a*d)/d) + 2*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)
```

```
/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x), x)
```

Giac [C] time = 1.20794, size = 768, normalized size = 11.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(a)^2*tan(b*c/d) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*
tan(b*c/d)^2 - 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d
)^2 - imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*sin_integral(2*(b*d*x + b*c)/d)*tan(a
)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*sin_integral(2*
(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) - imag_part(cos_integral(2*b*x + 2*b*c/d
))*tan(b*c/d)^2 + imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 -
2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a
) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*real_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(2*b*x + 2*
b*c/d)) - imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 2*sin_integral(2*(b*d
*x + b*c)/d))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*tan(b*c/d)^2 + d)
```

3.7 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=85

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}$$

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - Sin[2*a + 2*b
*x]/(2*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x
])/d^2
```

Rubi [A] time = 0.148834, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - Sin[2*a + 2*b
*x]/(2*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x
])/d^2
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx \\
&= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx \\
&= -\frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d} \\
&= -\frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} - \frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} \\
&= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.316456, size = 80, normalized size = 0.94

$$\frac{2b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-2b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)-\frac{d\sin(2(a+bx))}{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (2*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - (d*Sin[2*(a + b*
x)])/(c + d*x) - 2*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(
2*d^2)
```

Maple [A] time = 0.02, size = 124, normalized size = 1.5

$$\frac{b}{4}\left(-2\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)d}+2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\sin\left(2\frac{-ad+bc}{d}\right)+2\frac{1}{d}\text{Ci}\left(2bx+2a+2\frac{-ad+b}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/4*b*(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*
c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)
/d)/d)/d)
```

Maxima [C] time = 1.45052, size = 221, normalized size = 2.6

$$-\frac{b^2\left(iE_2\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)-iE_2\left(-\frac{2ibc+2i(bx+a)d-2iad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^2\left(E_2\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)+E_2\left(-\frac{2ibc+2i(bx+}{d}\right.\right.}{4\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I
*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) + b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
```

```
+ exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c
- a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 0.493468, size = 333, normalized size = 3.92

$$-\frac{2\,d\cos\left(bx+a\right)\sin\left(bx+a\right)+2\left(bdx+bc\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)-\left(\left(bdx+bc\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\left(bdx+bc\right)\mathrm{Ci}\left(-\frac{2}{}\right.\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a)*sin(b*x + a) + 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)
*sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral(2*(b*d*x + b
*c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)
/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)\sin\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

3.8 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=114

$$-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{2d^2(c+dx)}-\frac{\sin(2a+2bx)}{4d(c+dx)^2}$$

```
[Out] -(b*Cos[2*a + 2*b*x])/(2*d^2*(c + d*x)) - (b^2*CosIntegral[(2*b*c)/d + 2*b*
x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(4*d*(c + d*x)^2) - (b^2*Co
s[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rubi [A] time = 0.174515, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{2d^2(c+dx)}-\frac{\sin(2a+2bx)}{4d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(2*d^2*(c + d*x)) - (b^2*CosIntegral[(2*b*c)/d + 2*b*
x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(4*d*(c + d*x)^2) - (b^2*Co
s[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx$$

$$= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\sin(2a+2bx)}{4d(c+dx)^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{2d}$$

$$= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2}$$

$$= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} - \frac{\left(b^2\sin\left(2a\right.\right.}{}$$

$$= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{b^2\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\cos\left(2a-\frac{2bc}{d}\right.}{d}$$

Mathematica [A] time = 1.09059, size = 102, normalized size = 0.89

$$-\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(4*b^2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c +
d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a -
(2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(4*d^3)
```

Maple [A] time = 0.02, size = 162, normalized size = 1.4

$$\frac{b^2}{4}\left(-\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)^2 d}+\frac{1}{d}\left(-2\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)d}-2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\cos\left(2\frac{-ad+bc}{d}\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/4*b^2*(-sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a
)*d-a*d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci
(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)
```

Maxima [C] time = 1.74066, size = 269, normalized size = 2.36

$$-\frac{b^3\left(iE_3\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)-iE_3\left(-\frac{2ibc+2i(bx+a)d-2iad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(E_3\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)+E_3\left(-\frac{2ibc+2i(bx+a)}{d}\right.\right.}{4\left(b^2c^2d-2abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I
*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
```

```
 a*d)/d) + b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c
- a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d
^2 - a*d^3)*(b*x + a))*b)
```

Fricas [B] time = 0.513249, size = 525, normalized size = 4.61

$$\frac{bd^2x - d^2\cos(bx+a)\sin(bx+a) + bcd - 2\left(bd^2x + bcd\right)\cos(bx+a)^2 - 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}}{2\left(d^5x^2 + 2\,cd^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*x - d^2*cos(b*x + a)*sin(b*x + a) + b*c*d - 2*(b*d^2*x + b*c*d)*
cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)
/d)*sin_integral(2*(b*d*x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*c
os_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*
x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**3, x)
```

Giac [C] time = 1.60525, size = 7287, normalized size = 63.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*real_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^
2 - 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*sin_integral(2*(b*
d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^2*d^2*x^2*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 8*b^2*d^2
*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b^2*c
*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d
) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)
^2*tan(b*c/d) - b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*c^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c^2*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^2*d^2
*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 2*b^2*c*
d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 2*b^2*c*
d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 4*b^2*c
*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b^2*d^2*x^
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 8*b^2*c
*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)
- 8*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*t
an(b*c/d) + 16*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*
```

```
tan(b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^
2*tan(b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
a)^2*tan(b*c/d) + 2*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*c*d*x*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*real_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*
x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*
d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 4*b^2
*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b*d^2*x*tan(
b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*x)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 4*
b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^
2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - b^2*d
^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b^2*d^2*x^2*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^2*d^2*x^2*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2 - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(a)^2 + b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2 - 2*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^
2*tan(a)^2 - 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^
2*tan(b*c/d) - 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
x)^2*tan(b*c/d) + 4*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)*tan(b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a)*tan(b*c/d) + 8*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan
(b*c/d) + 4*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(a)*tan(b*c/d) - 4*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
x)^2*tan(a)*tan(b*c/d) + 8*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)
^2*tan(a)*tan(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))
*tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(a)^2*tan(b*c/d) - b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - b^2
*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b^2
*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*
b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d
*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d
*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + b^2*c^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b^2*c^2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c^2*s
```

```
in_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b*c*d*tan(b*x)^2*tan
(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*x)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^
2 + 4*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 2*b^2*d^2*x^2*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b^2*d^2*x^2*real_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a) + 2*b^2*c^2*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^2*c^2*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*x)^2*tan(a) - 2*b^2*c*d*x*imag_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2 + 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(a)^2 - 4*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + b*d^2*x*
tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*c/d) - 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*c/d) - 2*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
b*c/d) - 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(b*c/d) + 8*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b
*c/d) - 8*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*
c/d) + 16*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) + 2*b
^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b^2
*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 2*b^2*
c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 2*b^2*c*d*x*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*b^2*c*d*x*sin_int
egral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - b*d^2*x*tan(b*x)^2*tan(b*c/d)^2 - 2
*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b*
d^2*x*tan(b*x)*tan(a)*tan(b*c/d)^2 - b*d^2*x*tan(a)^2*tan(b*c/d)^2 + b^2*d^
2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d)) - b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d)) + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)
 + b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - b^2*c^2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 2*b^2*c^2*sin_integral
(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 4*b^2*c*d*x*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a) - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b^2*c^2
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^2*c^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2 + b*c*d*tan(b*x)^2*tan(a)^2 - 4*b^2*c*d*x*rea
l_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*b^2*c*d*x*real_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^2*c^2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*b^2*c^2*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^2*c^2*sin_integral(2*(b*d*x + b*c)/
d)*tan(a)*tan(b*c/d) - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*c/d)^2 + b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2
- 2*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - b*c*d*tan(b*x)^2
*tan(b*c/d)^2 - 4*b*c*d*tan(b*x)*tan(a)*tan(b*c/d)^2 - d^2*tan(b*x)^2*tan(a
)*tan(b*c/d)^2 - b*c*d*tan(a)^2*tan(b*c/d)^2 - d^2*tan(b*x)*tan(a)^2*tan(b*
c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)) - 2*b^2*c*d*x
*imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 4*b^2*c*d*x*sin_integral(2*(b*
```

```
d*x + b*c)/d) - b*d^2*x*tan(b*x)^2 + 2*b^2*c^2*real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(a) + 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a) - 4*b*d^2*x*tan(b*x)*tan(a) - b*d^2*x*tan(a)^2 - 2*b^2*c^2*real_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b^2*c^2*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(b*c/d) + b*d^2*x*tan(b*c/d)^2 + b^2*c^2*imag_part(
cos_integral(2*b*x + 2*b*c/d)) - b^2*c^2*imag_part(cos_integral(-2*b*x - 2*
b*c/d)) + 2*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d) - b*c*d*tan(b*x)^2 - 4*
b*c*d*tan(b*x)*tan(a) - d^2*tan(b*x)^2*tan(a) - b*c*d*tan(a)^2 - d^2*tan(b*
x)*tan(a)^2 + b*c*d*tan(b*c/d)^2 + d^2*tan(b*x)*tan(b*c/d)^2 + d^2*tan(a)*t
an(b*c/d)^2 + b*d^2*x + b*c*d + d^2*tan(b*x) + d^2*tan(a))/(d^5*x^2*tan(b*x
)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^
5*x^2*tan(b*x)^2*tan(a)^2 + d^5*x^2*tan(b*x)^2*tan(b*c/d)^2 + d^5*x^2*tan(a
)^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan
(b*x)^2*tan(a)^2 + 2*c*d^4*x*tan(b*x)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(a)^2*t
an(b*c/d)^2 + d^5*x^2*tan(b*x)^2 + d^5*x^2*tan(a)^2 + c^2*d^3*tan(b*x)^2*ta
n(a)^2 + d^5*x^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(b*c/d)^2 + c^2*d^3*t
an(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2 + 2*c*d^4*x*tan(a)^2 + 2*c*d^4*
x*tan(b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(b*x)^2 + c^2*d^3*tan(a)^2 + c^2*d^3*
tan(b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

3.9 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=144

$$-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-$$

```
[Out] -(b*Cos[2*a + 2*b*x])/(6*d^2*(c + d*x)^2) - (2*b^3*Cos[2*a - (2*b*c)/d]*Cos
Integral[(2*b*c)/d + 2*b*x])/(3*d^4) - Sin[2*a + 2*b*x]/(6*d*(c + d*x)^3) +
 (b^2*Sin[2*a + 2*b*x])/(3*d^3*(c + d*x)) + (2*b^3*Sin[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rubi [A] time = 0.197655, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```


```
[Out] -(b*Cos[2*a + 2*b*x])/(6*d^2*(c + d*x)^2) - (2*b^3*Cos[2*a - (2*b*c)/d]*Cos
Integral[(2*b*c)/d + 2*b*x])/(3*d^4) - Sin[2*a + 2*b*x]/(6*d*(c + d*x)^3) +
 (b^2*Sin[2*a + 2*b*x])/(3*d^3*(c + d*x)) + (2*b^3*Sin[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/(3*d^4)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin(2a+2bx)}{2(c+dx)^4}\,dx$$
$$= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$
$$= -\frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{3d}$$
$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} - \frac{b^2\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx}{3d^2}$$
$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{3d^3(c+dx)} - \frac{(2b^3)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{3d^3}$$
$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{3d^3(c+dx)} - \frac{\left(2b^3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2\right.}{c+dx}}{3d^3}$$
$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2\sin(2a+2b}{3d^3(c+dx)}$$

Mathematica [A] time = 0.660525, size = 164, normalized size = 1.14

$$\frac{-4b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)\right)-d\cos(2bx)\left(\sin(2a)\left(d^2-2b^2(c+d\right.\right.}{6d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] (-(d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*a
])) + d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a])*Sin[
2*b*x] - 4*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x)
)/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]))/(6*d^4*(c + d*
x)^3)
```

Maple [A] time = 0.021, size = 200, normalized size = 1.4

$$\frac{b^3}{4}\left(-\frac{2\sin(2bx+2a)}{3((bx+a)d-ad+bc)^3 d} + \frac{2}{3d}\left(-\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)^2 d} - \frac{1}{d}\left(-2\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)d} + 2\frac{1}{d}\left(2\frac{1}{d}\operatorname{Si}\left(2bx+2\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x)
```

```
[Out] 1/4*b^3*(-2/3*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^3/d+2/3*(-cos(2*b*x+2*a)/(
(b*x+a)*d-a*d+b*c)^2/d-(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b
*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)
*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)
```

Maxima [C] time = 2.11298, size = 336, normalized size = 2.33

$$-\frac{b^4\left(i\,E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-i\,E_4\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_4\left(-\frac{2i\,bc+2i\,(bx+}{d}\right.\right.}{4\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\left(bcd^3-ad^4\right)(bx+a)^2+3\left(b^2c^2d^2-2\,abcd^3+a^2d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(b^4*(I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I
*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) + b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c
- a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4
- a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ a^2*d^4)*(b*x + a))*b)
```

Fricas [B] time = 0.53332, size = 703, normalized size = 4.88

$$\frac{bd^3x+bcd^2-2\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2+2\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+4\left(b^3d^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*d^3*x + b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 2*(2*b^2*d^
3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 4*(b
```

```
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/
d)*sin_integral(2*(b*d*x + b*c)/d) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*c
os(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**4, x)
```

Giac [C] time = 1.80927, size = 10249, normalized size = 71.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*d^3*x
^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d
^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d
)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^
2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^3*d^3*x^3*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 8*b^3*d^3*x
^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) +
```

8*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*t
an(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b
*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c*d^2*x^2*sin_integral(
2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 12*b^3*c*d
^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d
)^2 - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2
*tan(a)*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan
(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3
*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^3
*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^
3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 6*b^3*c*d^2*x
^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c*d
^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^
3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/
d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) +
24*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)
*tan(b*c/d) + 24*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(a)^2*tan(b*c/d) - 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c
)/d)*tan(a)^2*tan(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c^2*d*x*sin_
integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 6*b^3*c*d^2*x^
2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*
c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2
 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c
/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 +
 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*
tan(b*c/d)^2 - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(a)*tan(b*c/d)^2 + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d
)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^

```
3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^3*d^3*x
^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c*d^2*x^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 12*b^3*c*d^2*x
^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 24*b^3*c*d
^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^3*d^3*x^3*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^3*d^3*x^3*real_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 12*b^3*c*d^2*x^2*imag_part(c
os_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 24*b^3*c*d^2*
x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) + 8*b^3*d^3*x^3*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^3*d^3*x^3*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 24*b^3*c^2*d*x
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 24
*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*ta
n(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2*tan(b*c/d) - 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
a)^2*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 12*b^3*c*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 12*b^3*c*d^2*x^2*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 4*b^2*d^3*x^2*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^3*c^3*sin_integral(2*(b*d*x + b*
c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 4*b^2*d^3*x^2*tan(b*x)*tan(
a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) +
4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 8*b^3*d^3*
x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a) - 12*b^3*c^2*d*x*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 12*b^3*c^2*d*x*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 24*b^3*c^2*d*x*sin_integral
(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 6*b^3*c*d^2*x^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(a)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*
```

b*x - 2*b*c/d))*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(b*x)^2*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*c/d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 12*b^
3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c
/d) + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)
+ 24*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/
d) + 24*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(
b*c/d) + 8*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
a)*tan(b*c/d) + 8*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(a)*tan(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(a)^2*tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(a)^2*tan(b*c/d) - 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/
d)*tan(a)^2*tan(b*c/d) - 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 + 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*
c/d))*tan(a)*tan(b*c/d)^2 - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(a)*tan(b*c/d)^2 + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)
/d)*tan(a)*tan(b*c/d)^2 + 8*b^2*c*d^2*x*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*
b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*
b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 8
*b^2*c*d^2*x*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b*d^3*x*tan(b*x)^2*tan(a)^2*t
an(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d)) + 2*b^
3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d)) + 6*b^3*c^2*d*x*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 6*b^3*c^2*d*x*real_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2
*b*x - 2*b*c/d))*tan(a) - 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*
tan(a) + 4*b^2*d^3*x^2*tan(b*x)^2*tan(a) - 4*b^3*c^3*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^3*c^3*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/
d)*tan(b*x)^2*tan(a) - 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a
)^2 + 4*b^2*d^3*x^2*tan(b*x)*tan(a)^2 + 12*b^3*c*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*c/d) + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b
*c)/d)*tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(b*c/d) + 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*t
an(b*c/d) + 24*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*
tan(b*c/d) + 24*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a

```
)*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*
tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*t
an(b*c/d) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) -
 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 6*b^
3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*b^2*d^
3*x^2*tan(b*x)*tan(b*c/d)^2 - 4*b^2*d^3*x^2*tan(a)*tan(b*c/d)^2 + 4*b^3*c^3
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b^3*c^3*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 8*b^3*c^3*si
n_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*d*tan(b*x)^2*
tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*d*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b*c*d^2*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2
*b*x + 2*b*c/d)) + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d)
) + 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^3*c
^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c^2*d*x*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 12*b^3*c^2*d*x*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(a) - 24*b^3*c^2*d*x*sin_integral(2*(b*d*x
 + b*c)/d)*tan(a) + 8*b^2*c*d^2*x*tan(b*x)^2*tan(a) - 2*b^3*c^3*real_part(c
os_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(a)^2 + 8*b^2*c*d^2*x*tan(b*x)*tan(a)^2 + b*d^3*x*tan(
b*x)^2*tan(a)^2 + 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
c/d) + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 8*b^3*c^
3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^3*c^3*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - 2*b^3*c^3*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 8*b^2*c*d^2*x*tan(b*x)*tan(b*c/d
)^2 - b*d^3*x*tan(b*x)^2*tan(b*c/d)^2 - 8*b^2*c*d^2*x*tan(a)*tan(b*c/d)^2 -
 4*b*d^3*x*tan(b*x)*tan(a)*tan(b*c/d)^2 - b*d^3*x*tan(a)^2*tan(b*c/d)^2 + 6
*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)) + 6*b^3*c^2*d*x*real_
part(cos_integral(-2*b*x - 2*b*c/d)) - 4*b^2*d^3*x^2*tan(b*x) - 4*b^2*d^3*x
^2*tan(a) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 4*b
^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 8*b^3*c^3*sin_int
egral(2*(b*d*x + b*c)/d)*tan(a) + 4*b^2*c^2*d*tan(b*x)^2*tan(a) + 4*b^2*c^2
*d*tan(b*x)*tan(a)^2 + b*c*d^2*tan(b*x)^2*tan(a)^2 + 4*b^3*c^3*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*c/d) + 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*t
an(b*c/d) - 4*b^2*c^2*d*tan(b*x)*tan(b*c/d)^2 - b*c*d^2*tan(b*x)^2*tan(b*c/
d)^2 - 4*b^2*c^2*d*tan(a)*tan(b*c/d)^2 - 4*b*c*d^2*tan(b*x)*tan(a)*tan(b*c/
d)^2 - 2*d^3*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - b*c*d^2*tan(a)^2*tan(b*c/d)^2
 - 2*d^3*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_part(cos_integral(
2*b*x + 2*b*c/d)) + 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d)) - 8
*b^2*c*d^2*x*tan(b*x) - b*d^3*x*tan(b*x)^2 - 8*b^2*c*d^2*x*tan(a) - 4*b*d^3
*x*tan(b*x)*tan(a) - b*d^3*x*tan(a)^2 + b*d^3*x*tan(b*c/d)^2 - 4*b^2*c^2*d*
tan(b*x) - b*c*d^2*tan(b*x)^2 - 4*b^2*c^2*d*tan(a) - 4*b*c*d^2*tan(b*x)*tan
(a) - 2*d^3*tan(b*x)^2*tan(a) - b*c*d^2*tan(a)^2 - 2*d^3*tan(b*x)*tan(a)^2
```

```
+ b*c*d^2*tan(b*c/d)^2 + 2*d^3*tan(b*x)*tan(b*c/d)^2 + 2*d^3*tan(a)*tan(b*c
/d)^2 + b*d^3*x + b*c*d^2 + 2*d^3*tan(b*x) + 2*d^3*tan(a))/(d^7*x^3*tan(b*x
)^2*tan(a)^2*tan(b*c/d)^2 + 3*c*d^6*x^2*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 +
d^7*x^3*tan(b*x)^2*tan(a)^2 + d^7*x^3*tan(b*x)^2*tan(b*c/d)^2 + d^7*x^3*tan
(a)^2*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 3*c*d^6
*x^2*tan(b*x)^2*tan(a)^2 + 3*c*d^6*x^2*tan(b*x)^2*tan(b*c/d)^2 + 3*c*d^6*x^
2*tan(a)^2*tan(b*c/d)^2 + c^3*d^4*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^7*x^
3*tan(b*x)^2 + d^7*x^3*tan(a)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(a)^2 + d^7*x^3
*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(a)^2*
tan(b*c/d)^2 + 3*c*d^6*x^2*tan(b*x)^2 + 3*c*d^6*x^2*tan(a)^2 + c^3*d^4*tan(
b*x)^2*tan(a)^2 + 3*c*d^6*x^2*tan(b*c/d)^2 + c^3*d^4*tan(b*x)^2*tan(b*c/d)^
2 + c^3*d^4*tan(a)^2*tan(b*c/d)^2 + d^7*x^3 + 3*c^2*d^5*x*tan(b*x)^2 + 3*c^
2*d^5*x*tan(a)^2 + 3*c^2*d^5*x*tan(b*c/d)^2 + 3*c*d^6*x^2 + c^3*d^4*tan(b*x
)^2 + c^3*d^4*tan(a)^2 + c^3*d^4*tan(b*c/d)^2 + 3*c^2*d^5*x + c^3*d^4)
```

3.10 $\int \frac{\cos(x)\sin(x)}{x}\,dx$

Optimal. Leaf size=8

$$\frac{\mathrm{Si}(2x)}{2}$$

[Out] SinIntegral[2*x]/2

Rubi [A] time = 0.0285319, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4406, 12, 3299}

$$\frac{\mathrm{Si}(2x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/x,x]

[Out] SinIntegral[2*x]/2

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \int \frac{\sin(2x)}{2x}\,dx$$
$$= \frac{1}{2}\int \frac{\sin(2x)}{x}\,dx$$
$$= \frac{\text{Si}(2x)}{2}$$

Mathematica [A] time = 0.0058604, size = 8, normalized size = 1.

$$\frac{\text{Si}(2x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x,x]
```

```
[Out] SinIntegral[2*x]/2
```

Maple [A] time = 0.026, size = 7, normalized size = 0.9

$$\frac{\text{Si}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x,x)
```

```
[Out] 1/2*Si(2*x)
```

Maxima [C] time = 1.21864, size = 18, normalized size = 2.25

$$-\frac{1}{4}i\,\text{Ei}(2ix) + \frac{1}{4}i\,\text{Ei}(-2ix)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="maxima")
```

```
[Out] -1/4*I*Ei(2*I*x) + 1/4*I*Ei(-2*I*x)
```

Fricas [A] time = 0.451929, size = 31, normalized size = 3.88

$$\frac{1}{2}\operatorname{Si}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sin_integral(2*x)
```

Sympy [A] time = 1.07542, size = 5, normalized size = 0.62

$$\frac{\operatorname{Si}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x)
```

```
[Out] Si(2*x)/2
```

Giac [A] time = 1.11652, size = 8, normalized size = 1.

$$\frac{1}{2}\operatorname{Si}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="giac")
```

```
[Out] 1/2*sin_integral(2*x)
```

3.11 $\int \frac{\cos(x)\sin(x)}{x^2}\,dx$

Optimal. Leaf size=16

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

```
[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)
```

Rubi [A] time = 0.0457541, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4406, 12, 3297, 3302}

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/x^2,x]
```

```
[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{x^2}\,dx &= \int \frac{\sin(2x)}{2x^2}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2x)}{x^2}\,dx\\ &= -\frac{\sin(2x)}{2x}+\int \frac{\cos(2x)}{x}\,dx\\ &= \text{Ci}(2x)-\frac{\sin(2x)}{2x}\end{aligned}$$

Mathematica [A] time = 0.0060185, size = 16, normalized size = 1.

$$\text{CosIntegral}(2x)-\frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x^2,x]
```

```
[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)
```

Maple [A] time = 0.026, size = 15, normalized size = 0.9

$$\text{Ci}\left(2\,x\right)-\frac{\sin\left(2\,x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x^2,x)
```

```
[Out] Ci(2*x)-1/2*sin(2*x)/x
```

Maxima [C] time = 1.20794, size = 20, normalized size = 1.25

$$\frac{1}{2}\Gamma(-1, 2i x) + \frac{1}{2}\Gamma(-1, -2i x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*gamma(-1, 2*I*x) + 1/2*gamma(-1, -2*I*x)
```

Fricas [A] time = 0.459234, size = 95, normalized size = 5.94

$$\frac{x \operatorname{Ci}(2x) + x \operatorname{Ci}(-2x) - 2 \cos(x) \sin(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos_integral(2*x) + x*cos_integral(-2*x) - 2*cos(x)*sin(x))/x
```

Sympy [A] time = 2.15956, size = 22, normalized size = 1.38

$$-\log(x) + \frac{\log(x^2)}{2} + \operatorname{Ci}(2x) - \frac{\sin(2x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x**2,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(2*x) - sin(2*x)/(2*x)
```

Giac [A] time = 1.12137, size = 26, normalized size = 1.62

$$\frac{2x \operatorname{Ci}(2x) - \sin(2x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*x*cos_integral(2*x) - sin(2*x))/x
```

3.12 $\int \frac{\cos(x)\sin(x)}{x^3}\,dx$

Optimal. Leaf size=29

$$-\text{Si}(2x)-\frac{\sin(2x)}{4x^2}-\frac{\cos(2x)}{2x}$$

```
[Out] -Cos[2*x]/(2*x) - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

Rubi [A] time = 0.0589954, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4406, 12, 3297, 3299}

$$-\text{Si}(2x)-\frac{\sin(2x)}{4x^2}-\frac{\cos(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/x^3,x]
```

```
[Out] -Cos[2*x]/(2*x) - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\sin(x)}{x^3}\,dx &= \int \frac{\sin(2x)}{2x^3}\,dx \\
&= \frac{1}{2}\int \frac{\sin(2x)}{x^3}\,dx \\
&= -\frac{\sin(2x)}{4x^2} + \frac{1}{2}\int \frac{\cos(2x)}{x^2}\,dx \\
&= -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \int \frac{\sin(2x)}{x}\,dx \\
&= -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \text{Si}(2x)
\end{aligned}$$

Mathematica [A] time = 0.0075177, size = 29, normalized size = 1.

$$-\text{Si}(2x) - \frac{\sin(2x)}{4x^2} - \frac{\cos(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x^3,x]
```

```
[Out] -Cos[2*x]/(2*x) - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

Maple [A] time = 0.029, size = 26, normalized size = 0.9

$$-\frac{\cos(2\,x)}{2\,x} - \text{Si}\,(2\,x) - \frac{\sin(2\,x)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x^3,x)
```

```
[Out] -1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2
```

Maxima [C] time = 1.26484, size = 20, normalized size = 0.69

$$i\Gamma(-2, 2ix) - i\Gamma(-2, -2ix)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="maxima")
```

```
[Out] I*gamma(-2, 2*I*x) - I*gamma(-2, -2*I*x)
```

Fricas [A] time = 0.469186, size = 96, normalized size = 3.31

$$-\frac{2x\cos(x)^2 + 2x^2\,\mathrm{Si}(2x) + \cos(x)\sin(x) - x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cos(x)^2 + 2*x^2*sin_integral(2*x) + cos(x)*sin(x) - x)/x^2
```

Sympy [A] time = 1.59581, size = 24, normalized size = 0.83

$$-\mathrm{Si}(2x) - \frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x**3,x)
```

```
[Out] -Si(2*x) - cos(2*x)/(2*x) - sin(2*x)/(4*x**2)
```

Giac [A] time = 1.13422, size = 35, normalized size = 1.21

$$-\frac{4x^2\,\mathrm{Si}(2x) + 2x\cos(2x) + \sin(2x)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*x^2*sin_integral(2*x) + 2*x*cos(2*x) + sin(2*x))/x^2
```

3.13 $\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=275

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{8b}$$

```
[Out] ((-I/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d
*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*
E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(
b*(((-I)*b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((
3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.32994, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3307, 2181}

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```



```
[Out] ((-I/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d
*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*
E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(
b*(((-I)*b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((
3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^m\cos(a+bx) - \frac{1}{4}(c+dx)^m\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^m\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^m\cos(3a+3bx)\,dx$$

$$= \frac{1}{8}\int e^{-i(a+bx)}(c+dx)^m\,dx + \frac{1}{8}\int e^{i(a+bx)}(c+dx)^m\,dx - \frac{1}{8}\int e^{-i(3a+3bx)}(c+dx)^{m}$$

$$= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c}{}\right.}{}$$

Mathematica [A] time = 0.722672, size = 237, normalized size = 0.86

$$-\frac{ie^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(\left(\frac{ib(c+dx)}{d}\right)^{-m}\left(3^{-m}\left(e^{\frac{6ibc}{d}}\text{Gamma}\left(m+1,\frac{3ib(c+dx)}{d}\right)-e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^{2m}\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\text{Gamma}\left(m+1,-\right.\right.\right.\right.}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/24)*(c + d*x)^m*((3*E^((2*I)*(2*a + (b*c)/d))*Gamma[1 + m, ((-I)*b*(c
+ d*x))/d])/(((-I)*b*(c + d*x))/d)^m + (-3*E^((2*I)*a + ((4*I)*b*c)/d)*Gamm
a[1 + m, (I*b*(c + d*x))/d] + (-((E^((6*I)*a)*((I*b*(c + d*x))/d)^(2*m)*Gam
ma[1 + m, ((-3*I)*b*(c + d*x))/d])/((b^2*(c + d*x)^2)/d^2)^m) + E^(((6*I)*b
*c)/d)*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/3^m)/((I*b*(c + d*x))/d)^m))/(b
*E^(((3*I)*(b*c + a*d))/d))
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a) (\sin(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a) \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

Fricas [A] time = 0.553723, size = 471, normalized size = 1.71

$$\frac{-i e^{\left(-\frac{dm \log\left(\frac{3i b}{d}\right)-3i bc+3i ad}{d}\right)} \Gamma\left(m+1, \frac{3i bdx+3i bc}{d}\right) + 3i e^{\left(-\frac{dm \log\left(\frac{i b}{d}\right)-i bc+i ad}{d}\right)} \Gamma\left(m+1, \frac{i bdx+i bc}{d}\right) - 3i e^{\left(-\frac{dm \log\left(-\frac{i b}{d}\right)+i bc-i ad}{d}\right)} \Gamma\left(m+1, \right.}{24 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, (3*I*b*
d*x + 3*I*b*c)/d) + 3*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1
, (I*b*d*x + I*b*c)/d) - 3*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma
(m + 1, (-I*b*d*x - I*b*c)/d) + I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*
d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)\sin(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

3.14 $\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx) \, dx$

Optimal. Leaf size=205

$$-\frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3}-\frac{8d^2(c+dx)^2\sin(a+bx)}{3b^3}-\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin^2(a+bx)\cos(a+}{27b^4}$$

```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(27*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])
/(9*b^2) + (160*d^4*Sin[a + b*x])/(27*b^5) - (8*d^2*(c + d*x)^2*Sin[a + b*x
])/(3*b^3) - (8*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(27*b^4) + (4*d*
(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) + (8*d^4*Sin[a + b*x]^3)/(
81*b^5) - (4*d^2*(c + d*x)^2*Sin[a + b*x]^3)/(9*b^3) + ((c + d*x)^4*Sin[a +
 b*x]^3)/(3*b)
```

Rubi [A] time = 0.199716, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4404, 3311, 3296, 2637, 3310}

$$-\frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3}-\frac{8d^2(c+dx)^2\sin(a+bx)}{3b^3}-\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin^2(a+bx)\cos(a+}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```


```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(27*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])
/(9*b^2) + (160*d^4*Sin[a + b*x])/(27*b^5) - (8*d^2*(c + d*x)^2*Sin[a + b*x
])/(3*b^3) - (8*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(27*b^4) + (4*d*
(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) + (8*d^4*Sin[a + b*x]^3)/(
81*b^5) - (4*d^2*(c + d*x)^2*Sin[a + b*x]^3)/(9*b^3) + ((c + d*x)^4*Sin[a +
 b*x]^3)/(3*b)
```


Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^4\sin^3(a+bx)}{3b} - \frac{(4d)\int (c+dx)^3\sin^3(a+bx)\,dx}{3b}$$

$$= \frac{4d(c+dx)^3\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3} + \frac{(c+dx)^4\,\text{si}}{3}$$

$$= \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} - \frac{8d^3(c+dx)\cos(a+bx)\sin^2(a+bx)}{27b^4} + \frac{4d(c+dx)^3\,\text{c}}{}$$

$$= -\frac{16d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} - \frac{8d^2(c+dx)^2\sin(a+bx}{3b^3}$$

$$= -\frac{160d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} + \frac{16d^4\sin(a+bx)}{27b^5} - 8a$$

$$= -\frac{160d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} + \frac{160d^4\sin(a+bx)}{27b^5} - 8$$

Mathematica [A] time = 1.5138, size = 385, normalized size = 1.88

$$\frac{486b^4c^2d^2x^2\sin(a+bx)-162b^4c^2d^2x^2\sin(3(a+bx))-972b^2c^2d^2\sin(a+bx)+36b^2c^2d^2\sin(3(a+bx))+324b^4c^3dx\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (324*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 12*b*d*(c + d*
x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*b^4*c^4*Sin[a + b*x]
- 972*b^2*c^2*d^2*Sin[a + b*x] + 1944*d^4*Sin[a + b*x] + 324*b^4*c^3*d*x*Si
n[a + b*x] - 1944*b^2*c*d^3*x*Sin[a + b*x] + 486*b^4*c^2*d^2*x^2*Sin[a + b*
x] - 972*b^2*d^4*x^2*Sin[a + b*x] + 324*b^4*c*d^3*x^3*Sin[a + b*x] + 81*b^4
*d^4*x^4*Sin[a + b*x] - 27*b^4*c^4*Sin[3*(a + b*x)] + 36*b^2*c^2*d^2*Sin[3*
(a + b*x)] - 8*d^4*Sin[3*(a + b*x)] - 108*b^4*c^3*d*x*Sin[3*(a + b*x)] + 72
*b^2*c*d^3*x*Sin[3*(a + b*x)] - 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)] + 36*b
^2*d^4*x^2*Sin[3*(a + b*x)] - 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] - 27*b^4*d
^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

Maple [B] time = 0.058, size = 835, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/3*(b*x+a)^4*sin(b*x+a)^3+4/9*(b*x+a)^3*(2+sin(b*x+a)^2)*c
os(b*x+a)-8/3*(b*x+a)^2*sin(b*x+a)+160/27*sin(b*x+a)-16/3*(b*x+a)*cos(b*x+a
)-4/9*(b*x+a)^2*sin(b*x+a)^3-8/27*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+8/81*
sin(b*x+a)^3)-4/b^4*a*d^4*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(
b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(
b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+4/b^3*c*d^3*(1/3*(b*x+a)^3*sin(b
*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a
)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+6/b
^4*a^2*d^4*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x
+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*sin(b*x
+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x
+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)
*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*si
n(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*s
```

```
in(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-12/b^2*a*c^2*d^2*(1/3*(b*x+a)*
sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+4/b*c^3*d*(1/3*(b*x+a)*sin(b*
x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+1/3/b^4*a^4*d^4*sin(b*x+a)^3-4/3/b^
3*a^3*c*d^3*sin(b*x+a)^3+2/b^2*a^2*c^2*d^2*sin(b*x+a)^3-4/3/b*a*c^3*d*sin(b
*x+a)^3+1/3*c^4*sin(b*x+a)^3)
```

Maxima [B] time = 1.35791, size = 1188, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/324*(108*c^4*sin(b*x + a)^3 - 432*a*c^3*d*sin(b*x + a)^3/b + 648*a^2*c^2*
d^2*sin(b*x + a)^3/b^2 - 432*a^3*c*d^3*sin(b*x + a)^3/b^3 + 108*a^4*d^4*sin
(b*x + a)^3/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x +
a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c^3*d/b + 108*(3*(b*x + a)*sin(3*b*
x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*
c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*c*d^3/b^3 + 36*(3*(b*x + a)*sin(3*
b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a)
+ (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))
*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a)
 + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a)
)*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a
) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a
))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2
 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*
((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 + 12*((9*(b*x + a)^2 -
2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3
- 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x +
a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 324*((
b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) + (27*(b*x + a)^4 - 36*(b*x + a)^2 +
 8)*sin(3*b*x + 3*a) - 81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*sin(b*x + a))
*d^4/b^4)/b
```

Fricas [A] time = 0.520872, size = 759, normalized size = 3.7

$$-\frac{12\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d-2\,bcd^3+\left(9\,b^3c^2d^2-2\,bd^4\right)x\right)\cos\left(bx+a\right)^3-36\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*b
^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 - 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x
^2 + 3*b^3*c^3*d - 14*b*c*d^3 + (9*b^3*c^2*d^2 - 14*b*d^4)*x)*cos(b*x + a)
- (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 252*b^2*c^2*d^2 + 488*
d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3
*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)
*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d -
 14*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [A] time = 11.3384, size = 646, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)**3/(3*b) +
2*c**2*d**2*x**2*sin(a + b*x)**3/b + 4*c*d**3*x**3*sin(a + b*x)**3/(3*b) +
d**4*x**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*sin(a + b*x)**2*cos(a + b*x)/(3*
b**2) + 8*c**3*d*cos(a + b*x)**3/(9*b**2) + 4*c**2*d**2*x*sin(a + b*x)**2*c
os(a + b*x)/b**2 + 8*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*s
in(a + b*x)**2*cos(a + b*x)/b**2 + 8*c*d**3*x**2*cos(a + b*x)**3/(3*b**2) +
 4*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 8*d**4*x**3*cos(a + b*
x)**3/(9*b**2) - 28*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 8*c**2*d**2*sin(a
+ b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**3*x*sin(a + b*x)**3/(9*b**3) - 16
*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 28*d**4*x**2*sin(a + b*x)
**3/(9*b**3) - 8*d**4*x**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**
3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*c*d**3*cos(a + b*x)**3/(27*b*
*4) - 56*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*d**4*x*cos(a +
b*x)**3/(27*b**4) + 488*d**4*sin(a + b*x)**3/(81*b**5) + 160*d**4*sin(a + b
*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2
```

```
*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a), True))
```

Giac [A] time = 1.12878, size = 473, normalized size = 2.31

$$-\frac{\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3\right)\cos\left(3\,bx+3\,a\right)}{27\,b^5}+\frac{\left(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c\right.}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*
b*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
+ 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1
/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^
3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d
^4)*sin(3*b*x + 3*a)/b^5 + 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d
^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2
*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

3.15 $\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\,dx$

Optimal. Leaf size=151

$$-\frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3}-\frac{4d^2(c+dx)\sin(a+bx)}{3b^3}+\frac{2d(c+dx)^2\cos(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b^2}+$$

```
[Out] (-14*d^3*Cos[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Cos[a + b*x])/(3*b^2) + (
2*d^3*Cos[a + b*x]^3)/(27*b^4) - (4*d^2*(c + d*x)*Sin[a + b*x])/(3*b^3) + (
d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b^2) - (2*d^2*(c + d*x)*Sin[a
 + b*x]^3)/(9*b^3) + ((c + d*x)^3*Sin[a + b*x]^3)/(3*b)
```

Rubi [A] time = 0.134578, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4404, 3311, 3296, 2638, 2633}

$$-\frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3}-\frac{4d^2(c+dx)\sin(a+bx)}{3b^3}+\frac{2d(c+dx)^2\cos(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (-14*d^3*Cos[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Cos[a + b*x])/(3*b^2) + (
2*d^3*Cos[a + b*x]^3)/(27*b^4) - (4*d^2*(c + d*x)*Sin[a + b*x])/(3*b^3) + (
d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b^2) - (2*d^2*(c + d*x)*Sin[a
 + b*x]^3)/(9*b^3) + ((c + d*x)^3*Sin[a + b*x]^3)/(3*b)
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\int (c+dx)^2\sin^3(a+bx)\,dx}{b}$$
$$= \frac{d(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b^2} - \frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3} + \frac{(c+dx)^3\sin^3}{3b}$$
$$= \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{d(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b^2} - \frac{2d^2(c+dx)\sin}{9b^3}$$
$$= -\frac{2d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{2d^3\cos^3(a+bx)}{27b^4} - \frac{4d^2(c+dx)}{3b}$$
$$= -\frac{14d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{2d^3\cos^3(a+bx)}{27b^4} - \frac{4d^2(c+dx)}{3}$$

Mathematica [A] time = 0.991882, size = 121, normalized size = 0.8

$$-\frac{-81d\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)+d\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)+6b(c+dx)\sin(a+bx)\left(\cos(2(a+bx))\right)\left(3\right.}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -(-81*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*(c + d*
x)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(26*d^2 - 3*b^2*(c + d*x)^2 + (-2*d^
2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^4)
```

Maple [B] time = 0.019, size = 447, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*c
os(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/
27*(2+sin(b*x+a)^2)*cos(b*x+a))-3/b^3*a*d^3*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9
*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+3/b^
2*c*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)
-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+3/b^3*a^2*d^3*(1/3*(b*x+a)*sin(b*x+a)^3+
1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-6/b^2*a*c*d^2*(1/3*(b*x+a)*sin(b*x+a)^3+1/
9*(2+sin(b*x+a)^2)*cos(b*x+a))+3/b*c^2*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+s
in(b*x+a)^2)*cos(b*x+a))-1/3/b^3*a^3*d^3*sin(b*x+a)^3+1/b^2*a^2*c*d^2*sin(b
*x+a)^3-1/b*a*c^2*d*sin(b*x+a)^3+1/3*c^3*sin(b*x+a)^3)
```

Maxima [B] time = 1.11837, size = 674, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/108*(36*c^3*sin(b*x + a)^3 - 108*a*c^2*d*sin(b*x + a)^3/b + 108*a^2*c*d^2
*sin(b*x + a)^3/b^2 - 36*a^3*d^3*sin(b*x + a)^3/b^3 - 9*(3*(b*x + a)*sin(3*
b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos
(3*b*x + 3*a) - 9*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*sin(3*b*x + 3*
a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*d^3/
b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x
 + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^2/b^
```

```
2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x +
 a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^3/b^3
- ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a)
 + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b
*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A] time = 0.502087, size = 491, normalized size = 3.25

$$-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3-3\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-14\,d^3\right)\cos\left(bx+a\right)-3\left(3\,b^3d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^
3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 14*d^3)*cos(b*x + a)
- 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 14*b*c*d^2 - (3*b^3*d^3*
x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)*x)*
cos(b*x + a)^2 + (9*b^3*c^2*d - 14*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [A] time = 5.26756, size = 391, normalized size = 2.59

$$\begin{cases} \frac{c^3\sin^3(a+bx)}{3b}+\frac{c^2dx\sin^3(a+bx)}{b}+\frac{cd^2x^2\sin^3(a+bx)}{b}+\frac{d^3x^3\sin^3(a+bx)}{3b}+\frac{c^2d\sin^2(a+bx)\cos(a+bx)}{b^2}+\frac{2c^2d\cos^3(a+bx)}{3b^2}+\frac{2cd^2x\sin^2(a+bx)\cos(a+}{b^2} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^2(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**3/(3*b) + c**2*d*x*sin(a + b*x)**3/b + c*d**2
*x**2*sin(a + b*x)**3/b + d**3*x**3*sin(a + b*x)**3/(3*b) + c**2*d*sin(a +
b*x)**2*cos(a + b*x)/b**2 + 2*c**2*d*cos(a + b*x)**3/(3*b**2) + 2*c*d**2*x*
sin(a + b*x)**2*cos(a + b*x)/b**2 + 4*c*d**2*x*cos(a + b*x)**3/(3*b**2) + d
**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 2*d**3*x**2*cos(a + b*x)**3/(3
*b**2) - 14*c*d**2*sin(a + b*x)**3/(9*b**3) - 4*c*d**2*sin(a + b*x)*cos(a +
 b*x)**2/(3*b**3) - 14*d**3*x*sin(a + b*x)**3/(9*b**3) - 4*d**3*x*sin(a + b
*x)*cos(a + b*x)**2/(3*b**3) - 14*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4
```

```
) - 40*d**3*cos(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/
2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a), True))
```

Giac [A] time = 1.16585, size = 312, normalized size = 2.07

$$-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(3\,bx+3\,a\right)}{108\,b^4}+\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)}{4\,b^4}-\frac{\left(3\,b^3d^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3
*a)/b^4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a
)/b^4 - 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 -
 2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/4*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

3.16 $\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{4d(c+dx)\cos(a+bx)}{9b^2}+\frac{2d(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^2}-\frac{2d^2\sin^3(a+bx)}{27b^3}-\frac{4d^2\sin(a+bx)}{9b^3}+\frac{(c+dx)^2\sin^3(a+b$$

```
[Out] (4*d*(c + d*x)*Cos[a + b*x])/(9*b^2) - (4*d^2*Sin[a + b*x])/(9*b^3) + (2*d*
(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) - (2*d^2*Sin[a + b*x]^3)/(27
*b^3) + ((c + d*x)^2*Sin[a + b*x]^3)/(3*b)
```

Rubi [A] time = 0.0772006, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4404, 3310, 3296, 2637}

$$\frac{4d(c+dx)\cos(a+bx)}{9b^2}+\frac{2d(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^2}-\frac{2d^2\sin^3(a+bx)}{27b^3}-\frac{4d^2\sin(a+bx)}{9b^3}+\frac{(c+dx)^2\sin^3(a+b$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```


```
[Out] (4*d*(c + d*x)*Cos[a + b*x])/(9*b^2) - (4*d^2*Sin[a + b*x])/(9*b^3) + (2*d*
(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) - (2*d^2*Sin[a + b*x]^3)/(27
*b^3) + ((c + d*x)^2*Sin[a + b*x]^3)/(3*b)
```


Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{(c+dx)^2\sin^3(a+bx)}{3b}-\frac{(2d)\int (c+dx)\sin^3(a+bx)\,dx}{3b}\\ &= \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2}-\frac{2d^2\sin^3(a+bx)}{27b^3}+\frac{(c+dx)^2\sin^3(a+b}{3b}\\ &= \frac{4d(c+dx)\cos(a+bx)}{9b^2}+\frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2}-\frac{2d^2\sin^3(a+}{27b^3}\\ &= \frac{4d(c+dx)\cos(a+bx)}{9b^2}-\frac{4d^2\sin(a+bx)}{9b^3}+\frac{2d(c+dx)\cos(a+bx)\sin^2(a+b}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.6145, size = 93, normalized size = 0.9

$$\frac{-2\sin(a+bx)\left(\cos(2(a+bx))\left(9b^2(c+dx)^2-2d^2\right)-9b^2(c+dx)^2+26d^2\right)+54bd(c+dx)\cos(a+bx)-6bd(c+dx)\text{co}}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (54*b*d*(c + d*x)*Cos[a + b*x] - 6*b*d*(c + d*x)*Cos[3*(a + b*x)] - 2*(26*d
^2 - 9*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin
[a + b*x])/(108*b^3)
```

Maple [B] time = 0.016, size = 204, normalized size = 2.

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(\frac{(bx+a)^2(\sin(bx+a))^3}{3}+\frac{(2bx+2a)\left(2+(\sin(bx+a))^2\right)\cos(bx+a)}{9}-\frac{2(\sin(bx+a))^3}{27}-\frac{4\sin(bx+a)}{9}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos
(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-2/b^2*a*d^2*(1/3*(b*x+a)*sin(b*x+
a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+2/b*c*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9
*(2+sin(b*x+a)^2)*cos(b*x+a))+1/3/b^2*a^2*d^2*sin(b*x+a)^3-2/3/b*a*c*d*sin(
b*x+a)^3+1/3*c^2*sin(b*x+a)^3)
```

Maxima [B] time = 1.10982, size = 324, normalized size = 3.15

$$36\,c^2 \sin(bx+a)^3 - \frac{72\,acd\sin(bx+a)^3}{b} + \frac{36\,a^2d^2\sin(bx+a)^3}{b^2} - \frac{6\,(3\,(bx+a)\sin(3\,bx+3\,a) - 9\,(bx+a)\sin(bx+a) + \cos(3\,bx+3\,a) - 9\,\cos(bx+a))cd}{b} + \frac{6\,(3\,(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/108*(36*c^2*sin(b*x + a)^3 - 72*a*c*d*sin(b*x + a)^3/b + 36*a^2*d^2*sin(b
*x + a)^3/b^2 - 6*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) - 9*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x + 3*a
) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*d^2/b^2
 - (6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x + a)
^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A] time = 0.479784, size = 296, normalized size = 2.87

$$-\frac{6\left(bd^2x + bcd\right)\cos(bx+a)^3 - 18\left(bd^2x + bcd\right)\cos(bx+a) - \left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - \left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b\right.\right.}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 18*(b*d^2*x + b*c*d)*cos(b*x +
a) - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - (9*b^2*d^2*x^2 + 18*b^2*c*
d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 14*d^2)*sin(b*x + a))/b^3
```

Sympy [A] time = 2.62016, size = 216, normalized size = 2.1

$$\begin{cases} \frac{c^2\sin^3(a+bx)}{3b} + \frac{2cdx\sin^3(a+bx)}{3b} + \frac{d^2x^2\sin^3(a+bx)}{3b} + \frac{2cd\sin^2(a+bx)\cos(a+bx)}{3b^2} + \frac{4cd\cos^3(a+bx)}{9b^2} + \frac{2d^2x\sin^2(a+bx)\cos(a+bx)}{3b^2} + \frac{4d^2x\cos^3(a+ \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)**3/(3*b) + d**
2*x**2*sin(a + b*x)**3/(3*b) + 2*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2)
+ 4*c*d*cos(a + b*x)**3/(9*b**2) + 2*d**2*x*sin(a + b*x)**2*cos(a + b*x)/(3
*b**2) + 4*d**2*x*cos(a + b*x)**3/(9*b**2) - 14*d**2*sin(a + b*x)**3/(27*b*
*3) - 4*d**2*sin(a + b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c
*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a), True))
```

Giac [A] time = 1.1409, size = 185, normalized size = 1.8

$$-\frac{\left(bd^2x + bcd\right)\cos\left(3\,bx + 3\,a\right)}{18\,b^3} + \frac{\left(bd^2x + bcd\right)\cos\left(bx + a\right)}{2\,b^3} - \frac{\left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 2\,d^2\right)\sin\left(3\,bx + 3\,a\right)}{108\,b^3} + \frac{(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*cos(b*
x + a)/b^3 - 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3
*b*x + 3*a)/b^3 + 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x
 + a)/b^3
```

3.17 $\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=51

$$-\frac{d\cos^3(a+bx)}{9b^2}+\frac{d\cos(a+bx)}{3b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

```
[Out] (d*Cos[a + b*x])/(3*b^2) - (d*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)*Sin[a +
b*x]^3)/(3*b)
```

Rubi [A] time = 0.0332234, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4404, 2633}

$$-\frac{d\cos^3(a+bx)}{9b^2}+\frac{d\cos(a+bx)}{3b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (d*Cos[a + b*x])/(3*b^2) - (d*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)*Sin[a +
b*x]^3)/(3*b)
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)\sin^3(a+bx)}{3b} - \frac{d\int \sin^3(a+bx)\,dx}{3b}$$
$$= \frac{(c+dx)\sin^3(a+bx)}{3b} + \frac{d\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(a+bx)\right)}{3b^2}$$
$$= \frac{d\cos(a+bx)}{3b^2} - \frac{d\cos^3(a+bx)}{9b^2} + \frac{(c+dx)\sin^3(a+bx)}{3b}$$

Mathematica [A] time = 0.181565, size = 44, normalized size = 0.86

$$\frac{12b(c+dx)\sin^3(a+bx)+9d\cos(a+bx)-d\cos(3(a+bx))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (9*d*Cos[a + b*x] - d*Cos[3*(a + b*x)] + 12*b*(c + d*x)*Sin[a + b*x]^3)/(36
*b^2)
```

Maple [A] time = 0.018, size = 71, normalized size = 1.4

$$\frac{1}{b}\left(\frac{d}{b}\left(\frac{(bx+a)(\sin(bx+a))^3}{3}+\frac{\left(2+(\sin(bx+a))^2\right)\cos(bx+a)}{9}\right)-\frac{ad(\sin(bx+a))^3}{3b}+\frac{c(\sin(bx+a))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(d/b*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-1/3/b*a
*d*sin(b*x+a)^3+1/3*c*sin(b*x+a)^3)
```

Maxima [A] time = 1.13164, size = 115, normalized size = 2.25

$$\frac{12\,c\sin(bx+a)^3-\frac{12\,ad\sin(bx+a)^3}{b}-\frac{(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(bx+a))d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/36*(12*c*sin(b*x + a)^3 - 12*a*d*sin(b*x + a)^3/b - (3*(b*x + a)*sin(3*b*
x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*d/
b)/b
```

Fricas [A] time = 0.471744, size = 149, normalized size = 2.92

$$-\frac{d\cos(bx+a)^3 - 3\,d\cos(bx+a) - 3\left(bdx - (bdx+bc)\cos(bx+a)^2 + bc\right)\sin(bx+a)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(d*cos(b*x + a)^3 - 3*d*cos(b*x + a) - 3*(b*d*x - (b*d*x + b*c)*cos(b*
x + a)^2 + b*c)*sin(b*x + a))/b^2
```

Sympy [A] time = 1.17803, size = 85, normalized size = 1.67

$$\begin{cases} \frac{c\sin^3(a+bx)}{3b} + \frac{dx\sin^3(a+bx)}{3b} + \frac{d\sin^2(a+bx)\cos(a+bx)}{3b^2} + \frac{2d\cos^3(a+bx)}{9b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)**3/(3*b) + d*sin(a +
b*x)**2*cos(a + b*x)/(3*b**2) + 2*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)), ((
c*x + d*x**2/2)*sin(a)**2*cos(a), True))
```

Giac [A] time = 1.10169, size = 93, normalized size = 1.82

$$-\frac{d\cos(3\,bx+3\,a)}{36\,b^2} + \frac{d\cos(bx+a)}{4\,b^2} - \frac{(bdx+bc)\sin(3\,bx+3\,a)}{12\,b^2} + \frac{(bdx+bc)\sin(bx+a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/36*d*cos(3*b*x + 3*a)/b^2 + 1/4*d*cos(b*x + a)/b^2 - 1/12*(b*d*x + b*c)*
sin(3*b*x + 3*a)/b^2 + 1/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

3.18 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\sin\left(3a-3b\right.$$

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rubi [A] time = 0.26993, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\sin\left(3a-3b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)} - \frac{\cos(3a+3bx)}{4(c+dx)}\right)dx \\
&= \frac{1}{4}\int \frac{\cos(a+bx)}{c+dx}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{c+dx}\,dx \\
&= -\left(\frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx\right) + \frac{1}{4}\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx + \frac{1}{4} \\
&= \frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{4d}
\end{aligned}
$$

Mathematica [A] time = 0.335325, size = 102, normalized size = 0.84

$$
\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - \cos\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - \sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right) + \sin\left(3a-\right.}{4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosIntegr
al[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A] time = 0.017, size = 166, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b}{4}\left(\frac{1}{d}\mathrm{Si}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)+\frac{1}{d}\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)\right)-\frac{b}{12}\left(3\frac{1}{d}\mathrm{Si}\left(3bx+3a+3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/b*(1/4*b*(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/
d)*cos((-a*d+b*c)/d)/d)-1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+
b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

Maxima [C] time = 1.47769, size = 370, normalized size = 3.06

$$-\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_1\left(-\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integra
l_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp_in
tegral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(-I*exp_
integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -(I*
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(I*exp_integral_e(
1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral_e(1, -(3*I*b*c
 + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 0.482637, size = 404, normalized size = 3.34

$$\frac{\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)+2\,\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bd}{}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*cos(-
(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x
+ b*c)/d))*cos(-3*(b*c - a*d)/d) + 2*sin(-3*(b*c - a*d)/d)*sin_integral(3*(
b*d*x + b*c)/d) - 2*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x), x)
```

Giac [C] time = 1.73713, size = 8180, normalized size = 67.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(co
s_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b
*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
```

```
 - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
+ 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-b*x - b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*real_part(cos_
integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) - 4*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*real_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*t
an(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*ta
n(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_i
ntegral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))
*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*
a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*sin_int
egral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*imag_
part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) -
2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2
```

```
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(b*x + b*c/d))*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(-b*x
 - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*sin_integral((b
*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(c
os_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*imag
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2
- 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2
- 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/
2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2
*a)^2*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*
(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^
2 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)
*tan(1/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*
b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/
2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*sin_integral(3*(
b*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(
cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + r
eal_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - real_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + 4*real_part(cos
_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 4*real
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d
) + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2
- real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - real
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + real_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - real_part(cos_integral(
-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*real_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*real_part(c
os_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^
2 + real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + re
al_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - real_pa
```

```
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(
cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + real_part(cos_integra
l(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)
^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a) - 2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 4*sin_integr
al((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a) - 2*imag_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 + 2*imag_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*a)*tan(1/2*a)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2
*tan(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*
tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c
/d) + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d
) - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)
 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 - 2*imag_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integ
ral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2 + 2*imag_part(cos_integr
al(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*t
an(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
*a)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2
*tan(1/2*b*c/d) - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/
d) + 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*sin_
integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*sin_integral((b*d*x
+ b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*
tan(3/2*a)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/
2*a)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*
tan(1/2*b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d
)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^
2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2 - real_part(cos_i
```

```
ntegral(b*x + b*c/d))*tan(3/2*a)^2 - real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2 + rea
l_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2 + real_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)^2 + real_part(cos_integral(-b*x - b*c/d))*tan(1/2
*a)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2 + 4*real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 4*real_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - real_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c
/d))*tan(3/2*b*c/d)^2 - real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d
)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 4*real_p
art(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 4*real_part(cos_
integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + real_part(cos_integral(
3*b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 + real_part(cos_integral(b*x + b*c/d))*t
an(1/2*b*c/d)^2 + real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 +
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 - 2*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a) + 2*im
ag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) - 2*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a) + 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a) + 2*im
ag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) + 4*sin_integral(3*(b*d*x + b*c)/d)
*tan(3/2*b*c/d) - 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) + 2
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) - 4*sin_integral((b*d
*x + b*c)/d)*tan(1/2*b*c/d) + real_part(cos_integral(3*b*x + 3*b*c/d)) - re
al_part(cos_integral(b*x + b*c/d)) - real_part(cos_integral(-b*x - b*c/d))
+ real_part(cos_integral(-3*b*x - 3*b*c/d)))/(d*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d*tan
(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2
+ d*tan(1/2*a)^2 + d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
```

3.19 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=168

$$\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos}{}$$

```
[Out] -Cos[a + b*x]/(4*d*(c + d*x)) + Cos[3*a + 3*b*x]/(4*d*(c + d*x)) + (3*b*Cos
Integral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d^2) - (b*CosIntegral[
(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d^2) - (b*Cos[a - (b*c)/d]*SinIntegral[
(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d +
 3*b*x])/(4*d^2)
```

Rubi [A] time = 0.301291, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```


```
[Out] -Cos[a + b*x]/(4*d*(c + d*x)) + Cos[3*a + 3*b*x]/(4*d*(c + d*x)) + (3*b*Cos
Integral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d^2) - (b*CosIntegral[
(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d^2) - (b*Cos[a - (b*c)/d]*SinIntegral[
(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d +
 3*b*x])/(4*d^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)^2}-\frac{\cos(3a+3bx)}{4(c+dx)^2}\right)dx\\
&= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx-\frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx\\
&= -\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}-\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{4d}+\frac{(3b)\int \frac{\sin(3a+3bx)}{c+dx}\,dx}{4d}\\
&= -\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}+\frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right.\right.}{}\\
&= -\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}+\frac{3b\,\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2}-\frac{b\,\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin}{4d^2}
\end{aligned}
$$

Mathematica [A] time = 1.43532, size = 139, normalized size = 0.83

$$-\frac{-3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)-3b\cos\ldots}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -((d*Cos[a + b*x])/(c + d*x) - (d*Cos[3*(a + b*x)])/(c + d*x) - 3*b*CosInte
gral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b*CosIntegral[b*(c/d + x)]*S
in[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3*b*Cos[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d^2)
```

Maple [A] time = 0.023, size = 242, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^2}{4}\left(-\frac{\cos(bx+a)}{((bx+a)d-ad+bc)d}-\frac{1}{d}\left(\frac{1}{d}\text{Si}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)-\frac{1}{d}\text{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+b\ldots}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/4*b^2*(-cos(b*x+a)/((b*x+a)*d-a*d+b*c)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos
((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/12*b^2*(-
3*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*co
s(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d
))
```

Maxima [C] time = 1.77077, size = 408, normalized size = 2.43

$$-\frac{8192\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-8192\,b^2\left(E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_2\left(-\frac{3i\,bc+3i\,(bx+a)\ldots}{d}\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/65536*(8192*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- 8192*b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + ex
p_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^2*(-8192*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 8192*I*exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c -
a*d)/d) + b^2*(8192*I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*
d)/d) - 8192*I*exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 0.54075, size = 593, normalized size = 3.53

$$\frac{8\,d\cos\left(bx+a\right)^{3}+6\left(bdx+bc\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-2\left(bdx+bc\right)\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)-8\,d\cos\left(bx+a\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*d*cos(b*x + a)^3 + 6*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) - 2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*
d*x + b*c)/d) - 8*d*cos(b*x + a) - ((b*d*x + b*c)*cos_integral((b*d*x + b*c
)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) +
3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integr
al(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\sin(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sin(b*x + a)^2/(d*x + c)^2, x)
```

3.20 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=221

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-$$

```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2
*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^3) + (b*Sin[a + b*x])/(8*d^2*
(c + d*x)) - (3*b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)) + (b^2*Sin[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Sin[3*a - (3*b*c)/d]*SinInt
egral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rubi [A] time = 0.358156, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```


```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2
*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^3) + (b*Sin[a + b*x])/(8*d^2*
(c + d*x)) - (3*b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)) + (b^2*Sin[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Sin[3*a - (3*b*c)/d]*SinInt
egral[(3*b*c)/d + 3*b*x])/(8*d^3)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)^3}-\frac{\cos(3a+3bx)}{4(c+dx)^3}\right)dx \\
&= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx-\frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx \\
&= -\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2}-\frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{8d}+\frac{(3b)\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx}{8d} \\
&= -\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2}+\frac{b\sin(a+bx)}{8d^2(c+dx)}-\frac{3b\sin(3a+3bx)}{8d^2(c+dx)}-\frac{b^2\int \frac{\cos(a+bx)}{c+dx}\,dx}{8d^2}+ \\
&= -\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2}+\frac{b\sin(a+bx)}{8d^2(c+dx)}-\frac{3b\sin(3a+3bx)}{8d^2(c+dx)}+\frac{\left(9b^2\cos\left(3a-\frac{3bc}{d}\right.\right.}{8} \\
&= -\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Ci}\left(\right.}{8d^3}
\end{aligned}
$$

Mathematica [A] time = 2.2384, size = 183, normalized size = 0.83

$$\frac{b^2\left(-\cos\left(a-\frac{bc}{d}\right)\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] (-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + 9*b^2*Cos[3*a - (3*b*c)
/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Si
n[a + b*x]))/(c + d*x)^2 + (d*(d*Cos[3*(a + b*x)] - 3*b*(c + d*x)*Sin[3*(a
+ b*x)]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b
^2*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(8*d^3)
```

Maple [A] time = 0.021, size = 311, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^3}{4}\left(-\frac{\cos(bx+a)}{2\left((bx+a)d-ad+bc\right)^2 d}-\frac{1}{2d}\left(-\frac{\sin(bx+a)}{((bx+a)d-ad+bc)d}+\frac{1}{d}\left(\frac{1}{d}\text{Si}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)+\frac{1}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/4*b^3*(-1/2*cos(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d-1/2*(-sin(b*x+a)/((b*
x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*
d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/12*b^3*(-3/2*cos(3*b*x+3*a)/((b*x+a)
*d-a*d+b*c)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*Si(3*b*x+
3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*co
s(3*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] time = 2.27331, size = 455, normalized size = 2.06

$$-\frac{8192\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-8192\,b^3\left(E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_3\left(-\frac{3i\,bc+3i\,(bx+\right.\right.}{d}\right.}{65}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*(8192*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- 8192*b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + ex
p_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^3*(-8192*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 8192*I*exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c -
a*d)/d) + b^3*(8192*I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*
d)/d) - 8192*I*exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A] time = 0.574991, size = 914, normalized size = 4.14

$$\frac{8\,d^2\cos\left(bx+a\right)^3-8\,d^2\cos\left(bx+a\right)-18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right)+2\left(b^2d^2x^2+2\,b^2cdx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(8*d^2*cos(b*x + a)^3 - 8*d^2*cos(b*x + a) - 18*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c - a*d)/d)*sin_integral((b*d*
x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x +
b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)
/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 8*(b*d^2*x + b*c*d - 3*(b*d
^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.21 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=270

$$-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}-\ldots$$

```
[Out] -Cos[a + b*x]/(12*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(24*d^3*(c + d*x)) +
Cos[3*a + 3*b*x]/(12*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(8*d^3*(c +
d*x)) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^4)
 + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(24*d^4) + (b*Sin[a +
b*x])/(24*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)^2) + (b^
3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^3*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Rubi [A] time = 0.419572, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -Cos[a + b*x]/(12*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(24*d^3*(c + d*x)) +
Cos[3*a + 3*b*x]/(12*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(8*d^3*(c +
d*x)) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^4)
 + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(24*d^4) + (b*Sin[a +
b*x])/(24*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)^2) + (b^
3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^3*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\cos(a+bx)}{4(c+dx)^4} - \frac{\cos(3a+3bx)}{4(c+dx)^4}\right) dx$$
$$= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^4}\,dx$$
$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{12d} + \frac{b\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx}{4d}$$
$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} + \frac{b\sin(a+bx)}{24d^2(c+dx)^2} - \frac{b\sin(3a+3bx)}{8d^2(c+dx)^2} - \frac{b^2\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{24d^2}$$
$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} + \frac{b\sin(a+bx)}{24d^2(c+dx)^2}$$
$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} + \frac{b\sin(a+bx)}{24d^2(c+dx)^2}$$
$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} - \frac{9b^3\text{Ci}\left(\frac{3bc}{d}+3\right.}{}$$

Mathematica [A] time = 1.75246, size = 298, normalized size = 1.1

$$\frac{b^3(c+dx)^3\left(\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)-27b^3(c+dx)^3\left(\sin\left(3a-\frac{3bc}{d}\right)\text{CosInteg}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] (d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) - d*
Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a
]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x]
- d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Sin[3*a])*Sin[
3*b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a
 - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 27*b^3*(c + d*x)^3*(CosIntegral[(3*
b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a - (3*b*c)/d]*SinIntegral[(3*
b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

Maple [A] time = 0.024, size = 384, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^4}{4}\left(-\frac{\cos(bx+a)}{3\,((bx+a)\,d-ad+bc)^3\,d}-\frac{1}{3\,d}\left(-\frac{\sin(bx+a)}{2\,((bx+a)\,d-ad+bc)^2\,d}+\frac{1}{2\,d}\left(-\frac{\cos(bx+a)}{((bx+a)\,d-ad+bc)\,d}-\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(bx+\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/4*b^4*(-1/3*cos(b*x+a)/((b*x+a)*d-a*d+b*c)^3/d-1/3*(-1/2*sin(b*x+a)/
((b*x+a)*d-a*d+b*c)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-a*d+b*c)/d-(Si(b*x+a+(-
a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)
/d)/d)/d)-1/12*b^4*(-cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^3/d-(-3/2*sin(3*b*x
+3*a)/((b*x+a)*d-a*d+b*c)^2/d+3/2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d-
3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-
a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)/d))
```

Maxima [C] time = 2.80831, size = 522, normalized size = 1.93

$$-\frac{8192\,b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-8192\,b^4\left(E_4\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_4\left(-\frac{3i\,bc+3i\,(bx+\right.\right.}{65536\left(b^3c^3d-3\,ab^2c^2d^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/65536*(8192*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- 8192*b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + ex
p_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^4*(-8192*I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 8192*I*exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c -
a*d)/d) + b^4*(8192*I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*
d)/d) - 8192*I*exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] time = 0.637271, size = 1243, normalized size = 4.6

$$-\frac{8\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3+54\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(8*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a
)^3 + 54*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(
b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x +
b*c)/d) - 8*(7*b^2*d^3*x^2 + 14*b^2*c*d^2*x + 7*b^2*c^2*d - 2*d^3)*cos(b*x
+ a) - 8*(b*d^3*x + b*c*d^2 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x
 + a) - ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_inte
gral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 27*((b^3*d^3*x
^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c
)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integr
al(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c
^2*d^5*x + c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.22 $\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=271

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(n\right.}{b}$$

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) + (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rubi [A] time = 0.33105, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3308, 2181}

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(n\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) + (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^m \sin(2a+2bx) - \frac{1}{8}(c+dx)^m \sin(4a+4bx)\right) dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^m \sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^m \sin(2a+2bx)\,dx$$

$$= -\left(\frac{1}{16}i\int e^{-i(4a+4bx)}(c+dx)^m\,dx\right) + \frac{1}{16}i\int e^{i(4a+4bx)}(c+dx)^m\,dx + \frac{1}{8}i\int e^{-i(2a}$$

$$= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-4-m}e^{-2i\left(a-\frac{bc}{d}\right)}}{}$$

Mathematica [A] time = 0.299893, size = 246, normalized size = 0.91

$$\frac{4^{-m-3}e^{-\frac{4i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-2^{m+2}e^{2i\left(a+\frac{3bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^m \text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right) - 2^{m+2}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (4^(-3 - m)*(c + d*x)^m*(-(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c + d
*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 2^(2 + m)*E^((2*I)*(a + (
3*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d] + E
^((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] + E^(
((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d]
```

```
))/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F] time = 0.265, size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)(\sin(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

Fricas [A] time = 0.551049, size = 486, normalized size = 1.79

$$\frac{e^{\left(-\frac{dm\log\left(\frac{4ib}{d}\right)-4ibc+4iad}{d}\right)}\Gamma\left(m+1,\frac{4ibdx+4ibc}{d}\right)-4e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,\frac{2ibdx+2ibc}{d}\right)-4e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(m+1,\right.}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m + 1, (4*I*b*d*x
 + 4*I*b*c)/d) - 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
1, (2*I*b*d*x + 2*I*b*c)/d) - 4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)
```

```
/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*b
*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

3.23 $\int (c+dx)^4 \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=260

$$-\frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3}-\frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3}-\frac{3d^3(c+dx)\sin^3(a+bx)\cos(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)}{64b^4}$$

```
[Out] (45*c*d^3*x)/(64*b^3) + (45*d^4*x^2)/(128*b^3) - (3*(c + d*x)^4)/(32*b) - (
45*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^2) + (45*d^4*Sin[a + b*x]^2)/(128*b^5) - (9*d^
2*(c + d*x)^2*Sin[a + b*x]^2)/(16*b^3) - (3*d^3*(c + d*x)*Cos[a + b*x]*Sin[
a + b*x]^3)/(32*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3)/(4*b^2)
+ (3*d^4*Sin[a + b*x]^4)/(128*b^5) - (3*d^2*(c + d*x)^2*Sin[a + b*x]^4)/(16
*b^3) + ((c + d*x)^4*Sin[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.241104, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4404, 3311, 32, 3310}

$$-\frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3}-\frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3}-\frac{3d^3(c+dx)\sin^3(a+bx)\cos(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)}{64b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```


```
[Out] (45*c*d^3*x)/(64*b^3) + (45*d^4*x^2)/(128*b^3) - (3*(c + d*x)^4)/(32*b) - (
45*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^2) + (45*d^4*Sin[a + b*x]^2)/(128*b^5) - (9*d^
2*(c + d*x)^2*Sin[a + b*x]^2)/(16*b^3) - (3*d^3*(c + d*x)*Cos[a + b*x]*Sin[
a + b*x]^3)/(32*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3)/(4*b^2)
+ (3*d^4*Sin[a + b*x]^4)/(128*b^5) - (3*d^2*(c + d*x)^2*Sin[a + b*x]^4)/(16
*b^3) + ((c + d*x)^4*Sin[a + b*x]^4)/(4*b)
```


Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)^4\sin^4(a+bx)}{4b} - \frac{d\int (c+dx)^3\sin^4(a+bx)\,dx}{b}$$
$$= \frac{d(c+dx)^3\cos(a+bx)\sin^3(a+bx)}{4b^2} - \frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3} + \frac{(c+dx)^4 \text{s}}{4}$$
$$= \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{8b^2} - \frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3} - \frac{3d^3(c+dx}{}$$
$$= -\frac{3(c+dx)^4}{32b} - \frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4} + \frac{3d(c+dx)^3\cos(a+bx}{8b^2}$$
$$= \frac{45cd^3x}{64b^3} + \frac{45d^4x^2}{128b^3} - \frac{3(c+dx)^4}{32b} - \frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4} + \frac{3d(}{}$$

Mathematica [A] time = 1.81049, size = 158, normalized size = 0.61

$$\frac{-64\cos(2(a+bx))\left(-6b^2d^2(c+dx)^2+2b^4(c+dx)^4+3d^4\right)+\cos(4(a+bx))\left(-24b^2d^2(c+dx)^2+32b^4(c+dx)^4+3d^4\right)}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
 (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*x)] - 8
*b*d*(c + d*x)*(-16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c + d*x
)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/(1024*b^5)
```

Maple [B] time = 0.063, size = 1143, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/4*(b*x+a)^4*sin(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/
2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*
x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+27/128*(
b*x+a)^2+3/128*sin(b*x+a)^4+45/128*sin(b*x+a)^2+9/16*(b*x+a)^2*cos(b*x+a)^2
-9/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*(b*x+a)^4)-4/b^
4*a*d^4*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*s
in(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b
*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*cos(b*x
+a)^2-9/64*cos(b*x+a)*sin(b*x+a)+3/16*(b*x+a)^3)+4/b^3*c*d^3*(1/4*(b*x+a)^3
*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+
3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b*x+a)^3+3/2*sin(b*x+a)
)*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*
sin(b*x+a)+3/16*(b*x+a)^3)+6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b
*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b
*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)-12/b^3*a*c*d^3*(1/4*(b*x+a)^2*
sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8
*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)+6/b^2*c^2*d
^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+
a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x
+a)^2)-4/b^4*a^3*d^4*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b
*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+12/b^3*a^2*c*d^3*(1/4*(b*x+a)*sin(b*x+a)
^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)-12/b^2*a*
c^2*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)-3/32*b*x-3/32*a)+4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+1/4/b^4*a^4*d^4*sin(b*x+a)^4-
1/b^3*a^3*c*d^3*sin(b*x+a)^4+3/2/b^2*a^2*c^2*d^2*sin(b*x+a)^4-1/b*a*c^3*d*s
in(b*x+a)^4+1/4*c^4*sin(b*x+a)^4)
```

Maxima [B] time = 1.313, size = 1305, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(256*c^4*sin(b*x + a)^4 - 1024*a*c^3*d*sin(b*x + a)^4/b + 1536*a^2*c
^2*d^2*sin(b*x + a)^4/b^2 - 1024*a^3*c*d^3*sin(b*x + a)^4/b^3 + 256*a^4*d^4
*sin(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2
*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x +
 a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8
*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b
*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^2*c*d^3
/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - s
in(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1)
*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*s
in(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x
+ a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*
(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 2
4*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
+ 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a^2*
d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x
+ a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x +
4*a) + 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x +
a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos
(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2
- 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*c
os(4*b*x + 4*a) - 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) -
 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) + 128*(2*(b*x + a)^3 - 3*
b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

Fricas [A] time = 0.540208, size = 936, normalized size = 3.6

$$\frac{20\,b^4d^4x^4 + 80\,b^4cd^3x^3 + \left(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 32\,b^4c^4 - 24\,b^2c^2d^2 + 3\,d^4 + 24\left(8\,b^4c^2d^2 - b^2d^4\right)x^2 + 16\left(8\,b^4c^3d \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(20*b^4*d^4*x^4 + 80*b^4*c*d^3*x^3 + (32*b^4*d^4*x^4 + 128*b^4*c*d^3*
x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^
2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 3*(40*b^4*c^2*d^2 -
17*b^2*d^4)*x^2 - (64*b^4*d^4*x^4 + 256*b^4*c*d^3*x^3 + 64*b^4*c^4 - 120*b^
2*c^2*d^2 + 51*d^4 + 24*(16*b^4*c^2*d^2 - 5*b^2*d^4)*x^2 + 16*(16*b^4*c^3*d
 - 15*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(40*b^4*c^3*d - 51*b^2*c*d^3)*x - 2*
(2*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c
^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - (40*b^3*d^4*x^3 + 120*b^3*c*d^3*x^2 + 4
0*b^3*c^3*d - 51*b*c*d^3 + 3*(40*b^3*c^2*d^2 - 17*b*d^4)*x)*cos(b*x + a))*s
in(b*x + a))/b^5
```

Sympy [A] time = 18.5406, size = 976, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) - c**4*cos(a + b*x)*
*4/(4*b) + 5*c**3*d*x*sin(a + b*x)**4/(8*b) - 3*c**3*d*x*sin(a + b*x)**2*co
s(a + b*x)**2/(4*b) - 3*c**3*d*x*cos(a + b*x)**4/(8*b) + 15*c**2*d**2*x**2*
sin(a + b*x)**4/(16*b) - 9*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(
8*b) - 9*c**2*d**2*x**2*cos(a + b*x)**4/(16*b) + 5*c*d**3*x**3*sin(a + b*x)
**4/(8*b) - 3*c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 3*c*d**3*
x**3*cos(a + b*x)**4/(8*b) + 5*d**4*x**4*sin(a + b*x)**4/(32*b) - 3*d**4*x*
*4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d**4*x**4*cos(a + b*x)**4/(32
*b) + 5*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 3*c**3*d*sin(a + b*x
)*cos(a + b*x)**3/(8*b**2) + 15*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(8
*b**2) + 9*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c*d**3*x*
*2*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c*d**3*x**2*sin(a + b*x)*cos(a
 + b*x)**3/(8*b**2) + 5*d**4*x**3*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 3
*d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c**2*d**2*sin(a + b*x
)**2*cos(a + b*x)**2/(16*b**3) + 3*c**2*d**2*cos(a + b*x)**4/(4*b**3) - 51*
c*d**3*x*sin(a + b*x)**4/(64*b**3) + 9*c*d**3*x*sin(a + b*x)**2*cos(a + b*x
)**2/(32*b**3) + 45*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 51*d**4*x**2*sin(a
 + b*x)**4/(128*b**3) + 9*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**
3) + 45*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 51*c*d**3*sin(a + b*x)**3*co
s(a + b*x)/(64*b**4) - 45*c*d**3*sin(a + b*x)*cos(a + b*x)**3/(64*b**4) - 5
1*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*d**4*x*sin(a + b*x)*co
s(a + b*x)**3/(64*b**4) - 51*d**4*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**5
) - 3*d**4*cos(a + b*x)**4/(8*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 +
```

```
2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a), True))
```

Giac [A] time = 1.08493, size = 487, normalized size = 1.87

$$\frac{\left(32\,b^4d^4x^4+128\,b^4cd^3x^3+192\,b^4c^2d^2x^2+128\,b^4c^3dx+32\,b^4c^4-24\,b^2d^4x^2-48\,b^2cd^3x-24\,b^2c^2d^2+3\,d^4\right)\cos\left(4\,bx+\right.}{1024\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*
c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3
*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x -
 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/256*(8*b^3*d^4*x^3 + 24*b^
3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(4
*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

3.24 $\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx$

Optimal. Leaf size=196

$$-\frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3}-\frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a+bx)\cos(a+bx)}{32b^2}$$

```
[Out] (45*d^3*x)/(256*b^3) - (3*(c + d*x)^3)/(32*b) - (45*d^3*Cos[a + b*x]*Sin[a
+ b*x])/(256*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) -
(9*d^2*(c + d*x)*Sin[a + b*x]^2)/(32*b^3) - (3*d^3*Cos[a + b*x]*Sin[a + b*x
]^3)/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3)/(16*b^2) - (
3*d^2*(c + d*x)*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^3*Sin[a + b*x]^4)/(4*
b)
```

Rubi [A] time = 0.165402, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4404, 3311, 32, 2635, 8}

$$-\frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3}-\frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a+bx)\cos(a+bx)}{32b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```


```
[Out] (45*d^3*x)/(256*b^3) - (3*(c + d*x)^3)/(32*b) - (45*d^3*Cos[a + b*x]*Sin[a
+ b*x])/(256*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) -
(9*d^2*(c + d*x)*Sin[a + b*x]^2)/(32*b^3) - (3*d^3*Cos[a + b*x]*Sin[a + b*x
]^3)/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3)/(16*b^2) - (
3*d^2*(c + d*x)*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^3*Sin[a + b*x]^4)/(4*
b)
```


Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{(3d)\int (c+dx)^2\sin^4(a+bx)\,dx}{4b}$$

$$= \frac{3d(c+dx)^2\cos(a+bx)\sin^3(a+bx)}{16b^2} - \frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3} + \frac{(c+dx)^3 s\ldots}{4\ldots}$$

$$= \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{32b^2} - \frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3} - \frac{3d^3\cos(a+\ldots}{1\ldots}$$

$$= -\frac{3(c+dx)^3}{32b} - \frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4} + \frac{9d(c+dx)^2\cos(a+bx)\sin(a+\ldots}{32b^2}$$

$$= \frac{45d^3x}{256b^3} - \frac{3(c+dx)^3}{32b} - \frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4} + \frac{9d(c+dx)^2\cos(a+bx\ldots}{32b^2}$$

Mathematica [A] time = 0.969691, size = 135, normalized size = 0.69

$$\frac{-64b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+4b(c+dx)\cos(4(a+bx))\left(8b^2(c+dx)^2-3d^2\right)-6d\sin(2(a+bx))\left(\ldots\right.}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 4*b*(c + d
*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 6*d*(-16*(-d^2 + 2*b^2*
(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)
])/(1024*b^4)
```

Maple [B] time = 0.019, size = 594, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128
*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*
cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)+3/16*(b*x+a)^3)-3/b^3*a*d^3*(1/4*(b
*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)+3/b
^2*c*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*si
n(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*s
in(b*x+a)^2)+3/b^3*a^2*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2
*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)-6/b^2*a*c*d^2*(1/4*(b*x+a)*sin(b*x
+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+3/b*c^
2*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)
-3/32*b*x-3/32*a)-1/4/b^3*a^3*d^3*sin(b*x+a)^4+3/4/b^2*a^2*c*d^2*sin(b*x+a)
^4-3/4/b*a*c^2*d*sin(b*x+a)^4+1/4*c^3*sin(b*x+a)^4)
```

Maxima [B] time = 1.21312, size = 741, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(256*c^3*sin(b*x + a)^4 - 768*a*c^2*d*sin(b*x + a)^4/b + 768*a^2*c*d
^2*sin(b*x + a)^4/b^2 - 256*a^3*d^3*sin(b*x + a)^4/b^3 + 24*(4*(b*x + a)*co
```

```
s(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2
*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(
2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24*(4*(
b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a
) + 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4
*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a
) + 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2 - 1)*cos(
4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*
b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b*x + a)^3 -
 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x
 + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2 - 1)*s
in(2*b*x + 2*a))*d^3/b^3)/b
```

Fricas [A] time = 0.517854, size = 621, normalized size = 3.17

$$\frac{40\,b^3d^3x^3 + 120\,b^3cd^2x^2 + 8\left(8\,b^3d^3x^3 + 24\,b^3cd^2x^2 + 8\,b^3c^3 - 3\,bcd^2 + 3\left(8\,b^3c^2d - bd^3\right)x\right)\cos\left(bx + a\right)^4 - 8\left(16\,b^3d^3x^3 \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(40*b^3*d^3*x^3 + 120*b^3*c*d^2*x^2 + 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2
*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 -
8*(16*b^3*d^3*x^3 + 48*b^3*c*d^2*x^2 + 16*b^3*c^3 - 15*b*c*d^2 + 3*(16*b^3*
c^2*d - 5*b*d^3)*x)*cos(b*x + a)^2 + 3*(40*b^3*c^2*d - 17*b*d^3)*x - 3*(2*(
8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - (40*b^
2*d^3*x^2 + 80*b^2*c*d^2*x + 40*b^2*c^2*d - 17*d^3)*cos(b*x + a))*sin(b*x +
 a))/b^4
```

Sympy [A] time = 10.4897, size = 634, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) - c**3*cos(a + b*x)*
*4/(4*b) + 15*c**2*d*x*sin(a + b*x)**4/(32*b) - 9*c**2*d*x*sin(a + b*x)**2*
```

```
cos(a + b*x)**2/(16*b) - 9*c**2*d*x*cos(a + b*x)**4/(32*b) + 15*c*d**2*x**2
*sin(a + b*x)**4/(32*b) - 9*c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16
*b) - 9*c*d**2*x**2*cos(a + b*x)**4/(32*b) + 5*d**3*x**3*sin(a + b*x)**4/(3
2*b) - 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d**3*x**3*cos
(a + b*x)**4/(32*b) + 15*c**2*d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 9*
c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) + 15*c*d**2*x*sin(a + b*x)**3
*cos(a + b*x)/(16*b**2) + 9*c*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**2)
 + 15*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 9*d**3*x**2*sin(a
+ b*x)*cos(a + b*x)**3/(32*b**2) + 15*c*d**2*sin(a + b*x)**2*cos(a + b*x)**
2/(32*b**3) + 3*c*d**2*cos(a + b*x)**4/(8*b**3) - 51*d**3*x*sin(a + b*x)**4
/(256*b**3) + 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**3) + 45*d**3
*x*cos(a + b*x)**4/(256*b**3) - 51*d**3*sin(a + b*x)**3*cos(a + b*x)/(256*b
**4) - 45*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b, 0)), ((c**3*x
 + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3*cos(a), True))
```

Giac [A] time = 1.13294, size = 325, normalized size = 1.66

$$\frac{\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^4}-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c\right.}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*
d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^2
*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/
b^4 - 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*x
 + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(
2*b*x + 2*a)/b^4
```

3.25 $\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx$

Optimal. Leaf size=134

$$\frac{d(c+dx)\sin^3(a+bx)\cos(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}-\frac{d^2\sin^4(a+bx)}{32b^3}-\frac{3d^2\sin^2(a+bx)}{32b^3}+\frac{(c+dx}{}$$

```
[Out] (-3*c*d*x)/(16*b) - (3*d^2*x^2)/(32*b) + (3*d*(c + d*x)*Cos[a + b*x]*Sin[a
+ b*x])/(16*b^2) - (3*d^2*Sin[a + b*x]^2)/(32*b^3) + (d*(c + d*x)*Cos[a + b
*x]*Sin[a + b*x]^3)/(8*b^2) - (d^2*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^2*
Sin[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.0924045, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4404, 3310}

$$\frac{d(c+dx)\sin^3(a+bx)\cos(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}-\frac{d^2\sin^4(a+bx)}{32b^3}-\frac{3d^2\sin^2(a+bx)}{32b^3}+\frac{(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*c*d*x)/(16*b) - (3*d^2*x^2)/(32*b) + (3*d*(c + d*x)*Cos[a + b*x]*Sin[a
+ b*x])/(16*b^2) - (3*d^2*Sin[a + b*x]^2)/(32*b^3) + (d*(c + d*x)*Cos[a + b
*x]*Sin[a + b*x]^3)/(8*b^2) - (d^2*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^2*
Sin[a + b*x]^4)/(4*b)
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

]

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)^2\sin^4(a+bx)}{4b} - \frac{d\int (c+dx)\sin^4(a+bx)\,dx}{2b}$$
$$= \frac{d(c+dx)\cos(a+bx)\sin^3(a+bx)}{8b^2} - \frac{d^2\sin^4(a+bx)}{32b^3} + \frac{(c+dx)^2\sin^4(a+bx)}{4b} -$$
$$= \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} - \frac{3d^2\sin^2(a+bx)}{32b^3} + \frac{d(c+dx)\cos(a+bx)\,\mathrm{s}}{8b^2}$$
$$= -\frac{3cdx}{16b} - \frac{3d^2x^2}{32b} + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} - \frac{3d^2\sin^2(a+bx)}{32b^3} + \frac{d(c}{}$$

Mathematica [A] time = 0.524018, size = 91, normalized size = 0.68

$$\frac{-16\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+\cos(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)-4bd(c+dx)(\sin(4(a+bx))-8\sin(2(a+bx}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 8*b^2*(c + d*x)^
2)*Cos[4*(a + b*x)] - 4*b*d*(c + d*x)*(-8*Sin[2*(a + b*x)] + Sin[4*(a + b*x
)]))/(256*b^3)
```

Maple [B] time = 0.02, size = 260, normalized size = 1.9

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(\frac{(bx+a)^2(\sin(bx+a))^4}{4} - \frac{bx+a}{2}\left(-\frac{\cos(bx+a)}{4}\left((\sin(bx+a))^3+\frac{3\sin(bx+a)}{2}\right)+\frac{3bx}{8}+\frac{3a}{8}\right)+\frac{3(bx+a)^2}{32} -\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+
3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-
3/32*sin(b*x+a)^2)-2/b^2*a*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3
```

```
+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+2/b*c*d*(1/4*(b*x+a)*sin(b*x+a
)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+1/4/b^2*
a^2*d^2*sin(b*x+a)^4-1/2/b*a*c*d*sin(b*x+a)^4+1/4*c^2*sin(b*x+a)^4)
```

Maxima [B] time = 1.1864, size = 355, normalized size = 2.65

$$64\,c^2\sin\left(bx+a\right)^4 - \frac{128\,acd\sin(bx+a)^4}{b} + \frac{64\,a^2d^2\sin(bx+a)^4}{b^2} + \frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))cd}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(64*c^2*sin(b*x + a)^4 - 128*a*c*d*sin(b*x + a)^4/b + 64*a^2*d^2*sin(
b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x +
 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(4
*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*
x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x +
a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*si
n(2*b*x + 2*a))*d^2/b^2)/b
```

Fricas [A] time = 0.486331, size = 359, normalized size = 2.68

$$\frac{5\,b^2d^2x^2 + 10\,b^2cdx + \left(8\,b^2d^2x^2 + 16\,b^2cdx + 8\,b^2c^2 - d^2\right)\cos\left(bx+a\right)^4 - \left(16\,b^2d^2x^2 + 32\,b^2cdx + 16\,b^2c^2 - 5\,d^2\right)\cos\left(b\right.}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(5*b^2*d^2*x^2 + 10*b^2*c*d*x + (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*
c^2 - d^2)*cos(b*x + a)^4 - (16*b^2*d^2*x^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 5
*d^2)*cos(b*x + a)^2 - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 5*(b*d^2*x +
 b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [A] time = 5.11579, size = 350, normalized size = 2.61

$$\begin{cases} -\frac{c^2\sin^2(a+bx)\cos^2(a+bx)}{2b} - \frac{c^2\cos^4(a+bx)}{4b} + \frac{5cdx\sin^4(a+bx)}{16b} - \frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b} - \frac{3cdx\cos^4(a+bx)}{16b} + \frac{5d^2x^2\sin^4(a+bx)}{32b} - \frac{3d^2x^2\sin^2 \ldots \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) - c**2*cos(a + b*x)*
*4/(4*b) + 5*c*d*x*sin(a + b*x)**4/(16*b) - 3*c*d*x*sin(a + b*x)**2*cos(a +
 b*x)**2/(8*b) - 3*c*d*x*cos(a + b*x)**4/(16*b) + 5*d**2*x**2*sin(a + b*x)*
*4/(32*b) - 3*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d**2*x**
2*cos(a + b*x)**4/(32*b) + 5*c*d*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 3
*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) + 5*d**2*x*sin(a + b*x)**3*cos(
a + b*x)/(16*b**2) + 3*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) + 5*d*
*2*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) + d**2*cos(a + b*x)**4/(8*b**3
), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a), True))
```

Giac [A] time = 1.13904, size = 196, normalized size = 1.46

$$\frac{\left(8\,b^2d^2x^2 + 16\,b^2cdx + 8\,b^2c^2 - d^2\right)\cos\left(4\,bx + 4\,a\right)}{256\,b^3} - \frac{\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(2\,bx + 2\,a\right)}{16\,b^3} - \frac{\left(bd^2x + bcd\right)\ldots}{64\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^3
 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^
3 - 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*sin
(2*b*x + 2*a)/b^3
```

3.26 $\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=72

$$\frac{d\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}+\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{3dx}{32b}$$

```
[Out] (-3*d*x)/(32*b) + (3*d*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) + (d*Cos[a + b*x
]*Sin[a + b*x]^3)/(16*b^2) + ((c + d*x)*Sin[a + b*x]^4)/(4*b)
```

Rubi [A] time = 0.04547, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4404, 2635, 8}

$$\frac{d\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}+\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{3dx}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```


```
[Out] (-3*d*x)/(32*b) + (3*d*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) + (d*Cos[a + b*x
]*Sin[a + b*x]^3)/(16*b^2) + ((c + d*x)*Sin[a + b*x]^4)/(4*b)
```


Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx &= \frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{d\int \sin^4(a+bx)\,dx}{4b}\\
&= \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} + \frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{(3d)\int \sin^2(a+bx)\,dx}{16b}\\
&= \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} + \frac{(c+dx)\sin^4(a+bx)}{4b}\\
&= -\frac{3dx}{32b} + \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} + \frac{(c+dx)\sin^4}{4b}
\end{aligned}
$$

Mathematica [A] time = 0.116165, size = 75, normalized size = 1.04

$$
\frac{d(\sin(2(a+bx)) - 2bx\cos(2(a+bx)))}{16b^2} - \frac{d(\sin(4(a+bx)) - 4bx\cos(4(a+bx)))}{128b^2} + \frac{c\sin^4(a+bx)}{4b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sin[a + b*x]^4)/(4*b) + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]))
/(16*b^2) - (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

Maple [A] time = 0.021, size = 85, normalized size = 1.2

$$
\frac{1}{b}\left(\frac{d}{b}\left(\frac{(bx+a)(\sin(bx+a))^4}{4} + \frac{\cos(bx+a)}{16}\left((\sin(bx+a))^3 + \frac{3\sin(bx+a)}{2}\right) - \frac{3bx}{32} - \frac{3a}{32}\right) - \frac{ad(\sin(bx+a))^4}{4b} + \frac{c(\mathrm{si}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(d/b*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b
*x+a)-3/32*b*x-3/32*a)-1/4/b*a*d*sin(b*x+a)^4+1/4*c*sin(b*x+a)^4)
```

Maxima [A] time = 1.09336, size = 124, normalized size = 1.72

$$\frac{32\,c\sin\left(bx+a\right)^4-\frac{32\,ad\sin(bx+a)^4}{b}+\frac{\left(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a)\right)d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(32*c*sin(b*x + a)^4 - 32*a*d*sin(b*x + a)^4/b + (4*(b*x + a)*cos(4*b
*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x
+ 2*a))*d/b)/b
```

Fricas [A] time = 0.473922, size = 192, normalized size = 2.67

$$\frac{8\,(bdx+bc)\cos\left(bx+a\right)^4+5\,bdx-16\,(bdx+bc)\cos\left(bx+a\right)^2-\left(2\,d\cos\left(bx+a\right)^3-5\,d\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 + 5*b*d*x - 16*(b*d*x + b*c)*cos(b*x +
 a)^2 - (2*d*cos(b*x + a)^3 - 5*d*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A] time = 2.37126, size = 160, normalized size = 2.22

$$\begin{cases}-\frac{c\sin^2(a+bx)\cos^2(a+bx)}{2b}-\frac{c\cos^4(a+bx)}{4b}+\frac{5dx\sin^4(a+bx)}{32b}-\frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b}-\frac{3dx\cos^4(a+bx)}{32b}+\frac{5d\sin^3(a+bx)\cos(a+bx)}{32b^2}+\frac{3d\sin\ldots}{} \\ \left(cx+\frac{dx^2}{2}\right)\sin^3(a)\cos(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**2/(2*b) - c*cos(a + b*x)**4/(4*
b) + 5*d*x*sin(a + b*x)**4/(32*b) - 3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(
16*b) - 3*d*x*cos(a + b*x)**4/(32*b) + 5*d*sin(a + b*x)**3*cos(a + b*x)/(32
```

```
*b**2) + 3*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2), Ne(b, 0)), ((c*x + d*x
**2/2)*sin(a)**3*cos(a), True))
```

Giac [A] time = 1.11426, size = 101, normalized size = 1.4

$$\frac{(bdx+bc)\cos(4\,bx+4\,a)}{32\,b^2}-\frac{(bdx+bc)\cos(2\,bx+2\,a)}{8\,b^2}-\frac{d\sin(4\,bx+4\,a)}{128\,b^2}+\frac{d\sin(2\,bx+2\,a)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*a
)/b^2 - 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

3.27 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=129

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}-$$

```
[Out] -(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral
[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/(4*d) - (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b
*c)/d + 4*b*x])/(8*d)
```

Rubi [A] time = 0.232436, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```


```
[Out] -(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral
[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/(4*d) - (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b
*c)/d + 4*b*x])/(8*d)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)} - \frac{\sin(4a+4bx)}{8(c+dx)}\right)dx \\ &= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{c+dx}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{c+dx}\,dx \\ &= -\left(\frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx\right) + \frac{1}{4}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx - \\ &= -\frac{\text{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d} + \frac{\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{4d}\end{aligned}$$

Mathematica [A] time = 0.416103, size = 110, normalized size = 0.85

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) - 2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(4\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] - 2*CosIntegral[(2*b*
(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*
b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d
)
```

Maple [A] time = 0.021, size = 178, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b}{8}\left(2\,\frac{1}{d}\mathrm{Si}\left(2\,bx+2\,a+2\,\frac{-ad+bc}{d}\right)\cos\left(2\,\frac{-ad+bc}{d}\right)-2\,\frac{1}{d}\mathrm{Ci}\left(2\,bx+2\,a+2\,\frac{-ad+bc}{d}\right)\sin\left(2\,\frac{-ad+bc}{d}\right)\right)-\frac{b}{32}\left(4\,\frac{1}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(1/8*b*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x
+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)-1/32*b*(4*Si(4*b*x+4*a+4*(-a*d+
b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*
c)/d)/d))
```

Maxima [C] time = 1.43695, size = 370, normalized size = 2.87

$$\frac{b\left(-2i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_1\left(-\frac{4i\,bc+4i\,(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(b*(-2*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
2*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*
c - a*d)/d) + b*(I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/
d) - I*exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*
(b*c - a*d)/d) - 2*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*
d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
*(b*c - a*d)/d) + b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d
)/d) + exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*
(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 0.480233, size = 421, normalized size = 3.26

$$\frac{2\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-\left(\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-2\,\cos\left(-\frac{4\,(bc-ad}{d}\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) - (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-
4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) - 2*cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x), x)
```

Giac [C] time = 1.72027, size = 8162, normalized size = 63.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2
+ 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(
b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/
d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*t
an(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integr
```

al(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - 8*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 8*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 16*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d

```
)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*
c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(
b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)
^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(
2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2
*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*
c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(-4
*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2
*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/
d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan
(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)
^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a
)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2 + imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*sin_integral(4*(b*d*x
+ b*c)/d)*tan(2*a)^2*tan(a)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^
2*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*t
an(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2
*tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b
*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2
- 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - imag_
part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*sin_inte
gral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - 4*sin_integral(2*(b*d*x
 + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c
/d))*tan(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)
^2*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(
2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 + 4*
sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 - 8*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) + 8*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) - 16*sin_integral(2*
(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)*tan(b*c/d) - 8*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 8*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 16*sin_integral(2*(b
*d*x + b*c)/d)*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x +
```

```
2*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*ta
n(2*a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*
c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c/d)^2 + imag_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x -
 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)
^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 +
 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(b*c/d
)^2 - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan
(b*c/d)^2 + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)*tan(b*c
/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^
2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2
- 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integra
l(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a) - 4*real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(2*a)^2*tan(a) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*a)*tan(a)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)
^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2
*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*real
_part(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos_integ
ral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2*real_part(cos_integral(-4
*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(a)*tan(2*b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
2*a)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*
tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)
 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) + 4*real
_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 + 2*real_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b
*c/d)*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 -
2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2 + 2*imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(2*a)^2 + imag_part(cos_integral(-4*b*x - 4*b*c
/d))*tan(2*a)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 - 4*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(2*a)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(a)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*imag_
```

```
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - imag_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(a)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2 + 4*si
n_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*a)*tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*
b*c/d) - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 - 2*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c
/d))*tan(2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 - 4*
sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 - 8*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)*tan(b*c/d) - 16*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c
/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(b*c/d)^2 + 2*imag_part(c
os_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 4*sin_integral(
2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(2*a) + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - 4*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(a) - 4*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c
/d) - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + 4*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 4*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d)) - 2*im
ag_part(cos_integral(2*b*x + 2*b*c/d)) + 2*imag_part(cos_integral(-2*b*x -
2*b*c/d)) - imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*sin_integral(4*(b
*d*x + b*c)/d) - 4*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(2*a)^2*tan(a)^2*
tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + d*tan(
2*a)^2*tan(a)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d
*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2 + d*tan(2*a)^
2*tan(2*b*c/d)^2 + d*tan(a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2*tan(b*c/d)^2 +
d*tan(a)^2*tan(b*c/d)^2 + d*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2 + d*
tan(a)^2 + d*tan(2*b*c/d)^2 + d*tan(b*c/d)^2 + d)
```

3.28 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=179

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}$$

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) + Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rubi [A] time = 0.280256, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```


```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) + Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2} - \frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx \\
&= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx \\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)} + \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{2d} - \frac{b\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{2d} \\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)} - \frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d} + \frac{\left(b\cos\left(2a-\frac{2\ldots}{}\right.\right.}{} \\
&= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2} - \frac{b\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2} - \frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin\ldots}{8\ldots}
\end{aligned}
$$

Mathematica [A] time = 1.29574, size = 151, normalized size = 0.84

$$\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-4b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)-4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+4}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*a - (4
*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - (2*d*Sin[2*(a + b*x)])/(c + d*x)
+ (d*Sin[4*(a + b*x)])/(c + d*x) - 4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*
b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/
(8*d^2)
```

Maple [A] time = 0.023, size = 256, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^2}{8}\left(-2\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)d}+2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\sin\left(2\frac{-ad+bc}{d}\right)+2\frac{1}{d}\text{Ci}\left(2bx+2a+2\frac{-ad}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/8*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b*x+2*a+2*(
-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a
*d+b*c)/d)/d)/d)-1/32*b^2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d+4*(4*Si(
4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)
/d)*cos(4*(-a*d+b*c)/d)/d)/d))
```

Maxima [C] time = 1.77806, size = 406, normalized size = 2.27

$$\frac{b^2\left(-2i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_2\left(-\frac{4i\,bc+4}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(b^2*(-2*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 2*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) - 2*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 0.553995, size = 621, normalized size = 3.47

$$\frac{2\,(bdx+bc)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)-2\,(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d)
- 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + (
(b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(
-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - ((b*d*x + b*c)*cos_integral(4*
(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(
b*c - a*d)/d) + 4*(d*cos(b*x + a)^3 - d*cos(b*x + a))*sin(b*x + a))/(d^3*x
+ c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\sin(bx+a)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sin(b*x + a)^3/(d*x + c)^2, x)
```

3.29 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=229

$$\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}$$

```
[Out] -(b*Cos[2*a + 2*b*x])/(4*d^2*(c + d*x)) + (b*Cos[4*a + 4*b*x])/(4*d^2*(c +
d*x)) + (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (b^
2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a +
2*b*x]/(8*d*(c + d*x)^2) + Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos[2
*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) + (b^2*Cos[4*a - (4
*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Rubi [A] time = 0.34384, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```


```
[Out] -(b*Cos[2*a + 2*b*x])/(4*d^2*(c + d*x)) + (b*Cos[4*a + 4*b*x])/(4*d^2*(c +
d*x)) + (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (b^
2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a +
2*b*x]/(8*d*(c + d*x)^2) + Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos[2
*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) + (b^2*Cos[4*a - (4
*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3} - \frac{\sin(4a+4bx)}{8(c+dx)^3}\right) dx$$

$$= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{4d} - \frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{4d}$$

$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} - \frac{b^2\int \frac{\sin(2a+2b}{c+dx}}{2d^2}$$

$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{\left(b^2\cos\left(4a\right.\right.}{}$$

$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} + \frac{b^2\,\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3} - \frac{b^2\,\mathrm{Ci}\left(\frac{2bc}{d}+\right.}{}$$

Mathematica [A] time = 2.82636, size = 199, normalized size = 0.87

$$\frac{-2\left(4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}\right)+16b^2}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*(4*b*(c +
d*x)*Cos[4*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 - 2*(4*b^2*CosInte
gral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c + d*x)*Cos[2*(a +
 b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*(c + d*x))/d]) + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*
(c + d*x))/d])/(16*d^3)
```

Maple [A] time = 0.024, size = 329, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^3}{8}\left(-\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)^2 d}+\frac{1}{d}\left(-2\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)d}-2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\cos\left(2\frac{-ad+b}{d}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/8*b^3*(-sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d+(-2*cos(2*b*x+2*a)/((
b*x+a)*d-a*d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d
-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)-1/32*b^3*(-2*s
in(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)^2/d+2*(-4*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b
*c)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*
a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] time = 2.17134, size = 454, normalized size = 1.98

$$\frac{b^3\left(-2iE_3\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)+2iE_3\left(-\frac{2ibc+2i(bx+a)d-2iad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(iE_3\left(\frac{4ibc+4i(bx+a)d-4iad}{d}\right)-iE_3\left(-\frac{4ibc+4i\right.\right.}{16\left(b^2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b^3*(-2*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 2*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) - 2*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A] time = 0.617182, size = 976, normalized size = 4.26

$$\frac{8\left(bd^2x + bcd\right)\cos\left(bx + a\right)^4 + 2\,bd^2x + 2\,bcd - 10\left(bd^2x + bcd\right)\cos\left(bx + a\right)^2 + 4\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cos\left(-\frac{4\,(bc-a}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(b*d^2*x + b*c*d)*cos(b*x + a)^4 + 2*b*d^2*x + 2*b*c*d - 10*(b*d^2*x
 + b*c*d)*cos(b*x + a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-4*(
b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + 2*(d^2
*cos(b*x + a)^3 - d^2*cos(b*x + a))*sin(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) + 2*((b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d) + (b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(
b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3\left(a + bx\right)\cos\left(a + bx\right)}{\left(c + dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.30 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{3d^4}$$

```
[Out] -(b*Cos[2*a + 2*b*x])/(12*d^2*(c + d*x)^2) + (b*Cos[4*a + 4*b*x])/(12*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) + (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
- Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c +
d*x)) + Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Sin[4*a + 4*b*x])/(3*d^3
*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) - (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rubi [A] time = 0.389443, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```


```
[Out] -(b*Cos[2*a + 2*b*x])/(12*d^2*(c + d*x)^2) + (b*Cos[4*a + 4*b*x])/(12*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) + (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
- Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c +
d*x)) + Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Sin[4*a + 4*b*x])/(3*d^3
*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) - (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

tQ[p, 0]

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4} - \frac{\sin(4a+4bx)}{8(c+dx)^4}\right) dx$$

$$= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^4}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{6d} - \frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\int \frac{\sin(2a+2b}{(c+dx)^2}}{6d^2}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} + \frac{\sin(4a+}{24d(c+d}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} + \frac{\sin(4a+}{24d(c+d}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4} + \frac{4b^3\cos\left(4a\right.}{}$$

Mathematica [A] time = 2.40638, size = 316, normalized size = 1.1

$$\frac{-8b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - \sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)\right) + 32b^3(c+dx)^3\left(\cos\left(4a-\frac{4bc}{d}\right)\text{Co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*
a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^2)*Si
n[4*a]) + 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a]
)*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x)*Sin
[4*a])*Sin[4*b*x] - 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*
b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]) + 32
*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - Sin
[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

Maple [A] time = 0.024, size = 404, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^4}{8}\left(-\frac{2\sin(2bx+2a)}{3((bx+a)d-ad+bc)^3d}+\frac{2}{3d}\left(-\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)^2d}-\frac{1}{d}\left(-2\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)d}+2\frac{1}{d}\left(2\frac{1}{d}\mathrm{Si}\left(2bx\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/8*b^4*(-2/3*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^3/d+2/3*(-cos(2*b*x+2
*a)/((b*x+a)*d-a*d+b*c)^2/d-(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*S
i(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*
c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/32*b^4*(-4/3*sin(4*b*x+4*a)/((b*x+a
)*d-a*d+b*c)^3/d+4/3*(-2*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)^2/d-2*(-4*sin(4
*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d+4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a
*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)/d)
)
```

Maxima [C] time = 3.09143, size = 521, normalized size = 1.82

$$\frac{b^4\left(-2iE_4\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)+2iE_4\left(-\frac{2ibc+2i(bx+a)d-2iad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^4\left(iE_4\left(\frac{4ibc+4i(bx+a)d-4iad}{d}\right)-iE_4\left(-\frac{4ibc+4i\right.\right.}{16\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/16*(b^4*(-2*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 2*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) - 2*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] time = 0.684968, size = 1293, normalized size = 4.51

$$bd^3x + 4\left(bd^3x + bcd^2\right)\cos\left(bx + a\right)^4 + bcd^2 - 5\left(bd^3x + bcd^2\right)\cos\left(bx + a\right)^2 - 8\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3\right)\,\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*d^3*x + 4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 + b*c*d^2 - 5*(b*d^3*x
+ b*c*d^2)*cos(b*x + a)^2 - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)
*sin_integral(2*(b*d*x + b*c)/d) - ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-
2*(b*c - a*d)/d) + 4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*cos_integral(4*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b
^3*c^2*d*x + b^3*c^3)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/
d) - 2*((8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3
 - (5*b^2*d^3*x^2 + 10*b^2*c*d^2*x + 5*b^2*c^2*d - d^3)*cos(b*x + a))*sin(b
*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.31 $\int (c+dx)^m \cot(a+bx)\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\cot(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Cot[a + b*x], x]
```

Rubi [A] time = 0.0182283, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\,dx$$

Mathematica [A] time = 2.53905, size = 0, normalized size = 0.

$$\int (c+dx)^m \cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x], x]
```

Maple [A] time = 0.234, size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \cos(bx+a)\csc(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

3.32 $\int (c+dx)^4 \cot(a+bx)\,dx$

Optimal. Leaf size=151

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{3d^4\text{PolyL}}{}$$

```
[Out] ((-I/5)*(c + d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - ((2
*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2
*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^(
(2*I)*(a + b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rubi [A] time = 0.220252, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{3d^4\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x],x]
```

```
[Out] ((-I/5)*(c + d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - ((2
*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2
*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^(
(2*I)*(a + b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4\cot(a+bx)\,dx = -\frac{i(c+dx)^5}{5d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}}\,dx$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(4d)\int (c+dx)^3\log\left(1-e^{2i(a+bx)}\right)dx}{b}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{\left(6id^2\right)\int (c+dx)^2\text{L}}{b^2}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+dx)^2\text{Li}_3\left(e\right.}{b^3}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+dx)^2\text{Li}_3\left(e\right.}{b^3}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+dx)^2\text{Li}_3\left(e\right.}{b^3}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+dx)^2\text{Li}_3\left(e\right.}{b^3}$$

<u>Mathematica</u> [B] time = 5.76902, size = 799, normalized size = 5.29

$$\frac{1}{5}id^4x^5 + icd^3x^4 + \frac{d^4\log\left(1-e^{-i(a+bx)}\right)x^4}{b} + \frac{d^4\log\left(1+e^{-i(a+bx)}\right)x^4}{b} + 2ic^2d^2x^3 + \frac{4cd^3\log\left(1-e^{-i(a+bx)}\right)x^3}{b} + \frac{4cd^3\log\left(1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x],x]
```

```
[Out] ((2*I)*c^3*d*Pi*x)/b + (2*I)*c^2*d^2*x^3 + I*c*d^3*x^4 + (I/5)*d^4*x^5 - ((
4*I)*c^3*d*x*ArcTan[Tan[a]])/b + 2*c^3*d*x^2*Cot[a] + (2*c^3*d*Pi*Log[1 + E
^((-2*I)*b*x)])/b^2 + (6*c^2*d^2*x^2*Log[1 - E^((-I)*(a + b*x))])/b + (4*c*
d^3*x^3*Log[1 - E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 - E^((-I)*(a + b*x)
)])/b + (6*c^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))])/b + (4*c*d^3*x^3*Log[1
+ E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 + E^((-I)*(a + b*x))])/b + (4*c^3
*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b + (4*c^3*d*ArcTan[Tan[a]]
*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b^2 - (2*c^3*d*Pi*Log[Cos[b*x]]
)/b^2 + (c^4*Log[Sin[a + b*x]])/b - (4*c^3*d*ArcTan[Tan[a]]*Log[Sin[b*x + A
rcTan[Tan[a]]]])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*PolyLog[2,
-E^((-I)*(a + b*x))])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*PolyLo
g[2, E^((-I)*(a + b*x))])/b^2 - ((2*I)*c^3*d*PolyLog[2, E^((2*I)*(b*x + Arc
Tan[Tan[a]]))])/b^2 + (12*c^2*d^2*PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (2
4*c*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (12*d^4*x^2*PolyLog[3, -E^
((-I)*(a + b*x))])/b^3 + (12*c^2*d^2*PolyLog[3, E^((-I)*(a + b*x))])/b^3 +
```

```
(24*c*d^3*x*PolyLog[3, E^((-I)*(a + b*x))])/b^3 + (12*d^4*x^2*PolyLog[3, E^
((-I)*(a + b*x))])/b^3 - ((24*I)*c*d^3*PolyLog[4, -E^((-I)*(a + b*x))])/b^4
 - ((24*I)*d^4*x*PolyLog[4, -E^((-I)*(a + b*x))])/b^4 - ((24*I)*c*d^3*PolyL
og[4, E^((-I)*(a + b*x))])/b^4 - ((24*I)*d^4*x*PolyLog[4, E^((-I)*(a + b*x)
)])/b^4 - (24*d^4*PolyLog[5, -E^((-I)*(a + b*x))])/b^5 - (24*d^4*PolyLog[5,
 E^((-I)*(a + b*x))])/b^5 - 2*c^3*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Se
c[a]^2]
```

Maple [B] time = 0.329, size = 1150, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] -1/5*I*d^4*x^5+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+1/b^5*d^4*a^4*ln(ex
p(I*(b*x+a))-1)-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,e
xp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-12*I/b^2*c^2*d^2*p
olylog(2,-exp(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-8*
I/b*c^3*d*a*x-12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+12*I/b^2*c^2*d^
2*a^2*x-8*I/b^3*c*d^3*a^3*x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+I*
c^4*x-2*I*c^3*d*x^2-I*c*d^3*x^4-2*I*c^2*d^2*x^3+8/5*I/b^5*d^4*a^5+12/b^3*d^
4*polylog(3,-exp(I*(b*x+a)))*x^2+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)+24/
b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b
*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a-4*I/b^2*d^
4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*
I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x
+a)))*x+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,-
exp(I*(b*x+a)))+2*I/b^4*d^4*a^4*x+8*I/b^3*c^2*d^2*a^3-6*I/b^4*c*d^3*a^4-4*I
/b^2*c^3*d*a^2-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+1/b*d^4*ln(1-exp(I*(
b*x+a)))*x^4-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+1/b*d^4*ln(exp(I*(b*x+a))+1
)*x^4+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b*c^2*d^2*ln(exp(I*(b*x+a)
)+1)*x^2+1/b*c^4*ln(exp(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-2/b*c^4*
ln(exp(I*(b*x+a)))-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,e
xp(I*(b*x+a)))/b^5-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+6/b*c^2*d^2*ln(
1-exp(I*(b*x+a)))*x^2-6/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2+8/b^4*c*d^3*a^
3*ln(exp(I*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)+8/b^2*c^3*d*a*ln(ex
p(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-12/b^3*c^2*d^2*a^2*ln(ex
p(I*(b*x+a)))+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*c*d^3*ln(1-exp(I*(b*x+
a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3
```

Maxima [B] time = 1.97812, size = 1704, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/10*(10*c^4*log(sin(b*x + a)) - 40*a*c^3*d*log(sin(b*x + a))/b + 60*a^2*c^
2*d^2*log(sin(b*x + a))/b^2 - 40*a^3*c*d^3*log(sin(b*x + a))/b^3 + 10*a^4*d
^4*log(sin(b*x + a))/b^4 + (-2*I*(b*x + a)^5*d^4 + (-10*I*b*c*d^3 + 10*I*a*
d^4)*(b*x + a)^4 - 240*d^4*polylog(5, -e^(I*b*x + I*a)) - 240*d^4*polylog(5
, e^(I*b*x + I*a)) + (-20*I*b^2*c^2*d^2 + 40*I*a*b*c*d^3 - 20*I*a^2*d^4)*(b
*x + a)^3 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 - 60*I*a^2*b*c*d^3 + 20*I
*a^3*d^4)*(b*x + a)^2 + (10*I*(b*x + a)^4*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)
*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + 60*I*a^2*d^4)*(b*x + a
)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + 120*I*a^2*b*c*d^3 - 40*I*a^3*
d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (-10*I*(b*x + a)^
4*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^3 + (-60*I*b^2*c^2*d^2 + 120
*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a)^2 + (-40*I*b^3*c^3*d + 120*I*a*b^2*c
^2*d^2 - 120*I*a^2*b*c*d^3 + 40*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a),
 -cos(b*x + a) + 1) + (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 120*I*a^2*b*
c*d^3 - 40*I*(b*x + a)^3*d^4 + 40*I*a^3*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4
)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*a^2*d^4)*(b*x
 + a))*dilog(-e^(I*b*x + I*a)) + (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 1
20*I*a^2*b*c*d^3 - 40*I*(b*x + a)^3*d^4 + 40*I*a^3*d^4 + (-120*I*b*c*d^3 +
120*I*a*d^4)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*a^
2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) + 5*((b*x + a)^4*d^4 + 4*(b*c*d^3
- a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2
+ 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 5*((b*x + a)^4*d^4
+ 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (240*I
*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*polylog(4, -e^(I*b*x + I*a))
+ (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*polylog(4, e^(I*b*x +
 I*a)) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*
c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 120*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*p
olylog(3, e^(I*b*x + I*a)))/b^4)/b
```

Fricas [C] time = 0.694346, size = 2938, normalized size = 19.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*
x + a)) + 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - (-4*I*b^3*d^4
*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^
3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (4*I*b^
3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 -
12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
 (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b*
x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*
x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4
*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*
c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4
*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*
b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a
)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x + a
)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x +
a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x +
a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a
) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylo
g(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2
*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \cos(bx+a)\csc(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a), x)
```

3.33 $\int (c + dx)^3 \cot(a + bx)\, dx$

Optimal. Leaf size=127

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{(c+dx)^3\log\left(1-\right.}{b}$$

```
[Out] ((-I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((
3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*
x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^(
(2*I)*(a + b*x))])/b^4
```

Rubi [A] time = 0.192486, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{(c+dx)^3\log\left(1-\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x],x]
```


```
[Out] ((-I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((
3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*
x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^(
(2*I)*(a + b*x))])/b^4
```


Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \cot(a+bx)\,dx = -\frac{i(c+dx)^4}{4d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-e^{2i(a+bx)}\right)dx}{b}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{\left(3id^2\right)\int (c+dx)\mathrm{I}}{b^2}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+dx)\mathrm{Li}_3\left(e\right.}{2b^3}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+dx)\mathrm{Li}_3\left(e\right.}{2b^3}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+dx)\mathrm{Li}_3\left(e\right.}{2b^3}$$

Mathematica [B] time = 2.75977, size = 560, normalized size = 4.41

$$\frac{-6ib^2c^2d\mathrm{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right) + 12ib^2d^2x(2c+dx)\mathrm{PolyLog}\left(2,-e^{-i(a+bx)}\right) + 12ib^2d^2x(2c+dx)\mathrm{PolyLog}\left(2,e\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x],x]
```

```
[Out] ((6*I)*b^3*c^2*d*Pi*x + (4*I)*b^4*c*d^2*x^3 + I*b^4*d^3*x^4 - (12*I)*b^3*c^
2*d*x*ArcTan[Tan[a]] + 6*b^4*c^2*d*x^2*Cot[a] + 6*b^2*c^2*d*Pi*Log[1 + E^((
-2*I)*b*x)] + 12*b^3*c*d^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 4*b^3*d^3*x^3*
Log[1 - E^((-I)*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 + E^((-I)*(a + b*x))]
 + 4*b^3*d^3*x^3*Log[1 + E^((-I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*
I)*(b*x + ArcTan[Tan[a]]))] + 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[1 - E^((2*I)*
(b*x + ArcTan[Tan[a]]))] - 6*b^2*c^2*d*Pi*Log[Cos[b*x]] + 4*b^3*c^3*Log[Sin
[a + b*x]] - 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + (
12*I)*b^2*d^2*x*(2*c + d*x)*PolyLog[2, -E^((-I)*(a + b*x))] + (12*I)*b^2*d^
2*x*(2*c + d*x)*PolyLog[2, E^((-I)*(a + b*x))] - (6*I)*b^2*c^2*d*PolyLog[2,
 E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 24*b*c*d^2*PolyLog[3, -E^((-I)*(a + b*
x))] + 24*b*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))] + 24*b*c*d^2*PolyLog[3, E
^((-I)*(a + b*x))] + 24*b*d^3*x*PolyLog[3, E^((-I)*(a + b*x))] - (24*I)*d^3
*PolyLog[4, -E^((-I)*(a + b*x))] - (24*I)*d^3*PolyLog[4, E^((-I)*(a + b*x))
] - 6*b^4*c^2*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2])/(4*b^4)
```

Maple [B] time = 0.296, size = 783, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] 6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/
b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x-3/2*I/b^4*d^3*a^4+6*I/b^4*d^3*polylog(
4,-exp(I*(b*x+a)))+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,-
exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-1/4*I*d^3*x^4+6*I/b^2
*c*d^2*a^2*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polyl
og(2,exp(I*(b*x+a)))*x-6*I/b*c^2*d*a*x-I*c*d^2*x^3-3/2*I*c^2*d*x^2+6*I*d^3*
polylog(4,exp(I*(b*x+a)))/b^4+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c^2*d*
ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+3/b*c*d^2*ln(1-ex
p(I*(b*x+a)))*x^2-2/b*c^3*ln(exp(I*(b*x+a)))+1/b*c^3*ln(exp(I*(b*x+a))-1)+1
/b*c^3*ln(exp(I*(b*x+a))+1)-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+3/b*c^2*d*
ln(exp(I*(b*x+a))+1)*x+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(
I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/b^3*c*d^2*a^2*ln(exp(I*(
b*x+a))-1)+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))
-1)-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)
))*x^2-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*c^2*d*polylog(2,ex
p(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+4*I/b^3*c*d^2*a^3-2*
I/b^3*d^3*a^3*x-3*I/b^2*c^2*d*a^2+I*c^3*x
```

Maxima [B] time = 1.70128, size = 1008, normalized size = 7.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(4*c^3*log(sin(b*x + a)) - 12*a*c^2*d*log(sin(b*x + a))/b + 12*a^2*c*d^
2*log(sin(b*x + a))/b^2 - 4*a^3*d^3*log(sin(b*x + a))/b^3 + (-I*(b*x + a)^4
*d^3 + (-4*I*b*c*d^2 + 4*I*a*d^3)*(b*x + a)^3 + 24*I*d^3*polylog(4, -e^(I*b
*x + I*a)) + 24*I*d^3*polylog(4, e^(I*b*x + I*a)) + (-6*I*b^2*c^2*d + 12*I*
a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a)^2 + (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2
 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^
3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (-4*I*(b*x + a)^3*d
^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b
```

```
*c*d^2 - 12*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1)
+ (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 +
 (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (-12*I*b
^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*
c*d^2 + 24*I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + 2*((b*x + a)^3*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(
b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 2*((b
*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a
) + 1) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a))
+ 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3)/b
```

Fricas [C] time = 0.626955, size = 2056, normalized size = 16.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*
d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x +
 a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(cos(b*x +
a) - I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*
dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x
 - 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a)
+ 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*
x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
+ a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x + b*c*d^2)*
polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3
, cos(b*x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*
x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \cos(bx+a) \csc(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a), x)
```

3.34 $\int (c + dx)^2 \cot(a + bx) \, dx$

Optimal. Leaf size=93

$$-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^3}{3d}$$

```
[Out] ((-I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b - (I*
d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, E^((2*I)
*(a + b*x))])/(2*b^3)
```

Rubi [A] time = 0.166433, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3717, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x],x]
```

```
[Out] ((-I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b - (I*
d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, E^((2*I)
*(a + b*x))])/(2*b^3)
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\cot(a+bx)\,dx &= -\frac{i(c+dx)^3}{3d}-2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(2d)\int (c+dx)\log\left(1-e^{2i(a+bx)}\right)dx}{b}\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{\left(id^2\right)\int \text{Li}_2\left(e^{2i(a+bx)}\right)\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\,\text{Subst}\left(\int \frac{\text{Li}_2(x)}{x}\,dx\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}
\end{aligned}$$

Mathematica [B] time = 1.42455, size = 356, normalized size = 3.83

$$-3ibcd\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+6ibd^2x\text{PolyLog}\left(2,-e^{-i(a+bx)}\right)+6ibd^2x\text{PolyLog}\left(2,e^{-i(a+bx)}\right)+6d^2\text{PolyLog}\left(3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x],x]
```

```
[Out] ((3*I)*b^2*c*d*Pi*x + I*b^3*d^2*x^3 - (6*I)*b^2*c*d*x*ArcTan[Tan[a]] + 3*b^
3*c*d*x^2*Cot[a] + 3*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 3*b^2*d^2*x^2*Log[1
 - E^((-I)*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 6*b^2*
c*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 6*b*c*d*ArcTan[Tan[a]]*Lo
g[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 3*b*c*d*Pi*Log[Cos[b*x]] + 3*b^2*
c^2*Log[Sin[a + b*x]] - 6*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]
]] + (6*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*d^2*x*PolyLog[
2, E^((-I)*(a + b*x))] - (3*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[
a]]))] + 6*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 6*d^2*PolyLog[3, E^((-I)*(
a + b*x))] - 3*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2])/(3*b
^3)
```

Maple [B] time = 0.27, size = 468, normalized size = 5.

$$-2\frac{a^2d^2\ln\left(e^{i(bx+a)}\right)}{b^3}+\frac{a^2d^2\ln\left(e^{i(bx+a)}-1\right)}{b^3}-\frac{d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{b^3}+\frac{d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{b}+\frac{d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{b}+\frac{4}{3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] -2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-1/b^3*
d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d^2*ln(1-
exp(I*(b*x+a)))*x^2+4/3*I/b^3*d^2*a^3-I*c*d*x^2-4*I/b*c*d*a*x+2*I/b^2*d^2*a
^2*x-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,-exp(I*(b*
x+a)))-2*I/b^2*c*d*a^2-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*d^2*
polylog(2,-exp(I*(b*x+a)))*x+I*c^2*x+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+2/b*c*d
*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-2/b^2*c*d*a*ln(exp
(I*(b*x+a))-1)+4/b^2*c*d*a*ln(exp(I*(b*x+a)))+2*d^2*polylog(3,-exp(I*(b*x+a
)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+1/b*c^2*ln(exp(I*(b*x+a))-1)-2/
b*c^2*ln(exp(I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/3*I*d^2*x^3
```

Maxima [B] time = 1.52594, size = 545, normalized size = 5.86

$$6\,c^2\log\left(\sin\left(bx+a\right)\right)-\frac{12\,acd\log(\sin(bx+a))}{b}+\frac{6\,a^2d^2\log(\sin(bx+a))}{b^2}+\frac{-2i\,(bx+a)^3d^2+\left(-6i\,bcd+6i\,ad^2\right)(bx+a)^2+12\,d^2\mathrm{Li}_3(-e^{(i\,bx+i\,a)})+12\,d^2\mathrm{Li}_3(e^{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^2*log(sin(b*x + a)) - 12*a*c*d*log(sin(b*x + a))/b + 6*a^2*d^2*log
(sin(b*x + a))/b^2 + (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x
+ a)^2 + 12*d^2*polylog(3, -e^(I*b*x + I*a)) + 12*d^2*polylog(3, e^(I*b*x +
 I*a)) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*arctan
2(sin(b*x + a), cos(b*x + a) + 1) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d +
12*I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-12*I*b*
c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (-12*I*b*c
*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*dilog(e^(I*b*x + I*a)) + 3*((b*x + a)
^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
 2*cos(b*x + a) + 1) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

Fricas [C] time = 0.566539, size = 1330, normalized size = 14.3

$$\frac{2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)+2\,d^2\mathrm{polylog}\left(3,-\cos\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) + 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*
I*b*c*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*d
ilog(cos(b*x + a) - I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-cos(
b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-cos(b*x + a)
 - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
 + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x
+ a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b
*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cos(a + bx) \csc(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \cos(bx + a) \csc(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a), x)
```

3.35 $\int (c + dx) \cot(a + bx) \, dx$

Optimal. Leaf size=65

$$-\frac{id\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{i(c+dx)^2}{2d}$$

```
[Out] ((-I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b - ((I/2
)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rubi [A] time = 0.0963027, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3717, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x],x]
```

```
[Out] ((-I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b - ((I/2
)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot(a+bx)\,dx &= -\frac{i(c+dx)^2}{2d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx\\ &= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-e^{2i(a+bx)}\right)dx}{b}\\ &= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{(id)\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^2}\\ &= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id\,\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}\end{aligned}$$

Mathematica [B] time = 5.18029, size = 188, normalized size = 2.89

$$-\frac{d\csc(a)\sec(a)\left(\frac{\tan(a)\left(i\mathrm{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+2\tan^{-1}(\tan(a\cdots}{\sqrt{\tan^2(a)+1}}\right.}{2b^2\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Cot[a + b*x],x]
```

```
[Out] (d*x^2*Cot[a])/2 + (c*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b - (d*Csc[a
]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) -
 Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*
x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + A
rcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/S
qrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.248, size = 215, normalized size = 3.3

$$-\frac{i}{2}dx^2 + icx + \frac{c\ln\left(e^{i(bx+a)}-1\right)}{b} - 2\frac{c\ln\left(e^{i(bx+a)}\right)}{b} + \frac{c\ln\left(e^{i(bx+a)}+1\right)}{b} - \frac{2\,idax}{b} - \frac{ida^2}{b^2} + \frac{d\ln\left(1-e^{i(bx+a)}\right)x}{b} + \frac{d\ln\left(1-e\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] -1/2*I*d*x^2+I*c*x+1/b*c*ln(exp(I*(b*x+a))-1)-2/b*c*ln(exp(I*(b*x+a)))+1/b*
c*ln(exp(I*(b*x+a))+1)-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(1-exp(I*(b*x+a)))*x
+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/b*d*ln(
exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(
b*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 1.53232, size = 255, normalized size = 3.92

$$\frac{-i\,b^2dx^2 - 2i\,b^2cx - 2i\,bdx\arctan\left(\sin\left(bx+a\right), -\cos\left(bx+a\right)+1\right) + 2i\,bc\arctan\left(\sin\left(bx+a\right), \cos\left(bx+a\right)-1\right) + (2i\,bc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x
+ a) + 1) + 2*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x +
2*I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*I*d*dilog(-e^(I*b*x +
I*a)) - 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*cos(b*x + a) + 1) + (b*d*x + b*c)*log(cos(b*x + a)^2 + si
n(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

Fricas [B] time = 0.546684, size = 721, normalized size = 11.09

$$\frac{-i\,d\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right) + i\,d\mathrm{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right) + i\,d\mathrm{Li}_2\left(-\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right) - i\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*dilog(-cos
(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x +
a) + 1) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b*c - a*d
)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*
cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a)
+ I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) +
1))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \cos (a + bx) \csc (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \cos (bx + a) \csc (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)*csc(b*x + a), x)
```

3.36 $\int \frac{\cot(a+bx)}{c+dx} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\cot(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Cot[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.0208909, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Cot[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{c+dx} dx = \int \frac{\cot(a+bx)}{c+dx} dx$$

Mathematica [A] time = 3.58517, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Cot[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.244, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

3.37 $\int \frac{\cot(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\cot(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable[Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.0207621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 7.06491, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Cot[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.279, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

3.38 $\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\cot(a+bx)\csc(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi [A] time = 0.203924, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Mathematica [A] time = 1.24198, size = 0, normalized size = 0.

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Maple [A] time = 0.155, size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a) (\csc(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a) \csc(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \cos(bx+a) \csc(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a) \csc(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

3.39 $\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=208

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{12i}{}$$

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5
```

Rubi [A] time = 0.171711, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4410, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{12i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```


```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5
```


Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^4\csc(a+bx)}{b} + \frac{(4d)\int (c+dx)^3\csc(a+bx)\,dx}{b}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} - \frac{\left(12d^2\right)\int (c+dx)^2 \log}{b^2}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} + \frac{12id^2(c+dx)^2\text{Li}_2\left(-\right.}{b^3}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} + \frac{12id^2(c+dx)^2\text{Li}_2\left(-\right.}{b^3}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} + \frac{12id^2(c+dx)^2\text{Li}_2\left(-\right.}{b^3}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} + \frac{12id^2(c+dx)^2\text{Li}_2\left(-\right.}{b^3}$$

Mathematica [A] time = 1.37479, size = 308, normalized size = 1.48

$$8id\left(\frac{3d\left(b^2(c+dx)^2\text{PolyLog}(2,-\cos(a+bx)-i\sin(a+bx))+2ibd(c+dx)\text{PolyLog}(3,-\cos(a+bx)-i\sin(a+bx))-2d^2\text{PolyLog}(4,-\cos(a+bx)-i\sin(a+bx))\right)}{b^3} - \frac{3d\left(b\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)^4*Csc[a] + (8*I)*d*((2*I)*(c + d*x)^3*ArcTanh[Cos[a + b*x]
+ I*Sin[a + b*x]] + (3*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[
a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]
- 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]))/b^3 - (3*d*(b^2*(c + d
*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyL
og[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Si
n[a + b*x]]))/b^3) + b*(c + d*x)^4*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] -
 b*(c + d*x)^4*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^2)
```

Maple [B] time = 0.28, size = 716, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] -24*I*d^3/b^3*c*polylog(2,exp(I*(b*x+a)))*x+24*I*d^3/b^3*c*polylog(2,-exp(I
*(b*x+a)))*x+4*d^4/b^2*ln(1-exp(I*(b*x+a)))*x^3+8*d^4/b^5*a^3*arctanh(exp(I
*(b*x+a)))+24*d^4/b^4*polylog(3,exp(I*(b*x+a)))*x-24*d^4/b^4*polylog(3,-exp
(I*(b*x+a)))*x-24*d^3/b^4*c*polylog(3,-exp(I*(b*x+a)))+24*d^3/b^4*c*polylog
(3,exp(I*(b*x+a)))-8*d/b^2*c^3*arctanh(exp(I*(b*x+a)))+24*I*d^4*polylog(4,e
xp(I*(b*x+a)))/b^5-12*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)*a-12*d^3/b^2*c*ln(ex
p(I*(b*x+a))+1)*x^2+12*d^3/b^4*c*ln(exp(I*(b*x+a))+1)*a^2+12*d^3/b^2*c*ln(1
-exp(I*(b*x+a)))*x^2-12*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a^2+12*I*d^2/b^3*c^2
*polylog(2,-exp(I*(b*x+a)))-12*I*d^2/b^3*c^2*polylog(2,exp(I*(b*x+a)))+12*I
*d^4/b^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I*d^4/b^3*polylog(2,exp(I*(b*x+a
)))*x^2-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5-2*I*(d^4*x^4+4*c*d^3*x^3+6*
c^2*d^2*x^2+4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+4*d^4/b^5*
ln(1-exp(I*(b*x+a)))*a^3-4*d^4/b^2*ln(exp(I*(b*x+a))+1)*x^3-24*d^3/b^4*c*a^
2*arctanh(exp(I*(b*x+a)))+24*d^2/b^3*c^2*a*arctanh(exp(I*(b*x+a)))-4*d^4/b^
5*ln(exp(I*(b*x+a))+1)*a^3+12*d^2/b^2*c^2*ln(1-exp(I*(b*x+a)))*x+12*d^2/b^3
*c^2*ln(1-exp(I*(b*x+a)))*a-12*d^2/b^2*c^2*ln(exp(I*(b*x+a))+1)*x
```

Maxima [B] time = 2.49972, size = 3974, normalized size = 19.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*
a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*c^
3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b)
- 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*
a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*a*
c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b^2) + 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*
x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*
b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
```

(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x +
a))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b^3) - 2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*co
s(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
+ 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(
b*x + a))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*b^4) + c^4/sin(b*x + a) - 4*a*c^3*d/(b*sin(b*x + a)) + 6*a^2*c^2
*d^2/(b^2*sin(b*x + a)) - 4*a^3*c*d^3/(b^3*sin(b*x + a)) + a^4*d^4/(b^4*sin
(b*x + a)) - ((4*(b*x + a)^3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - 4*((b*x + a)^3*d^4 + 3*(b*c
*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x +
a))*cos(2*b*x + 2*a) - (4*I*(b*x + a)^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(
b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a))*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*(b*x + a)^3*
d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3
*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (4*I*(
b*x + a)^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^
2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin
(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b
x + a)^3 + 6(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*cos(b*x +
a) - (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^4 + 12*a^2*d^4 + 24*
(b*c*d^3 - a*d^4)*(b*x + a) - 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d
^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*b^2
*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*(b*x + a)^2*d^4 - 12*I*a^2*d^4 + (-24*I*b*
c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) +
(12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^4 + 12*a^2*d^4 + 24*(b*c*
d^3 - a*d^4)*(b*x + a) - 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 +
a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (12*I*b^2*c^2*d
^2 - 24*I*a*b*c*d^3 + 12*I*(b*x + a)^2*d^4 + 12*I*a^2*d^4 + (24*I*b*c*d^3 -
24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (2*I*(b*
x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 1
2*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a) + (-2*I*(b*x + a)^3*d^4 + (-6*I*b*c*
d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2
d^4)(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4
)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-2*I
*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2*d
^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a) + (2*I*(b*x + a)^3*d^4 + (6*I*
b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*
a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*
d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2

```
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 24
*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog(4, -e^(I*b*x
 + I*a)) + 24*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog
(4, e^(I*b*x + I*a)) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4 + (-
24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a*d^4)*cos(2*b*x + 2*a) + 24*(b*c*
d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a))
 - (-24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a*d^4 + (24*I*b*c*d^3 + 24*I*
(b*x + a)*d^4 - 24*I*a*d^4)*cos(2*b*x + 2*a) - 24*(b*c*d^3 + (b*x + a)*d^4
- a*d^4)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - (2*I*(b*x + a)^4*d
^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c
*d^3 + 12*I*a^2*d^4)*(b*x + a)^2)*sin(b*x + a))/(-I*b^4*cos(2*b*x + 2*a) +
b^4*sin(2*b*x + 2*a) + I*b^4))/b
```

Fricas [C] time = 0.699808, size = 2569, normalized size = 12.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4 - 12*I*d^4*polylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I
*d^4*polylog(4, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*poly
log(4, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*
c*d^3*x - 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - (6*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(cos(b*x +
a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x -
6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*
b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*s
in(b*x + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^
2*x + b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b
^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a
) - I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin
(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3
+ 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3
*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3
 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^
3*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x +
```

```
 b*c*d^3)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^
4*x + b*c*d^3)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*
(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a)
 + 12*(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*
x + a))/(b^5*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \cos(bx+a) \csc(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^2, x)
```

3.40 $\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=146

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}\left(3,e^{i}\right.}{b^4}$$

```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x]
)/b + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*
x))])/b^4 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4
```

Rubi [A] time = 0.115982, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4410, 4183, 2531, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}\left(3,e^{i}\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```


```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x]
)/b + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*
x))])/b^4 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4
```


Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
```

```
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\csc(a+bx)\,dx}{b}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}-\frac{\left(6d^2\right)\int (c+dx)\log\left(\right.}{b^2}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)\mathrm{Li}_2\left(-e^{i(\right.}}{b^3}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)\mathrm{Li}_2\left(-e^{i(\right.}}{b^3}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)\mathrm{Li}_2\left(-e^{i(\right.}}{b^3}$$

Mathematica [B] time = 1.18697, size = 311, normalized size = 2.13

$$-\frac{-6ibd^2(c+dx)\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)+6ibd^2(c+dx)\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)+6d^3\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)-6d^3\mathrm{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((b^3*c^3*Csc[a + b*x] + 3*b^3*c^2*d*x*Csc[a + b*x] + 3*b^3*c*d^2*x^2*Csc[
a + b*x] + b^3*d^3*x^3*Csc[a + b*x] - 3*b^2*c^2*d*Log[1 - E^(I*(a + b*x))]
- 6*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Log[1 - E^(I*(a +
b*x))] + 3*b^2*c^2*d*Log[1 + E^(I*(a + b*x))] + 6*b^2*c*d^2*x*Log[1 + E^(I*
(a + b*x))] + 3*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))] - (6*I)*b*d^2*(c + d*x
)*PolyLog[2, -E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a +
 b*x))] + 6*d^3*PolyLog[3, -E^(I*(a + b*x))] - 6*d^3*PolyLog[3, E^(I*(a + b
*x))])/b^4)
```

Maple [B] time = 0.256, size = 433, normalized size = 3.

$$\frac{-6\,id^3\text{polylog}\left(2,\mathrm{e}^{i(bx+a)}\right)x}{b^3}-6\,\frac{d^3a^2\text{Artanh}\left(\mathrm{e}^{i(bx+a)}\right)}{b^4}-6\,\frac{d^2c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x}{b^2}-6\,\frac{d^2c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)a}{b^3}-6\,\frac{c^2d\text{Artanh}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] -6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-6*d^3/b^4*a^2*arctanh(exp(I*(b*x+a
)))-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-6*d^2/b^3*c*ln(exp(I*(b*x+a))+1)*a-6
*d/b^2*c^2*arctanh(exp(I*(b*x+a)))+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x+6*d^2
/b^3*c*ln(1-exp(I*(b*x+a)))*a-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+6*I*d^2/b^
3*c*polylog(2,-exp(I*(b*x+a)))-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/b^4
*ln(exp(I*(b*x+a))+1)*a^2+12*d^2/b^3*c*a*arctanh(exp(I*(b*x+a)))+6*I*d^3/b^
3*polylog(2,-exp(I*(b*x+a)))*x+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2+6*d^3*pol
ylog(3,exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4-6*I*d^2/b^3
*c*polylog(2,exp(I*(b*x+a)))-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*
(b*x+a))/b/(exp(2*I*(b*x+a))-1)
```

Maxima [B] time = 1.81669, size = 2390, normalized size = 16.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x
+ 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a)
)*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b) - 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x
+ 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a)
)*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) +
1)*b^2) + 3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*
b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x
+ a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b^3) + 2*c^3/sin(b*x + a) - 6*a*c^2*d/(b*sin(b*x + a)) + 6*a^2*c*d^2
/(b^2*sin(b*x + a)) - 2*a^3*d^3/(b^3*sin(b*x + a)) - 2*((6*(b*x + a)^2*d^3
+ 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)
*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*
a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1)
 + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x +
 a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a
)^2)*cos(b*x + a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2
 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a
)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (12*b*c*d^2
 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2
*b*x + 2*a) - (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x +
2*a))*dilog(e^(I*b*x + I*a)) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*
d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (3*I*(b*x
 + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b
*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-12*I*d^3*cos(2*b*x + 2*
a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*polylog(3, -e^(I*b*x + I*a)) - (12
*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, e^
```

```
(I*b*x + I*a)) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x +
a)^2)*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) + 2
*I*b^3))/b
```

Fricas [C] time = 0.605109, size = 1740, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*d^3*p
olylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, co
s(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x
 + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(cos(b*x + a) + I*s
in(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(cos(b*x + a)
- I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(-cos(b*
x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(-
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2
*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1
)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I
*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*
d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^4*s
in(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3\cos(bx+a)\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^2, x)
```

3.41 $\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=90

$$\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/
b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((2*I)*d^2*PolyLog[2, E^
(I*(a + b*x))])/b^3
```

Rubi [A] time = 0.0622306, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4410, 4183, 2279, 2391}

$$\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/
b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((2*I)*d^2*PolyLog[2, E^
(I*(a + b*x))])/b^3
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2\cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{(2d)\int (c+dx)\csc(a+bx)\,dx}{b}$$
$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}-\frac{\left(2d^2\right)\int \log\left(1-e^{i(a+bx}\right.}{b^2}$$
$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{\left(2id^2\right)\mathrm{Subst}\left(\int \frac{\log(1-}{x}\right.}{b^3}$$
$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2id^2\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-$$

Mathematica [B] time = 2.0513, size = 234, normalized size = 2.6

$$4d^2\left(2\tan^{-1}(\tan(a))\tanh^{-1}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right)+\frac{\sec(a)\left(i\mathrm{PolyLog}\left(2,-e^{i\left(\tan^{-1}(\tan(a))+bx\right)}\right)-i\mathrm{PolyLog}\left(2,e^{i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+(\mathrm{ta}\right.}{\sqrt{\sec^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] - 2*b^2*(c + d*x)^2*Csc[a]
+ 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*x +
ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x
+ ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*Poly
Log[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + b^2*(c + d*
x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] - b^2*(c + d*x)^2*Sec[a/2]*Sec[
```

```
(a + b*x)/2]*Sin[(b*x)/2])/(2*b^3)
```

Maple [B] time = 0.205, size = 212, normalized size = 2.4

$$\frac{-2\,i\left(d^2x^2+2\,cdx+c^2\right)\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2\,i(bx+a)}-1\right)}-4\,\frac{cd\mathrm{Artanh}\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}+2\,\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x}{b^2}+2\,\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a}{b^3}-\frac{2\,id^2\mathrm{polylog}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] -2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-4*d/b^2*c*
arctanh(exp(I*(b*x+a)))+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-exp
(I*(b*x+a)))*a-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-2*d^2/b^2*ln(exp(I*(b*
x+a))+1)*x-2*d^2/b^3*ln(exp(I*(b*x+a))+1)*a+2*I*d^2*polylog(2,-exp(I*(b*x+a
)))/b^3+4*d^2/b^3*a*arctanh(exp(I*(b*x+a)))
```

Maxima [B] time = 1.63384, size = 751, normalized size = 8.34

$$\frac{\left(2\,bd^2x+2\,bcd-2\left(bd^2x+bcd\right)\cos\left(2\,bx+2\,a\right)-\left(2i\,bd^2x+2i\,bcd\right)\sin\left(2\,bx+2\,a\right)\right)\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] ((2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (2*I*b*d^2*x
 + 2*I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (
2*b*c*d*cos(2*b*x + 2*a) + 2*I*b*c*d*sin(2*b*x + 2*a) - 2*b*c*d)*arctan2(si
n(b*x + a), cos(b*x + a) - 1) - (2*b*d^2*x*cos(2*b*x + 2*a) + 2*I*b*d^2*x*s
in(2*b*x + 2*a) - 2*b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + 2*(d^2*cos(2*b*x + 2*a)
 + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 2*(d^2*cos(2*b*x
 + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x
 + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)*si
n(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
 (-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*a) - (b*d^2*x
+ b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x
+ a) + 1) - (2*I*b^2*d^2*x^2 + 4*I*b^2*c*d*x + 2*I*b^2*c^2)*sin(b*x + a))/(
```

```
-I*b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)
```

Fricas [B] time = 0.558857, size = 1006, normalized size = 11.18

$$-\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\mathrm{Li}_2\left(\cos\left(bx+a\right) + i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right) - i\,d^2\mathrm{Li}_2\left(\cos\left(bx+a\right) - i\,\sin\left(bx+a\right)\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*sin(b*
x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*dilo
g(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(
b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x
 + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*c
os(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(
-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^3*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \cos\left(bx+a\right)\csc\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a)^2, x)
```

3.42 $\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=30

$$-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}$$

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - ((c + d*x)*Csc[a + b*x])/b
```

Rubi [A] time = 0.0195758, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4410, 3770}

$$-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - ((c + d*x)*Csc[a + b*x])/b
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx &= -\frac{(c+dx)\csc(a+bx)}{b}+\frac{d\int\csc(a+bx)\,dx}{b}\\ &= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.0584151, size = 131, normalized size = 4.37

$$\frac{d\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{c\csc(a+bx)}{b}-\frac{dx\csc(a)}{b}+\frac{dx\csc\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)}{2b}-\frac{dx\sec\left(\frac{a}{2}\right)s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((d*x*Csc[a])/b) - (c*Csc[a + b*x])/b - (d*Log[Cos[a/2 + (b*x)/2]])/b^2 +
(d*Log[Sin[a/2 + (b*x)/2]])/b^2 + (d*x*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x
)/2])/(2*b) - (d*x*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b)
```

Maple [A] time = 0.024, size = 52, normalized size = 1.7

$$-\frac{dx}{b\sin(bx+a)}+\frac{d\ln(\csc(bx+a)-\cot(bx+a))}{b^2}-\frac{c}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] -1/b*d/sin(b*x+a)*x+1/b^2*d*ln(csc(b*x+a)-cot(b*x+a))-1/b*c/sin(b*x+a)
```

Maxima [B] time = 1.09171, size = 350, normalized size = 11.67

$$-\frac{\left(4(bx+a)\cos(bx+a)\sin(2bx+2a)-4(bx+a)\cos(2bx+2a)\sin(bx+a)+\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos\right.}{\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x +
2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
+ 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*
```

```
d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) +
2*c/sin(b*x + a) - 2*a*d/(b*sin(b*x + a)))/b
```

Fricas [B] time = 0.488275, size = 181, normalized size = 6.03

$$-\frac{2\,bdx + d\log\left(\frac{1}{2}\,\cos\left(bx + a\right) + \frac{1}{2}\right)\sin\left(bx + a\right) - d\log\left(-\frac{1}{2}\,\cos\left(bx + a\right) + \frac{1}{2}\right)\sin\left(bx + a\right) + 2\,bc}{2\,b^{2}\sin\left(bx + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos
(b*x + a) + 1/2)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c + dx\right)\cos\left(a + bx\right)\csc^{2}\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**2, x)
```

Giac [B] time = 1.52748, size = 1081, normalized size = 36.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 +
b*d*x*tan(1/2*b*x)^2 - d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a
)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b
```

```
*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))*tan(1/2*b*x)^2*tan(1/2*a) + d*log(4
*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2
*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2
*a)^2))*tan(1/2*b*x)^2*tan(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(4*(tan(1/2*a
)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1
/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))*t
an(1/2*b*x)*tan(1/2*a)^2 + d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*t
an(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2
*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*t
an(1/2*b*x)^2 + b*c*tan(1/2*a)^2 + b*d*x + d*log(4*(tan(1/2*a)^2 + 1)/(tan(
1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(
1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))*tan(1/2*b*x) -
d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) +
tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) +
tan(1/2*a)^2))*tan(1/2*b*x) + d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*ta
n(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + ta
n(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))*tan(1/2*a) - d*log(4*(tan(1/
2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*
tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*
tan(1/2*a) + b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2
*a)^2 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

3.43 $\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

Rubi [A] time = 0.114518, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 16.7603, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

Maple [A] time = 0.295, size = 0, normalized size = 0.

$$\int \frac{\cos (bx + a) (\csc (bx + a))^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos (bx + a) \csc (bx + a)^2}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (a + bx) \csc^2 (a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

3.44 $\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi [A] time = 0.152228, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 20.204, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.411, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)(\csc(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(bx+a\right)\csc\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c)^2, x)
```

3.45 $\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\cot(a+bx)\csc^2(a+bx)(c+dx)^m, x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

Rubi [A] time = 0.21658, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

Mathematica [A] time = 3.29169, size = 0, normalized size = 0.

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a) (\csc(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a) \csc(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \cos(bx+a) \csc(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

3.46 $\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\,dx$

Optimal. Leaf size=137

$$-\frac{6id^3(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4}+\frac{3d^4\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^5}+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\cot(a+b}{b^2}$$

```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (2*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x
)^4*Csc[a + b*x]^2)/(2*b) + (6*d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*
PolyLog[3, E^((2*I)*(a + b*x))])/b^5
```

Rubi [A] time = 0.256067, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4410, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{6id^3(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4}+\frac{3d^4\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^5}+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\cot(a+b}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```


```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (2*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x
)^4*Csc[a + b*x]^2)/(2*b) + (6*d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*
PolyLog[3, E^((2*I)*(a + b*x))])/b^5
```


Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4\cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^4\csc^2(a+bx)}{2b}+\frac{(2d)\int (c+dx)^3\csc^2(a+bx)\,dx}{b}$$

$$= -\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+\frac{\left(6d^2\right)\int (c+dx)^2\cot(a+}{b^2}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}-\frac{\left(12id^2\right)\int}{}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+\frac{6d^2(c+dx}{}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+\frac{6d^2(c+dx}{}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+\frac{6d^2(c+dx}{}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+\frac{6d^2(c+dx}{}$$

Mathematica [B] time = 6.60207, size = 504, normalized size = 3.68

$$-\frac{6cd^3\csc(a)\sec(a)\left(\frac{\tan(a)\left(i\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+2\tan^{-1}(\tan}{\sqrt{\tan^2(a)+1}}\right.}{b^4\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -((c + d*x)^4*Csc[a + b*x]^2)/(2*b) - (d^4*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((
2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3
*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2
*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b
*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*
x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^5 + (6*c^2*d^2*Cs
c[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*
(Cos[a]^2 + Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2
*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 - (6*c*d^3*Csc[
a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]])
- Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b
```

```
*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x +
ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/
Sqrt[1 + Tan[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.188, size = 716, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] 6*d^4/b^5*a^2*ln(exp(I*(b*x+a))-1)-12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))-12*d^2
/b^3*c^2*ln(exp(I*(b*x+a)))+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)+6*d^2/b^3*c^
2*ln(exp(I*(b*x+a))-1)+6*d^4/b^3*ln(1-exp(I*(b*x+a)))*x^2-6*d^4/b^5*ln(1-ex
p(I*(b*x+a)))*a^2+6*d^4/b^3*ln(exp(I*(b*x+a))+1)*x^2-4*I*d^4/b^2*x^3+8*I*d^
4/b^5*a^3+12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(3,exp(I*(b*x
+a)))/b^5+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*
c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2
*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d
^2*x*exp(2*I*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))+6*I*c*d^3*x^2+
6*I*c^2*d^2*x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2+12*d^3/b^3*c*ln(1-exp(I
*(b*x+a)))*x+12*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a+12*d^3/b^3*c*ln(exp(I*(b*x
+a))+1)*x-12*I*d^4/b^4*polylog(2,exp(I*(b*x+a)))*x-12*I*d^4/b^4*polylog(2,-
exp(I*(b*x+a)))*x-12*I*d^3/b^4*c*polylog(2,exp(I*(b*x+a)))-12*I*d^3/b^4*c*p
olylog(2,-exp(I*(b*x+a)))+12*I*d^4/b^4*a^2*x-12*I*d^3/b^4*c*a^2-12*I*d^3/b^
2*c*x^2-12*d^3/b^4*c*a*ln(exp(I*(b*x+a))-1)+24*d^3/b^4*c*a*ln(exp(I*(b*x+a)
))-24*I*d^3/b^3*c*a*x
```

Maxima [B] time = 2.47741, size = 6129, normalized size = 44.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
```

```
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) - 1)*
cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) - 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*si
n(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*
b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) -
 cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*c^2*d^2/((2*(
2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x
 + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*si
n(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + 24*(4*(b*x + a)*cos(2*b*x
 + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a)
+ sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b
*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b
*x + 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*
b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*
a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^3) -
 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b
*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)
*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*s
in(4*b*x + 4*a) + sin(2*b*x + 2*a))*a^3*d^4/((2*(2*cos(2*b*x + 2*a) - 1)*co
s(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*
a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2
*b*x + 2*a) - 1)*b^4) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2
*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2
*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*
x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2
- sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x +
2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x
+ 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*s
in(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x
+ 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x +
a)*sin(2*b*x + 2*a))*c^2*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a)
- cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1
)*b^2) - 12*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2
*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) +
(b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*
cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1
) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*c
os(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
```

) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin (b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin (2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^3) + 6*(8* (b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2 *b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2 *b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a) ^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2 *cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/ ((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^4) - c^4/sin(b*x + a)^2 + 4*a*c^3*d/(b*sin(b*x + a)^2) - 6*a^2*c^2*d^2/(b^2*sin(b*x + a)^2) + 4*a^3* c*d^3/(b^3*sin(b*x + a)^2) - a^4*d^4/(b^4*sin(b*x + a)^2) + 2*((6*(b*x + a) ^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 12*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a* d^4)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*d^3 - 1 2*I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*(b*x + a)^2*d^4 + (-24*I*b* c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*(b*x + a)^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b *x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 12*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (12* I*(b*x + a)^2*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^ 3 - a*d^4)*(b*x + a)^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^4*d^4 + (-8*I*b* c*d^3 - 4*(-2*I*a - 1)*d^4)*(b*x + a)^3 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2) *cos(2*b*x + 2*a) - (12*b*c*d^3 + 12*(b*x + a)*d^4 - 12*a*d^4 + 12*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-12*I*b*c*d^3 - 12*I*(b*x + a)*d^4 + 12*I*a*d^4) *sin(4*b*x + 4*a) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4)*sin(2* b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (12*b*c*d^3 + 12*(b*x + a)*d^4 - 12*a *d^4 + 12*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-12*I*b*c*d^3 - 12*I*(b*x + a) *d^4 + 12*I*a*d^4)*sin(4*b*x + 4*a) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-3*I*(b*x + a)^2*d^ 4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*

```
c*d^3 + 6*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^4 + (12
*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 +
 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^2*d^4 + 2*(
b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*
a*d^4)*(b*x + a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x
+ a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*
x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x +
a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
+ 1) + (-12*I*d^4*cos(4*b*x + 4*a) + 24*I*d^4*cos(2*b*x + 2*a) + 12*d^4*sin
(4*b*x + 4*a) - 24*d^4*sin(2*b*x + 2*a) - 12*I*d^4)*polylog(3, -e^(I*b*x +
I*a)) + (-12*I*d^4*cos(4*b*x + 4*a) + 24*I*d^4*cos(2*b*x + 2*a) + 12*d^4*si
n(4*b*x + 4*a) - 24*d^4*sin(2*b*x + 2*a) - 12*I*d^4)*polylog(3, e^(I*b*x +
I*a)) + (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*s
in(4*b*x + 4*a) + (2*(b*x + a)^4*d^4 + (8*b*c*d^3 - (8*a - 4*I)*d^4)*(b*x +
 a)^3 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-I*b^4*
cos(4*b*x + 4*a) + 2*I*b^4*cos(2*b*x + 2*a) + b^4*sin(4*b*x + 4*a) - 2*b^4*
sin(2*b*x + 2*a) - I*b^4))/b
```

Fricas [C] time = 0.694654, size = 2562, normalized size = 18.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*cos
(b*x + a)*sin(b*x + a) + (12*I*b*d^4*x + 12*I*b*c*d^3 + (-12*I*b*d^4*x - 12
*I*b*c*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (-12*I*b
*d^4*x - 12*I*b*c*d^3 + (12*I*b*d^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2)*dilog
(cos(b*x + a) - I*sin(b*x + a)) + (-12*I*b*d^4*x - 12*I*b*c*d^3 + (12*I*b*d
^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))
+ (12*I*b*d^4*x + 12*I*b*c*d^3 + (-12*I*b*d^4*x - 12*I*b*c*d^3)*cos(b*x + a
)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x
 + b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2
)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
 b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2)*
log(cos(b*x + a) - I*sin(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b
```

```
*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^
4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c
*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(
b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^
2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*
d^3 - a^2*d^4)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 12
*(d^4*cos(b*x + a)^2 - d^4)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 12*
(d^4*cos(b*x + a)^2 - d^4)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(
d^4*cos(b*x + a)^2 - d^4)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 12*(
d^4*cos(b*x + a)^2 - d^4)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^5*
cos(b*x + a)^2 - b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \cos(bx+a)\csc(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^3, x)
```

3.47 $\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx) \, dx$

Optimal. Leaf size=115

$$-\frac{3id^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}-\frac{3id(c+\ldots}{2b\ldots}$$

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - (
(c + d*x)^3*Csc[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a +
b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4
```

Rubi [A] time = 0.173609, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4410, 4184, 3717, 2190, 2279, 2391}

$$-\frac{3id^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}-\frac{3id(c+\ldots}{2b\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - (
(c + d*x)^3*Csc[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a +
b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^3 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{(3d)\int (c+dx)^2\csc^2(a+bx)\,dx}{2b}$$

$$= -\frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{\left(3d^2\right)\int (c+dx)\cot(a+ \cdots}{b^2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} - \frac{\left(6id^2\right)\int \cdots}{\cdots}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{3d^2(c+ \cdots}{\cdots}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{3d^2(c+ \cdots}{\cdots}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{3d^2(c+ \cdots}{\cdots}$$

Mathematica [B] time = 6.41234, size = 277, normalized size = 2.41

$$-\frac{3d^3\csc(a)\sec(a)\left(\frac{\tan(a)\left(i\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+2\tan^{-1}(\tan(\right.}{\sqrt{\tan^2(a)+1}}\right.}{2b^4\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -((c + d*x)^3*Csc[a + b*x]^2)/(2*b) + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[
Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (
3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*
x]))/(2*b^2) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x
*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Ta
n[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*Arc
Tan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + A
rcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]
^2 + Sin[a]^2)])
```

Maple [B] time = 0.164, size = 409, normalized size = 3.6

$$\frac{2bd^3x^3e^{2i(bx+a)}-3id^3x^2e^{2i(bx+a)}+6bcd^2x^2e^{2i(bx+a)}-6icd^2xe^{2i(bx+a)}+6bc^2dxe^{2i(bx+a)}-3ic^2de^{2i(bx+a)}+3id^3x^2+2bc^3}{b^2\left(e^{2i(bx+a)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] (2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*ex
p(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*
I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3
*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2+3*d^2/b^3*c*ln(exp(I*(b*x+a))-1)-6*d^2
/b^3*c*ln(exp(I*(b*x+a)))+3*d^2/b^3*c*ln(exp(I*(b*x+a))+1)-3*I*d^3/b^2*x^2-
6*I*d^3/b^3*a*x-3*I*d^3/b^4*a^2+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*
ln(1-exp(I*(b*x+a)))*a-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+3*d^3/b^3*ln(e
xp(I*(b*x+a))+1)*x-3*I*d^3/b^4*polylog(2,-exp(I*(b*x+a)))-3*d^3/b^4*a*ln(ex
p(I*(b*x+a))-1)+6*d^3/b^4*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 2.27893, size = 1411, normalized size = 12.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (6*b^2*c^2*d + (6*b*d^3*x + 6*b*c*d^2 + 6*(b*d^3*x + b*c*d^2)*cos(4*b*x + 4
*a) - 12*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)
*sin(4*b*x + 4*a) + (-12*I*b*d^3*x - 12*I*b*c*d^2)*sin(2*b*x + 2*a))*arctan
2(sin(b*x + a), cos(b*x + a) + 1) + (6*b*c*d^2*cos(4*b*x + 4*a) - 12*b*c*d^
2*cos(2*b*x + 2*a) + 6*I*b*c*d^2*sin(4*b*x + 4*a) - 12*I*b*c*d^2*sin(2*b*x
+ 2*a) + 6*b*c*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (6*b*d^3*x*co
s(4*b*x + 4*a) - 12*b*d^3*x*cos(2*b*x + 2*a) + 6*I*b*d^3*x*sin(4*b*x + 4*a)
 - 12*I*b*d^3*x*sin(2*b*x + 2*a) + 6*b*d^3*x)*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)*cos(4*b*x + 4*a) + (-4*I*b^3*
d^3*x^3 - 4*I*b^3*c^3 - 6*b^2*c^2*d + (-12*I*b^3*c*d^2 + 6*b^2*d^3)*x^2 - 1
2*(I*b^3*c^2*d - b^2*c*d^2)*x)*cos(2*b*x + 2*a) - (6*d^3*cos(4*b*x + 4*a) -
 12*d^3*cos(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) - 12*I*d^3*sin(2*b*x +
2*a) + 6*d^3)*dilog(-e^(I*b*x + I*a)) - (6*d^3*cos(4*b*x + 4*a) - 12*d^3*co
s(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) - 12*I*d^3*sin(2*b*x + 2*a) + 6*d
^3)*dilog(e^(I*b*x + I*a)) + (-3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x -
3*I*b*c*d^2)*cos(4*b*x + 4*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(2*b*x + 2*a
) + 3*(b*d^3*x + b*c*d^2)*sin(4*b*x + 4*a) - 6*(b*d^3*x + b*c*d^2)*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (-3*I
*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x - 3*I*b*c*d^2)*cos(4*b*x + 4*a) + (6
*I*b*d^3*x + 6*I*b*c*d^2)*cos(2*b*x + 2*a) + 3*(b*d^3*x + b*c*d^2)*sin(4*b*
x + 4*a) - 6*(b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x)*s
in(4*b*x + 4*a) + (4*b^3*d^3*x^3 + 4*b^3*c^3 - 6*I*b^2*c^2*d + 6*(2*b^3*c*d
^2 + I*b^2*d^3)*x^2 + (12*b^3*c^2*d + 12*I*b^2*c*d^2)*x)*sin(2*b*x + 2*a))/
(-2*I*b^4*cos(4*b*x + 4*a) + 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin(4*b*x + 4
*a) - 4*b^4*sin(2*b*x + 2*a) - 2*I*b^4)
```

Fricas [B] time = 0.609607, size = 1445, normalized size = 12.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + (-3*I*d^3*cos(b
*x + a)^2 + 3*I*d^3)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*d^3*cos(b*
x + a)^2 - 3*I*d^3)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (3*I*d^3*cos(b*x
 + a)^2 - 3*I*d^3)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*d^3*cos(b*
x + a)^2 + 3*I*d^3)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d^3*x + b*
c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x +
a) + 1) - 3*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(co
s(b*x + a) - I*sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3 - (b*c*d^2 - a*d^3)*c
os(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*(b*c*d
^2 - a*d^3 - (b*c*d^2 - a*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*
I*sin(b*x + a) + 1/2) - 3*(b*d^3*x + a*d^3 - (b*d^3*x + a*d^3)*cos(b*x + a)
^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3 - (b*d^3*x
 + a*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/(b^4*cos
(b*x + a)^2 - b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cos(bx + a) \csc(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^3, x)
```

3.48 $\int (c+dx)^2 \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=54

$$-\frac{d(c+dx)\cot(a+bx)}{b^2}+\frac{d^2\log(\sin(a+bx))}{b^3}-\frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

```
[Out] -((d*(c + d*x)*Cot[a + b*x])/b^2) - ((c + d*x)^2*Csc[a + b*x]^2)/(2*b) + (d
^2*Log[Sin[a + b*x]])/b^3
```

Rubi [A] time = 0.0653742, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4410, 4184, 3475}

$$-\frac{d(c+dx)\cot(a+bx)}{b^2}+\frac{d^2\log(\sin(a+bx))}{b^3}-\frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(c + d*x)*Cot[a + b*x])/b^2) - ((c + d*x)^2*Csc[a + b*x]^2)/(2*b) + (d
^2*Log[Sin[a + b*x]])/b^3
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d\int (c+dx)\csc^2(a+bx)\,dx}{b}$$
$$= -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d^2\int \cot(a+bx)\,dx}{b^2}$$
$$= -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d^2\log(\sin(a+bx))}{b^3}$$

Mathematica [C] time = 0.918487, size = 94, normalized size = 1.74

$$\frac{-b^2(c+dx)^2\csc^2(a+bx) + 2bd\csc(a)\sin(bx)(c+dx)\csc(a+bx) - 2id^2\tan^{-1}(\tan(a+bx)) - 2bd^2x\cot(a) + d^2\log\left(\sin\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] ((2*I)*b*d^2*x - (2*I)*d^2*ArcTan[Tan[a + b*x]] - 2*b*d^2*x*Cot[a] - b^2*(c
 + d*x)^2*Csc[a + b*x]^2 + d^2*Log[Sin[a + b*x]^2] + 2*b*d*(c + d*x)*Csc[a]
*Csc[a + b*x]*Sin[b*x])/(2*b^3)
```

Maple [A] time = 0.029, size = 95, normalized size = 1.8

$$-\frac{d^2x^2}{2b(\sin(bx+a))^2} - \frac{d^2\cot(bx+a)x}{b^2} + \frac{d^2\ln(\sin(bx+a))}{b^3} - \frac{cdx}{b(\sin(bx+a))^2} - \frac{cd\cot(bx+a)}{b^2} - \frac{c^2}{2b(\sin(bx+a))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] -1/2/b*d^2/sin(b*x+a)^2*x^2-1/b^2*d^2*cot(b*x+a)*x+d^2*ln(sin(b*x+a))/b^3-1
/b*c*d/sin(b*x+a)^2*x-1/b^2*c*d*cot(b*x+a)-1/2/b*c^2/sin(b*x+a)^2
```

Maxima [B] time = 1.3463, size = 1526, normalized size = 28.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) - 1)*co
s(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*
a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2
*b*x + 2*a) - 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2
*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x
 + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*d^2/((2*(2*cos(2
*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)
^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a)^
2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*(
(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*co
s(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(
b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x +
4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a
) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x
+ a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x +
 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(
4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)
^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b
*x + 2*a) - 1)*b^2) - c^2/sin(b*x + a)^2 + 2*a*c*d/(b*sin(b*x + a)^2) - a^2
*d^2/(b^2*sin(b*x + a)^2))/b
```

Fricas [A] time = 0.504084, size = 231, normalized size = 4.28

$$\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\left(bd^2x + bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right) + 2\left(d^2\cos\left(bx+a\right)^2 - d^2\right)\log\left(\frac{1}{2}\,\sin\left(bx+a\right)\right)}{2\left(b^3\cos\left(bx+a\right)^2 - b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)
*sin(b*x + a) + 2*(d^2*cos(b*x + a)^2 - d^2)*log(1/2*sin(b*x + a)))/(b^3*co
s(b*x + a)^2 - b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.5771, size = 4701, normalized size = 87.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*ta
n(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c*d
*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*
b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)
^4 + b^2*d^2*x^2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 2*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a
)^3 + b^2*d^2*x^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*
b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 + 4*b*d^2*x*
tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*
d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 4*d^2*log(16*(tan(1/2*a)^4 + 2*tan(1/2*
a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + ta
n(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*
tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^
4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2
*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan
(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 -
```

2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^4*tan(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 8*d^2*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 2*b^2*c*d*x*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^4 - 4*d^2*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^2 + b^2*c^2*tan(1/2*b*x)^4 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^2*d^2*x^2*tan(1/2*a)^2 + 4*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 24*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + b^2*c^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*b*x)*tan(1/2*a)^4 + 4*b^2*c*d*x*tan(1/2*b*x)^2 - 4*b*d^2*x*tan(1/2*b*x)^3 - 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a) + 8*d^2*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^3*tan(1/2*a) + 4*b^2*c*d*x*tan(1/2*a)^2 - 24*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 + 16*d^2*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*a)^3 + 8*d^2*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(

```
1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^
5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^
3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)*tan(1/2
*a)^3 + b^2*d^2*x^2 + 2*b^2*c^2*tan(1/2*b*x)^2 - 4*b*c*d*tan(1/2*b*x)^3 - 2
4*b*c*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c^2*tan(1/2*a)^2 - 24*b*c*d*tan(1
/2*b*x)*tan(1/2*a)^2 - 4*b*c*d*tan(1/2*a)^3 + 2*b^2*c*d*x + 4*b*d^2*x*tan(1
/2*b*x) - 4*d^2*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*
tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4
- 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b
*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan
(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*t
an(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*
x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)
^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*
x)^2 + 4*b*d^2*x*tan(1/2*a) - 8*d^2*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 +
 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*
b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + ta
n(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2
- 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b
*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan
(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1
/2*a)^2))*tan(1/2*b*x)*tan(1/2*a) - 4*d^2*log(16*(tan(1/2*a)^4 + 2*tan(1/2*
a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + ta
n(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*
tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^
4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2
*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan
(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 -
 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) +
 tan(1/2*a)^2))*tan(1/2*a)^2 + b^2*c^2 + 4*b*c*d*tan(1/2*b*x) + 4*b*c*d*tan
(1/2*a))/(b^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*tan(1/2*b*x)^3*tan(1/2*a)
^3 + b^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^3*tan(1/2*b*x)^3*tan(1/2*a) - 4*
b^3*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^3*tan(1/2*b*x)*tan(1/2*a)^3 + b^3*tan
(1/2*b*x)^2 + 2*b^3*tan(1/2*b*x)*tan(1/2*a) + b^3*tan(1/2*a)^2)
```

3.49 $\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=35

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

```
[Out] -(d*Cot[a + b*x])/(2*b^2) - ((c + d*x)*Csc[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.0314137, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4410, 3767, 8}

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -(d*Cot[a + b*x])/(2*b^2) - ((c + d*x)*Csc[a + b*x]^2)/(2*b)
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)\csc^2(a+bx)}{2b}+\frac{d\int \csc^2(a+bx)\,dx}{2b}$$
$$= -\frac{(c+dx)\csc^2(a+bx)}{2b}-\frac{d\,\text{Subst}(\int 1\,dx,x,\cot(a+bx))}{2b^2}$$
$$= -\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

Mathematica [A] time = 0.0738112, size = 48, normalized size = 1.37

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{c\csc^2(a+bx)}{2b}-\frac{dx\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -(d*Cot[a + b*x])/(2*b^2) - (c*Csc[a + b*x]^2)/(2*b) - (d*x*Csc[a + b*x]^2)
/(2*b)
```

Maple [A] time = 0.029, size = 61, normalized size = 1.7

$$\frac{1}{b}\left(\frac{d}{b}\left(-\frac{bx+a}{2\left(\sin(bx+a)\right)^2}-\frac{\cot(bx+a)}{2}\right)+\frac{ad}{2b\left(\sin(bx+a)\right)^2}-\frac{c}{2\left(\sin(bx+a)\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] 1/b*(d/b*(-1/2*(b*x+a)/sin(b*x+a)^2-1/2*cot(b*x+a))+1/2/b*d*a/sin(b*x+a)^2-
1/2*c/sin(b*x+a)^2)
```

Maxima [B] time = 1.16603, size = 387, normalized size = 11.06

$$\frac{\frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2+4\,(bx+a)\sin(2\,bx+2\,a)^2-(2\,(bx+a)\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a))\cos(4\,bx+4\,a)-2\,(bx+a)\cos(2\,bx+2\,a)-(2\,(bx+a)\sin(2\,bx+2\,a)-\cos(2}{\left(2\,(2\,\cos(2\,bx+2\,a)-1)\cos(4\,bx+4\,a)-\cos(4\,bx+4\,a)^2-4\,\cos(2\,bx+2\,a)^2-\sin(4\,bx+4\,a)^2+4\,\sin(4\,bx+4\,a)\sin(2\,bx+2\,a)-4\,\sin(2\,bx+2\,a)^2+4}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*d/((2*(2*cos(2*b*x + 2*a) - 1)*cos(
4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)
^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b
*x + 2*a) - 1)*b) - c/sin(b*x + a)^2 + a*d/(b*sin(b*x + a)^2))/b
```

Fricas [A] time = 0.456576, size = 103, normalized size = 2.94

$$\frac{bdx + d\cos(bx + a)\sin(bx + a) + bc}{2\left(b^2\cos(bx + a)^2 - b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20892, size = 710, normalized size = 20.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4
+ 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*ta
n(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b
*c*tan(1/2*b*x)^4 + 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^2*tan(1/2
*a)^3 + b*c*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*
b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2*b*x)^3 -
 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)*ta
n(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + b*c + 2*d*tan(1/2*b*x) + 2*d*tan(1/
2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3
+ b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*b^2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/
2*b*x)^2 + 2*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2*tan(1/2*a)^2)
```

3.50 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

Rubi [A] time = 0.136579, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 11.5526, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

Maple [A] time = 0.445, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)(\csc(bx+a))^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\csc^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

3.51 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} \, dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [A] time = 0.166483, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} \, dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} \, dx$$

Mathematica [A] time = 11.1508, size = 0, normalized size = 0.

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A] time = 0.595, size = 0, normalized size = 0.

$$\int \frac{\cos \left(bx+a\right) \left(\csc \left(bx+a\right)\right)^{3}}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos \left(bx+a\right) \csc \left(bx+a\right)^{3}}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos \left(a+bx\right) \csc^{3}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\csc(bx+a)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)^3/(d*x + c)^2, x)
```

3.52 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=196

$$-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rubi [A] time = 0.447523, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx &= \int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)\,dx \\
&= \frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx \\
&= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{(5d)\int (c+dx)^{3/2}\cos(2a+2bx)\,dx}{8b} \\
&= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2} - \frac{(15d^2)\int \sqrt{c+dx}}{32} \\
&= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2}{16b^2} \\
&= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2}{16b^2} \\
&= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2}{16b^2} \\
&= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a\right.}{128}
\end{aligned}
$$

<u>Mathematica</u> [A] time = 2.36958, size = 179, normalized size = 0.91

$$
\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(20bd(c+dx)\sin(2(a+bx)) - \cos(2(a+bx))\left(16b^2(c+dx)^2 - 15d^2\right)\right) - 15\sqrt{\pi}d^2\cos\left(2a - \frac{2bc}{d}\right)\text{FresnelC}}{128b^3\sqrt{\frac{b}{d}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-((-15*d^2 + 16*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)]) + 20*b*d*(c + d*x)*Sin[2*(a + b*x)]))/(128*b^3*S
qrt[b/d])
```

Maple [A] time = 0.037, size = 234, normalized size = 1.2

$$2\frac{1}{d}\left(-1/8\frac{d(dx+c)^{5/2}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+5/8\frac{d}{b}\left(1/4\frac{d(dx+c)^{3/2}}{b}\sin\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)-3/4\frac{d}{b}\left(-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b
*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)
^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d
-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 1.95552, size = 910, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*sin(2
*((d*x + c)*b - b*c + a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*sqrt(ab
s(b)/abs(d)) - 15*sqrt(2)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d)))*cos(2*((d*
x + c)*b - b*c + a*d)/d) - ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*cos(-2*(b*c - a*d)/d) - (15*I*sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 1
5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))
*d^3*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((15*sqrt(pi
)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(p
i)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sq
rt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d
```

```
^3*cos(-2*(b*c - a*d)/d) - (-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(-2*I*b/d)))/(b^3*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 0.550609, size = 540, normalized size = 2.76

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\left(16\,b^3d^2x^2+32\,b^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 -
 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.31783, size = 1327, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(16*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(
d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*
d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c^2 + d^2*((I*sqrt(pi)*(-
16*I*b^2*c^2*d + 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*
b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sqrt
(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c
- 2*I*a*d)/d)/b^3)/d^2 + (I*sqrt(pi)*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d
^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b
*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*
x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*
c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x
 + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) + 8*c*(I*s
qrt(pi)*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) + I*sqrt(pi)*(4*I*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*
d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2 -
2*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^
2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2))/d
```

3.53 $\int (c+dx)^{3/2} \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=168

$$-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2}-\frac{(c}{}$$

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(4*b) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```

Rubi [A] time = 0.29454, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2}-\frac{(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```


```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(4*b) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{(3d)\int \sqrt{c+dx}\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\right)\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,d}{32b^2}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\cos\left(2a-\frac{2bc}{d}\right)\right.}{32b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2} - \frac{\left(3d\cos\left(2a-\frac{2bc}{d}\right)\right)}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}}{}$$

Mathematica [A] time = 0.886386, size = 157, normalized size = 0.93

$$\frac{-3\sqrt{\pi}d\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 3\sqrt{\pi}d\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\sqrt{c+dx}(4b(c+dx)\cos(2(a+}{32d^2\left(\frac{b}{d}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sq
rt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2
*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)]
 - 3*d*Sin[2*(a + b*x)]))/(32*(b/d)^(5/2)*d^2)
```

Maple [A] time = 0.025, size = 187, normalized size = 1.1

$$2\frac{1}{d}\left(-1/8\frac{d(dx+c)^{3/2}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+3/8\frac{d}{b}\left(1/4\frac{d\sqrt{dx+c}}{b}\sin\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)-1/8\frac{d\sqrt{\pi}}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b
*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1
/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+
sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 1.88766, size = 869, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*cos(2*((
d*x + c)*b - b*c + a*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d
))*sin(2*((d*x + c)*b - b*c + a*d)/d) - ((-3*I*sqrt(pi)*cos(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-2*(b*c - a*d)/d) -
 (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d^2*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((3*I*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*
I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))
*d^2*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(-2*I*b/d)))/(b^2*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 0.531847, size = 413, normalized size = 2.46

$$-\frac{3\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^{2}\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(2\,b^{2}dx+3\,bd\cos\left(b\right.\right.}{32\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x + c))
/b^3
```

Sympy [B] time = 130.886, size = 665, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -5*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sq
rt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1/4)/(32*d*gamma(9/4)) + sqrt(pi)*sqr
t(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)
*d*sqrt(b/d)))/(2*d) - 21*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d
)*fresnels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(32*d*gam
ma(11/4)) + sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d)*fresnels(2*b
*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) - 15*sqrt(pi)*d*sqrt(d/b)*sin(
2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1
/4)/(512*b**2*gamma(9/4)) - 63*sqrt(pi)*d*sqrt(d/b)*cos(2*a - 2*b*c/d)*fres
nels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(512*b**2*gamma
(11/4)) + 5*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*sin(2*b*c/d + 2*b
*x)*gamma(1/4)/(64*sqrt(b)*sqrt(d)*gamma(9/4)) - 21*sqrt(d/b)*(c + d*x)**(3
/2)*cos(2*a - 2*b*c/d)*cos(2*b*c/d + 2*b*x)*gamma(3/4)/(64*sqrt(b)*sqrt(d)*
gamma(11/4)) + 15*sqrt(d)*sqrt(d/b)*sqrt(c + d*x)*sin(2*a - 2*b*c/d)*cos(2*
b*c/d + 2*b*x)*gamma(1/4)/(256*b**(3/2)*gamma(9/4)) + 63*sqrt(d)*sqrt(d/b)*
sqrt(c + d*x)*sin(2*b*c/d + 2*b*x)*cos(2*a - 2*b*c/d)*gamma(3/4)/(256*b**(3
/2)*gamma(11/4))
```

Giac [C] time = 1.22975, size = 732, normalized size = 4.36

$$-\frac{4\left(\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2i\,bc+2i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{2\sqrt{dx+c}de^{\left(\frac{2i\,(dx+c)b-2i\,bc+2i\,ad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(\frac{-2i\,(dx}{}\right.}}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(4*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + s
qrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*
x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*
e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c + I*sqrt(pi)*(4*I*b*c*d -
 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I
*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(pi)*(
4*I*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2)
- 2*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*
d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2 - 2*I*(4*I*(d*x + c)^(
3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c
)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)/d
```

3.54 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(4*b) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```

Rubi [A] time = 0.230695, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```


```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(4*b) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}\sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{8b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx}{8b} - \frac{\left(d\sin\left(2a-\right.\right.}{}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Subst}\left(\int\cos\left(\frac{2bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{4b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}S\left(\frac{2\sqrt{b}}{\sqrt{}}\right.}{}$$

Mathematica [A] time = 0.269331, size = 134, normalized size = 0.94

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-2\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqr
t[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

Maple [A] time = 0.023, size = 142, normalized size = 1.

$$2\frac{1}{d}\left(-1/8\frac{d\sqrt{dx+c}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+1/16\frac{d\sqrt{\pi}}{b}\left(\cos\left(2\frac{ad-bc}{d}\right)\text{FresnelC}\left(2\frac{\sqrt{dx+c}b}{d\sqrt{\pi}}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1
/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))
```

Maxima [C] time = 2.06039, size = 788, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*cos(2*((d*x +
c)*b - b*c + a*d)/d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(2*I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)
/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/(b*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 0.521148, size = 308, normalized size = 2.17

$$\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\left(2b\cos(bx+a)^2-b\right)\sqrt{dx+c}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))/b^
2
```

Sympy [B] time = 5.90256, size = 389, normalized size = 2.74

$$-\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4}\\ \frac{1}{2},\end{matrix}\right.}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/
4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*
gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c
/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x
)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*
a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqr
t(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d
*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*s
qrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b
*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

Giac [C] time = 1.16851, size = 316, normalized size = 2.23

$$-\frac{\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{2\sqrt{dx+c}de^{\left(\frac{2i(dx+c)b-2ibc+2iad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(\frac{-2i(dx+\cdots}{}\right)}}{b}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt
(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*
I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x +
 c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^(
(-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)/d
```

3.55 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(4*b) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```

Rubi [A] time = 0.22096, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```


```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(4*b) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}\sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b} - \frac{\left(d\sin\left(2a-\right.\right.}{}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Subst}\left(\int\cos\left(\frac{2bx^2}{d}\right)dx, x, \sqrt{c+dx}\right)}{4b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}S\left(\frac{2\sqrt{b}}{\sqrt{}}\right.}{}$$

Mathematica [A] time = 0.026048, size = 134, normalized size = 0.94

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - \sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqr
t[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

Maple [A] time = 0.023, size = 142, normalized size = 1.

$$2\,\frac{1}{d}\left(-1/8\,\frac{d\sqrt{dx+c}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+1/16\,\frac{d\sqrt{\pi}}{b}\left(\cos\left(2\,\frac{ad-bc}{d}\right)\mathrm{FresnelC}\left(2\,\frac{\sqrt{dx+c}b}{d\sqrt{\pi}}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\,\frac{a}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1
/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))
```

Maxima [C] time = 1.97827, size = 788, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*cos(2*((d*x +
c)*b - b*c + a*d)/d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(2*I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-2*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-2*(b*c - a*d)
/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/(b*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 0.519941, size = 308, normalized size = 2.17

$$\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\left(2b\cos(bx+a)^{2}-b\right)\sqrt{dx+c}}{8b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))/b^
2
```

Sympy [B] time = 5.84613, size = 389, normalized size = 2.74

$$-\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\middle|-\frac{b^2(c+dx)^2}{d^2}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right){}_2F_3\left(\begin{matrix}\frac{1}{4}\\ \frac{1}{2},\end{matrix}\right.}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/
4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*
gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c
/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x
)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*
a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqr
t(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d
*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*s
qrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b
*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

Giac [C] time = 1.15843, size = 316, normalized size = 2.23

$$-\frac{\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2i\,bc+2i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{2\sqrt{dx+c}\,de^{\left(\frac{2i\,(dx+c)b-2i\,bc+2i\,ad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}\,de^{\left(\frac{-2i\,(dx+}{}\right.}}{b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt
(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*
I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x +
 c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^(
(-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)/d
```

3.56 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=168

$$-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2}-\frac{(c$$

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(4*b) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```

Rubi [A] time = 0.275287, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2}-\frac{(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(4*b) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{(3d)\int \sqrt{c+dx}\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2} - \frac{(3d^2)\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}\,d}{32b^2}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\cos\left(2a-\frac{2bc}{d}\right)\right.}{32b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2} - \frac{\left(3d\cos\left(2a-\frac{2bc}{d}\right)\right)}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}}{}$$

Mathematica [A] time = 0.0417029, size = 157, normalized size = 0.93

$$\frac{-3\sqrt{\pi}d\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 3\sqrt{\pi}d\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\sqrt{c+dx}(4b(c+dx)\cos(2(a+}{32d^2\left(\frac{b}{d}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sq
rt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2
*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)]
 - 3*d*Sin[2*(a + b*x)]))/(32*(b/d)^(5/2)*d^2)
```

Maple [A] time = 0.026, size = 187, normalized size = 1.1

$$2\,\frac{1}{d}\left(-1/8\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+3/8\,\frac{d}{b}\left(1/4\,\frac{d\sqrt{dx+c}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)-1/8\,\frac{d\sqrt{\pi}}{b}\right(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b
*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1
/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+
sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 1.94144, size = 869, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*cos(2*((
d*x + c)*b - b*c + a*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d
))*sin(2*((d*x + c)*b - b*c + a*d)/d) - ((-3*I*sqrt(pi)*cos(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-2*(b*c - a*d)/d) -
 (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d^2*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((3*I*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*
I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))
*d^2*cos(-2*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(-2*I*b/d)))/(b^2*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 0.530726, size = 413, normalized size = 2.46

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\left(2\,b^2dx+3\,bd\cos(b\ldots\right.}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x + c))
/b^3
```

Sympy [B] time = 132.766, size = 665, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -5*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sq
rt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1/4)/(32*d*gamma(9/4)) + sqrt(pi)*sqr
t(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)
*d*sqrt(b/d)))/(2*d) - 21*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d
)*fresnels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(32*d*gam
ma(11/4)) + sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d)*fresnels(2*b
*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) - 15*sqrt(pi)*d*sqrt(d/b)*sin(
2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1
/4)/(512*b**2*gamma(9/4)) - 63*sqrt(pi)*d*sqrt(d/b)*cos(2*a - 2*b*c/d)*fres
nels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(512*b**2*gamma
(11/4)) + 5*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*sin(2*b*c/d + 2*b
*x)*gamma(1/4)/(64*sqrt(b)*sqrt(d)*gamma(9/4)) - 21*sqrt(d/b)*(c + d*x)**(3
/2)*cos(2*a - 2*b*c/d)*cos(2*b*c/d + 2*b*x)*gamma(3/4)/(64*sqrt(b)*sqrt(d)*
gamma(11/4)) + 15*sqrt(d)*sqrt(d/b)*sqrt(c + d*x)*sin(2*a - 2*b*c/d)*cos(2*
b*c/d + 2*b*x)*gamma(1/4)/(256*b**(3/2)*gamma(9/4)) + 63*sqrt(d)*sqrt(d/b)*
sqrt(c + d*x)*sin(2*b*c/d + 2*b*x)*cos(2*a - 2*b*c/d)*gamma(3/4)/(256*b**(3
/2)*gamma(11/4))
```

Giac [C] time = 1.26103, size = 732, normalized size = 4.36

$$-\frac{4\left(\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2i\,bc+2i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{2\sqrt{dx+c}de^{\left(\frac{2i\,(dx+c)b-2i\,bc+2i\,ad}{d}\right)}}{b}+\frac{2\sqrt{dx+c}de^{\left(\frac{-2i\,(dx}{}\right.}}{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(4*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + s
qrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*
x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*
e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c + I*sqrt(pi)*(4*I*b*c*d -
 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I
*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(pi)*(
4*I*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2)
- 2*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*
d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2 - 2*I*(4*I*(d*x + c)^(
3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c
)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)/d
```

3.57 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=196

$$-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rubi [A] time = 0.334144, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```


```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{(5d)\int (c+dx)^{3/2}\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2} - \frac{\left(15d^2\right)\int \sqrt{c+dx}}{32}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2}{16b^2}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2}{16b^2}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2}{16b^2}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a\right.}{128}$$

Mathematica [A] time = 0.0810628, size = 179, normalized size = 0.91

$$\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(20bd(c+dx)\sin(2(a+bx)) - \cos(2(a+bx))\left(16b^2(c+dx)^2-15d^2\right)\right) - 15\sqrt{\pi}d^2\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}}{128b^3\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-((-15*d^2 + 16*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)]) + 20*b*d*(c + d*x)*Sin[2*(a + b*x)]))/(128*b^3*S
qrt[b/d])
```

Maple [A] time = 0.024, size = 234, normalized size = 1.2

$$2\frac{1}{d}\left(-1/8\,\frac{d\,(dx+c)^{5/2}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+5/8\,\frac{d}{b}\left(1/4\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)-3/4\,\frac{d}{b}\left(-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b
*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)
^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d
-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.03432, size = 910, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*sin(2
*((d*x + c)*b - b*c + a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*sqrt(ab
s(b)/abs(d)) - 15*sqrt(2)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d)))*cos(2*((d*
x + c)*b - b*c + a*d)/d) - ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*cos(-2*(b*c - a*d)/d) - (15*I*sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 1
5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))
*d^3*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - ((15*sqrt(pi
)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(p
i)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sq
rt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d
```

```
^3*cos(-2*(b*c - a*d)/d) - (-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(-2*I*b/d)))/(b^3*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 0.54649, size = 540, normalized size = 2.76

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\left(16\,b^3d^2x^2+32\,b^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 -
 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.31464, size = 1327, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(16*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(
d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*
d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c^2 + d^2*((I*sqrt(pi)*(-
16*I*b^2*c^2*d + 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*
b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sqrt
(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c
- 2*I*a*d)/d)/b^3)/d^2 + (I*sqrt(pi)*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d
^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b
*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*
x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*
c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x
 + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) + 8*c*(I*s
qrt(pi)*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) + I*sqrt(pi)*(4*I*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*
d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^2 -
2*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^
2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2))/d
```

3.58 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=406

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\right.\cdots}{16\cdots}$$

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rubi [A] time = 1.13674, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\right.\cdots}{16\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b} + \frac{(5d)\int (c+dx)^{3/2}\sin(3}{24b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{(c+dx)^{5/2}\sin(a}{4b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d^2\sqrt{c+dx}\sin(}{16b^3}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d^2\sqrt{c+dx}\sin(}{16b^3}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d^2\sqrt{c+dx}\sin(}{16b^3}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a\right.}{1}$$

Mathematica [C] time = 15.5226, size = 1171, normalized size = 2.88

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b} - \frac{\left(-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) - \sqrt{2\pi}\text{F}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/S
qrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d
])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*Sq
rt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d
] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*
Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(8*b^3) + ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^
```

```
2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*
b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(c
- 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(32*b^5) - (c^2
*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c +
d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a -
(3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[
3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + S
qrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos
[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Co
s[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) - ((b/d)^(3/2)*
d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2
 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c
)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*S
qrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin[3
*(a + b*x)])))/(288*Sqrt[3]*b^5)
```

Maple [A] time = 0.04, size = 474, normalized size = 1.2

$$2\,\frac{1}{d}\left(1/8\,\frac{d\,(dx+c)^{5/2}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-5/8\,\frac{d}{b}\left(-1/2\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+3/2\,\frac{d}{b}\left(1/2\,\frac{d\sqrt{dx}}{b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/2
4/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)*
sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.5027, size = 1866, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456*sqrt(3)*(80*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*cos(3*((d*x + c)*b
 - b*c + a*d)/d)/abs(d) - 720*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*cos(((d*
x + c)*b - b*c + a*d)/d)/abs(d) + ((5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*
c - a*d)/d) + (5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3)*(-135*I*sqrt(pi)*cos(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*I*sqrt(pi)*cos(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*sqrt(pi)*sin(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*
cos(-(b*c - a*d)/d) - sqrt(3)*(135*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 135*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*I*sqrt(pi)*sin(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)*(135*I*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*I*sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*sqrt(
pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*sqr
t(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*s
qrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(135*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*sqrt(pi)*cos(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*I*sqrt(pi)*sin(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*I*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs
(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((-5*I*s
qrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
```

```
5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*
d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) + (5*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d
))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 8*(12*sqrt(3)
*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 5*sqrt(3)*sqrt(d*x + c)*d^3*abs(b)/a
bs(d))*sin(3*((d*x + c)*b - b*c + a*d)/d) - 72*(4*sqrt(3)*(d*x + c)^(5/2)*b
^2*d*abs(b)/abs(d) - 15*sqrt(3)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*sin(((d*x
+ c)*b - b*c + a*d)/d))*abs(d)/(b^3*d*abs(b))
```

Fricas [A] time = 0.643364, size = 918, normalized size = 2.26

$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*
c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/
b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

[Out] Timed out

Giac [C] time = 1.58877, size = 2722, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/1728*(12*(I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*d^2*erf(-1/
2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*
c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c
)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^
((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I
*b*c - 3*I*a*d)/d)/b)*c^2 - d^2*((sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*d + 12*b*
c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^3) - 6*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^2*c*d + 12*
I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c
*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/
b^3)/d^2 + 27*(sqrt(2)*sqrt(pi)*(4*I*b^2*c^2*d - 12*b*c*d^2 - 15*I*d^3)*d*e
rf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x +
c)^(5/2)*b^2*d + 8*I*(d*x + c)^(3/2)*b^2*c*d - 4*I*sqrt(d*x + c)*b^2*c^2*d
- 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*
d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 27*(sqrt(2)*sqrt(pi)
*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 8*I*(d*x +
c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 +
 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b
*c + I*a*d)/d)/b^3)/d^2 + (sqrt(6)*sqrt(pi)*(12*I*b^2*c^2*d + 12*b*c*d^2 -
5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3
```

```
) - 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 24*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sq
rt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2
 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/
d^2) - 12*(sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*sqrt(2)*sqrt(pi)*(-2*I*b*c*d + 3*d^
2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*sqrt(2)
*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) + sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I*(d*x + c)^(3/2)
*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3
*I*b*c + 3*I*a*d)/d)/b^2 - 18*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*
b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 - 18
*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)
*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 6*(2*I*(d*x + c)^(3/2)*b*d -
2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c
 - 3*I*a*d)/d)/b^2)*c)/d
```

3.59 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3b}{d}\right.}{24b^{5}}$$

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```

Rubi [A] time = 0.684237, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3b}{d}\right.}{24b^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```


```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int (c+dx)^{3/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx$$
$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b} + \frac{d\int \sqrt{c+dx}\sin(3a+3}{8b}$$
$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{4b}$$
$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{4b}$$
$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{4b}$$
$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C}{8b^{5/2}}$$

Mathematica [C] time = 9.30698, size = 677, normalized size = 1.92

$$-\frac{ic\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{8b} + \frac{d\left(\sqrt{2\pi}\sqrt{\frac{b}{d}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)\left(2bc\sin\left(a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqr
t[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])
/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d] +
2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi
]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sqrt
[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) - (c*(-(Sqrt[2*P
i]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] +
 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[3]*b*Sqrt[b/
```

```
d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]
*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqrt[
2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)
/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x)]
 + 2*b*x*Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

Maple [A] time = 0.036, size = 386, normalized size = 1.1

$$2\frac{1}{d}\left(1/8\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-3/8\,\frac{d}{b}\left(-1/2\,\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+1/4\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.53624, size = 1787, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576*sqrt(3)*(16*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*sin(3*((d*x + c)*b -
b*c + a*d)/d)/abs(d) - 48*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*sin(((d*x + c)
*b - b*c + a*d)/d)/abs(d) + 8*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*cos(3*((d*x
+ c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*cos(((d
*x + c)*b - b*c + a*d)/d)/abs(d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
```

```
) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d
)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d^2*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) + (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sq
rt(3)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(I*b/d)) + (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d)
+ sqrt(3)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(-I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (-I*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(
pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(
abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*ab
s(d)/(b^2*d*abs(b))
```

Fricas [A] time = 0.612319, size = 756, normalized size = 2.14

$$\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}S\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 -
3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2)
*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.37037, size = 1517, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/288*(2*(I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
```

```
rt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
+ 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*
d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((
I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b
*c - 3*I*a*d)/d)/b)*c - sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(
6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*sqrt(2)*sqrt(pi)*(-2*I
*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2
) - 9*sqrt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(-2*I*(d
*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I*(d
*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 18*(2*I*(d*x + c)^(3/2)*b*d - 2*I*s
qrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d
)/d)/b^2 + 18*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(
d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 + 6*(2*I*(d*x + c)
^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c
)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d
```

3.60 $\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\cdots}{\cdots}\right)}{4b^{3/2}}$$

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rubi [A] time = 0.469995, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\cdots}{\cdots}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```


```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx) - \frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int \sqrt{c+dx}\cos(a+bx)\,dx - \frac{1}{4}\int \sqrt{c+dx}\cos(3a+3bx)\,dx$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}dx}{24b} - \frac{d\int \frac{\sin(a+bx}{\sqrt{c+dx}}}{8b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right.}{\sqrt{c+dx}}}{24b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\left(\int \sin\left(\right.\right.}{12b}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} +$$

Mathematica [C] time = 6.56647, size = 280, normalized size = 0.92

$$-\frac{-\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - \sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) + 2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3(a-}{24\sqrt{3}b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[
((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - (-(Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[3]*
Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)])/(24*Sqrt[3]*b*Sqrt[b/d])
```

Maple [A] time = 0.036, size = 294, normalized size = 1.

$$2\frac{1}{d}\left(1/8\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)b}{d}+\frac{ad-bc}{d}\right) - 1/16\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right) + \sin\left(\frac{ad-bc}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d
)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.31223, size = 1648, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288*sqrt(3)*(8*sqrt(3)*sqrt(d*x + c)*d*abs(b)*sin(3*((d*x + c)*b - b*c +
 a*d)/d)/abs(d) - 24*sqrt(3)*sqrt(d*x + c)*d*abs(b)*sin(((d*x + c)*b - b*c
+ a*d)/d)/abs(d) + ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*co
s(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b
)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3
)*(3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(3*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(ab
s(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)
```

```
*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(3*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(ab
s(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I*sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqr
t(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(p
i)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(a
bs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 0.583368, size = 639, normalized size = 2.1

$$\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{d\right.}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt(d*x +
 c)*sin(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

Giac [C] time = 1.24356, size = 662, normalized size = 2.18

$$-\frac{\frac{i\sqrt{6}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{3ibc-3iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{9i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\right.}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144*(I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*d*e
^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c
- 3*I*a*d)/d)/b)/d
```

3.61 $\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\ldots}{}\right.}{4b^{3/2}}$$

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rubi [A] time = 0.46789, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\ldots}{}\right.}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```


```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx) - \frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int \sqrt{c+dx}\cos(a+bx)\,dx - \frac{1}{4}\int \sqrt{c+dx}\cos(3a+3bx)\,dx$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}dx}{24b} - \frac{d\int \frac{\sin(a+bx}{\sqrt{c+dx}}}{8b}$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right.}{\sqrt{c+dx}}}{24b}$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\left(\int \sin\left(\right.\right.}{12b}$$
$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} +$$

Mathematica [C] time = 6.48926, size = 280, normalized size = 0.92

$$-\frac{-\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - \sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) + 2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3(a-}{24\sqrt{3}b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[
((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - (-(Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[3]*
Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)])/(24*Sqrt[3]*b*Sqrt[b/d])
```

Maple [A] time = 0.035, size = 294, normalized size = 1.

$$2\frac{1}{d}\left(1/8\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)b}{d}+\frac{ad-bc}{d}\right) - 1/16\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right) + \sin\left(\frac{ad-bc}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d
)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.43116, size = 1648, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288*sqrt(3)*(8*sqrt(3)*sqrt(d*x + c)*d*abs(b)*sin(3*((d*x + c)*b - b*c +
 a*d)/d)/abs(d) - 24*sqrt(3)*sqrt(d*x + c)*d*abs(b)*sin(((d*x + c)*b - b*c
+ a*d)/d)/abs(d) + ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*co
s(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b
)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3
)*(3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(3*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(ab
s(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)
```

```
*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sqrt(3)*(3*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(ab
s(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I*sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqr
t(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(p
i)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(a
bs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 0.586352, size = 639, normalized size = 2.1

$$\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{d\right.}{72\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt(d*x +
 c)*sin(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

Giac [C] time = 1.27978, size = 662, normalized size = 2.18

$$-\frac{\frac{i\sqrt{6}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{3ibc-3iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{9i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(-\cdots\right.}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144*(I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*d*e
^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(
d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c
- 3*I*a*d)/d)/b)/d
```

3.62 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}}$$

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```

Rubi [A] time = 0.57135, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```


```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b} + \frac{d\int \sqrt{c+dx}\sin(3a+3b}{8b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} -$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} -$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} -$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\Big(}{8b^{5/2}}$$

Mathematica [C] time = 9.30628, size = 677, normalized size = 1.92

$$-\frac{ic\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{8b}+\frac{d\left(\sqrt{2\pi}\sqrt{\frac{b}{d}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)\left(2bc\sin\left(a-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqr
t[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])
/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d] +
2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi
]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sqrt
[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) - (c*(-(Sqrt[2*P
i]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] +
 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[3]*b*Sqrt[b/
```

```
d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]
*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqrt[
2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)
/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x)]
 + 2*b*x*Sin[3*(a + b*x)]))))/(48*Sqrt[3]*b^3)
```

Maple [A] time = 0.04, size = 386, normalized size = 1.1

$$2\,\frac{1}{d}\left(1/8\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-3/8\,\frac{d}{b}\left(-1/2\,\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+1/4\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.37528, size = 1787, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576*sqrt(3)*(16*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*sin(3*((d*x + c)*b -
b*c + a*d)/d)/abs(d) - 48*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*sin(((d*x + c)
*b - b*c + a*d)/d)/abs(d) + 8*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*cos(3*((d*x
+ c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*cos(((d
*x + c)*b - b*c + a*d)/d)/abs(d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
```

```
) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d
)/d) - (I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d^2*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) + (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) + sq
rt(3)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(I*b/d)) + (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d)
+ sqrt(3)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(-I*b/d)) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (-I*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(
pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(
abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*ab
s(d)/(b^2*d*abs(b))
```

Fricas [A] time = 0.607138, size = 756, normalized size = 2.14

$$\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\mathrm{S}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 -
3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2)
*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.37434, size = 1517, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/288*(2*(I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
```

```
rt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
+ 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*
d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((
I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b
*c - 3*I*a*d)/d)/b)*c - sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(
6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*sqrt(2)*sqrt(pi)*(-2*I
*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2
) - 9*sqrt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(-2*I*(d
*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I*(d
*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 18*(2*I*(d*x + c)^(3/2)*b*d - 2*I*s
qrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d
)/d)/b^2 + 18*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(
d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 + 6*(2*I*(d*x + c)
^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c
)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d
```

3.63 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=406

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\right.}{16}$$

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rubi [A] time = 0.667682, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\right.}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```


```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right) dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b} + \frac{(5d)\int (c+dx)^{3/2}\sin(3}{24b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{(c+dx)^{5/2}\sin(a}{4b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d^2\sqrt{c+dx}\sin(}{16b^3}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d^2\sqrt{c+dx}\sin(}{16b^3}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d^2\sqrt{c+dx}\sin(}{16b^3}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a\right.}{1}$$

<u>Mathematica</u> [C] time = 14.6632, size = 1171, normalized size = 2.88

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b} - \frac{\left(-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) - \sqrt{2\pi}\mathrm{F}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/S
qrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d
])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*Sq
rt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d
] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*
Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(8*b^3) + ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^
```

```
2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*
b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(c
- 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(32*b^5) - (c^2
*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c +
d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a -
(3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[
3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + S
qrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos
[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Co
s[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) - ((b/d)^(3/2)*
d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2
 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c
)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*S
qrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin[3
*(a + b*x)])))/(288*Sqrt[3]*b^5)
```

Maple [A] time = 0.036, size = 474, normalized size = 1.2

$$2\,\frac{1}{d}\left(1/8\,\frac{d\,(dx+c)^{5/2}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-5/8\,\frac{d}{b}\left(-1/2\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+3/2\,\frac{d}{b}\left(1/2\,\frac{d\sqrt{dx}}{b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/2
4/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)*
sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.45579, size = 1866, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456*sqrt(3)*(80*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*cos(3*((d*x + c)*b
 - b*c + a*d)/d)/abs(d) - 720*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*cos(((d*
x + c)*b - b*c + a*d)/d)/abs(d) + ((5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*
c - a*d)/d) + (5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(3*I*b/d)) + (sqrt(3)*(-135*I*sqrt(pi)*cos(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*I*sqrt(pi)*cos(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*sqrt(pi)*sin(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*
cos(-(b*c - a*d)/d) - sqrt(3)*(135*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 135*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*I*sqrt(pi)*sin(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(3)*(135*I*sqrt(pi)*cos
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*I*sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*sqrt(
pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*sqr
t(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*s
qrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(135*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*sqrt(pi)*cos(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*I*sqrt(pi)*sin(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 135*I*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs
(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((-5*I*s
qrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
```

```
5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*
d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) + (5*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d
))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 8*(12*sqrt(3)
*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 5*sqrt(3)*sqrt(d*x + c)*d^3*abs(b)/a
bs(d))*sin(3*((d*x + c)*b - b*c + a*d)/d) - 72*(4*sqrt(3)*(d*x + c)^(5/2)*b
^2*d*abs(b)/abs(d) - 15*sqrt(3)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*sin(((d*x
+ c)*b - b*c + a*d)/d))*abs(d)/(b^3*d*abs(b))
```

Fricas [A] time = 0.638412, size = 918, normalized size = 2.26

$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*
c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/
b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

[Out] Timed out

Giac [C] time = 1.5621, size = 2722, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/1728*(12*(I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*d^2*erf(-1/
2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*
c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c
)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^
((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I
*b*c - 3*I*a*d)/d)/b)*c^2 - d^2*((sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*d + 12*b*
c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^3) - 6*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^2*c*d + 12*
I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c
*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/
b^3)/d^2 + 27*(sqrt(2)*sqrt(pi)*(4*I*b^2*c^2*d - 12*b*c*d^2 - 15*I*d^3)*d*e
rf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x +
c)^(5/2)*b^2*d + 8*I*(d*x + c)^(3/2)*b^2*c*d - 4*I*sqrt(d*x + c)*b^2*c^2*d
- 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*
d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 27*(sqrt(2)*sqrt(pi)
*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 8*I*(d*x +
c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 +
 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b
*c + I*a*d)/d)/b^3)/d^2 + (sqrt(6)*sqrt(pi)*(12*I*b^2*c^2*d + 12*b*c*d^2 -
5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3
```

```
) - 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 24*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sq
rt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2
 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/
d^2) - 12*(sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*sqrt(2)*sqrt(pi)*(-2*I*b*c*d + 3*d^
2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*sqrt(2)
*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) + sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I*(d*x + c)^(3/2)
*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3
*I*b*c + 3*I*a*d)/d)/b^2 - 18*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*
b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 - 18
*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)
*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 6*(2*I*(d*x + c)^(3/2)*b*d -
2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c
 - 3*I*a*d)/d)/b^2)*c)/d
```

3.64 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a\cdots\right.}{40\cdots}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rubi [A] time = 1.05142, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a\cdots\right.}{40\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
```

```
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)-\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx\right)+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}-\frac{(5d)\int (c+dx)^{3/2}\,\mathrm{c}}{64}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c+dx)^{3/2}\sin(}{32b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\cos(}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\cos(}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\cos(}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\cos(}{2048b^3}$$

Mathematica [A] time = 15.219, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\cos(2(a+bx))+256b^3c^2\sqrt{c+dx}\cos(4(a+bx))-1024b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx))+256b^3d^2x^2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] -
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

Maple [A] time = 0.036, size = 470, normalized size = 1.2

$$2\frac{1}{d}\left(-1/16\frac{d(dx+c)^{5/2}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+\frac{5d}{16b}\left(1/4\frac{d(dx+c)^{3/2}}{b}\sin\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)-3/4\frac{d}{b}\left(-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.36066, size = 1874, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*sqrt(2)*(640*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(4*((d*x + c)
*b - b*c + a*d)/d)/abs(d) - 5120*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2
*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 16*(64*sqrt(2)*(d*x + c)^(5/2)*b^2*d
*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(4*((d*x +
```

```
c)*b - b*c + a*d)/d) + 256*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d)
 - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*
d)/d) + ((480*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 480*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) - 480*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 480*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (-48
0*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 480*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-4*(b*c -
a*d)/d) - sqrt(2)*(15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*
erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d
))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-
4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((480*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sq
rt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (480*I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs
(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b^3*
d*abs(b))
```

Fricas [A] time = 0.693492, size = 1011, normalized size = 2.48

$$15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-480\pi d^3\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) -
 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +
 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*c
os(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^
2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.86076, size = 2681, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

[Out] 1/16384*(64*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2 - d^2*((I*sqrt(2)*sqrt(pi)*(64*I*b^2*c^2*d - 48*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 128*I*(d*x + c)^(3/2)*b^2*c*d - 64*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 48*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*I*b^2*c^2*d + 48*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 128*I*(d*x + c)^(3/2)*b^2*c*d - 64*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 48*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^3)/d^2 + 32*(I*sqrt(pi)*(-16*I*b^2*c^2*d + 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 32*(I*sqrt(pi)*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) - 16*(I*sqrt(2)*sqrt(pi)*(-8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(2)*sqrt(pi)*(-8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x

```
 + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*s
qrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2
*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d +
3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2 - 4*I
*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)
*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)*c)/d
```

3.65 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\right.}{512}$$

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rubi [A] time = 0.673582, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\right.}{512}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```


```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right) dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{(3d)\int \sqrt{c+dx}\,\mathrm{co}}{64b}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}\sin(2a}{32b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}\sin(2a}{32b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}\sin(2a}{32b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a\right.}{5}$$

Mathematica [A] time = 3.34361, size = 393, normalized size = 1.12

$$\frac{3\sqrt{2\pi}d\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - 48\sqrt{\pi}d\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + 3\sqrt{2\pi}d\cos\left(4a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
+ 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

Maple [A] time = 0.039, size = 376, normalized size = 1.1

$$2\frac{1}{d}\left(-1/16\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+3/16\,\frac{d}{b}\left(1/4\,\frac{d\sqrt{dx+c}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)-1/8\,\frac{d\sqrt{\pi}}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.31838, size = 1804, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8192*sqrt(2)*(128*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(4*((d*x + c)*b -
 b*c + a*d)/d)/abs(d) - 512*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(2*((d*x
+ c)*b - b*c + a*d)/d)/abs(d) - 48*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(4*(
(d*x + c)*b - b*c + a*d)/d)/abs(d) + 384*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*s
in(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((48*I*sqrt(pi)*cos(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d
))*cos(-2*(b*c - a*d)/d) + (48*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
```

```
 + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c -
a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (sqrt(2)*(-3*I*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)
/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(3*I*sqrt(pi
)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(
pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sq
rt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(3*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs
(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((-48*I*sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 4
8*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
48*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))
*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (48*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/
abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b
^2*d*abs(b))
```

Fricas [A] time = 0.646811, size = 802, normalized size = 2.28

$$\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-48\pi d^2\sqrt{\frac{b}{\pi d}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 4
8*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
+ 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*co
s(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.55835, size = 1485, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(8*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b -
16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 16*sqrt(
d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)
```

```
*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c - I*sqrt(2)*sqrt(pi)*(
-8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) - I*sqrt(2)*sqrt(pi)*(-8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 16*I*sqrt(pi)*(4*I*b*c
*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*I*(
-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e
^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 + 32*I*(4*I*(d*x + c)^(3/2)*
b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b -
2*I*b*c + 2*I*a*d)/d)/b^2 + 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x +
c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2 + 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*
x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.66 $\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S}{64b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rubi [A] time = 0.499154, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S}{64b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}dx}{64b} + \frac{d\int \frac{\text{co}}{}}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4b}{d}\right.}{\sqrt{c}}}{64b}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Subst}\left(\int \text{c}\right.}{32}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)C\left(\right.}{64b^{3/2}}$$

Mathematica [A] time = 0.873951, size = 264, normalized size = 0.88

$$\frac{-\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) + 8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(\right.}{128b\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

Maple [A] time = 0.034, size = 286, normalized size = 1.

$$2\frac{1}{d}\left(-1/16\frac{d\sqrt{dx+c}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+1/32\frac{d\sqrt{\pi}}{b}\left(\cos\left(2\frac{ad-bc}{d}\right)\text{FresnelC}\left(2\frac{\sqrt{dx+c}\,b}{d\sqrt{\pi}}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\frac{a}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.30806, size = 1655, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(16*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(4*((d*x + c)*b - b*c
+ a*d)/d)/abs(d) - 64*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(2*((d*x + c)*b - b
*c + a*d)/d)/abs(d) + ((8*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) - 8*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + 8*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) +
(-8*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 8*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 8*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 8*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
2*I*b/d)) - (sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(I
*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sq
rt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sq
rt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) -
```

```
(sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/
abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((8*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d
*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (8*I*sqrt(pi)*cos(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*sin(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*si
n(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 0.607041, size = 630, normalized size = 2.11

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-8\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_si
n(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d*sq
rt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d
))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*b)*sqr
t(d*x + c))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.337, size = 643, normalized size = 2.15

$$\frac{\frac{\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{8\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b - 16*s
qrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 16*sqrt(d*x
+ c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e
^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

3.67 $\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S}{64b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rubi [A] time = 0.459801, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S}{64b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```


```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}\,dx}{64b} + \frac{d\int \text{co}}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4b}{d}\right.}{\sqrt{c}}}{64b}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Subst}\left(\int \text{c}\right.}{32}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)C\left(\right.}{64b^{3/2}}$$

Mathematica [A] time = 0.325591, size = 264, normalized size = 0.88

$$\frac{-\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) + 8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(\right.}{128b\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

Maple [A] time = 0.035, size = 286, normalized size = 1.

$$2\,\frac{1}{d}\left(-1/16\,\frac{d\sqrt{dx+c}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+1/32\,\frac{d\sqrt{\pi}}{b}\left(\cos\left(2\,\frac{ad-bc}{d}\right)\text{FresnelC}\left(2\,\frac{\sqrt{dx+c}b}{d\sqrt{\pi}}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\,\frac{ad}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.44712, size = 1655, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(16*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(4*((d*x + c)*b - b*c
+ a*d)/d)/abs(d) - 64*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(2*((d*x + c)*b - b
*c + a*d)/d)/abs(d) + ((8*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) - 8*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + 8*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) +
(-8*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 8*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 8*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 8*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
2*I*b/d)) - (sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(I
*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sq
rt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sq
rt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) -
```

```
(sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/
abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((8*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d
*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (8*I*sqrt(pi)*cos(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*sin(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*si
n(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 0.608352, size = 630, normalized size = 2.11

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-8\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_si
n(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d*sq
rt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d
))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*b)*sqr
t(d*x + c))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.36449, size = 643, normalized size = 2.15

$$\frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{8\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b - 16*s
qrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 16*sqrt(d*x
+ c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e
^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

3.68 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right.\cdots}{512b^{5}\cdots}$$

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rubi [A] time = 0.566206, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right.\cdots}{512b^{5}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```


```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{(3d)\int \sqrt{c+dx}\cos(}{64b}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}\sin(2a+}{32b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}\sin(2a+}{32b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}\sin(2a+}{32b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\right.}{512}$$

Mathematica [A] time = 3.12884, size = 393, normalized size = 1.12

$$\frac{3\sqrt{2\pi}d\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - 48\sqrt{\pi}d\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + 3\sqrt{2\pi}d\cos\left(4a-\frac{4bc}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
+ 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

Maple [A] time = 0.033, size = 376, normalized size = 1.1

$$2\frac{1}{d}\left(-1/16\frac{d\,(dx+c)^{3/2}}{b}\cos\left(2\frac{(dx+c)\,b}{d}+2\frac{ad-bc}{d}\right)+3/16\frac{d}{b}\left(1/4\frac{d\sqrt{dx+c}}{b}\sin\left(2\frac{(dx+c)\,b}{d}+2\frac{ad-bc}{d}\right)-1/8\frac{d\sqrt{\pi}}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.47109, size = 1804, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8192*sqrt(2)*(128*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(4*((d*x + c)*b -
 b*c + a*d)/d)/abs(d) - 512*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(2*((d*x
+ c)*b - b*c + a*d)/d)/abs(d) - 48*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(4*(
(d*x + c)*b - b*c + a*d)/d)/abs(d) + 384*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*s
in(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((48*I*sqrt(pi)*cos(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d
))*cos(-2*(b*c - a*d)/d) + (48*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
```

```
 + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c -
a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (sqrt(2)*(-3*I*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)
/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(3*I*sqrt(pi
)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(
pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sq
rt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(3*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs
(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((-48*I*sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 4
8*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
48*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))
*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (48*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/
abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b
^2*d*abs(b))
```

Fricas [A] time = 0.645748, size = 802, normalized size = 2.28

$$3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-48\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 4
8*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
+ 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*co
s(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.53918, size = 1485, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2048*(8*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b -
16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 16*sqrt(
d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)
```

```
*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c - I*sqrt(2)*sqrt(pi)*(
-8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) - I*sqrt(2)*sqrt(pi)*(-8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 16*I*sqrt(pi)*(4*I*b*c
*d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*I*(
-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e
^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 + 32*I*(4*I*(d*x + c)^(3/2)*
b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b -
2*I*b*c + 2*I*a*d)/d)/b^2 + 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x +
c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2 + 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*
x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.69 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a \right.}{4 \ldots}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rubi [A] time = 0.696998, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a \right.}{4 \ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
```

```
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)-\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx\right)+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}-\frac{(5d)\int (c+dx)^{3/2}\,c}{64}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c+dx)^{3/2}\sin(}{32b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\cos(}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\cos(}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\cos(}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{15d^2\sqrt{c+dx}\cos(}{2048b^3}$$

Mathematica [A] time = 11.7879, size = 550, normalized size = 1.35

$$-1024b^3c^2\sqrt{c+dx}\cos(2(a+bx))+256b^3c^2\sqrt{c+dx}\cos(4(a+bx))-1024b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx))+256b^3d^2x^2\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] -
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

Maple [A] time = 0.033, size = 470, normalized size = 1.2

$$2\frac{1}{d}\left(-1/16\frac{d(dx+c)^{5/2}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+\frac{5d}{16b}\left(1/4\frac{d(dx+c)^{3/2}}{b}\sin\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)-3/4\frac{d}{b}\left(-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.41457, size = 1874, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*sqrt(2)*(640*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(4*((d*x + c)
*b - b*c + a*d)/d)/abs(d) - 5120*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2
*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 16*(64*sqrt(2)*(d*x + c)^(5/2)*b^2*d
*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(4*((d*x +
```

c)*b - b*c + a*d)/d) + 256*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d)
- 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*
d)/d) + ((480*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 480*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) - 480*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 480*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (-48
0*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 480*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-4*(b*c -
a*d)/d) - sqrt(2)*(15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*
erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d
))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-
4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((480*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sq
rt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + (480*I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs
(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b^3*
d*abs(b))

Fricas [A] time = 0.68742, size = 1011, normalized size = 2.48

$$15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-480\pi d^3\sqrt{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) -
 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +
 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*c
os(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^
2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.79902, size = 2681, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16384*(64*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) - 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b
- 16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 16*sqr
t(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x +
c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2 - d^2*((I*sqrt(2)*
sqrt(pi)*(64*I*b^2*c^2*d - 48*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 12
8*I*(d*x + c)^(3/2)*b^2*c*d - 64*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(
3/2)*b*d^2 + 48*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d
*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*I*b^2*
c^2*d + 48*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 128*I*(d*x + c)^(3
/2)*b^2*c*d - 64*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 48*
sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I*b
*c + 4*I*a*d)/d)/b^3)/d^2 + 32*(I*sqrt(pi)*(-16*I*b^2*c^2*d + 24*b*c*d^2 +
15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2
*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*
(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b
^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(
d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 32*(I
*sqrt(pi)*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*(
d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*
b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x + c
)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) - 16*(I*sqrt(2)*sqrt(pi)*(-8*I*b*c*d
+ 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*s
qrt(2)*sqrt(pi)*(-8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d + 3*d^2)
*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c
+ 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x +
 c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x
```

```
 + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*s
qrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2
*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d +
3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2 - 4*I
*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)
*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)*c)/d
```

3.70 $\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=267

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-3\right.}{8b}$$

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8*b*
(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(
8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a -
(b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-I)*b*(
c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/
d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.282696, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3308, 2181}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-3\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```


```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8*b*
(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(
8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a -
(b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-I)*b*(
c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/
d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx &= \int \left(\frac{1}{4}(c+dx)^m \sin(a+bx)+\frac{1}{4}(c+dx)^m \sin(3a+3bx)\right)dx\\ &= \frac{1}{4}\int (c+dx)^m \sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^m \sin(3a+3bx)\,dx\\ &= \frac{1}{8}i\int e^{-i(a+bx)}(c+dx)^m\,dx-\frac{1}{8}i\int e^{i(a+bx)}(c+dx)^m\,dx+\frac{1}{8}i\int e^{-i(3a+3bx)}(c+dx\\ &= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+d}{d}\right.}{8}\end{aligned}$$

Mathematica [A] time = 0.476431, size = 250, normalized size = 0.94

$$\frac{e^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(3e^{\frac{2i(ad+bc)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{ib(c+dx)}{d}\right)\right.\right.}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((c + d*x)^m*(3*E^(((2*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((-I
)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1 +
 m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (E^((6*I)*a)*((I*b*(c + d*
x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*(((-I)*b*
(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/(3^m*((b^2*(c + d*x)^2
)/d^2)^m)))/(24*b*E^(((3*I)*(b*c + a*d))/d))
```

Maple [F] time = 0.239, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cos (bx+a))^2 \sin (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos (bx+a)^2 \sin (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

Fricas [A] time = 0.547447, size = 460, normalized size = 1.72

$$-\frac{e^{\left(-\frac{dm \log\left(\frac{3i b}{d}\right) - 3i bc + 3i ad}{d}\right)} \Gamma\left(m+1, \frac{3i bdx + 3i bc}{d}\right) + 3 e^{\left(-\frac{dm \log\left(\frac{i b}{d}\right) - i bc + i ad}{d}\right)} \Gamma\left(m+1, \frac{i bdx + i bc}{d}\right) + 3 e^{\left(-\frac{dm \log\left(-\frac{i b}{d}\right) + i bc - i ad}{d}\right)} \Gamma\left(m+1, \frac{-i bc \cdots}{\cdots}\right.}{24 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/24*(e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, (3*I*b*d*
x + 3*I*b*c)/d) + 3*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I
*b*d*x + I*b*c)/d) + 3*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1
, (-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*ga
mma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

3.71 $\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=205

$$-\frac{160d^3(c+dx)\sin(a+bx)}{27b^4}+\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^3(c+dx)\sin(a+bx)\cos^2(a+}{27b^4}$$

```
[Out] (-160*d^4*Cos[a + b*x])/(27*b^5) + (8*d^2*(c + d*x)^2*Cos[a + b*x])/(3*b^3)
 - (8*d^4*Cos[a + b*x]^3)/(81*b^5) + (4*d^2*(c + d*x)^2*Cos[a + b*x]^3)/(9*
b^3) - ((c + d*x)^4*Cos[a + b*x]^3)/(3*b) - (160*d^3*(c + d*x)*Sin[a + b*x]
)/(27*b^4) + (8*d*(c + d*x)^3*Sin[a + b*x])/(9*b^2) - (8*d^3*(c + d*x)*Cos[
a + b*x]^2*Sin[a + b*x])/(27*b^4) + (4*d*(c + d*x)^3*Cos[a + b*x]^2*Sin[a +
 b*x])/(9*b^2)
```

Rubi [A] time = 0.202537, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4405, 3311, 3296, 2638, 3310}

$$-\frac{160d^3(c+dx)\sin(a+bx)}{27b^4}+\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^3(c+dx)\sin(a+bx)\cos^2(a+}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```


```
[Out] (-160*d^4*Cos[a + b*x])/(27*b^5) + (8*d^2*(c + d*x)^2*Cos[a + b*x])/(3*b^3)
 - (8*d^4*Cos[a + b*x]^3)/(81*b^5) + (4*d^2*(c + d*x)^2*Cos[a + b*x]^3)/(9*
b^3) - ((c + d*x)^4*Cos[a + b*x]^3)/(3*b) - (160*d^3*(c + d*x)*Sin[a + b*x]
)/(27*b^4) + (8*d*(c + d*x)^3*Sin[a + b*x])/(9*b^2) - (8*d^3*(c + d*x)*Cos[
a + b*x]^2*Sin[a + b*x])/(27*b^4) + (4*d*(c + d*x)^3*Cos[a + b*x]^2*Sin[a +
 b*x])/(9*b^2)
```


Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^4\cos^3(a+bx)}{3b} + \frac{(4d)\int (c+dx)^3\cos^3(a+bx)\,dx}{3b}$$

$$= \frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3} - \frac{(c+dx)^4\cos^3(a+bx)}{3b} + \frac{4d(c+dx)^3\cos^2(a+bx)}{9b^2}$$

$$= -\frac{8d^4\cos^3(a+bx)}{81b^5} + \frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3} - \frac{(c+dx)^4\cos^3(a+bx)}{3b} + \frac{8d(}{}$$

$$= \frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3} - \frac{8d^4\cos^3(a+bx)}{81b^5} + \frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3} - \frac{(c}{}$$

$$= -\frac{16d^4\cos(a+bx)}{27b^5} + \frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3} - \frac{8d^4\cos^3(a+bx)}{81b^5} + \frac{4d^2(c+dx}{}$$

$$= -\frac{160d^4\cos(a+bx)}{27b^5} + \frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3} - \frac{8d^4\cos^3(a+bx)}{81b^5} + \frac{4d^2(c+}{}$$

Mathematica [A] time = 1.62255, size = 150, normalized size = 0.73

$$-\frac{81\cos(a+bx)\left(-12b^2d^2(c+dx)^2+b^4(c+dx)^4+24d^4\right)+\cos(3(a+bx))\left(-36b^2d^2(c+dx)^2+27b^4(c+dx)^4+8d^4\right)-}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(81*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] + (8*
d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)] - 24*b*
d*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d*x)^2)*Co
s[2*(a + b*x)])*Sin[a + b*x])/(324*b^5)
```

Maple [B] time = 0.043, size = 835, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/3*(b*x+a)^4*cos(b*x+a)^3+4/9*(b*x+a)^3*(2+cos(b*x+a)^2)*
sin(b*x+a)+8/3*(b*x+a)^2*cos(b*x+a)-160/27*cos(b*x+a)-16/3*(b*x+a)*sin(b*x+
a)+4/9*(b*x+a)^2*cos(b*x+a)^3-8/27*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)-8/81
*cos(b*x+a)^3)-4/b^4*a*d^4*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+co
s(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*co
s(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+4/b^3*c*d^3*(-1/3*(b*x+a)^3*co
s(b*x+a)^3+1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*
x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+
6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin
(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*co
s(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*co
s(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x
+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-4/b^4*a^3*d^4*(-1/3*(b*
x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+12/b^3*a^2*c*d^3*(-1/3*(
b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-12/b^2*a*c^2*d^2*(-1/3
*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+4/b*c^3*d*(-1/3*(b*x
+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-1/3/b^4*a^4*d^4*cos(b*x+a
)^3+4/3/b^3*a^3*c*d^3*cos(b*x+a)^3-2/b^2*a^2*c^2*d^2*cos(b*x+a)^3+4/3/b*a*c
^3*d*cos(b*x+a)^3-1/3*c^4*cos(b*x+a)^3)
```

Maxima [B] time = 1.39513, size = 1200, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/324*(108*c^4*cos(b*x + a)^3 - 432*a*c^3*d*cos(b*x + a)^3/b + 648*a^2*c^2
*d^2*cos(b*x + a)^3/b^2 - 432*a^3*c*d^3*cos(b*x + a)^3/b^3 + 108*a^4*d^4*co
s(b*x + a)^3/b^4 + 36*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x +
 a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c^3*d/b - 108*(3*(b*x + a)*cos(3*b
*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a
*c^2*d^2/b^2 + 108*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
 - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*c*d^3/b^3 - 36*(3*(b*x + a)*cos(3
*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 -
 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a)
)*c^2*d^2/b^2 - 36*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2
- 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a
))*a*c*d^3/b^3 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2
 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x +
a))*a^2*d^4/b^4 + 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27
*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x +
 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 12*(3*(3*(b*x + a)^3
 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x +
 a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x +
 a))*a*d^4/b^4 + ((27*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*cos(3*b*x + 3*a) +
81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*cos(b*x + a) - 12*(3*(b*x + a)^3 - 2
*b*x - 2*a)*sin(3*b*x + 3*a) - 324*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a)
)*d^4/b^4)/b
```

Fricas [A] time = 0.525951, size = 636, normalized size = 3.1

$$-\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4-36\,b^2c^2d^2+8\,d^4+18\left(9\,b^4c^2d^2-2\,b^2d^4\right)x^2+36\left(3\,b^4c^3d-2\,b^2cd^3\right)x\right)\cos\left(bx+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 +
8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)
*x)*cos(b*x + a)^3 - 24*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*d^2 - 2
0*d^4)*cos(b*x + a) - 12*(6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d -
40*b*c*d^3 + (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (
9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 2*(9*b^3*c^2*d^2 - 20*b*d^4)*x
)*sin(b*x + a))/b^5
```

Sympy [A] time = 10.4695, size = 646, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**3/(3*b) - 4*c**3*d*x*cos(a + b*x)**3/(3*b) -
 2*c**2*d**2*x**2*cos(a + b*x)**3/b - 4*c*d**3*x**3*cos(a + b*x)**3/(3*b) -
 d**4*x**4*cos(a + b*x)**3/(3*b) + 8*c**3*d*sin(a + b*x)**3/(9*b**2) + 4*c*
*3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)**3/
(3*b**2) + 4*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 8*c*d**3*x**2*
sin(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b**2
+ 8*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 4*d**4*x**3*sin(a + b*x)*cos(a + b
*x)**2/(3*b**2) + 8*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 28*c*
*2*d**2*cos(a + b*x)**3/(9*b**3) + 16*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)
/(3*b**3) + 56*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 8*d**4*x**2*sin(a + b*x)
**2*cos(a + b*x)/(3*b**3) + 28*d**4*x**2*cos(a + b*x)**3/(9*b**3) - 160*c*d
**3*sin(a + b*x)**3/(27*b**4) - 56*c*d**3*sin(a + b*x)*cos(a + b*x)**2/(9*b
**4) - 160*d**4*x*sin(a + b*x)**3/(27*b**4) - 56*d**4*x*sin(a + b*x)*cos(a
+ b*x)**2/(9*b**4) - 160*d**4*sin(a + b*x)**2*cos(a + b*x)/(27*b**5) - 488*
d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**
2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**2, True))
```

Giac [A] time = 1.20536, size = 473, normalized size = 2.31

$$-\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8\,d^4\right)\cos\left(3\,bx\right.}{324\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*
c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8
*d^4)*cos(3*b*x + 3*a)/b^5 - 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b
^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 + 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x
^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x + 3*a
)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*
d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

3.72 $\int (c+dx)^3 \cos^2(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=151

$$\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}+\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d(c+dx)^2\sin(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b^2}+$$

```
[Out] (4*d^2*(c + d*x)*Cos[a + b*x])/(3*b^3) + (2*d^2*(c + d*x)*Cos[a + b*x]^3)/(
9*b^3) - ((c + d*x)^3*Cos[a + b*x]^3)/(3*b) - (14*d^3*Sin[a + b*x])/(9*b^4)
 + (2*d*(c + d*x)^2*Sin[a + b*x])/(3*b^2) + (d*(c + d*x)^2*Cos[a + b*x]^2*S
in[a + b*x])/(3*b^2) + (2*d^3*Sin[a + b*x]^3)/(27*b^4)
```

Rubi [A] time = 0.13308, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4405, 3311, 3296, 2637, 2633}

$$\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}+\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d(c+dx)^2\sin(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```


```
[Out] (4*d^2*(c + d*x)*Cos[a + b*x])/(3*b^3) + (2*d^2*(c + d*x)*Cos[a + b*x]^3)/(
9*b^3) - ((c + d*x)^3*Cos[a + b*x]^3)/(3*b) - (14*d^3*Sin[a + b*x])/(9*b^4)
 + (2*d*(c + d*x)^2*Sin[a + b*x])/(3*b^2) + (d*(c + d*x)^2*Cos[a + b*x]^2*S
in[a + b*x])/(3*b^2) + (2*d^3*Sin[a + b*x]^3)/(27*b^4)
```


Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{d\int (c+dx)^2\cos^3(a+bx)\,dx}{b}\\ &= \frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{d(c+dx)^2\cos^2(a+bx)\sin}{3b^2}\\ &= \frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{2d(c+dx)^2\sin(a+bx)}{3b^2}+\\ &= \frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}-\\ &= \frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}-\end{aligned}$$

Mathematica [A] time = 0.960798, size = 127, normalized size = 0.84

$$\frac{-27b(c+dx)\cos(a+bx)\left(b^2(c+dx)^2-6d^2\right)-3b(c+dx)\cos(3(a+bx))\left(3b^2(c+dx)^2-2d^2\right)+2d\sin(a+bx)\left(\cos(2(a-\right.}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (-27*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 3*b*(c + d*x)*(-
2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 2*d*(-82*d^2 + 45*b^2*(c + d*
x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^
4)
```

Maple [B] time = 0.018, size = 447, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*
sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2
/27*(2+cos(b*x+a)^2)*sin(b*x+a))-3/b^3*a*d^3*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2
/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+3/
b^2*c*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x
+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+3/b^3*a^2*d^3*(-1/3*(b*x+a)*cos(b*x+a
)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-6/b^2*a*c*d^2*(-1/3*(b*x+a)*cos(b*x+a)
^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b*c^2*d*(-1/3*(b*x+a)*cos(b*x+a)^3+1/
9*(2+cos(b*x+a)^2)*sin(b*x+a))+1/3/b^3*a^3*d^3*cos(b*x+a)^3-1/b^2*a^2*c*d^2
*cos(b*x+a)^3+1/b*a*c^2*d*cos(b*x+a)^3-1/3*c^3*cos(b*x+a)^3)
```

Maxima [B] time = 1.17515, size = 682, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/108*(36*c^3*cos(b*x + a)^3 - 108*a*c^2*d*cos(b*x + a)^3/b + 108*a^2*c*d^
2*cos(b*x + a)^3/b^2 - 36*a^3*d^3*cos(b*x + a)^3/b^3 + 9*(3*(b*x + a)*cos(3
*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*c^2*d/b - 18*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - si
n(3*b*x + 3*a) - 9*sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*cos(3*b*x + 3
*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*d^3
/b^3 + 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b
```

```
*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*c*d^2/b
^2 - 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b*x
 + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*a*d^3/b^3
 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*
b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x +
 a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

Fricas [A] time = 0.496975, size = 404, normalized size = 2.68

$$-\frac{3\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+\left(9\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^3-36\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)-\left(18\,b^2d^3x\right.}{27\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*
c^2*d - 2*b*d^3)*x)*cos(b*x + a)^3 - 36*(b*d^3*x + b*c*d^2)*cos(b*x + a) -
(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - 40*d^3 + (9*b^2*d^3*x^2 +
 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

Sympy [A] time = 5.38453, size = 391, normalized size = 2.59

$$\begin{cases} -\frac{c^3\cos^3(a+bx)}{3b}-\frac{c^2dx\cos^3(a+bx)}{b}-\frac{cd^2x^2\cos^3(a+bx)}{b}-\frac{d^3x^3\cos^3(a+bx)}{3b}+\frac{2c^2d\sin^3(a+bx)}{3b^2}+\frac{c^2d\sin(a+bx)\cos^2(a+bx)}{b^2}+\frac{4cd^2x\sin^3(a+bx)}{3b^2}+ \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)**3/(3*b) - c**2*d*x*cos(a + b*x)**3/b - c*d**
2*x**2*cos(a + b*x)**3/b - d**3*x**3*cos(a + b*x)**3/(3*b) + 2*c**2*d*sin(a
 + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**2/b**2 + 4*c*d**2*x
*sin(a + b*x)**3/(3*b**2) + 2*c*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 +
2*d**3*x**2*sin(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b*x)*
*2/b**2 + 4*c*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*c*d**2*cos(a
+ b*x)**3/(9*b**3) + 4*d**3*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*d*
*3*x*cos(a + b*x)**3/(9*b**3) - 40*d**3*sin(a + b*x)**3/(27*b**4) - 14*d**3
```

```
*sin(a + b*x)*cos(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2
/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a)**2, True))
```

Giac [A] time = 1.20255, size = 312, normalized size = 2.07

$$-\frac{\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2\right)\cos\left(3\,bx+3\,a\right)}{36\,b^4}-\frac{\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^
3*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 1/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 + 1/108
*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^
4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

3.73 $\int (c+dx)^2 \cos^2(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=103

$$\frac{4d(c+dx)\sin(a+bx)}{9b^2}+\frac{2d(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^2}+\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d^2\cos(a+bx)}{9b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

```
[Out] (4*d^2*Cos[a + b*x])/(9*b^3) + (2*d^2*Cos[a + b*x]^3)/(27*b^3) - ((c + d*x)
^2*Cos[a + b*x]^3)/(3*b) + (4*d*(c + d*x)*Sin[a + b*x])/(9*b^2) + (2*d*(c +
 d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^2)
```

Rubi [A] time = 0.0790108, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4405, 3310, 3296, 2638}

$$\frac{4d(c+dx)\sin(a+bx)}{9b^2}+\frac{2d(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^2}+\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d^2\cos(a+bx)}{9b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```


```
[Out] (4*d^2*Cos[a + b*x])/(9*b^3) + (2*d^2*Cos[a + b*x]^3)/(27*b^3) - ((c + d*x)
^2*Cos[a + b*x]^3)/(3*b) + (4*d*(c + d*x)*Sin[a + b*x])/(9*b^2) + (2*d*(c +
 d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^2)
```


Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^2\cos^3(a+bx)}{3b}+\frac{(2d)\int (c+dx)\cos^3(a+bx)\,dx}{3b}$$
$$=\frac{2d^2\cos^3(a+bx)}{27b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}+\frac{2d(c+dx)\cos^2(a+bx)\sin(a+}{9b^2}$$
$$=\frac{2d^2\cos^3(a+bx)}{27b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}+\frac{4d(c+dx)\sin(a+bx)}{9b^2}+\frac{2d(c+}{}$$
$$=\frac{4d^2\cos(a+bx)}{9b^3}+\frac{2d^2\cos^3(a+bx)}{27b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}+\frac{4d(c+dx)\sin}{9b^2}$$

Mathematica [A] time = 0.509733, size = 86, normalized size = 0.83

$$-\frac{27\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)+\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)-6bd(c+dx)(9\sin(a+bx)+\sin(3(a+bx)))}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(27*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c + d*x)^2)
*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(9*Sin[a + b*x] + Sin[3*(a + b*x)]))/(1
08*b^3)
```

Maple [B] time = 0.018, size = 204, normalized size = 2.

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(-\frac{(bx+a)^2(\cos(bx+a))^3}{3}+\frac{(2bx+2a)\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{9}+\frac{2(\cos(bx+a))^3}{27}+\frac{4\cos(bx+a)}{9}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*si
n(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-2/b^2*a*d^2*(-1/3*(b*x+a)*cos(b*
x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+2/b*c*d*(-1/3*(b*x+a)*cos(b*x+a)^3+
1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-1/3/b^2*a^2*d^2*cos(b*x+a)^3+2/3/b*a*c*d*c
os(b*x+a)^3-1/3*c^2*cos(b*x+a)^3)
```

Maxima [B] time = 1.20785, size = 328, normalized size = 3.18

$$-\frac{36\,c^2\cos\left(bx+a\right)^3-\frac{72\,acd\cos(bx+a)^3}{b}+\frac{36\,a^2d^2\cos(bx+a)^3}{b^2}+\frac{6\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))cd}{b}-\frac{6\,(3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/108*(36*c^2*cos(b*x + a)^3 - 72*a*c*d*cos(b*x + a)^3/b + 36*a^2*d^2*cos(
b*x + a)^3/b^2 + 6*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
 - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x + 3*
a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a*d^2/b^
2 + ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b*x +
a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A] time = 0.482611, size = 236, normalized size = 2.29

$$-\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^3-12\,d^2\cos\left(bx+a\right)-6\left(2\,bd^2x+2\,bcd+\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2\right)}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 -
12*d^2*cos(b*x + a) - 6*(2*b*d^2*x + 2*b*c*d + (b*d^2*x + b*c*d)*cos(b*x +
a)^2)*sin(b*x + a))/b^3
```

Sympy [A] time = 2.62664, size = 216, normalized size = 2.1

$$\begin{cases} -\frac{c^2\cos^3(a+bx)}{3b} - \frac{2cdx\cos^3(a+bx)}{3b} - \frac{d^2x^2\cos^3(a+bx)}{3b} + \frac{4cd\sin^3(a+bx)}{9b^2} + \frac{2cd\sin(a+bx)\cos^2(a+bx)}{3b^2} + \frac{4d^2x\sin^3(a+bx)}{9b^2} + \frac{2d^2x\sin(a+bx)\cos^2(a}{3b^2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)**3/(3*b) - 2*c*d*x*cos(a + b*x)**3/(3*b) - d*
*2*x**2*cos(a + b*x)**3/(3*b) + 4*c*d*sin(a + b*x)**3/(9*b**2) + 2*c*d*sin(
a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*x*sin(a + b*x)**3/(9*b**2) + 2*d
**2*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*sin(a + b*x)**2*cos(a
+ b*x)/(9*b**3) + 14*d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x +
c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a)**2, True))
```

Giac [A] time = 1.15306, size = 185, normalized size = 1.8

$$-\frac{\left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 2\,d^2\right)\cos(3\,bx + 3\,a)}{108\,b^3} - \frac{\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,d^2\right)\cos(bx + a)}{4\,b^3} + \frac{\left(bd^2x + bcd\right)s}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)/
b^3 - 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 +
1/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*sin(b*x
 + a)/b^3
```

3.74 $\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=51

$$-\frac{d\sin^3(a+bx)}{9b^2}+\frac{d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

```
[Out] -((c + d*x)*Cos[a + b*x]^3)/(3*b) + (d*Sin[a + b*x])/(3*b^2) - (d*Sin[a + b
*x]^3)/(9*b^2)
```

Rubi [A] time = 0.0342699, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4405, 2633}

$$-\frac{d\sin^3(a+bx)}{9b^2}+\frac{d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)*Cos[a + b*x]^3)/(3*b) + (d*Sin[a + b*x])/(3*b^2) - (d*Sin[a + b
*x]^3)/(9*b^2)
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)\cos^3(a+bx)}{3b} + \frac{d\int \cos^3(a+bx)\,dx}{3b}$$
$$= -\frac{(c+dx)\cos^3(a+bx)}{3b} - \frac{d\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(a+bx)\right)}{3b^2}$$
$$= -\frac{(c+dx)\cos^3(a+bx)}{3b} + \frac{d\sin(a+bx)}{3b^2} - \frac{d\sin^3(a+bx)}{9b^2}$$

Mathematica [A] time = 0.153405, size = 71, normalized size = 1.39

$$\frac{d(\sin(a+bx)-bx\cos(a+bx))}{4b^2} + \frac{d(\sin(3(a+bx))-3bx\cos(3(a+bx)))}{36b^2} - \frac{c\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(c*Cos[a + b*x]^3)/(3*b) + (d*(-(b*x*Cos[a + b*x]) + Sin[a + b*x]))/(4*b^2
) + (d*(-3*b*x*Cos[3*(a + b*x)] + Sin[3*(a + b*x)]))/(36*b^2)
```

Maple [A] time = 0.018, size = 71, normalized size = 1.4

$$\frac{1}{b}\left(\frac{d}{b}\left(-\frac{(bx+a)(\cos(bx+a))^3}{3} + \frac{\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{9}\right) + \frac{ad(\cos(bx+a))^3}{3b} - \frac{(\cos(bx+a))^3c}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(d/b*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+1/3/b*
d*a*cos(b*x+a)^3-1/3*cos(b*x+a)^3*c)
```

Maxima [A] time = 1.11705, size = 116, normalized size = 2.27

$$-\frac{12\,c\cos(bx+a)^3 - \frac{12\,ad\cos(bx+a)^3}{b} + \frac{(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/36*(12*c*cos(b*x + a)^3 - 12*a*d*cos(b*x + a)^3/b + (3*(b*x + a)*cos(3*b
*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*d
/b)/b
```

Fricas [A] time = 0.474018, size = 112, normalized size = 2.2

$$-\frac{3\,(bdx+bc)\cos\left(bx+a\right)^{3}-\left(d\cos\left(bx+a\right)^{2}+2\,d\right)\sin\left(bx+a\right)}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/9*(3*(b*d*x + b*c)*cos(b*x + a)^3 - (d*cos(b*x + a)^2 + 2*d)*sin(b*x + a
))/b^2
```

Sympy [A] time = 1.13508, size = 85, normalized size = 1.67

$$\begin{cases} -\frac{c\cos^{3}(a+bx)}{3b}-\frac{dx\cos^{3}(a+bx)}{3b}+\frac{2d\sin^{3}(a+bx)}{9b^{2}}+\frac{d\sin(a+bx)\cos^{2}(a+bx)}{3b^{2}} & \text{for } b\neq 0\\ \left(cx+\frac{dx^{2}}{2}\right)\sin\left(a\right)\cos^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)**3/(3*b) - d*x*cos(a + b*x)**3/(3*b) + 2*d*sin(a
 + b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b, 0)), (
(c*x + d*x**2/2)*sin(a)*cos(a)**2, True))
```

Giac [A] time = 1.20394, size = 93, normalized size = 1.82

$$-\frac{(bdx+bc)\cos\left(3\,bx+3\,a\right)}{12\,b^{2}}-\frac{(bdx+bc)\cos\left(bx+a\right)}{4\,b^{2}}+\frac{d\sin\left(3\,bx+3\,a\right)}{36\,b^{2}}+\frac{d\sin\left(bx+a\right)}{4\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 1/4*(b*d*x + b*c)*cos(b*x + a)/b
^2 + 1/36*d*sin(3*b*x + 3*a)/b^2 + 1/4*d*sin(b*x + a)/b^2
```

3.75 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-3\right.}{}$$

```
[Out] (CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (CosIntegral[
(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rubi [A] time = 0.22444, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (CosIntegral[
(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)}+\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx \\
&= \frac{1}{4}\int \frac{\sin(a+bx)}{c+dx}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{c+dx}\,dx \\
&= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx+\frac{1}{4}\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\frac{1}{4}\sin \\
&= \frac{\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d}+\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.304082, size = 100, normalized size = 0.83

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + CosIntegral[b*(c/d +
 x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A] time = 0.02, size = 167, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b}{12}\left(3\frac{1}{d}\mathrm{Si}\left(3bx+3a+3\frac{-ad+bc}{d}\right)\cos\left(3\frac{-ad+bc}{d}\right)-3\frac{1}{d}\mathrm{Ci}\left(3bx+3a+3\frac{-ad+bc}{d}\right)\sin\left(3\frac{-ad+bc}{d}\right)\right)+\frac{b}{4}\left(\frac{1}{d}\mathrm{Si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x)
```

```
[Out] 1/b*(1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*
x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+1/4*b*(Si(b*x+a+(-a*d+b*c)/d)*
cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d))
```

Maxima [C] time = 1.38316, size = 369, normalized size = 3.05

$$-\frac{b\left(iE_1\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_1\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(iE_1\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right)-iE_1\left(-\frac{3ibc+3i(bx+a)d-3iad}{d}\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_int
egral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(I*
exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral
_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*
(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -
(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_integral_e
(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(3*I*b*c
+ 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 0.481807, size = 404, normalized size = 3.34

$$\frac{\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ci}\left(\frac{3(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3(bc-ad)}{d}\right)+2\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3(bd}{}\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*sin(-
(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x
+ b*c)/d))*sin(-3*(b*c - a*d)/d) + 2*cos(-3*(b*c - a*d)/d)*sin_integral(3*(
b*d*x + b*c)/d) + 2*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x), x)
```

Giac [C] time = 1.77063, size = 8477, normalized size = 70.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_inte
gral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
```

```
)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - i
mag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*imag_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
+ 8*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((
b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2
*b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*d*x + b*c)/d)*ta
n(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + imag_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b
*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(c
os_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*real_par
t(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*r
eal_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)
^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan
(3/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(
1/2*a)^2*tan(3/2*b*c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*
```

a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x + b
*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
+ 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) - 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a
)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*t
an(1/2*a)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan
(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c
/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*rea
l_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan
(1/2*a)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2
+ imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + imag_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 2*sin_integr
al(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2 - 2*sin_integral((b*d*x + b
*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2 + 4*imag_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 4*imag_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 8*sin_integral(3*(b*d
*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + imag_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + imag_part(cos_integral(
b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*t
an(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2
*tan(3/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*t
an(3/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2
*b*c/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*s
in_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*imag_part(co
s_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*s
in_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 4*ima
g_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
) - 4*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan

```
(1/2*b*c/d) + 8*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1
/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c
/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2
 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2
- 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 2*sin_i
ntegral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integr
al(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d
)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 + 4*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a
)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c
/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - imag_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integ
ral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integra
l(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral
(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x +
b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(b*x +
b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 2*real_part(cos_integral(-b*x - b*c/d))*t
an(3/2*a)^2*tan(1/2*a) + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*a)*tan(1/2*a)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*t
an(1/2*a)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3
/2*b*c/d) + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/
2*b*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*
b*c/d) - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b
*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d
)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)
^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) - 2*real_part(
cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integra
l(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x
 + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c
/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))
```

```
*tan(3/2*a)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2 - imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2 + imag_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*a)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2
+ 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2 + imag_part(cos_integral(3*b
*x + 3*b*c/d))*tan(1/2*a)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(1/2*
a)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - imag_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)
*tan(1/2*a)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - 4*imag_part(cos_
integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 8*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d) - imag_part(cos_integral(3*b*x + 3
*b*c/d))*tan(3/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*
c/d)^2 - imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2 + imag_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x
 + b*c)/d)*tan(3/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*c/d
)^2 + 4*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 4*
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 8*sin_int
egral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d) + imag_part(cos_integral(3
*b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*ta
n(1/2*b*c/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 + 2*sin_integral(
3*(b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(1
/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*rea
l_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a) + 2*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*a) + 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/
2*a) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) - 2*real_part(cos_integr
al(b*x + b*c/d))*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*c/d) + imag_part(cos_integral(3*b*x + 3*b*c/d)) + imag_part(cos_in
tegral(b*x + b*c/d)) - imag_part(cos_integral(-b*x - b*c/d)) - imag_part(co
s_integral(-3*b*x - 3*b*c/d)) + 2*sin_integral(3*(b*d*x + b*c)/d) + 2*sin_i
ntegral((b*d*x + b*c)/d))/(d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^
2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b
*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2 + d*tan(1/2*a)^2 +
d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
```

3.76 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=168

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\sin}{}$$

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]/(4*d*(c + d
*x)) - Sin[3*a + 3*b*x]/(4*d*(c + d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(
b*c)/d + b*x])/(4*d^2) - (3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d +
3*b*x])/(4*d^2)
```

Rubi [A] time = 0.266386, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]/(4*d*(c + d
*x)) - Sin[3*a + 3*b*x]/(4*d*(c + d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(
b*c)/d + b*x])/(4*d^2) - (3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d +
3*b*x])/(4*d^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)^2}+\frac{\sin(3a+3bx)}{4(c+dx)^2}\right)dx \\
&= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx \\
&= -\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}+\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{4d}+\frac{(3b)\int \frac{\cos(3a+3bx)}{c+dx}\,dx}{4d} \\
&= -\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}+\frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right.\right.}{} \\
&= \frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a}{4d(c}
\end{aligned}
$$

Mathematica [A] time = 1.13457, size = 139, normalized size = 0.83

$$-\frac{-b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+3b}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -(-(b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) - 3*b*Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*(c + d*x))/d] + (d*Sin[a + b*x])/(c + d*x) + (d*Sin[3*(a
+ b*x)])/(c + d*x) + b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*Sin[
3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d^2)
```

Maple [A] time = 0.022, size = 240, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^2}{12}\left(-3\frac{\sin(3bx+3a)}{((bx+a)d-ad+bc)d}+3\frac{1}{d}\left(3\frac{1}{d}\text{Si}\left(3bx+3a+3\frac{-ad+bc}{d}\right)\sin\left(3\frac{-ad+bc}{d}\right)+3\frac{1}{d}\text{Ci}\left(3bx+3a+3\frac{-ad+}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/12*b^2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*Si(3*b*x+3*a+3*
(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-
a*d+b*c)/d)/d)/d)+1/4*b^2*(-sin(b*x+a)/((b*x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a*
d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d
))
```

Maxima [C] time = 1.72511, size = 405, normalized size = 2.41

$$-\frac{b^2\left(iE_2\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_2\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(iE_2\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right)-iE_2\left(-\frac{3ibc+3i(bx+a)d-3iad}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2
*(I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
+ b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(exp_in
tegral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(2, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (
b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 0.538292, size = 586, normalized size = 3.49

$$-\frac{8\,d\cos\left(bx+a\right)^2\sin\left(bx+a\right)+6\left(bdx+bc\right)\sin\left(-\frac{3\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{3\left(bdx+bc\right)}{d}\right)+2\left(bdx+bc\right)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-\left((bd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*d*cos(b*x + a)^2*sin(b*x + a) + 6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/
d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*si
n_integral((b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)
+ (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) - 3*((b
*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-3
*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**2, x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2\sin(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sin(b*x + a)/(d*x + c)^2, x)
```

3.77 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=221

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-$$

```
[Out] -(b*Cos[a + b*x])/(8*d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(8*d^2*(c + d*
x)) - (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^3) -
 (b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - Sin[a + b*x]/(
8*d*(c + d*x)^2) - Sin[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]*SinInt
egral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rubi [A] time = 0.324418, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```


```
[Out] -(b*Cos[a + b*x])/(8*d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(8*d^2*(c + d*
x)) - (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^3) -
 (b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - Sin[a + b*x]/(
8*d*(c + d*x)^2) - Sin[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]*SinInt
egral[(3*b*c)/d + 3*b*x])/(8*d^3)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(a+bx)}{4(c+dx)^3}+\frac{\sin(3a+3bx)}{4(c+dx)^3}\right)dx$$

$$= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx + \frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{8d} + \frac{(3b)\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx}{8d}$$

$$= -\frac{b\cos(a+bx)}{8d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{8d^2(c+dx)} - \frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} - \frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{8d^2} -$$

$$= -\frac{b\cos(a+bx)}{8d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{8d^2(c+dx)} - \frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} - \frac{\left(9b^2\cos\left(3a-\frac{3bc}{d}\right.\right.}{8}$$

$$= -\frac{b\cos(a+bx)}{8d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{8d^2(c+dx)} - \frac{9b^2\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3} - \frac{b^2\text{Ci}\left(\frac{bc}{d}+bx\right.}{8d}$$

Mathematica [A] time = 2.6147, size = 181, normalized size = 0.82

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(9*b^2*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b^2*CosIntegr
al[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin[a +
 b*x]))/(c + d*x)^2 + (d*(3*b*(c + d*x)*Cos[3*(a + b*x)] + d*Sin[3*(a + b*x
)]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 9*b^2*Co
s[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(8*d^3)
```

Maple [A] time = 0.02, size = 313, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^3}{12}\left(-\frac{3\sin(3bx+3a)}{2\left((bx+a)d-ad+bc\right)^2 d}+\frac{3}{2d}\left(-3\frac{\cos(3bx+3a)}{\left((bx+a)d-ad+bc\right)d}-3\frac{1}{d}\left(3\frac{1}{d}\text{Si}\left(3bx+3a+3\frac{-ad+bc}{d}\right)\cos\left(3\frac{-a}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/12*b^3*(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^2/d+3/2*(-3*cos(3*b*
x+3*a)/((b*x+a)*d-a*d+b*c)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+
b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)+1/4*b
^3*(-1/2*sin(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-a*d
+b*c)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*
sin((-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] time = 2.17134, size = 452, normalized size = 2.05

$$-\frac{b^3\left(iE_3\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_3\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(iE_3\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right)-iE_3\left(-\frac{3ibc+3i(bx+a)d-3ia}{d}\right.\right.}{8\left(b^2c^2d-2ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3
*(I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
+ b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(exp_in
tegral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(3, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*b)
```

Fricas [A] time = 0.589684, size = 907, normalized size = 4.1

$$-\frac{8\,d^2\cos(bx+a)^2\sin(bx+a) + 24\left(bd^2x + bcd\right)\cos(bx+a)^3 + 18\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(8*d^2*cos(b*x + a)^2*sin(b*x + a) + 24*(b*d^2*x + b*c*d)*cos(b*x + a
)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin_in
tegral(3*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b
*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(b*d^2*x + b*c*d)*cos(b*x +
 a) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)/d))*sin
(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(3*
(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-3*(b
*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.78 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4} dx$

Optimal. Leaf size=270

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}+\cdots$$

```
[Out] -(b*Cos[a + b*x])/(24*d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(8*d^2*(c + d
*x)^2) - (b^3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^
3*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^4) - Sin[a + b*
x]/(12*d*(c + d*x)^3) + (b^2*Sin[a + b*x])/(24*d^3*(c + d*x)) - Sin[3*a + 3
*b*x]/(12*d*(c + d*x)^3) + (3*b^2*Sin[3*a + 3*b*x])/(8*d^3*(c + d*x)) + (b^
3*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) + (9*b^3*Sin[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Rubi [A] time = 0.377452, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```


```
[Out] -(b*Cos[a + b*x])/(24*d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(8*d^2*(c + d
*x)^2) - (b^3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^
3*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^4) - Sin[a + b*
x]/(12*d*(c + d*x)^3) + (b^2*Sin[a + b*x])/(24*d^3*(c + d*x)) - Sin[3*a + 3
*b*x]/(12*d*(c + d*x)^3) + (3*b^2*Sin[3*a + 3*b*x])/(8*d^3*(c + d*x)) + (b^
3*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) + (9*b^3*Sin[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)^4} + \frac{\sin(3a+3bx)}{4(c+dx)^4}\right) dx \\
&= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^4}\,dx + \frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^4}\,dx \\
&= -\frac{\sin(a+bx)}{12d(c+dx)^3} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx}{12d} + \frac{b\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx}{4d} \\
&= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{\sin(a+bx)}{12d(c+dx)^3} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} - \frac{b^2\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{24d^2} - \\
&= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{\sin(a+bx)}{12d(c+dx)^3} + \frac{b^2\sin(a+bx)}{24d^3(c+dx)} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} + \\
&= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{\sin(a+bx)}{12d(c+dx)^3} + \frac{b^2\sin(a+bx)}{24d^3(c+dx)} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} + \\
&= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{24d^4} - \frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{C}}{8d^4}
\end{aligned}
$$

Mathematica [A] time = 1.93298, size = 300, normalized size = 1.11

$$
-\frac{b^3(c+dx)^3\left(\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)+27b^3(c+dx)^3\left(\cos\left(3a-\frac{3bc}{d}\right)\text{CosInt}\right.}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) + d
*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Sin[3*
a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a])*Sin[b*x]
 - d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a])*Sin
[3*b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Sin[
a - (b*c)/d]*SinIntegral[b*(c/d + x)]) + 27*b^3*(c + d*x)^3*(Cos[3*a - (3*b
*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*b*c)/d]*SinIntegral[(3
*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

Maple [A] time = 0.023, size = 381, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^4}{12}\left(-\frac{\sin(3bx+3a)}{((bx+a)d-ad+bc)^3 d}+\frac{1}{d}\left(-\frac{3\cos(3bx+3a)}{2((bx+a)d-ad+bc)^2 d}-\frac{3}{2d}\left(-3\frac{\sin(3bx+3a)}{((bx+a)d-ad+bc)d}+3\frac{1}{d}\left(3\frac{1}{d}\mathrm{Si}\left(3b\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/12*b^4*(-sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^3/d+(-3/2*cos(3*b*x+3*a)
/((b*x+a)*d-a*d+b*c)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*
Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b
*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)/d)+1/4*b^4*(-1/3*sin(b*x+a)/((b*x+a)*d-
a*d+b*c)^3/d+1/3*(-1/2*cos(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d-1/2*(-sin(b*x+a)/
((b*x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+
(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d))
```

Maxima [C] time = 2.78691, size = 520, normalized size = 1.93

$$-\frac{b^4\left(iE_4\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_4\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(iE_4\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right)-iE_4\left(-\frac{3ibc+3i(bx+a)d-3ia}{d}\right.\right.}{8\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3+(bx+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/8*(b^4*(I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^4
*(I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
+ b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(exp_in
tegral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(4, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d
 - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3
 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*
b)
```

Fricas [B] time = 0.625508, size = 1231, normalized size = 4.56

$$-\frac{24\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^3-54\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-2\left(b^3d^3x^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - 54*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*
x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*s
in(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(b*d^3*x + b*c*d^2)*c
os(b*x + a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*co
s_integral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 27*((b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x
 + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_
integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 8*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d -
 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x
+ c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**4, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.79 $\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=162

$$\frac{i2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{i2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(\right.}{b}$$

```
[Out] (c + d*x)^(1 + m)/(8*d*(1 + m)) + (I*E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)
^m) - (I*(c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*
E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.2048, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3307, 2181}

$$\frac{i2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{i2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(8*d*(1 + m)) + (I*E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)
^m) - (I*(c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*
E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\cos^2(a+bx)\sin^2(a+bx)\,dx &= \int\left(\frac{1}{8}(c+dx)^m-\frac{1}{8}(c+dx)^m\cos(4a+4bx)\right)dx\\ &= \frac{(c+dx)^{1+m}}{8d(1+m)}-\frac{1}{8}\int(c+dx)^m\cos(4a+4bx)\,dx\\ &= \frac{(c+dx)^{1+m}}{8d(1+m)}-\frac{1}{16}\int e^{-i(4a+4bx)}(c+dx)^m\,dx-\frac{1}{16}\int e^{i(4a+4bx)}(c+dx)^m\,dx\\ &= \frac{(c+dx)^{1+m}}{8d(1+m)}+\frac{i4^{-3-m}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib(c+dx)}{d}\right)}{b}-\end{aligned}$$

Mathematica [A] time = 1.10385, size = 213, normalized size = 1.31

$$\frac{4^{-m-3}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-id(m+1)\left(-\frac{ib(c+dx)}{d}\right)^m\left(\cos\left(4a-\frac{4bc}{d}\right)-i\sin\left(4a-\frac{4bc}{d}\right)\right)\text{Gamma}\left(m+1,\frac{4ib(c+dx)}{d}\right)+id}{bd(m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (4^(-3 - m)*(c + d*x)^m*(2^(3 + 2*m)*b*(c + d*x)*((b^2*(c + d*x)^2)/d^2)^m
- I*d*(1 + m)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d]*
(Cos[4*a - (4*b*c)/d] - I*Sin[4*a - (4*b*c)/d]) + I*d*(1 + m)*((I*b*(c + d*
x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d]*(Cos[4*a - (4*b*c)/d] + I*Sin
[4*a - (4*b*c)/d])))/(b*d*(1 + m)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F] time = 0.209, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cos (bx+a))^2 (\sin (bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(dm+d)\int (dx+c)^m \cos (4\,bx+4\,a) \, dx - e^{(m\log(dx+c)+\log(dx+c))}}{8\,(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((d*m + d)*integrate((d*x + c)^m*cos(4*b*x + 4*a), x) - e^(m*log(d*x +
 c) + log(d*x + c)))/(d*m + d)
```

Fricas [A] time = 0.526546, size = 342, normalized size = 2.11

$$\frac{(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{4i\,b}{d}\right)-4i\,bc+4i\,ad}{d}\right)}\Gamma\left(m+1,\frac{4i\,bdx+4i\,bc}{d}\right)+(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{4i\,b}{d}\right)+4i\,bc-4i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-4i\,bdx-4i\,bc}{d}\right)+8\,(bdx+}{64\,(bdm+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*((-I*d*m - I*d)*e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m
+ 1, (4*I*b*d*x + 4*I*b*c)/d) + (I*d*m + I*d)*e^(-(d*m*log(-4*I*b/d) + 4*I*
b*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d) + 8*(b*d*x + b*c)*
(d*x + c)^m)/(b*d*m + b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \sin^2(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^2, x)
```

3.80 $\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx$

Optimal. Leaf size=131

$$\frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{3d^4\sin(4a+4bx)}{1024b^5}-\frac{(c+dx)^4\sin(}{32b}$$

```
[Out] (c + d*x)^5/(40*d) + (3*d^3*(c + d*x)*Cos[4*a + 4*b*x])/(256*b^4) - (d*(c +
 d*x)^3*Cos[4*a + 4*b*x])/(32*b^2) - (3*d^4*Sin[4*a + 4*b*x])/(1024*b^5) +
(3*d^2*(c + d*x)^2*Sin[4*a + 4*b*x])/(128*b^3) - ((c + d*x)^4*Sin[4*a + 4*b
*x])/(32*b)
```

Rubi [A] time = 0.1644, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$\frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{3d^4\sin(4a+4bx)}{1024b^5}-\frac{(c+dx)^4\sin(}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^5/(40*d) + (3*d^3*(c + d*x)*Cos[4*a + 4*b*x])/(256*b^4) - (d*(c +
 d*x)^3*Cos[4*a + 4*b*x])/(32*b^2) - (3*d^4*Sin[4*a + 4*b*x])/(1024*b^5) +
(3*d^2*(c + d*x)^2*Sin[4*a + 4*b*x])/(128*b^3) - ((c + d*x)^4*Sin[4*a + 4*b
*x])/(32*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\sin^2(a+bx)\,dx=\int\left(\frac{1}{8}(c+dx)^4-\frac{1}{8}(c+dx)^4\cos(4a+4bx)\right)dx$$
$$=\frac{(c+dx)^5}{40d}-\frac{1}{8}\int (c+dx)^4\cos(4a+4bx)\,dx$$
$$=\frac{(c+dx)^5}{40d}-\frac{(c+dx)^4\sin(4a+4bx)}{32b}+\frac{d\int (c+dx)^3\sin(4a+4bx)\,dx}{8b}$$
$$=\frac{(c+dx)^5}{40d}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{(c+dx)^4\sin(4a+4bx)}{32b}+\frac{\left(3d^2\right)\int (c$$
$$=\frac{(c+dx)^5}{40d}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}+\frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3}-\frac{(c+d$$
$$=\frac{(c+dx)^5}{40d}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}+\frac{3d^2(c+$$
$$=\frac{(c+dx)^5}{40d}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{3d^4\sin}{1}$$

Mathematica [A] time = 1.36655, size = 132, normalized size = 1.01

$$\frac{-5\sin(4(a+bx))\left(-24b^2d^2(c+dx)^2+32b^4(c+dx)^4+3d^4\right)+20bd(c+dx)\cos(4(a+bx))\left(3d^2-8b^2(c+dx)^2\right)+128b^5}{5120b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (128*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) +
20*b*d*(c + d*x)*(3*d^2 - 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 5*(3*d^4 -
24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Sin[4*(a + b*x)])/(5120*b^5)
```

Maple [B] time = 0.076, size = 1915, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*((b*x+a)^4*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b
*x+a)^3*cos(b*x+a)^2+3/4*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a
)+3/32*(b*x+a)*cos(b*x+a)^2-3/64*cos(b*x+a)*sin(b*x+a)-21/256*b*x-21/256*a-
7/16*(b*x+a)^3-1/10*(b*x+a)^5-(b*x+a)^4*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))
*cos(b*x+a)+3/8*b*x+3/8*a)-1/4*(b*x+a)^3*sin(b*x+a)^4+3/4*(b*x+a)^2*(-1/4*(
sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*sin(b*x
+a)^4+3/128*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))-4/b^4*a*d^4*((b*x+a)^
3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)^2*cos(b*x+a)^2+3/
8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/128*(b*x+a)^2-3/128*
sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*c
os(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*x+a)*(-1/4*(sin
(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/128*sin(b*x+a)^4)+4/b
^3*c*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)
^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/12
8*(b*x+a)^2-3/128*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3
+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*
(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/128
*sin(b*x+a)^4)+6/b^4*a^2*d^4*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x
+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a
-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/
8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*co
s(b*x+a))-12/b^3*a*c*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2
*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/1
2*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*
x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a))+6/b^2*c^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*
x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8
*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))
-4/b^4*a^3*d^4*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*
x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+12/b^3*a^2*c*d^3*((b*x+a)*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(
-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a
)^4)-12/b^2*a*c^2*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))
*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+4/b*c^3*d*((b*x+a)*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(
-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a
)^4)+1/b^4*a^4*d^4*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+
1/8*b*x+1/8*a)-4/b^3*a^3*c*d^3*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)
*sin(b*x+a)+1/8*b*x+1/8*a)+6/b^2*a^2*c^2*d^2*(-1/4*sin(b*x+a)*cos(b*x+a)^3+
```

```
1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-4/b*a*c^3*d*(-1/4*sin(b*x+a)*cos(b
*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c^4*(-1/4*sin(b*x+a)*cos(b
*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

Maxima [B] time = 1.37141, size = 992, normalized size = 7.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5120*(160*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^4 - 640*(4*b*x + 4*a - sin(4
*b*x + 4*a))*a*c^3*d/b + 960*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c^2*d^2/b
^2 - 640*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*c*d^3/b^3 + 160*(4*b*x + 4*a
- sin(4*b*x + 4*a))*a^4*d^4/b^4 + 160*(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*
x + 4*a) - cos(4*b*x + 4*a))*c^3*d/b - 480*(8*(b*x + a)^2 - 4*(b*x + a)*sin
(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c^2*d^2/b^2 + 480*(8*(b*x + a)^2 - 4*(b
*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*c*d^3/b^3 - 160*(8*(b*x +
a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^3*d^4/b^4 + 40*(3
2*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4
*b*x + 4*a))*c^2*d^2/b^2 - 80*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*
a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*c*d^3/b^3 + 40*(32*(b*x + a)
^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a)
)*a^2*d^4/b^4 + 20*(32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a)
 - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a))*c*d^3/b^3 - 20*(32*(b*
x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*
x - 3*a)*sin(4*b*x + 4*a))*a*d^4/b^4 + (128*(b*x + a)^5 - 20*(8*(b*x + a)^3
 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 5*(32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*
sin(4*b*x + 4*a))*d^4/b^4)/b
```

Fricas [B] time = 0.531492, size = 986, normalized size = 7.53

$$\frac{32\,b^5d^4x^5 + 160\,b^5cd^3x^4 - 40\left(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 8\,b^3c^3d - 3\,bcd^3 + 3\left(8\,b^3c^2d^2 - bd^4\right)x\right)\cos\left(bx + a\right)^4 + 40\left(8\,b^5c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1280*(32*b^5*d^4*x^5 + 160*b^5*c*d^3*x^4 - 40*(8*b^3*d^4*x^3 + 24*b^3*c*d
^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a
)^4 + 40*(8*b^5*c^2*d^2 - b^3*d^4)*x^3 + 40*(8*b^5*c^3*d - 3*b^3*c*d^3)*x^2
 + 40*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^
3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(32*b^5*c^4 - 24*b^3*c^2*d^2 + 3*b
*d^4)*x - 5*(2*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4 - 24*b^2*c^
2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^3*d - 3*b^2*
c*d^3)*x)*cos(b*x + a)^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4
 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^
3*d - 3*b^2*c*d^3)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

Sympy [A] time = 17.6012, size = 1209, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sin(a + b*x)**4/8 + c**4*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**4*x*cos(a + b*x)**4/8 + c**3*d*x**2*sin(a + b*x)**4/4 + c**3*d*x**
2*sin(a + b*x)**2*cos(a + b*x)**2/2 + c**3*d*x**2*cos(a + b*x)**4/4 + c**2*
d**2*x**3*sin(a + b*x)**4/4 + c**2*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**
2/2 + c**2*d**2*x**3*cos(a + b*x)**4/4 + c*d**3*x**4*sin(a + b*x)**4/8 + c*
d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*d**3*x**4*cos(a + b*x)**4/8
 + d**4*x**5*sin(a + b*x)**4/40 + d**4*x**5*sin(a + b*x)**2*cos(a + b*x)**2
/20 + d**4*x**5*cos(a + b*x)**4/40 + c**4*sin(a + b*x)**3*cos(a + b*x)/(8*b
) - c**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c**3*d*x*sin(a + b*x)**3*cos(
a + b*x)/(2*b) - c**3*d*x*sin(a + b*x)*cos(a + b*x)**3/(2*b) + 3*c**2*d**2*
x**2*sin(a + b*x)**3*cos(a + b*x)/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x)*cos
(a + b*x)**3/(4*b) + c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)/(2*b) - c*d**
3*x**3*sin(a + b*x)*cos(a + b*x)**3/(2*b) + d**4*x**4*sin(a + b*x)**3*cos(a
 + b*x)/(8*b) - d**4*x**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c**3*d*sin(a
 + b*x)**2*cos(a + b*x)**2/(4*b**2) - 3*c**2*d**2*x*sin(a + b*x)**4/(32*b**
2) + 9*c**2*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - 3*c**2*d**2*
x*cos(a + b*x)**4/(32*b**2) - 3*c*d**3*x**2*sin(a + b*x)**4/(32*b**2) + 9*c
*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - 3*c*d**3*x**2*cos(a
+ b*x)**4/(32*b**2) - d**4*x**3*sin(a + b*x)**4/(32*b**2) + 3*d**4*x**3*sin
(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - d**4*x**3*cos(a + b*x)**4/(32*b**2
) - 3*c**2*d**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**3) + 3*c**2*d**2*sin(a
+ b*x)*cos(a + b*x)**3/(32*b**3) - 3*c*d**3*x*sin(a + b*x)**3*cos(a + b*x)/
(16*b**3) + 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**3) - 3*d**4*x**2
```

```
*sin(a + b*x)**3*cos(a + b*x)/(32*b**3) + 3*d**4*x**2*sin(a + b*x)*cos(a +
b*x)**3/(32*b**3) - 3*c*d**3*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**4) + 3*
d**4*x*sin(a + b*x)**4/(256*b**4) - 9*d**4*x*sin(a + b*x)**2*cos(a + b*x)**
2/(128*b**4) + 3*d**4*x*cos(a + b*x)**4/(256*b**4) + 3*d**4*sin(a + b*x)**3
*cos(a + b*x)/(256*b**5) - 3*d**4*sin(a + b*x)*cos(a + b*x)**3/(256*b**5),
Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4
*x**5/5)*sin(a)**2*cos(a)**2, True))
```

Giac [A] time = 1.16027, size = 302, normalized size = 2.31

$$\frac{1}{40}\,d^4x^5 + \frac{1}{8}\,cd^3x^4 + \frac{1}{4}\,c^2d^2x^3 + \frac{1}{4}\,c^3dx^2 + \frac{1}{8}\,c^4x - \frac{\left(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 24\,b^3c^2d^2x + 8\,b^3c^3d - 3\,bd^4x - 3\,bcd^3\right)\cos}{256\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/40*d^4*x^5 + 1/8*c*d^3*x^4 + 1/4*c^2*d^2*x^3 + 1/4*c^3*d*x^2 + 1/8*c^4*x
- 1/256*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d
- 3*b*d^4*x - 3*b*c*d^3)*cos(4*b*x + 4*a)/b^5 - 1/1024*(32*b^4*d^4*x^4 + 12
8*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*c^3*d*x + 32*b^4*c^4 - 24*b
^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3*d^4)*sin(4*b*x + 4*a)/b^5
```

3.81 $\int (c+dx)^3 \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=105

$$\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}+\frac{3d^3\cos(4a+4bx)}{1024b^4}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(c+dx)^4}{32d}$$

```
[Out] (c + d*x)^4/(32*d) + (3*d^3*Cos[4*a + 4*b*x])/(1024*b^4) - (3*d*(c + d*x)^2
*Cos[4*a + 4*b*x])/(128*b^2) + (3*d^2*(c + d*x)*Sin[4*a + 4*b*x])/(256*b^3)
 - ((c + d*x)^3*Sin[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.130323, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}+\frac{3d^3\cos(4a+4bx)}{1024b^4}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(c+dx)^4}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^4/(32*d) + (3*d^3*Cos[4*a + 4*b*x])/(1024*b^4) - (3*d*(c + d*x)^2
*Cos[4*a + 4*b*x])/(128*b^2) + (3*d^2*(c + d*x)*Sin[4*a + 4*b*x])/(256*b^3)
 - ((c + d*x)^3*Sin[4*a + 4*b*x])/(32*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^3-\frac{1}{8}(c+dx)^3\cos(4a+4bx)\right)dx\\ &= \frac{(c+dx)^4}{32d}-\frac{1}{8}\int (c+dx)^3\cos(4a+4bx)\,dx\\ &= \frac{(c+dx)^4}{32d}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(3d)\int (c+dx)^2\sin(4a+4bx)\,dx}{32b}\\ &= \frac{(c+dx)^4}{32d}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{\left(3d^2\right)\int (\ldots}{\ldots}\\ &= \frac{(c+dx)^4}{32d}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}+\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{(c+d\ldots}{\ldots}\\ &= \frac{(c+dx)^4}{32d}+\frac{3d^3\cos(4a+4bx)}{1024b^4}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}+\frac{3d^2(c+dx)\sin\ldots}{256\ldots}\end{aligned}$$

Mathematica [A] time = 0.685076, size = 106, normalized size = 1.01

$$\frac{-4b(c+dx)\sin(4(a+bx))\left(8b^2(c+dx)^2-3d^2\right)-3d\cos(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)+32b^4x\left(6c^2dx+4c^3+4cd^2x^2\ldots\right.}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (32*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 8*b^2*(
c + d*x)^2)*Cos[4*(a + b*x)] - 4*b*(c + d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*S
in[4*(a + b*x)])/(1024*b^4)
```

Maple [B] time = 0.022, size = 1074, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(
b*x+a)^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
-21/128*(b*x+a)^2-3/128*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*
x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^
4+3/8*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
+3/128*sin(b*x+a)^4)-3/b^3*a*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2
*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/
64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a
)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a)
)*cos(b*x+a))+3/b^2*c*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/
2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/
12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b
*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b
*x+a))+3/b^3*a^2*d^3*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/
16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*
cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-6/b^2*a*c*d^2*((b*x+a)*(-1/2*c
os(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a
)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*
x+a)^4)+3/b*c^2*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*
(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos
(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-1/b^3*a^3*d^3*(-1/4*sin(b*x+a)*co
s(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+3/b^2*a^2*c*d^2*(-1/4*s
in(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-3/b*a*c^2*d
*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c^3
*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

Maxima [B] time = 1.30413, size = 597, normalized size = 5.69

$$\frac{32\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^3-\frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ac^2d}{b}+\frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2cd^2}{b^2}-\frac{32\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^3d^3}{b^3}+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1024*(32*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^3 - 96*(4*b*x + 4*a - sin(4*b
*x + 4*a))*a*c^2*d/b + 96*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c*d^2/b^2 -
32*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*d^3/b^3 + 24*(8*(b*x + a)^2 - 4*(b*
x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c^2*d/b - 48*(8*(b*x + a)^2 - 4
*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c*d^2/b^2 + 24*(8*(b*x +
a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*d^3/b^3 + 4*(32
*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*
```

```
b*x + 4*a))*c*d^2/b^2 - 4*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) -
 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*d^3/b^3 + (32*(b*x + a)^4 - 3*(8
*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*
b*x + 4*a))*d^3/b^3)/b
```

Fricas [B] time = 0.513568, size = 647, normalized size = 6.16

$$\frac{4\,b^4d^3x^4+16\,b^4cd^2x^3-3\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^4+3\left(8\,b^4c^2d-b^2d^3\right)x^2+3\left(8\,b^2d^3x^2+16\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(4*b^4*d^3*x^4 + 16*b^4*c*d^2*x^3 - 3*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x
 + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 + 3*(8*b^4*c^2*d - b^2*d^3)*x^2 + 3*(8
*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(8*b^
4*c^3 - 3*b^2*c*d^2)*x - 2*(2*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 8*b^3*c^3
 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^3 - (8*b^3*d^3*x^3 +
 24*b^3*c*d^2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(
b*x + a))*sin(b*x + a))/b^4
```

Sympy [A] time = 9.97599, size = 813, normalized size = 7.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sin(a + b*x)**4/8 + c**3*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**3*x*cos(a + b*x)**4/8 + 3*c**2*d*x**2*sin(a + b*x)**4/16 + 3*c**2*
d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/8 + 3*c**2*d*x**2*cos(a + b*x)**4/16
 + c*d**2*x**3*sin(a + b*x)**4/8 + c*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)
**2/4 + c*d**2*x**3*cos(a + b*x)**4/8 + d**3*x**4*sin(a + b*x)**4/32 + d**3
*x**4*sin(a + b*x)**2*cos(a + b*x)**2/16 + d**3*x**4*cos(a + b*x)**4/32 + c
**3*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**3*sin(a + b*x)*cos(a + b*x)**3/
(8*b) + 3*c**2*d*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*c**2*d*x*sin(a +
b*x)*cos(a + b*x)**3/(8*b) + 3*c*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*
```

```
b) - 3*c*d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d**3*x**3*sin(a + b
*x)**3*cos(a + b*x)/(8*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) +
3*c**2*d*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - 3*c*d**2*x*sin(a + b*x
)**4/(64*b**2) + 9*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) - 3*c
*d**2*x*cos(a + b*x)**4/(64*b**2) - 3*d**3*x**2*sin(a + b*x)**4/(128*b**2)
+ 9*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**2) - 3*d**3*x**2*cos(a
 + b*x)**4/(128*b**2) - 3*c*d**2*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + 3
*c*d**2*sin(a + b*x)*cos(a + b*x)**3/(64*b**3) - 3*d**3*x*sin(a + b*x)**3*c
os(a + b*x)/(64*b**3) + 3*d**3*x*sin(a + b*x)*cos(a + b*x)**3/(64*b**3) - 3
*d**3*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**4), Ne(b, 0)), ((c**3*x + 3*c
**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a)**2, True))
```

Giac [A] time = 1.14039, size = 207, normalized size = 1.97

$$\frac{1}{32}\,d^3x^4+\frac{1}{8}\,cd^2x^3+\frac{3}{16}\,c^2dx^2+\frac{1}{8}\,c^3x-\frac{3\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\cos\left(4\,bx+4\,a\right)}{1024\,b^4}-\frac{\left(8\,b^3d^3x^3+24\,b^3cd^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*d^3*x^4 + 1/8*c*d^2*x^3 + 3/16*c^2*d*x^2 + 1/8*c^3*x - 3/1024*(8*b^2*d
^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(4*b*x + 4*a)/b^4 - 1/256*(
8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*d^3*x -
 3*b*c*d^2)*sin(4*b*x + 4*a)/b^4
```

3.82 $\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx) \, dx$

Optimal. Leaf size=79

$$-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}+\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+\frac{(c+dx)^3}{24d}$$

```
[Out] (c + d*x)^3/(24*d) - (d*(c + d*x)*Cos[4*a + 4*b*x])/(64*b^2) + (d^2*Sin[4*a
 + 4*b*x])/(256*b^3) - ((c + d*x)^2*Sin[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.122533, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}+\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+\frac{(c+dx)^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^3/(24*d) - (d*(c + d*x)*Cos[4*a + 4*b*x])/(64*b^2) + (d^2*Sin[4*a
 + 4*b*x])/(256*b^3) - ((c + d*x)^2*Sin[4*a + 4*b*x])/(32*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\, dx = \int \left(\frac{1}{8}(c+dx)^2 - \frac{1}{8}(c+dx)^2 \cos(4a+4bx)\right) dx$$
$$= \frac{(c+dx)^3}{24d} - \frac{1}{8}\int (c+dx)^2 \cos(4a+4bx)\, dx$$
$$= \frac{(c+dx)^3}{24d} - \frac{(c+dx)^2 \sin(4a+4bx)}{32b} + \frac{d\int (c+dx)\sin(4a+4bx)\, dx}{16b}$$
$$= \frac{(c+dx)^3}{24d} - \frac{d(c+dx)\cos(4a+4bx)}{64b^2} - \frac{(c+dx)^2 \sin(4a+4bx)}{32b} + \frac{d^2 \int \cos(4a}{64b}$$
$$= \frac{(c+dx)^3}{24d} - \frac{d(c+dx)\cos(4a+4bx)}{64b^2} + \frac{d^2 \sin(4a+4bx)}{256b^3} - \frac{(c+dx)^2 \sin(4a+}{32b}$$

Mathematica [A] time = 0.43819, size = 77, normalized size = 0.97

$$\frac{-3\sin(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)-12bd(c+dx)\cos(4(a+bx))+32b^3x\left(3c^2+3cdx+d^2x^2\right)}{768b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (32*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 12*b*d*(c + d*x)*Cos[4*(a + b*x)] -
 3*(-d^2 + 8*b^2*(c + d*x)^2)*Sin[4*(a + b*x)])/(768*b^3)
```

Maple [B] time = 0.02, size = 519, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/8*(b
*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*x+a)^
3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1
/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))-2/b^
2*a*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+
```

```
1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/
8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+2/b*c*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a
)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)
^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+1/b^2*a^2*d
^2*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-2
/b*a*c*d*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/
8*a)+c^2*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/
8*a))
```

Maxima [B] time = 1.22758, size = 313, normalized size = 3.96

$$\frac{24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^2-\frac{48\,(4\,bx+4\,a-\sin(4\,bx+4\,a))acd}{b}+\frac{24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2d^2}{b^2}+\frac{12\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cdots\right.}{b}}{768\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/768*(24*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^2 - 48*(4*b*x + 4*a - sin(4*b*
x + 4*a))*a*c*d/b + 24*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*d^2/b^2 + 12*(8
*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c*d/b - 12*
(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*d^2/b^2
 + (32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*
sin(4*b*x + 4*a))*d^2/b^2)/b
```

Fricas [B] time = 0.500507, size = 398, normalized size = 5.04

$$\frac{8\,b^3d^2x^3+24\,b^3cdx^2-24\left(bd^2x+bcd\right)\cos(bx+a)^4+24\left(bd^2x+bcd\right)\cos(bx+a)^2+3\left(8\,b^3c^2-bd^2\right)x-3\left(2\left(8\,b^2d^2x\cdots\right.\right.}{192\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(8*b^3*d^2*x^3 + 24*b^3*c*d*x^2 - 24*(b*d^2*x + b*c*d)*cos(b*x + a)^4
 + 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(8*b^3*c^2 - b*d^2)*x - 3*(2*(8*
b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a)^3 - (8*b^2*d^2*x
^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [A] time = 5.12042, size = 484, normalized size = 6.13

$$\begin{cases} \frac{c^2x\sin^4(a+bx)}{8} + \frac{c^2x\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{c^2x\cos^4(a+bx)}{8} + \frac{cdx^2\sin^4(a+bx)}{8} + \frac{cdx^2\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{cdx^2\cos^4(a+bx)}{8} + \frac{d^2x^3\sin^4(a-}{24} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sin(a + b*x)**4/8 + c**2*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**2*x*cos(a + b*x)**4/8 + c*d*x**2*sin(a + b*x)**4/8 + c*d*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/4 + c*d*x**2*cos(a + b*x)**4/8 + d**2*x**3*sin(
a + b*x)**4/24 + d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**2/12 + d**2*x**3*c
os(a + b*x)**4/24 + c**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**2*sin(a +
b*x)*cos(a + b*x)**3/(8*b) + c*d*x*sin(a + b*x)**3*cos(a + b*x)/(4*b) - c*d
*x*sin(a + b*x)*cos(a + b*x)**3/(4*b) + d**2*x**2*sin(a + b*x)**3*cos(a + b
*x)/(8*b) - d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c*d*sin(a + b*x)
**2*cos(a + b*x)**2/(8*b**2) - d**2*x*sin(a + b*x)**4/(64*b**2) + 3*d**2*x*
sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) - d**2*x*cos(a + b*x)**4/(64*b**2
) - d**2*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + d**2*sin(a + b*x)*cos(a +
 b*x)**3/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2
*cos(a)**2, True))
```

Giac [A] time = 1.08375, size = 127, normalized size = 1.61

$$\frac{1}{24}\,d^2x^3 + \frac{1}{8}\,cdx^2 + \frac{1}{8}\,c^2x - \frac{\left(bd^2x + bcd\right)\cos\left(4\,bx + 4\,a\right)}{64\,b^3} - \frac{\left(8\,b^2d^2x^2 + 16\,b^2cdx + 8\,b^2c^2 - d^2\right)\sin\left(4\,bx + 4\,a\right)}{256\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/24*d^2*x^3 + 1/8*c*d*x^2 + 1/8*c^2*x - 1/64*(b*d^2*x + b*c*d)*cos(4*b*x +
 4*a)/b^3 - 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*sin(4*b*
x + 4*a)/b^3
```

3.83 $\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{(c+dx)^2}{16d}$$

```
[Out] (c + d*x)^2/(16*d) - (d*Cos[4*a + 4*b*x])/(128*b^2) - ((c + d*x)*Sin[4*a +
4*b*x])/(32*b)
```

Rubi [A] time = 0.0537823, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2638}

$$-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{(c+dx)^2}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^2/(16*d) - (d*Cos[4*a + 4*b*x])/(128*b^2) - ((c + d*x)*Sin[4*a +
4*b*x])/(32*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)-\frac{1}{8}(c+dx)\cos(4a+4bx)\right)dx\\ &= \frac{(c+dx)^2}{16d}-\frac{1}{8}\int (c+dx)\cos(4a+4bx)\,dx\\ &= \frac{(c+dx)^2}{16d}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{d\int \sin(4a+4bx)\,dx}{32b}\\ &= \frac{(c+dx)^2}{16d}-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}\end{aligned}$$

Mathematica [A] time = 0.297987, size = 54, normalized size = 1.02

$$-\frac{8(a+bx)(ad-2bc-bdx)+4b(c+dx)\sin(4(a+bx))+d\cos(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] -(8*(a + b*x)*(-2*b*c + a*d - b*d*x) + d*Cos[4*(a + b*x)] + 4*b*(c + d*x)*S
in[4*(a + b*x)])/(128*b^2)
```

Maple [B] time = 0.02, size = 194, normalized size = 3.7

$$\frac{1}{b}\left(\frac{d}{b}\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{16}+\frac{(\sin(bx+a))^2}{16}-(bx+a)\left(-\frac{\cos(bx+a)}{4}\left((\sin(bx+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(d/b*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2
+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3
/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-1/b*d*a*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*
cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c*(-1/4*sin(b*x+a)*cos(b*x+a)^3+1/8*co
s(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

Maxima [B] time = 1.25609, size = 130, normalized size = 2.45

$$\frac{4\,(4\,bx+4\,a-\sin\left(4\,bx+4\,a\right))c-\frac{4\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ad}{b}+\frac{\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cos(4\,bx+4\,a)\right)d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(4*(4*b*x + 4*a - sin(4*b*x + 4*a))*c - 4*(4*b*x + 4*a - sin(4*b*x +
4*a))*a*d/b + (8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4
*a))*d/b)/b
```

Fricas [A] time = 0.481737, size = 204, normalized size = 3.85

$$\frac{b^2dx^2-d\cos\left(bx+a\right)^4+2\,b^2cx+d\cos\left(bx+a\right)^2-2\left(2\,(bdx+bc)\cos\left(bx+a\right)^3-(bdx+bc)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(b^2*d*x^2 - d*cos(b*x + a)^4 + 2*b^2*c*x + d*cos(b*x + a)^2 - 2*(2*(b
*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A] time = 2.355, size = 231, normalized size = 4.36

$$\begin{cases}\frac{cx\sin^4(a+bx)}{8}+\frac{cx\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{cx\cos^4(a+bx)}{8}+\frac{dx^2\sin^4(a+bx)}{16}+\frac{dx^2\sin^2(a+bx)\cos^2(a+bx)}{8}+\frac{dx^2\cos^4(a+bx)}{16}+\frac{c\sin^3(a+bx)\cos}{8b} \\ \left(cx+\frac{dx^2}{2}\right)\sin^2(a)\cos^2(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sin(a + b*x)**4/8 + c*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
c*x*cos(a + b*x)**4/8 + d*x**2*sin(a + b*x)**4/16 + d*x**2*sin(a + b*x)**2*
cos(a + b*x)**2/8 + d*x**2*cos(a + b*x)**4/16 + c*sin(a + b*x)**3*cos(a + b
```

```
*x)/(8*b) - c*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d*x*sin(a + b*x)**3*cos(
a + b*x)/(8*b) - d*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d*sin(a + b*x)**2
*cos(a + b*x)**2/(16*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**2*cos(a)**
2, True))
```

Giac [A] time = 1.08702, size = 65, normalized size = 1.23

$$\frac{1}{16}\,dx^2 + \frac{1}{8}\,cx - \frac{d\cos\left(4\,bx + 4\,a\right)}{128\,b^2} - \frac{(bdx + bc)\sin\left(4\,bx + 4\,a\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*d*x^2 + 1/8*c*x - 1/128*d*cos(4*b*x + 4*a)/b^2 - 1/32*(b*d*x + b*c)*si
n(4*b*x + 4*a)/b^2
```

3.84 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=78

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}$$

```
[Out] -(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(8*d) + Log[c + d*x]
/(8*d) + (Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)
```

Rubi [A] time = 0.139741, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(8*d) + Log[c + d*x]
/(8*d) + (Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{1}{8(c+dx)} - \frac{\cos(4a+4bx)}{8(c+dx)}\right) dx \\
&= \frac{\log(c+dx)}{8d} - \frac{1}{8}\int \frac{\cos(4a+4bx)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{8d} - \frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx + \frac{1}{8}\sin\left(4a-\frac{4bc}{d}\right)\int \frac{\sin\left(\frac{4bc}{d}\right.}{c+}\\
&= -\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d} + \frac{\log(c+dx)}{8d} + \frac{\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.164507, size = 65, normalized size = 0.83

$$\frac{-\cos\left(4a-\frac{4bc}{d}\right)\mathrm{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4b(c+dx)}{d}\right)+\log(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (-(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d]) + Log[c + d*x] + Si
n[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

Maple [A] time = 0.026, size = 105, normalized size = 1.4

$$-\frac{1}{8\,d}\mathrm{Si}\left(4\,bx+4\,a+4\,\frac{-ad+bc}{d}\right)\sin\left(4\,\frac{-ad+bc}{d}\right)-\frac{1}{8\,d}\mathrm{Ci}\left(4\,bx+4\,a+4\,\frac{-ad+bc}{d}\right)\cos\left(4\,\frac{-ad+bc}{d}\right)+\frac{\ln\left((bx+a)\,d-\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] -1/8*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d-1/8*Ci(4*b*x+4*a+4*
(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d+1/8*ln((b*x+a)*d-a*d+b*c)/d
```

Maxima [C] time = 1.45638, size = 216, normalized size = 2.77

$$\frac{b\left(E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_1\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+b\left(-i\,E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+i\,E_1\left(-\frac{4i\,bc+4i\,(bx+a)}{d}\right.\right.}{16\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + exp_i
ntegral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a*d)/
d) + b*(-I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + I*e
xp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*(b*c - a
*d)/d) + 2*b*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

Fricas [A] time = 0.487259, size = 239, normalized size = 3.06

$$-\frac{\left(\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-2\,\sin\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)-2\,\log\left(dx+c\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/16*((cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-4*(b*d*x + b*c)/d))
*cos(-4*(b*c - a*d)/d) - 2*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*
c)/d) - 2*log(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x), x)
```

Giac [C] time = 1.19668, size = 903, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(d*x + c))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integra
l(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*a)^2*tan(2*b*c/d) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^
2*tan(2*b*c/d) - 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*
b*c/d)^2 + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d
)^2 - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 2*log(abs
(d*x + c))*tan(2*a)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2
 + real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*real_part(cos_i
ntegral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*real_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 2*log(abs(d*x + c))*tan(2*b*c/d)
^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 + real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a) - 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*a) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) - 2*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*c/d) + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*c/d) - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d) + 2*log(ab
s(d*x + c)) - real_part(cos_integral(4*b*x + 4*b*c/d)) - real_part(cos_inte
gral(-4*b*x - 4*b*c/d)))/(d*tan(2*a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2 + d*ta
n(2*b*c/d)^2 + d)
```

3.85 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=104

$$\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{\cos(4a+4bx)}{8d(c+dx)}-\frac{1}{8d(c+dx)}$$

```
[Out] -1/(8*d*(c + d*x)) + Cos[4*a + 4*b*x]/(8*d*(c + d*x)) + (b*CosIntegral[(4*b
*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*SinI
ntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rubi [A] time = 0.168982, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{\cos(4a+4bx)}{8d(c+dx)}-\frac{1}{8d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```


```
[Out] -1/(8*d*(c + d*x)) + Cos[4*a + 4*b*x]/(8*d*(c + d*x)) + (b*CosIntegral[(4*b
*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*SinI
ntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{1}{8(c+dx)^2}-\frac{\cos(4a+4bx)}{8(c+dx)^2}\right)dx\\
&= -\frac{1}{8d(c+dx)}-\frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx\\
&= -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{b\int \frac{\sin(4a+4bx)}{c+dx}\,dx}{2d}\\
&= -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d}+\frac{\left(b\sin\left(4a-\frac{4bc}{d}\right.\right.}{}\\
&= -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{b\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\right.}{2d^2}
\end{aligned}
$$

Mathematica [A] time = 0.462863, size = 81, normalized size = 0.78

$$
\frac{4b\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+4b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)+\frac{d(\cos(4(a+bx))-1)}{c+dx}}{8d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] ((d*(-1 + Cos[4*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(4*b*(c + d*x))/d]
*Sin[4*a - (4*b*c)/d] + 4*b*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x)
)/d])/(8*d^2)
```

Maple [A] time = 0.025, size = 156, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{b^2}{32}\left(-4\,\frac{\cos(4\,bx+4\,a)}{((bx+a)\,d-ad+bc)\,d}-4\,\frac{1}{d}\left(4\,\frac{1}{d}\mathrm{Si}\left(4\,bx+4\,a+4\,\frac{-ad+bc}{d}\right)\cos\left(4\,\frac{-ad+bc}{d}\right)-4\,\frac{1}{d}\mathrm{Ci}\left(4\,bx+4\,a+4\,\frac{-}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/32*b^2*(-4*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d-4*(4*Si(4*b*x+4*a+4
*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(
-a*d+b*c)/d)/d)/d)-1/8*b^2/((b*x+a)*d-a*d+b*c)/d)
```

Maxima [C] time = 1.46927, size = 231, normalized size = 2.22

$$\frac{64\,b^2\left(E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_2\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-b^2\left(64i\,E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-64i\,E_2\left(-\frac{4i\,bc}{}\right.\right.}{1024\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/1024*(64*b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d)
+ exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c
- a*d)/d) - b^2*(64*I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*
d)/d) - 64*I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*s
in(-4*(b*c - a*d)/d) - 128*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 0.54973, size = 346, normalized size = 3.33

$$\frac{4\,d\cos(bx+a)^4-4\,d\cos(bx+a)^2+2\,(bdx+bc)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+(bdx+bc\right.}{4\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*cos(b*x + a)^4 - 4*d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(-4*(b*c
- a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(4*(
b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b
*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2\sin(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sin(b*x + a)^2/(d*x + c)^2, x)
```

3.86 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=127

$$\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{1}{16d}$$

```
[Out] -1/(16*d*(c + d*x)^2) + Cos[4*a + 4*b*x]/(16*d*(c + d*x)^2) + (b^2*Cos[4*a
- (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d^3 - (b*Sin[4*a + 4*b*x])/(4*
d^2*(c + d*x)) - (b^2*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/
d^3
```

Rubi [A] time = 0.198285, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{1}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -1/(16*d*(c + d*x)^2) + Cos[4*a + 4*b*x]/(16*d*(c + d*x)^2) + (b^2*Cos[4*a
- (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d^3 - (b*Sin[4*a + 4*b*x])/(4*
d^2*(c + d*x)) - (b^2*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/
d^3
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{1}{8(c+dx)^3}-\frac{\cos(4a+4bx)}{8(c+dx)^3}\right)dx\\
&= -\frac{1}{16d(c+dx)^2}-\frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx\\
&= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}+\frac{b\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx}{4d}\\
&= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{b^2\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{d^2}\\
&= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\left(b^2\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}
\end{aligned}$$

$$= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}+\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}$$

Mathematica [A] time = 0.88326, size = 105, normalized size = 0.83

$$\frac{16b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)-16b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)+\frac{d(-4b(c+dx)\sin(4(a+bx))+d\cos(4(a+bx))-d)}{(c+dx)^2}}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] (16*b^2*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (d*(-d + d*Co
s[4*(a + b*x)] - 4*b*(c + d*x)*Sin[4*(a + b*x)]))/(c + d*x)^2 - 16*b^2*Sin[
4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(16*d^3)
```

Maple [A] time = 0.024, size = 193, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{b^3}{32}\left(-2\frac{\cos(4bx+4a)}{((bx+a)d-ad+bc)^2 d}-2\frac{1}{d}\left(-4\frac{\sin(4bx+4a)}{((bx+a)d-ad+bc)d}+4\frac{1}{d}\left(4\frac{1}{d}\mathrm{Si}\left(4bx+4a+4\frac{-ad+bc}{d}\right)\sin\left(4\frac{-}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/32*b^3*(-2*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)^2/d-2*(-4*sin(4*b*x+4
*a)/((b*x+a)*d-a*d+b*c)/d+4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c
)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)-1/16*b^3
/((b*x+a)*d-a*d+b*c)^2/d)
```

Maxima [C] time = 1.83377, size = 278, normalized size = 2.19

$$\frac{64\,b^3\left(E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_3\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-b^3\left(64i\,E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-64i\,E_3\left(-\frac{4i\,bc}{}\right.\right.}{1024\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(64*b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d)
+ exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c
- a*d)/d) - b^3*(64*I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*
d)/d) - 64*I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*s
in(-4*(b*c - a*d)/d) - 64*b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3
+ a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [B] time = 0.564002, size = 582, normalized size = 4.58

$$\frac{d^2\cos(bx+a)^4 - d^2\cos(bx+a)^2 - 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{4\,(bdx+bc)}{d}\right) + \left(\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d) + (b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c
- a*d)/d) - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - (b*d^2*x + b*c*d)*cos(b
*x + a))*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**3, x)
```

Giac [C] time = 1.64119, size = 7560, normalized size = 59.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b
```

*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*d^
2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan
(2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b^2*d^2*x^
2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 8*
b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*
tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(4
*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 4*b^2*d^2*x^2*real_part(cos_inte
gral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 16*b^2*d^2*x^2*real_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 16*b^2*
d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan
(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 32*b^2*c*d*x*sin_integral(
4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 4*b^2*d^2*x^2*rea
l_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 4*b^2*d
^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^
2 + 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(
2*a)*tan(2*b*c/d)^2 - 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 32*b^2*c*d*x*sin_integral(4*(b*d*x
 + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*rea
l_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^
2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*
c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*b*x)^2*tan(2*a) + 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x
- 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x +
b*c)/d)*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d*x*real_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 8*b^2*c*d*x*real_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 8*b^2*d^2*x^2*imag_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 8*b^2*d^2*x^2*imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 16*b^2*d^2*x^2*si
n_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 32*b^2*c*d*x*real
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 3
2*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)
*tan(2*b*c/d) - 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)
*tan(2*a)^2*tan(2*b*c/d) - 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*c^2*imag_part(cos_integral(

-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^2*c^2*sin_in
tegral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 8*b^2*c*d*
x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 8*
b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/
d)^2 + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(
2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
a)*tan(2*b*c/d)^2 + 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)
*tan(2*b*c/d)^2 + 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*c^2*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b^2*c^2*sin_integral(4*
(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d*x*r
eal_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*
d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 + 4*b^2*d^2*x
^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 16*b^2*c*d*x*im
ag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 16*b^2*c*d*x
*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 32*b^2*c
*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a) - 4*b^2*d^2*x^2*
real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 - 4*b^2*d^2*x^2*real_pa
rt(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 16*b^2*c*d*x*imag_part
(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 16*b^2*c*d*x*im
ag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 32*b^2*
c*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 16*b^2*d^
2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 16*b
^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)
+ 16*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)
*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 32*b^2*c*d*x*sin_integral(4*(b*d*
x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d) - 4*b^2*d^2*x^2*real_part(cos_integral(
4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 + 16*b^2*c*d*x*imag_part(cos_in
tegral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^2*c*d*x*imag_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 32*b^2*c*d*x*sin_i
ntegral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 4*b*d^2*x*tan(2*b*x)^2
*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b*d^2*x*tan(2*b*x)*tan(2*a)^2*tan(2*b
*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2
+ 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 8*b

```
^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 8*b^2*d^2*x^
2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - 16*b^2*d^2*x^2*sin_i
ntegral(4*(b*d*x + b*c)/d)*tan(2*a) - 8*b^2*c^2*imag_part(cos_integral(4*b*
x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 8*b^2*c^2*imag_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 16*b^2*c^2*sin_integral(4*(b*d*x + b
*c)/d)*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*a)^2 - 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*a)^2 + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
b*c/d) - 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/
d) + 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d) + 8*b^2*c^
2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 8*b^
2*c^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) +
 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 32*
b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) +
32*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d
) - 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c
/d) + 8*b^2*c^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*
b*c/d) - 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)
 - 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 - 8*
b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 8*b^2*
c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 8*b^
2*c^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 1
6*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 4*b*c*d
*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 4*b*c*d*tan(2*b*x)*tan(2*a)^2*tan(2
*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d)) + 4*b^2*
d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d)) + 4*b^2*c^2*real_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 + 4*b^2*c^2*real_part(cos_integral
(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 16*b^2*c*d*x*imag_part(cos_integral(4*b*
x + 4*b*c/d))*tan(2*a) + 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x - 4*b*c
/d))*tan(2*a) - 32*b^2*c*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) + 4*b
*d^2*x*tan(2*b*x)^2*tan(2*a) - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)^2 + 4*b*d^2*x*tan(2*b*x)*tan(2*a)^2 + 16*b^2*c*d*x*imag_part(cos_inte
gral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(-
4*b*x - 4*b*c/d))*tan(2*b*c/d) + 32*b^2*c*d*x*sin_integral(4*(b*d*x + b*c)/
d)*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*a)*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)*tan(2*b*c/d) - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c
/d)^2 - 4*b*d^2*x*tan(2*b*x)*tan(2*b*c/d)^2 - 4*b*d^2*x*tan(2*a)*tan(2*b*c/
d)^2 + 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d)) + 8*b^2*c*d*x*r
eal_part(cos_integral(-4*b*x - 4*b*c/d)) - 8*b^2*c^2*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*a) + 8*b^2*c^2*imag_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a) - 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) + 4*b
*c*d*tan(2*b*x)^2*tan(2*a) + 4*b*c*d*tan(2*b*x)*tan(2*a)^2 + 8*b^2*c^2*imag
```

```
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 8*b^2*c^2*imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + 16*b^2*c^2*sin_integral(4*(b*d*
x + b*c)/d)*tan(2*b*c/d) - 4*b*c*d*tan(2*b*x)*tan(2*b*c/d)^2 - d^2*tan(2*b*
x)^2*tan(2*b*c/d)^2 - 4*b*c*d*tan(2*a)*tan(2*b*c/d)^2 - 2*d^2*tan(2*b*x)*ta
n(2*a)*tan(2*b*c/d)^2 - d^2*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part
(cos_integral(4*b*x + 4*b*c/d)) + 4*b^2*c^2*real_part(cos_integral(-4*b*x -
 4*b*c/d)) - 4*b*d^2*x*tan(2*b*x) - 4*b*d^2*x*tan(2*a) - 4*b*c*d*tan(2*b*x)
 - d^2*tan(2*b*x)^2 - 4*b*c*d*tan(2*a) - 2*d^2*tan(2*b*x)*tan(2*a) - d^2*ta
n(2*a)^2)/(d^5*x^2*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2
*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + d^5*x^2*tan(2*b*x)^2*tan(2*a)^2 + d^5*x
^2*tan(2*b*x)^2*tan(2*b*c/d)^2 + d^5*x^2*tan(2*a)^2*tan(2*b*c/d)^2 + c^2*d^
3*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*b*x)^2*tan(2*a)^
2 + 2*c*d^4*x*tan(2*b*x)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*a)^2*tan(2*b*c/
d)^2 + d^5*x^2*tan(2*b*x)^2 + d^5*x^2*tan(2*a)^2 + c^2*d^3*tan(2*b*x)^2*tan
(2*a)^2 + d^5*x^2*tan(2*b*c/d)^2 + c^2*d^3*tan(2*b*x)^2*tan(2*b*c/d)^2 + c^
2*d^3*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*b*x)^2 + 2*c*d^4*x*tan(2*
a)^2 + 2*c*d^4*x*tan(2*b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(2*b*x)^2 + c^2*d^3*
tan(2*a)^2 + c^2*d^3*tan(2*b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

3.87 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=158

$$-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}$$

```
[Out] -1/(24*d*(c + d*x)^3) + Cos[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Cos[4*a
+ 4*b*x])/(3*d^3*(c + d*x)) - (4*b^3*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a
 - (4*b*c)/d])/(3*d^4) - (b*Sin[4*a + 4*b*x])/(12*d^2*(c + d*x)^2) - (4*b^3
*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rubi [A] time = 0.227787, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/(24*d*(c + d*x)^3) + Cos[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Cos[4*a
+ 4*b*x])/(3*d^3*(c + d*x)) - (4*b^3*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a
 - (4*b*c)/d])/(3*d^4) - (b*Sin[4*a + 4*b*x])/(12*d^2*(c + d*x)^2) - (4*b^3
*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
```

```
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{1}{8(c+dx)^4} - \frac{\cos(4a+4bx)}{8(c+dx)^4}\right) dx$$

$$= -\frac{1}{24d(c+dx)^3} - \frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^4}\,dx$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} + \frac{b\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} + \frac{b^2\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{3d^2}$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} - \frac{\left(4b^3\right)\int \frac{\sin(4a}{c+}}{3d^3}$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} - \frac{\left(4b^3\cos\left(4a\right.\right.}{}$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{4b^3\text{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right.}{3d^4}$$

Mathematica [A] time = 1.7543, size = 123, normalized size = 0.78

$$-\frac{32b^3\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\frac{d\left(\cos(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)+d(2b(c+dx)\sin(4(a+bx))+d)\right)}{(c+dx)^3}+32b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -(32*b^3*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*((-d^2 +
8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + d*(d + 2*b*(c + d*x)*Sin[4*(a + b*x)]
)))/(c + d*x)^3 + 32*b^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d
])/(24*d^4)
```

Maple [A] time = 0.026, size = 230, normalized size = 1.5

$$\frac{1}{b}\left(-\frac{b^4}{32}\left(-\frac{4\cos(4bx+4a)}{3((bx+a)d-ad+bc)^3d}-\frac{4}{3d}\left(-2\frac{\sin(4bx+4a)}{((bx+a)d-ad+bc)^2d}+2\frac{1}{d}\left(-4\frac{\cos(4bx+4a)}{((bx+a)d-ad+bc)d}-4\frac{1}{d}\left(4\frac{1}{d}\text{S}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/b*(-1/32*b^4*(-4/3*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)^3/d-4/3*(-2*sin(4*b
*x+4*a)/((b*x+a)*d-a*d+b*c)^2/d+2*(-4*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d-
4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-
a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)/d)-1/24*b^4/((b*x+a)*d-a*d+b*c)^3/
d)
```

Maxima [C] time = 2.338, size = 346, normalized size = 2.19

$$\frac{3b^4\left(E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_4\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4(bc-ad)}{d}\right)-b^4\left(3i\,E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-3i\,E_4\left(-\frac{4i\,bc+4i\,(\right.\right.}{48\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\left(bcd^3-ad^4\right)(bx+a)^2+3\left(b^2c^2d^2-2\,abcd^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^4*(exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + e
xp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a
*d)/d) - b^4*(3*I*exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d
) - 3*I*exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4
*(b*c - a*d)/d) - 2*b^4)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b
*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] time = 0.600936, size = 880, normalized size = 5.57

$$-\frac{b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d + \left(8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c^2d - d^3\right)\cos\left(bx+a\right)^4 - \left(8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c^2d - d^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (8*b^2*d^3*x^2 + 16*b^2*c*d
^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 - (8*b^2*d^3*x^2 + 16*b^2*c*d^2*x
+ 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + b^3*c^3)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d
) + (2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a
))*sin(b*x + a) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c
^3)*cos_integral(4*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d
))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**4, x)
```

Giac [C] time = 1.87002, size = 11486, normalized size = 72.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/12*(8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_integr
al(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x
^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b
*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)
^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 24*b^
3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^
2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*sin_integral(4
*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*im
ag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 8*b^3*d^3*
x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 16*
b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 + 32*b^
3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*ta
n(2*b*c/d) - 32*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*b*x)^2*tan(2*a)*tan(2*b*c/d) + 64*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)
/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 48*b^3*c*d^2*x^2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c*d^2
*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(
2*b*c/d) - 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x
)^2*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)
/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(cos_integral(4
*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*re
al_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^
2 + 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2
*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a
)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a
)^2*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c^
2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^
```

```
2 + 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(2*a) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*x)^2*tan(2*a)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x +
b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 - 16*b^3*d^3*x^3*real_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 96*b^3*c*d^2*x^2*imag_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 96
*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2
*a)*tan(2*b*c/d) + 192*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral
(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c^2*d*x*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c^
2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*x)^2*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d
*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
 - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a)*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*sin_integra
l(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^3*x^2*tan(2*b*x)^2
*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c
/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*c^3*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 8*
b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - 8*b^3*d
^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 16*b^3*d^3*
x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 48*b^3*c*d^2*x^2*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 48*b^3*c*d^2*x^2*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 8*b^3*d^3
*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 8*b^3*d^3*x^3*im
ag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 16*b^3*d^3*x^3*sin_int
egral(4*(b*d*x + b*c)/d)*tan(2*a)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 24*b^3*c^2*d*x*imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c^2*d*x*sin_int
egral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c*d^2*x^2*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 48*b^3*c*d^2*
x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 3
```

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2*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d
) - 32*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2
*b*c/d) + 64*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c
/d) + 96*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)*tan(2*b*c/d) - 96*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 192*b^3*c^2*d*x*sin_integral(4*(
b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 48*b^3*c*d^2*x^2*real_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c*d^2*
x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*
b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*ta
n(2*b*c/d) + 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d) - 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*x)^2*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*sin_integral(4*(b*d*x +
 b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(cos_integ
ral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*c^3*real_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 1
6*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*t
an(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b
*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d^2*x*tan(2*b*x)^2*tan(2*a)^2*ta
n(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*x)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*b*x)^2 + 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 +
 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 16*b^3*
d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 48*b^3*c^2*d*x
*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 48*b^3*c^
2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 24*
b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 24*b^3*
c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 48*b^3*c*d
^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 4*b^2*d^3*x^2*tan(2*b*x
)^2*tan(2*a)^2 - 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b
*x)^2*tan(2*a)^2 + 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2 - 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)
^2*tan(2*a)^2 - 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
b*c/d) - 48*b^3*c^2*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)
^2*tan(2*b*c/d) - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*b*x)^2*tan(2*b*c/d) + 96*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 96*b^3*c*d^2*x^2*imag_part(cos_integral(
```

```
-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 192*b^3*c*d^2*x^2*sin_integral(4
*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d) + 32*b^3*c^3*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 32*b^3*c^3*imag_par
t(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 64*b
^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) +
 48*b^3*c^2*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b
*c/d) + 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
*tan(2*b*c/d) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c
/d)^2 - 4*b^2*d^3*x^2*tan(2*b*x)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_pa
rt(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*c^3
*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c^2*d
*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^
3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2
 - 16*b^2*d^3*x^2*tan(2*b*x)*tan(2*a)*tan(2*b*c/d)^2 - 4*b^2*d^3*x^2*tan(2*
a)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*t
an(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*d*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)
^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d)) - 8*b^3*d^3*x^3
*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 16*b^3*d^3*x^3*sin_integral(4*
(b*d*x + b*c)/d) + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*
tan(2*b*x)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*b*x)^2 + 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 4
8*b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 48*b^3*
c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 16*b^3*c^3*r
eal_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 16*b^3*c^3*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 24*b^3*c^
2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 24*b^3*c^2*d*x*
imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 48*b^3*c^2*d*x*sin_i
ntegral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 8*b^2*c*d^2*x*tan(2*b*x)^2*tan(2*a)
^2 - 48*b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)
 - 48*b^3*c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)
- 16*b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*
c/d) - 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*ta
n(2*b*c/d) + 96*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
a)*tan(2*b*c/d) - 96*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*a)*tan(2*b*c/d) + 192*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*a)*tan(2*b*c/d) + 16*b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*a)^2*tan(2*b*c/d) + 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*a)^2*tan(2*b*c/d) - 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan
```

(2*b*c/d)^2 - 8*b^2*c*d^2*x*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*c^3*real_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*c^3*rea
l_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 32*b^2*c*d
^2*x*tan(2*b*x)*tan(2*a)*tan(2*b*c/d)^2 - 2*b*d^3*x*tan(2*b*x)^2*tan(2*a)*t
an(2*b*c/d)^2 - 8*b^2*c*d^2*x*tan(2*a)^2*tan(2*b*c/d)^2 - 2*b*d^3*x*tan(2*b
*x)*tan(2*a)^2*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b
*x + 4*b*c/d)) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))
 + 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d) - 4*b^2*d^3*x^2*tan(2*b
*x)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - 8
*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 16*b^3*c^
3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 48*b^3*c^2*d*x*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a) + 48*b^3*c^2*d*x*real_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*a) - 16*b^2*d^3*x^2*tan(2*b*x)*tan(2*a) - 4*b^
2*d^3*x^2*tan(2*a)^2 - 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*a)^2 + 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
 - 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 4*b^2*c^2*d*tan(2
*b*x)^2*tan(2*a)^2 - 48*b^3*c^2*d*x*real_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(2*b*c/d) - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*t
an(2*b*c/d) + 32*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*
tan(2*b*c/d) - 32*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a
)*tan(2*b*c/d) + 64*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*
b*c/d) + 4*b^2*d^3*x^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*c/d)^2 - 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2
*b*c/d)^2 - 4*b^2*c^2*d*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^2*c^2*d*tan(2*b*
x)*tan(2*a)*tan(2*b*c/d)^2 - 2*b*c*d^2*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
 - 4*b^2*c^2*d*tan(2*a)^2*tan(2*b*c/d)^2 - 2*b*c*d^2*tan(2*b*x)*tan(2*a)^2*
tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d)) -
24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 48*b^3*c^2*d*x*s
in_integral(4*(b*d*x + b*c)/d) - 8*b^2*c*d^2*x*tan(2*b*x)^2 + 16*b^3*c^3*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 16*b^3*c^3*real_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a) - 32*b^2*c*d^2*x*tan(2*b*x)*tan(2*a) -
 2*b*d^3*x*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d^2*x*tan(2*a)^2 - 2*b*d^3*x*tan
(2*b*x)*tan(2*a)^2 - 16*b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*c/d) - 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c
/d) + 8*b^2*c*d^2*x*tan(2*b*c/d)^2 + 2*b*d^3*x*tan(2*b*x)*tan(2*b*c/d)^2 +
2*b*d^3*x*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*d^3*x^2 + 8*b^3*c^3*imag_part(cos
_integral(4*b*x + 4*b*c/d)) - 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b
*c/d)) + 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d) - 4*b^2*c^2*d*tan(2*b*x
)^2 - 16*b^2*c^2*d*tan(2*b*x)*tan(2*a) - 2*b*c*d^2*tan(2*b*x)^2*tan(2*a) -
4*b^2*c^2*d*tan(2*a)^2 - 2*b*c*d^2*tan(2*b*x)*tan(2*a)^2 + 4*b^2*c^2*d*tan(
2*b*c/d)^2 + 2*b*c*d^2*tan(2*b*x)*tan(2*b*c/d)^2 + d^3*tan(2*b*x)^2*tan(2*b
*c/d)^2 + 2*b*c*d^2*tan(2*a)*tan(2*b*c/d)^2 + 2*d^3*tan(2*b*x)*tan(2*a)*tan
(2*b*c/d)^2 + d^3*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d^2*x + 2*b*d^3*x*tan
(2*b*x) + 2*b*d^3*x*tan(2*a) + 4*b^2*c^2*d + 2*b*c*d^2*tan(2*b*x) + d^3*tan

```
(2*b*x)^2 + 2*b*c*d^2*tan(2*a) + 2*d^3*tan(2*b*x)*tan(2*a) + d^3*tan(2*a)^2
)/(d^7*x^3*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*b*x)^
2*tan(2*a)^2*tan(2*b*c/d)^2 + d^7*x^3*tan(2*b*x)^2*tan(2*a)^2 + d^7*x^3*tan
(2*b*x)^2*tan(2*b*c/d)^2 + d^7*x^3*tan(2*a)^2*tan(2*b*c/d)^2 + 3*c^2*d^5*x*
tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*b*x)^2*tan(2*a)^
2 + 3*c*d^6*x^2*tan(2*b*x)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*a)^2*tan(2*
b*c/d)^2 + c^3*d^4*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + d^7*x^3*tan(2*b
*x)^2 + d^7*x^3*tan(2*a)^2 + 3*c^2*d^5*x*tan(2*b*x)^2*tan(2*a)^2 + d^7*x^3*
tan(2*b*c/d)^2 + 3*c^2*d^5*x*tan(2*b*x)^2*tan(2*b*c/d)^2 + 3*c^2*d^5*x*tan(
2*a)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*b*x)^2 + 3*c*d^6*x^2*tan(2*a)^2 +
 c^3*d^4*tan(2*b*x)^2*tan(2*a)^2 + 3*c*d^6*x^2*tan(2*b*c/d)^2 + c^3*d^4*tan
(2*b*x)^2*tan(2*b*c/d)^2 + c^3*d^4*tan(2*a)^2*tan(2*b*c/d)^2 + d^7*x^3 + 3*
c^2*d^5*x*tan(2*b*x)^2 + 3*c^2*d^5*x*tan(2*a)^2 + 3*c^2*d^5*x*tan(2*b*c/d)^
2 + 3*c*d^6*x^2 + c^3*d^4*tan(2*b*x)^2 + c^3*d^4*tan(2*a)^2 + c^3*d^4*tan(2
*b*c/d)^2 + 3*c^2*d^5*x + c^3*d^4)
```

3.88 $\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3}{}\right.}{32b}$$

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(16*b
*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/
(16*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a
- (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(32*b*(((-I)*
b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x
))/d])/(32*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (5^(-1 - m)*E
^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d])/(3
2*b*(((-I)*b*(c + d*x))/d)^m) + (5^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((5*I)
*b*(c + d*x))/d])/(32*b*E^((5*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.402562, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3308, 2181}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3}{}\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```


```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(16*b
*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/
(16*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a
- (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(32*b*(((-I)*
b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x
))/d])/(32*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (5^(-1 - m)*E
^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d])/(3
2*b*(((-I)*b*(c + d*x))/d)^m) + (5^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((5*I)
*b*(c + d*x))/d])/(32*b*E^((5*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^m \sin(a+bx) + \frac{1}{16}(c+dx)^m \sin(3a+3bx) - \frac{1}{16}(c+dx)^m \sin(5a\right. \\ &= \frac{1}{16}\int (c+dx)^m \sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)^m \sin(5a+5bx)\,dx + \frac{1}{8}\int (c \\ &= \frac{1}{32} i \int e^{-i(3a+3bx)}(c+dx)^m\,dx - \frac{1}{32} i \int e^{i(3a+3bx)}(c+dx)^m\,dx - \frac{1}{32} i \int e^{-i(5a+5b} \\ &= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+}{d}\right.}{1}\end{aligned}$$

Mathematica [A] time = 0.66986, size = 376, normalized size = 0.92

$$\frac{e^{-\frac{5i(ad+bc)}{d}}(c+dx)^m \left(-5\ 3^{-m} e^{\frac{2i(ad+bc)}{d}} \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^m \text{Gamma}\left(m+1,-\frac{3ib(c+dx)}{d}\right) + e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m \text{Gamma}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] ((c + d*x)^m*(30*E^(((4*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((-
I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1
+ m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (5*E^(((2*I)*(b*c + a*d))
/d)*(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]
 + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*
x))/d]))/(3^m*((b^2*(c + d*x)^2)/d^2)^m) + (3*(E^((10*I)*a)*((I*b*(c + d*x)
)/d)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d] + E^(((10*I)*b*c)/d)*(((-I)*b*(
c + d*x))/d)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(5^m*((b^2*(c + d*x)^2
)/d^2)^m)))/(480*b*E^(((5*I)*(b*c + a*d))/d))
```

Maple [F] time = 0.334, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cos(bx+a))^2 (\sin(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

Fricas [A] time = 0.569725, size = 709, normalized size = 1.74

$$3\, e^{\left(-\frac{dm\log\left(\frac{5ib}{d}\right)-5ibc+5iad}{d}\right)}\Gamma\left(m+1, \frac{5ibdx+5ibc}{d}\right) - 5\, e^{\left(-\frac{dm\log\left(\frac{3ib}{d}\right)-3ibc+3iad}{d}\right)}\Gamma\left(m+1, \frac{3ibdx+3ibc}{d}\right) - 30\, e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*e^(-(d*m*log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, (5*I*b*
d*x + 5*I*b*c)/d) - 5*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m
 + 1, (3*I*b*d*x + 3*I*b*c)/d) - 30*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)
*gamma(m + 1, (I*b*d*x + I*b*c)/d) - 30*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*
d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) - 5*e^(-(d*m*log(-3*I*b/d) + 3*I*b
*c - 3*I*a*d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d) + 3*e^(-(d*m*log(-5
*I*b/d) + 5*I*b*c - 5*I*a*d)/d)*gamma(m + 1, (-5*I*b*d*x - 5*I*b*c)/d))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

3.89 $\int (c+dx)^4 \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=330

$$-\frac{3d^3(c+dx)\sin(a+bx)}{b^4}-\frac{d^3(c+dx)\sin(3a+3bx)}{54b^4}+\frac{3d^3(c+dx)\sin(5a+5bx)}{1250b^4}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}+\frac{d^2(c+$$

```
[Out] (-3*d^4*Cos[a + b*x])/b^5 + (3*d^2*(c + d*x)^2*Cos[a + b*x])/(2*b^3) - ((c
+ d*x)^4*Cos[a + b*x])/(8*b) - (d^4*Cos[3*a + 3*b*x])/(162*b^5) + (d^2*(c +
 d*x)^2*Cos[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Cos[3*a + 3*b*x])/(48*b)
+ (3*d^4*Cos[5*a + 5*b*x])/(6250*b^5) - (3*d^2*(c + d*x)^2*Cos[5*a + 5*b*x]
)/(500*b^3) + ((c + d*x)^4*Cos[5*a + 5*b*x])/(80*b) - (3*d^3*(c + d*x)*Sin[
a + b*x])/b^4 + (d*(c + d*x)^3*Sin[a + b*x])/(2*b^2) - (d^3*(c + d*x)*Sin[3
*a + 3*b*x])/(54*b^4) + (d*(c + d*x)^3*Sin[3*a + 3*b*x])/(36*b^2) + (3*d^3*
(c + d*x)*Sin[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Sin[5*a + 5*b*x])/(
100*b^2)
```

Rubi [A] time = 0.391063, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$-\frac{3d^3(c+dx)\sin(a+bx)}{b^4}-\frac{d^3(c+dx)\sin(3a+3bx)}{54b^4}+\frac{3d^3(c+dx)\sin(5a+5bx)}{1250b^4}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}+\frac{d^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```


```
[Out] (-3*d^4*Cos[a + b*x])/b^5 + (3*d^2*(c + d*x)^2*Cos[a + b*x])/(2*b^3) - ((c
+ d*x)^4*Cos[a + b*x])/(8*b) - (d^4*Cos[3*a + 3*b*x])/(162*b^5) + (d^2*(c +
 d*x)^2*Cos[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Cos[3*a + 3*b*x])/(48*b)
+ (3*d^4*Cos[5*a + 5*b*x])/(6250*b^5) - (3*d^2*(c + d*x)^2*Cos[5*a + 5*b*x]
)/(500*b^3) + ((c + d*x)^4*Cos[5*a + 5*b*x])/(80*b) - (3*d^3*(c + d*x)*Sin[
a + b*x])/b^4 + (d*(c + d*x)^3*Sin[a + b*x])/(2*b^2) - (d^3*(c + d*x)*Sin[3
*a + 3*b*x])/(54*b^4) + (d*(c + d*x)^3*Sin[3*a + 3*b*x])/(36*b^2) + (3*d^3*
(c + d*x)*Sin[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Sin[5*a + 5*b*x])/(
100*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4\sin(a+bx)+\frac{1}{16}(c+dx)^4\sin(3a+3bx)-\frac{1}{16}(c+dx)^4\sin(5a\right.$$

$$=\frac{1}{16}\int (c+dx)^4\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^4\sin(5a+5bx)\,dx+\frac{1}{8}\int (c+$$

$$=-\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\cos(5a+5bx)}{80b}-$$

$$=-\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\cos(5a+5bx)}{80b}+$$

$$=\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)}{8b}+\frac{d^2(c+dx)^2\cos(3a+3bx)}{36b^3}$$

$$=\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)}{8b}+\frac{d^2(c+dx)^2\cos(3a+3bx)}{36b^3}$$

$$=-\frac{3d^4\cos(a+bx)}{b^5}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{d^4\cos}{}$$

Mathematica [A] time = 3.4162, size = 238, normalized size = 0.72

$$\frac{120bd(c+dx)\sin(a+bx)\left(16\cos(2(a+bx))\left(75b^2(c+dx)^2-68d^2\right)-27\cos(4(a+bx))\left(25b^2(c+dx)^2-6d^2\right)+17475b^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-506250*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] -
 3125*(8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)
] + 81*(24*d^4 - 300*b^2*d^2*(c + d*x)^2 + 625*b^4*(c + d*x)^4)*Cos[5*(a +
b*x)] + 120*b*d*(c + d*x)*(17475*b^2*c^2 - 101794*d^2 + 34950*b^2*c*d*x + 1
7475*b^2*d^2*x^2 + 16*(-68*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 27*
(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/(4050000*b^5)
```

Maple [B] time = 0.087, size = 1812, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/3*(b*x+a)^4*(2+sin(b*x+a)^2)*cos(b*x+a)+8/15*(b*x+a)^3*s
in(b*x+a)+8/5*(b*x+a)^2*cos(b*x+a)-3424/1125*cos(b*x+a)-3424/1125*(b*x+a)*s
in(b*x+a)+4/45*(b*x+a)^3*sin(b*x+a)^3+4/45*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b
*x+a)+88/3375*(b*x+a)*sin(b*x+a)^3+88/10125*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5
*(b*x+a)^4*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-4/25*(b*x+a)^3*si
n(b*x+a)^5-12/125*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+
24/625*(b*x+a)*sin(b*x+a)^5+24/3125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos
(b*x+a))-4/b^4*a*d^4*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a
)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*si
n(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/
5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*s
in(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/
625*sin(b*x+a)^5)+4/b^3*c*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2
/5*(b*x+a)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*
x+a)^2*sin(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*
x+a)^3+1/5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b
*x+a)^2*sin(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(
b*x+a)+6/625*sin(b*x+a)^5)+6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*c
os(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3
+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(
b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*
sin(b*x+a)^2)*cos(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*c
os(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3
+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(
b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*
sin(b*x+a)^2)*cos(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*co
s(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+
```

```
2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b
*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*s
in(b*x+a)^2)*cos(b*x+a))-4/b^4*a^3*d^4*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b
*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*s
in(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+12/b^3*a^2*c*d^3*(-1/3*(b*x+a)*(
2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8
/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)-12/b^2*a*c^
2*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(
b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(
b*x+a)^5)+4/b*c^3*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+
a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*
x+a)-1/25*sin(b*x+a)^5)+1/b^4*a^4*d^4*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*
cos(b*x+a)^3)-4/b^3*a^3*c*d^3*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+
a)^3)+6/b^2*a^2*c^2*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-
4/b*a*c^3*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+c^4*(-1/5*si
n(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))
```

Maxima [B] time = 1.42847, size = 1808, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4050000*(270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^4 - 1080000*(3*co
s(b*x + a)^5 - 5*cos(b*x + a)^3)*a*c^3*d/b + 1620000*(3*cos(b*x + a)^5 - 5*
cos(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*cos(b*x + a)^5 - 5*cos(b*x + a
)^3)*a^3*c*d^3/b^3 + 270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^4*d^4/b
^4 + 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) -
450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450
*sin(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a
)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*s
in(3*b*x + 3*a) + 450*sin(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x + a)*cos
(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a)
- 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*c*d^3/b^
3 - 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 4
50*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*
sin(b*x + a))*a^3*d^4/b^4 + 450*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) -
 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x
+ a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13
500*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 900*(27*(25*(b*x + a)^2 - 2)*cos(
```

```
5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2
 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b
*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 450*(27*(25*(b*x +
a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 675
0*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*
x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 60*(1
35*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 -
2*b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x +
 a) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*si
n(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 60*(135
*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 - 2*
b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a
) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*sin(
3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + (81*(625*
(b*x + a)^4 - 300*(b*x + a)^2 + 24)*cos(5*b*x + 5*a) - 3125*(27*(b*x + a)^4
 - 36*(b*x + a)^2 + 8)*cos(3*b*x + 3*a) - 506250*((b*x + a)^4 - 12*(b*x + a
)^2 + 24)*cos(b*x + a) - 1620*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*
a) + 37500*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 2025000*((b*x +
 a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [A] time = 0.610837, size = 1121, normalized size = 3.4

$$\frac{81\left(625\,b^4d^4x^4+2500\,b^4cd^3x^3+625\,b^4c^4-300\,b^2c^2d^2+24\,d^4+150\left(25\,b^4c^2d^2-2\,b^2d^4\right)x^2+100\left(25\,b^4c^3d-6\,b^2cd^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/253125*(81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*
c^2*d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d
 - 6*b^2*c*d^3)*x)*cos(b*x + a)^5 - 5*(16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*
x^3 + 16875*b^4*c^4 - 11700*b^2*c^2*d^2 + 1736*d^4 + 450*(225*b^4*c^2*d^2 -
 26*b^2*d^4)*x^2 + 900*(75*b^4*c^3*d - 26*b^2*c*d^3)*x)*cos(b*x + a)^3 + 12
0*(2925*b^2*d^4*x^2 + 5850*b^2*c*d^3*x + 2925*b^2*c^2*d^2 - 6284*d^4)*cos(b
*x + a) + 60*(1950*b^3*d^4*x^3 + 5850*b^3*c*d^3*x^2 + 1950*b^3*c^3*d - 1256
8*b*c*d^3 - 27*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d^
3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^4 + (975*b^3*d^4*x^3 + 292
5*b^3*c*d^3*x^2 + 975*b^3*c^3*d - 434*b*c*d^3 + (2925*b^3*c^2*d^2 - 434*b*d
^4)*x)*cos(b*x + a)^2 + 2*(2925*b^3*c^2*d^2 - 6284*b*d^4)*x)*sin(b*x + a))/
b^5
```

Sympy [A] time = 79.2152, size = 1098, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**4*cos(a + b*x
)**5/(15*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 8*c**3*d*x
*cos(a + b*x)**5/(15*b) - 2*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/
b - 4*c**2*d**2*x**2*cos(a + b*x)**5/(5*b) - 4*c*d**3*x**3*sin(a + b*x)**2*
cos(a + b*x)**3/(3*b) - 8*c*d**3*x**3*cos(a + b*x)**5/(15*b) - d**4*x**4*si
n(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**4*x**4*cos(a + b*x)**5/(15*b) +
104*c**3*d*sin(a + b*x)**5/(225*b**2) + 52*c**3*d*sin(a + b*x)**3*cos(a + b
*x)**2/(45*b**2) + 8*c**3*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2) + 104*c*
*2*d**2*x*sin(a + b*x)**5/(75*b**2) + 52*c**2*d**2*x*sin(a + b*x)**3*cos(a
+ b*x)**2/(15*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(5*b**2) +
 104*c*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 52*c*d**3*x**2*sin(a + b*x)**3
*cos(a + b*x)**2/(15*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**4/(5*
b**2) + 104*d**4*x**3*sin(a + b*x)**5/(225*b**2) + 52*d**4*x**3*sin(a + b*x
)**3*cos(a + b*x)**2/(45*b**2) + 8*d**4*x**3*sin(a + b*x)*cos(a + b*x)**4/(
15*b**2) + 104*c**2*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 676*c**2*
d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*c**2*d**2*cos(a + b*
x)**5/(1125*b**3) + 208*c*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 1
352*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 3424*c*d**3*x*cos
(a + b*x)**5/(1125*b**3) + 104*d**4*x**2*sin(a + b*x)**4*cos(a + b*x)/(75*b
**3) + 676*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*d**4
*x**2*cos(a + b*x)**5/(1125*b**3) - 50272*c*d**3*sin(a + b*x)**5/(16875*b**
4) - 20456*c*d**3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 3424*c*d**3
*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*x*sin(a + b*x)**5/(1
6875*b**4) - 20456*d**4*x*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 342
4*d**4*x*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*sin(a + b*x)
**4*cos(a + b*x)/(16875*b**5) - 303368*d**4*sin(a + b*x)**2*cos(a + b*x)**3
/(50625*b**5) - 760816*d**4*cos(a + b*x)**5/(253125*b**5), Ne(b, 0)), ((c**
4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*
*3*cos(a)**2, True))
```

Giac [A] time = 1.13131, size = 717, normalized size = 2.17

$$\frac{\left(625\,b^4d^4x^4 + 2500\,b^4cd^3x^3 + 3750\,b^4c^2d^2x^2 + 2500\,b^4c^3dx + 625\,b^4c^4 - 300\,b^2d^4x^2 - 600\,b^2cd^3x - 300\,b^2c^2d^2 + 24\,d^4\right) c}{50000\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/50000*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500
*b^4*c^3*d*x + 625*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^
2*d^2 + 24*d^4)*cos(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d
^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^
2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*cos(3*b*x + 3*a)/b^5 - 1/8*(b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 -
 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^
5 - 1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c
^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(5*b*x + 5*a)/b^5 + 1/108*(3*b^3*d^4*x^3 +
 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*s
in(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
+ b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

3.90 $\int (c+dx)^3 \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=259

$$\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-\frac{3d^2(c+dx)\cos(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{8b^2}+\frac{d(c+dx)^2}{4}$$

```
[Out] (3*d^2*(c + d*x)*Cos[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x])/(8*b) +
 (d^2*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Cos[3*a + 3*b*x])
/(48*b) - (3*d^2*(c + d*x)*Cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^3*Cos[
5*a + 5*b*x])/(80*b) - (3*d^3*Sin[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Sin[
a + b*x])/(8*b^2) - (d^3*Sin[3*a + 3*b*x])/(216*b^4) + (d*(c + d*x)^2*Sin[3
*a + 3*b*x])/(48*b^2) + (3*d^3*Sin[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x
)^2*Sin[5*a + 5*b*x])/(400*b^2)
```

Rubi [A] time = 0.279157, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-\frac{3d^2(c+dx)\cos(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{8b^2}+\frac{d(c+dx)^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```


```
[Out] (3*d^2*(c + d*x)*Cos[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x])/(8*b) +
 (d^2*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Cos[3*a + 3*b*x])
/(48*b) - (3*d^2*(c + d*x)*Cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^3*Cos[
5*a + 5*b*x])/(80*b) - (3*d^3*Sin[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Sin[
a + b*x])/(8*b^2) - (d^3*Sin[3*a + 3*b*x])/(216*b^4) + (d*(c + d*x)^2*Sin[3
*a + 3*b*x])/(48*b^2) + (3*d^3*Sin[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x
)^2*Sin[5*a + 5*b*x])/(400*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^3\sin(a+bx)+\frac{1}{16}(c+dx)^3\sin(3a+3bx)-\frac{1}{16}(c+dx)^3\sin(5a$$
$$= \frac{1}{16}\int (c+dx)^3\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^3\sin(5a+5bx)\,dx+\frac{1}{8}\int (c$$
$$= -\frac{(c+dx)^3\cos(a+bx)}{8b}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}+\frac{(c+dx)^3\cos(5a+5bx)}{80b}$$
$$= -\frac{(c+dx)^3\cos(a+bx)}{8b}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}+\frac{(c+dx)^3\cos(5a+5bx)}{80b}$$
$$= \frac{3d^2(c+dx)\cos(a+bx)}{4b^3}-\frac{(c+dx)^3\cos(a+bx)}{8b}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}$$
$$= \frac{3d^2(c+dx)\cos(a+bx)}{4b^3}-\frac{(c+dx)^3\cos(a+bx)}{8b}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}$$

Mathematica [A] time = 1.59901, size = 369, normalized size = 1.42

$$\frac{101250b^2c^2d\sin(a+bx)+5625b^2c^2d\sin(3(a+bx))-2025b^2c^2d\sin(5(a+bx))+10125b^3c^2dx\cos(5(a+bx))+3375b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-33750*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 1875*b*(c + d
*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 3375*b^3*c^3*Cos[5*(a +
 b*x)] - 810*b*c*d^2*Cos[5*(a + b*x)] + 10125*b^3*c^2*d*x*Cos[5*(a + b*x)]
- 810*b*d^3*x*Cos[5*(a + b*x)] + 10125*b^3*c*d^2*x^2*Cos[5*(a + b*x)] + 337
5*b^3*d^3*x^3*Cos[5*(a + b*x)] + 101250*b^2*c^2*d*Sin[a + b*x] - 202500*d^3
*Sin[a + b*x] + 202500*b^2*c*d^2*x*Sin[a + b*x] + 101250*b^2*d^3*x^2*Sin[a
+ b*x] + 5625*b^2*c^2*d*Sin[3*(a + b*x)] - 1250*d^3*Sin[3*(a + b*x)] + 1125
```

```
0*b^2*c*d^2*x*Sin[3*(a + b*x)] + 5625*b^2*d^3*x^2*Sin[3*(a + b*x)] - 2025*b
^2*c^2*d*Sin[5*(a + b*x)] + 162*d^3*Sin[5*(a + b*x)] - 4050*b^2*c*d^2*x*Sin
[5*(a + b*x)] - 2025*b^2*d^3*x^2*Sin[5*(a + b*x)])/(270000*b^4)
```

Maple [B] time = 0.024, size = 992, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a)^2*si
n(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*sin(b*x+
a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/5*(b*x
+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*sin(b*x
+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/625*si
n(b*x+a)^5)-3/b^3*a*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*co
s(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x
+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x
+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos
(b*x+a))+3/b^2*c*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b
*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)
^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*
x+a))+3/b^3*a^2*d^3*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+
a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*
x+a)-1/25*sin(b*x+a)^5)-6/b^2*a*c*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*
x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*si
n(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+3/b*c^2*d*(-1/3*(b*x+a)*(2+sin(b*
x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b
*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)-1/b^3*a^3*d^3*(-1/5
*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+3/b^2*a^2*c*d^2*(-1/5*sin(b*x
+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-3/b*a*c^2*d*(-1/5*sin(b*x+a)^2*cos(b*
x+a)^3-2/15*cos(b*x+a)^3)+c^3*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+
a)^3))
```

Maximа [B] time = 1.38457, size = 1034, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/270000*(18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^3 - 54000*(3*cos(b*
x + a)^5 - 5*cos(b*x + a)^3)*a*c^2*d/b + 54000*(3*cos(b*x + a)^5 - 5*cos(b*
x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^3*d
^3/b^3 + 225*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a)
 - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) +
450*sin(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x +
a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*
sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a)*cos(5*
b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9
*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*d^3/b^3 + 1
5*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3
*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x
 + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*c*
d^2/b^2 - 15*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2
 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a
)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b
*x + a))*a*d^3/b^3 + (135*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) -
 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 -
 6*b*x - 6*a)*cos(b*x + a) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625
*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x +
a))*d^3/b^3)/b
```

Fricas [A] time = 0.538011, size = 706, normalized size = 2.73

$$\frac{135\left(25\,b^3d^3x^3+75\,b^3cd^2x^2+25\,b^3c^3-6\,bcd^2+3\left(25\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^5-75\left(75\,b^3d^3x^3+225\,b^3cd^2x^2+7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(135*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 +
3*(25*b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^5 - 75*(75*b^3*d^3*x^3 + 225*b^3
*c*d^2*x^2 + 75*b^3*c^3 - 26*b*c*d^2 + (225*b^3*c^2*d - 26*b*d^3)*x)*cos(b*
x + a)^3 + 11700*(b*d^3*x + b*c*d^2)*cos(b*x + a) + (5850*b^2*d^3*x^2 + 117
00*b^2*c*d^2*x + 5850*b^2*c^2*d - 81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*
b^2*c^2*d - 2*d^3)*cos(b*x + a)^4 - 12568*d^3 + (2925*b^2*d^3*x^2 + 5850*b^
```

```
2*c*d^2*x + 2925*b^2*c^2*d - 434*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

Sympy [A] time = 23.6664, size = 690, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**3*cos(a + b*x
)**5/(15*b) - c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c**2*d*x*cos(a
 + b*x)**5/(5*b) - c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c*d**2
*x**2*cos(a + b*x)**5/(5*b) - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*
b) - 2*d**3*x**3*cos(a + b*x)**5/(15*b) + 26*c**2*d*sin(a + b*x)**5/(75*b**
2) + 13*c**2*d*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*c**2*d*sin(a +
 b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*x*sin(a + b*x)**5/(75*b**2) + 26
*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 4*c*d**2*x*sin(a + b*
x)*cos(a + b*x)**4/(5*b**2) + 26*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 13*d
**3*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*d**3*x**2*sin(a + b*
x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b*
*3) + 338*c*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*c*d**2*co
s(a + b*x)**5/(1125*b**3) + 52*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3
) + 338*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*d**3*x*cos(
a + b*x)**5/(1125*b**3) - 12568*d**3*sin(a + b*x)**5/(16875*b**4) - 5114*d*
*3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 856*d**3*sin(a + b*x)*cos(
a + b*x)**4/(1125*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**
3 + d**3*x**4/4)*sin(a)**3*cos(a)**2, True))
```

Giac [A] time = 1.11185, size = 474, normalized size = 1.83

$$\frac{\left(25\,b^3d^3x^3+75\,b^3cd^2x^2+75\,b^3c^2dx+25\,b^3c^3-6\,bd^3x-6\,bcd^2\right)\cos\left(5\,bx+5\,a\right)}{2000\,b^4}-\frac{\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\right.}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2*d*x + 25*b^3*c^3 - 6
*b*d^3*x - 6*b*c*d^2)*cos(5*b*x + 5*a)/b^4 - 1/144*(3*b^3*d^3*x^3 + 9*b^3*c
```

```
*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b*c*d^2)*cos(3*b*x + 3
*a)/b^4 - 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*
b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*
d^2*x + 25*b^2*c^2*d - 2*d^3)*sin(5*b*x + 5*a)/b^4 + 1/432*(9*b^2*d^3*x^2 +
 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*
x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

3.91 $\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx) \, dx$

Optimal. Leaf size=184

$$\frac{d(c+dx)\sin(a+bx)}{4b^2}+\frac{d(c+dx)\sin(3a+3bx)}{72b^2}-\frac{d(c+dx)\sin(5a+5bx)}{200b^2}+\frac{d^2\cos(a+bx)}{4b^3}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{d^2\cos}{1}$$

```
[Out] (d^2*Cos[a + b*x])/(4*b^3) - ((c + d*x)^2*Cos[a + b*x])/(8*b) + (d^2*Cos[3*
a + 3*b*x])/(216*b^3) - ((c + d*x)^2*Cos[3*a + 3*b*x])/(48*b) - (d^2*Cos[5*
a + 5*b*x])/(1000*b^3) + ((c + d*x)^2*Cos[5*a + 5*b*x])/(80*b) + (d*(c + d*
x)*Sin[a + b*x])/(4*b^2) + (d*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^2) - (d*(c
+ d*x)*Sin[5*a + 5*b*x])/(200*b^2)
```

Rubi [A] time = 0.197179, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$\frac{d(c+dx)\sin(a+bx)}{4b^2}+\frac{d(c+dx)\sin(3a+3bx)}{72b^2}-\frac{d(c+dx)\sin(5a+5bx)}{200b^2}+\frac{d^2\cos(a+bx)}{4b^3}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{d^2\cos}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (d^2*Cos[a + b*x])/(4*b^3) - ((c + d*x)^2*Cos[a + b*x])/(8*b) + (d^2*Cos[3*
a + 3*b*x])/(216*b^3) - ((c + d*x)^2*Cos[3*a + 3*b*x])/(48*b) - (d^2*Cos[5*
a + 5*b*x])/(1000*b^3) + ((c + d*x)^2*Cos[5*a + 5*b*x])/(80*b) + (d*(c + d*
x)*Sin[a + b*x])/(4*b^2) + (d*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^2) - (d*(c
+ d*x)*Sin[5*a + 5*b*x])/(200*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^2\sin(a+bx)+\frac{1}{16}(c+dx)^2\sin(3a+3bx)-\frac{1}{16}(c+dx)^2\sin(5a$$
$$=\frac{1}{16}\int (c+dx)^2\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^2\sin(5a+5bx)\,dx+\frac{1}{8}\int (c$$
$$=-\frac{(c+dx)^2\cos(a+bx)}{8b}-\frac{(c+dx)^2\cos(3a+3bx)}{48b}+\frac{(c+dx)^2\cos(5a+5bx)}{80b}$$
$$=-\frac{(c+dx)^2\cos(a+bx)}{8b}-\frac{(c+dx)^2\cos(3a+3bx)}{48b}+\frac{(c+dx)^2\cos(5a+5bx)}{80b}$$
$$=\frac{d^2\cos(a+bx)}{4b^3}-\frac{(c+dx)^2\cos(a+bx)}{8b}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{(c+dx)^2\cos(3}{48b}$$

Mathematica [A] time = 0.923184, size = 127, normalized size = 0.69

$$\frac{-6750\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)-125\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)+27\cos(5(a+bx))\left(25b^2(c+dx)^2-2\right.}{54000b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-6750*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 125*(-2*d^2 + 9*b^2*(c + d
*x)^2)*Cos[3*(a + b*x)] + 27*(-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*(a + b*x)]
 + 30*b*d*(c + d*x)*(450*Sin[a + b*x] + 25*Sin[3*(a + b*x)] - 9*Sin[5*(a +
b*x)]))/(54000*b^3)
```

Maple [B] time = 0.026, size = 466, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+
4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*co
s(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*
(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))-
2/b^2*a*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/1
5*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/2
5*sin(b*x+a)^5)+2/b*c*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(
b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*co
s(b*x+a)-1/25*sin(b*x+a)^5)+1/b^2*a^2*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2
/15*cos(b*x+a)^3)-2/b*a*c*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)
^3)+c^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))
```

Maxima [B] time = 1.25098, size = 506, normalized size = 2.75

$$\frac{3600\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)c^2-\frac{7200\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)acd}{b}+\frac{3600\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)a^2d^2}{b^2}+\frac{30(45(bx+a)\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/54000*(3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^2 - 7200*(3*cos(b*x +
 a)^5 - 5*cos(b*x + a)^3)*a*c*d/b + 3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)
^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*
x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x +
 3*a) + 450*sin(b*x + a))*c*d/b - 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b
*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a)
+ 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*d^2/b^2 + (27*(25*(b*x + a)^2 -
 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*
x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)
*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A] time = 0.505668, size = 406, normalized size = 2.21

$$\frac{27\left(25b^2d^2x^2+50b^2cdx+25b^2c^2-2d^2\right)\cos(bx+a)^5-5\left(225b^2d^2x^2+450b^2cdx+225b^2c^2-26d^2\right)\cos(bx+a)^3+\cdots}{3375\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x + a
)^5 - 5*(225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 - 26*d^2)*cos(b*x +
a)^3 + 780*d^2*cos(b*x + a) - 30*(9*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 26*b
*d^2*x - 26*b*c*d - 13*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/b^3
```

Sympy [A] time = 20.5763, size = 382, normalized size = 2.08

$$\begin{cases} -\frac{c^2 \sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{2c^2\cos^5(a+bx)}{15b} - \frac{2cdx\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{4cdx\cos^5(a+bx)}{15b} - \frac{d^2x^2\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{2d^2x^2\cos^5(a+bx)}{15b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**2*cos(a + b*x
)**5/(15*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 4*c*d*x*cos(a
 + b*x)**5/(15*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**
2*x**2*cos(a + b*x)**5/(15*b) + 52*c*d*sin(a + b*x)**5/(225*b**2) + 26*c*d*
sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*c*d*sin(a + b*x)*cos(a + b*x)
**4/(15*b**2) + 52*d**2*x*sin(a + b*x)**5/(225*b**2) + 26*d**2*x*sin(a + b*
x)**3*cos(a + b*x)**2/(45*b**2) + 4*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(15
*b**2) + 52*d**2*sin(a + b*x)**4*cos(a + b*x)/(225*b**3) + 338*d**2*sin(a +
 b*x)**2*cos(a + b*x)**3/(675*b**3) + 856*d**2*cos(a + b*x)**5/(3375*b**3),
 Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**2, True))
```

Giac [A] time = 1.12945, size = 282, normalized size = 1.53

$$\frac{\left(25\,b^2d^2x^2 + 50\,b^2cdx + 25\,b^2c^2 - 2\,d^2\right)\cos\left(5\,bx + 5\,a\right)}{2000\,b^3} - \frac{\left(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 2\,d^2\right)\cos\left(3\,bx + 3\,a\right)}{432\,b^3} - \frac{\left(b^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2000*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(5*b*x + 5*a
)/b^3 - 1/432*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x
+ 3*a)/b^3 - 1/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)
/b^3 - 1/200*(b*d^2*x + b*c*d)*sin(5*b*x + 5*a)/b^3 + 1/72*(b*d^2*x + b*c*d
)*sin(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*sin(b*x + a)/b^3
```

3.92 $\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{d\sin(a+bx)}{8b^2}+\frac{d\sin(3a+3bx)}{144b^2}-\frac{d\sin(5a+5bx)}{400b^2}-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+5}{80b}$$

```
[Out] -((c + d*x)*Cos[a + b*x])/(8*b) - ((c + d*x)*Cos[3*a + 3*b*x])/(48*b) + ((c
 + d*x)*Cos[5*a + 5*b*x])/(80*b) + (d*Sin[a + b*x])/(8*b^2) + (d*Sin[3*a +
3*b*x])/(144*b^2) - (d*Sin[5*a + 5*b*x])/(400*b^2)
```

Rubi [A] time = 0.0969841, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2637}

$$\frac{d\sin(a+bx)}{8b^2}+\frac{d\sin(3a+3bx)}{144b^2}-\frac{d\sin(5a+5bx)}{400b^2}-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+5}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)*Cos[a + b*x])/(8*b) - ((c + d*x)*Cos[3*a + 3*b*x])/(48*b) + ((c
 + d*x)*Cos[5*a + 5*b*x])/(80*b) + (d*Sin[a + b*x])/(8*b^2) + (d*Sin[3*a +
3*b*x])/(144*b^2) - (d*Sin[5*a + 5*b*x])/(400*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)\sin(a+bx)+\frac{1}{16}(c+dx)\sin(3a+3bx)-\frac{1}{16}(c+dx)\sin(5a+5$$
$$=\frac{1}{16}\int (c+dx)\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)\sin(5a+5bx)\,dx+\frac{1}{8}\int (c+d$$
$$=-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+5bx)}{80b}-\frac{d}{}$$
$$=-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+5bx)}{80b}+\frac{d}{}$$

Mathematica [A] time = 0.324346, size = 94, normalized size = 0.86

$$\frac{-450b(c+dx)\cos(a+bx)-75b(c+dx)\cos(3(a+bx))+45bc\cos(5(a+bx))+450d\sin(a+bx)+25d\sin(3(a+bx))}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-450*b*(c + d*x)*Cos[a + b*x] - 75*b*(c + d*x)*Cos[3*(a + b*x)] + 45*b*c*C
os[5*(a + b*x)] + 45*b*d*x*Cos[5*(a + b*x)] + 450*d*Sin[a + b*x] + 25*d*Sin
[3*(a + b*x)] - 9*d*Sin[5*(a + b*x)])/(3600*b^2)
```

Maple [A] time = 0.023, size = 163, normalized size = 1.5

$$\frac{1}{b}\left(\frac{d}{b}\left(-\frac{(bx+a)\left(2+(\sin(bx+a))^2\right)\cos(bx+a)}{3}+\frac{(\sin(bx+a))^3}{45}+\frac{2\sin(bx+a)}{15}+\frac{(bx+a)\cos(bx+a)}{5}\left(\frac{8}{3}+(\sin(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(d/b*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*s
in(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*s
```

```
in(b*x+a)^5)-1/b*d*a*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+c*(
-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))
```

Maxima [A] time = 1.23504, size = 188, normalized size = 1.72

$$\frac{240\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)c-\frac{240\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)ad}{b}+\frac{(45(bx+a)\cos(5bx+5a)-75(bx+a)\cos(3bx+3a)-450(bx+a)\cos}{b}}{3600b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3600*(240*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c - 240*(3*cos(b*x + a)^5
 - 5*cos(b*x + a)^3)*a*d/b + (45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*
cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin
(3*b*x + 3*a) + 450*sin(b*x + a))*d/b)/b
```

Fricas [A] time = 0.490192, size = 194, normalized size = 1.78

$$\frac{45(bdx+bc)\cos(bx+a)^5-75(bdx+bc)\cos(bx+a)^3-\left(9d\cos(bx+a)^4-13d\cos(bx+a)^2-26d\right)\sin(bx+a)}{225b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/225*(45*(b*d*x + b*c)*cos(b*x + a)^5 - 75*(b*d*x + b*c)*cos(b*x + a)^3 -
(9*d*cos(b*x + a)^4 - 13*d*cos(b*x + a)^2 - 26*d)*sin(b*x + a))/b^2
```

Sympy [A] time = 9.73825, size = 163, normalized size = 1.5

$$\begin{cases}-\frac{c\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2c\cos^5(a+bx)}{15b}-\frac{dx\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2dx\cos^5(a+bx)}{15b}+\frac{26d\sin^5(a+bx)}{225b^2}+\frac{13d\sin^3(a+bx)\cos^2(a+bx)}{45b^2}+\frac{2d\sin \\ \left(cx+\frac{dx^2}{2}\right)\sin^3(a)\cos^2(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c*cos(a + b*x)**5/(
15*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d*x*cos(a + b*x)**5/(
15*b) + 26*d*sin(a + b*x)**5/(225*b**2) + 13*d*sin(a + b*x)**3*cos(a + b*x)
**2/(45*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2), Ne(b, 0)), ((c*
x + d*x**2/2)*sin(a)**3*cos(a)**2, True))
```

Giac [A] time = 1.20442, size = 143, normalized size = 1.31

$$\frac{(bdx+bc)\cos(5\,bx+5\,a)}{80\,b^2}-\frac{(bdx+bc)\cos(3\,bx+3\,a)}{48\,b^2}-\frac{(bdx+bc)\cos(bx+a)}{8\,b^2}-\frac{d\sin(5\,bx+5\,a)}{400\,b^2}+\frac{d\sin(3\,bx+3\,a}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/80*(b*d*x + b*c)*cos(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*cos(3*b*x + 3*
a)/b^2 - 1/8*(b*d*x + b*c)*cos(b*x + a)/b^2 - 1/400*d*sin(5*b*x + 5*a)/b^2
+ 1/144*d*sin(3*b*x + 3*a)/b^2 + 1/8*d*sin(b*x + a)/b^2
```

3.93 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=185

$$-\frac{\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+b\right.}{8d}$$

```
[Out] -(CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(16*d) + (CosIntegra
l[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d) + (CosIntegral[(b*c)/d +
b*x]*Sin[a - (b*c)/d])/(8*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x]
)/(8*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (C
os[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rubi [A] time = 0.338659, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+b\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(16*d) + (CosIntegra
l[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d) + (CosIntegral[(b*c)/d +
b*x]*Sin[a - (b*c)/d])/(8*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x]
)/(8*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (C
os[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int\left(\frac{\sin(a+bx)}{8(c+dx)}+\frac{\sin(3a+3bx)}{16(c+dx)}-\frac{\sin(5a+5bx)}{16(c+dx)}\right)dx$$
$$= \frac{1}{16}\int\frac{\sin(3a+3bx)}{c+dx}\,dx-\frac{1}{16}\int\frac{\sin(5a+5bx)}{c+dx}\,dx+\frac{1}{8}\int\frac{\sin(a+bx)}{c+dx}\,dx$$
$$= -\left(\frac{1}{16}\cos\left(5a-\frac{5bc}{d}\right)\int\frac{\sin\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,dx\right)+\frac{1}{16}\cos\left(3a-\frac{3bc}{d}\right)\int\frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}$$
$$= -\frac{\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d}+\frac{\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{16d}+\frac{\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a\right.}{8d}$$

Mathematica [A] time = 0.523339, size = 154, normalized size = 0.83

$$\frac{\sin\left(5a-\frac{5bc}{d}\right)\left(-\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\right)+\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] (-(CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d]) + CosIntegral[(3*b*
(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + 2*CosIntegral[b*(c/d + x)]*Sin[a - (b*
c)/d] + 2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/d]*
SinIntegral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c +
```

```
d*x))/d])/(16*d)
```

Maple [A] time = 0.024, size = 253, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b}{80}\left(5\,\frac{1}{d}\mathrm{Si}\left(5\,bx+5\,a+5\,\frac{-ad+bc}{d}\right)\cos\left(5\,\frac{-ad+bc}{d}\right)-5\,\frac{1}{d}\mathrm{Ci}\left(5\,bx+5\,a+5\,\frac{-ad+bc}{d}\right)\sin\left(5\,\frac{-ad+bc}{d}\right)\right)+\frac{b}{8}\left(\frac{1}{d}\mathrm{S}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(-1/80*b*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b
*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)+1/8*b*(Si(b*x+a+(-a*d+b*c)/d)
*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)+1/48*b*(3*
Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b
*c)/d)*sin(3*(-a*d+b*c)/d)/d))
```

Maxima [C] time = 1.72738, size = 549, normalized size = 2.97

$$\frac{b\left(-2i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(-i\,E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_1\left(-\frac{3i\,bc+3i\,(bx+a)d-3i}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/32*(b*(-2*I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*ex
p_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) +
b*(-I*exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*exp_in
tegral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d
) + b*(I*exp_integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*exp
_integral_e(1, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d
)/d) - 2*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b*(exp
_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1,
 -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b*(exp_
integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(1,
-(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 0.50882, size = 612, normalized size = 3.31

$$\frac{2\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-\left(\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(\cdots\right.\right.}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/32*(2*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*si
n(-(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d
*x + b*c)/d))*sin(-3*(b*c - a*d)/d) - (cos_integral(5*(b*d*x + b*c)/d) + co
s_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d) - 2*cos(-5*(b*c - a*d
)/d)*sin_integral(5*(b*d*x + b*c)/d) + 2*cos(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) + 4*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/
d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.94 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=257

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5b}{d}\right.}{16d^2}$$

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) - (5*b*Cos[5*a - (5*b*c)/
d]*CosIntegral[(5*b*c)/d + 5*b*x])/(16*d^2) - Sin[a + b*x]/(8*d*(c + d*x))
- Sin[3*a + 3*b*x]/(16*d*(c + d*x)) + Sin[5*a + 5*b*x]/(16*d*(c + d*x)) - (
b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) - (3*b*Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rubi [A] time = 0.416478, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5b}{d}\right.}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```


```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) - (5*b*Cos[5*a - (5*b*c)/
d]*CosIntegral[(5*b*c)/d + 5*b*x])/(16*d^2) - Sin[a + b*x]/(8*d*(c + d*x))
- Sin[3*a + 3*b*x]/(16*d*(c + d*x)) + Sin[5*a + 5*b*x]/(16*d*(c + d*x)) - (
b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) - (3*b*Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)^2} + \frac{\sin(3a+3bx)}{16(c+dx)^2} - \frac{\sin(5a+5bx)}{16(c+dx)^2}\right) dx$$

$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx - \frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^2}\,dx + \frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx$$

$$= -\frac{\sin(a+bx)}{8d(c+dx)} - \frac{\sin(3a+3bx)}{16d(c+dx)} + \frac{\sin(5a+5bx)}{16d(c+dx)} + \frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{8d} + \frac{(3b)\int \frac{\cos(3a+3bx)}{c+dx}}{16d}$$

$$= -\frac{\sin(a+bx)}{8d(c+dx)} - \frac{\sin(3a+3bx)}{16d(c+dx)} + \frac{\sin(5a+5bx)}{16d(c+dx)} - \frac{\left(5b\cos\left(5a-\frac{5bc}{d}\right)\right)\int \frac{\cos\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,d}{16d}$$

$$= \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^2} + \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d^2} - \frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{C}}{16d^2}$$

Mathematica [A] time = 1.56163, size = 213, normalized size = 0.83

$$\frac{2b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5b(}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (2*b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*b*Cos[3*a - (3*b*c)/d]*C
osIntegral[(3*b*(c + d*x))/d] - 5*b*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(
c + d*x))/d] - (2*d*Sin[a + b*x])/(c + d*x) - (d*Sin[3*(a + b*x)])/(c + d*x
) + (d*Sin[5*(a + b*x)])/(c + d*x) - 2*b*Sin[a - (b*c)/d]*SinIntegral[b*(c/
d + x)] - 3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Sin
[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

Maple [A] time = 0.027, size = 365, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b^2}{80}\left(-5\frac{\sin(5bx+5a)}{((bx+a)d-ad+bc)d}+5\frac{1}{d}\left(5\frac{1}{d}\text{Si}\left(5bx+5a+5\frac{-ad+bc}{d}\right)\sin\left(5\frac{-ad+bc}{d}\right)+5\frac{1}{d}\text{Ci}\left(5bx+5a+5\frac{-ad}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/80*b^2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)/d+5*(5*Si(5*b*x+5*a+5
*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(
-a*d+b*c)/d)/d)/d)+1/8*b^2*(-sin(b*x+a)/((b*x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a
*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/
d)+1/48*b^2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*Si(3*b*x+3*a+3*(-
a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*
d+b*c)/d)/d)/d))
```

Maxima [C] time = 2.12228, size = 591, normalized size = 2.3

$$\frac{b^2\left(-2iE_2\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+2iE_2\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(-iE_2\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right)+iE_2\left(-\frac{3ibc+3i(bx+a)d-}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(b^2*(-2*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*
exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
+ b^2*(-I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*ex
p_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^2*(I*exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) -
 I*exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c
 - a*d)/d) - 2*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d)
- b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_int
egral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)
 + b^2*(exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_in
tegral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d
))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 0.64037, size = 883, normalized size = 3.44

$$10\,(bdx+bc)\sin\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right)-6\,(bdx+bc)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-4\,(bdx+bc)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx\ldots}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(10*(b*d*x + b*c)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d
) - 6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) -
 4*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2*((b*
d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d
*x + b*c)/d))*cos(-(b*c - a*d)/d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x
+ b*c)/d) + (b*d*x + b*c)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a
*d)/d) - 5*((b*d*x + b*c)*cos_integral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*c
os_integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) + 32*(d*cos(b*x + a)
^4 - d*cos(b*x + a)^2)*sin(b*x + a))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2 \sin(bx+a)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sin(b*x + a)^3/(d*x + c)^2, x)
```

3.95 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=338

$$\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosInteg}}{16d^3}$$

```
[Out] -(b*Cos[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)) + (5*b*Cos[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (25*b^2*CosIntegral[(5*
b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(32*d^3) - (9*b^2*CosIntegral[(3*b*c)
/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(32*d^3) - (b^2*CosIntegral[(b*c)/d + b*x
]*Sin[a - (b*c)/d])/(16*d^3) - Sin[a + b*x]/(16*d*(c + d*x)^2) - Sin[3*a +
3*b*x]/(32*d*(c + d*x)^2) + Sin[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rubi [A] time = 0.50475, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosInteg}}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)) + (5*b*Cos[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (25*b^2*CosIntegral[(5*
b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(32*d^3) - (9*b^2*CosIntegral[(3*b*c)
/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(32*d^3) - (b^2*CosIntegral[(b*c)/d + b*x
]*Sin[a - (b*c)/d])/(16*d^3) - Sin[a + b*x]/(16*d*(c + d*x)^2) - Sin[3*a +
3*b*x]/(32*d*(c + d*x)^2) + Sin[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)^3} + \frac{\sin(3a+3bx)}{16(c+dx)^3} - \frac{\sin(5a+5bx)}{16(c+dx)^3}\right)dx$$

$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx - \frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^3}\,dx + \frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin(3a+3bx)}{32d(c+dx)^2} + \frac{\sin(5a+5bx)}{32d(c+dx)^2} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{16d} + \frac{(3b)\int \frac{\cos(3a+3b}{(c+dx)^2}}{32d}$$

$$= -\frac{b\cos(a+bx)}{16d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b\cos(5a+5bx)}{32d^2(c+dx)} - \frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin(3a+3}{32d(c+dx}$$

$$= -\frac{b\cos(a+bx)}{16d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b\cos(5a+5bx)}{32d^2(c+dx)} - \frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin(3a+3}{32d(c+dx}$$

$$= -\frac{b\cos(a+bx)}{16d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b\cos(5a+5bx)}{32d^2(c+dx)} + \frac{25b^2\text{Ci}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a\right.}{32d^3}$$

Mathematica [A] time = 4.21656, size = 279, normalized size = 0.83

$$\frac{-2\left(b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d(b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^2}\right)+25b^2\sin\left(5\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (25*b^2*CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] - 9*b^2*CosInte
gral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - (d*(3*b*(c + d*x)*Cos[3*(a +
 b*x)] + d*Sin[3*(a + b*x)]))/(c + d*x)^2 + (d*(5*b*(c + d*x)*Cos[5*(a + b*
x)] + d*Sin[5*(a + b*x)]))/(c + d*x)^2 - 2*(b^2*CosIntegral[b*(c/d + x)]*Si
n[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin[a + b*x]))/(c + d*x)^
2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 9*b^2*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*(c + d*x))/d])/(32*d^3)
```

Maple [A] time = 0.027, size = 475, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b^3}{80}\left(-\frac{5\sin(5bx+5a)}{2\left((bx+a)d-ad+bc\right)^2d}+\frac{5}{2d}\left(-5\frac{\cos(5bx+5a)}{((bx+a)d-ad+bc)d}-5\frac{1}{d}\left(5\frac{1}{d}\text{Si}\left(5bx+5a+5\frac{-ad+bc}{d}\right)\cos\left(5-\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/80*b^3*(-5/2*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)^2/d+5/2*(-5*cos(5*b
*x+5*a)/((b*x+a)*d-a*d+b*c)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d
+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)+1/8*
b^3*(-1/2*sin(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-a*
d+b*c)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)
*sin((-a*d+b*c)/d)/d)/d)/d)+1/48*b^3*(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*
c)^2/d+3/2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d-3*(3*Si(3*b*x+3*a+3*(-a
*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d
+b*c)/d)/d)/d)/d))
```

Maxima [C] time = 3.00987, size = 639, normalized size = 1.89

$$\frac{b^3\left(-2i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(-i\,E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_3\left(-\frac{3i\,bc+3i\,(bx+a)d-}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(b^3*(-2*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*
exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
+ b^3*(-I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*ex
p_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^3*(I*exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) -
 I*exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c
 - a*d)/d) - 2*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d)
- b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_int
egral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)
 + b^3*(exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_in
tegral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d
))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d
^3)*(b*x + a))*b)
```

Fricas [A] time = 0.759665, size = 1358, normalized size = 4.02

$$160\left(bd^2x+bcd\right)\cos\left(bx+a\right)^5-224\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3+50\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(160*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 224*(b*d^2*x + b*c*d)*cos(b*x
+ a)^3 + 50*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-5*(b*c - a*d)/d)*sin
_integral(5*(b*d*x + b*c)/d) - 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 64*(b
*d^2*x + b*c*d)*cos(b*x + a) + 32*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)
*sin(b*x + a) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*
x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x +
b*c)/d))*sin(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_in
tegral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 25*((b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d))/(d
^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.96 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=413

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{CosInte}}{96d^4}$$

```
[Out] -(b*Cos[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)^2) + (5*b*Cos[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) - (b^3*Cos[a - (b*c)/
d]*CosIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*CosIn
tegral[(3*b*c)/d + 3*b*x])/(32*d^4) + (125*b^3*Cos[5*a - (5*b*c)/d]*CosInte
gral[(5*b*c)/d + 5*b*x])/(96*d^4) - Sin[a + b*x]/(24*d*(c + d*x)^3) + (b^2*
Sin[a + b*x])/(48*d^3*(c + d*x)) - Sin[3*a + 3*b*x]/(48*d*(c + d*x)^3) + (3
*b^2*Sin[3*a + 3*b*x])/(32*d^3*(c + d*x)) + Sin[5*a + 5*b*x]/(48*d*(c + d*x
)^3) - (25*b^2*Sin[5*a + 5*b*x])/(96*d^3*(c + d*x)) + (b^3*Sin[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) + (9*b^3*Sin[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Sin[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rubi [A] time = 0.5904, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{CosInte}}{96d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)^2) + (5*b*Cos[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) - (b^3*Cos[a - (b*c)/
d]*CosIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*CosIn
tegral[(3*b*c)/d + 3*b*x])/(32*d^4) + (125*b^3*Cos[5*a - (5*b*c)/d]*CosInte
gral[(5*b*c)/d + 5*b*x])/(96*d^4) - Sin[a + b*x]/(24*d*(c + d*x)^3) + (b^2*
Sin[a + b*x])/(48*d^3*(c + d*x)) - Sin[3*a + 3*b*x]/(48*d*(c + d*x)^3) + (3
*b^2*Sin[3*a + 3*b*x])/(32*d^3*(c + d*x)) + Sin[5*a + 5*b*x]/(48*d*(c + d*x
)^3) - (25*b^2*Sin[5*a + 5*b*x])/(96*d^3*(c + d*x)) + (b^3*Sin[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) + (9*b^3*Sin[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Sin[5*a - (5*b*c)/d]*SinIntegr
```

```
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)^4} + \frac{\sin(3a+3bx)}{16(c+dx)^4} - \frac{\sin(5a+5bx)}{16(c+dx)^4}\right) dx$$

$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^4}\,dx - \frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^4}\,dx + \frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(a+bx)}{24d(c+dx)^3} - \frac{\sin(3a+3bx)}{48d(c+dx)^3} + \frac{\sin(5a+5bx)}{48d(c+dx)^3} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx}{24d} + \frac{b\int \frac{\cos(3a+3bx)}{(c+dx)^3}}{16d}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} - \frac{\sin(3a+3b}{48d(c+dx}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} + \frac{b^2\sin(a+}{48d^3(c+d}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} + \frac{b^2\sin(a+}{48d^3(c+d}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{48d^4}$$

Mathematica [A] time = 3.17416, size = 457, normalized size = 1.11

$$-2\left(b^3(c+dx)^3\left(\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - \sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right) + d\cos(bx)\left(bd\cos(a)(c+dx) - \sin\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-(d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Si
n[3*a])) + d*Cos[5*b*x]*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2*(c + d
*x)^2)*Sin[5*a]) + d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*
x)*Sin[3*a])*Sin[3*b*x] - d*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d
*(c + d*x)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2
*d^2 + b^2*(c + d*x)^2)*Sin[a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*
d*(c + d*x)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegra
l[b*(c/d + x)] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) - 27*b^3*(c +
d*x)^3*(Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(Cos[5*a - (5
*b*c)/d]*CosIntegral[(5*b*(c + d*x))/d] - Sin[5*a - (5*b*c)/d]*SinIntegral[
(5*b*(c + d*x))/d]))/(96*d^4*(c + d*x)^3)
```

Maple [A] time = 0.028, size = 580, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b^4}{80}\left(-\frac{5\sin(5bx+5a)}{3\left((bx+a)d-ad+bc\right)^3 d}+\frac{5}{3d}\left(-\frac{5\cos(5bx+5a)}{2\left((bx+a)d-ad+bc\right)^2 d}-\frac{5}{2d}\left(-5\frac{\sin(5bx+5a)}{\left((bx+a)d-ad+bc\right)d}+5\frac{1}{d}\left(5\frac{1}{d}\mathrm{Si}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x)
```

```
[Out] 1/b*(-1/80*b^4*(-5/3*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)^3/d+5/3*(-5/2*cos(5
*b*x+5*a)/((b*x+a)*d-a*d+b*c)^2/d-5/2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c
)/d+5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+
5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-1/3*sin(b*x+a)/((
b*x+a)*d-a*d+b*c)^3/d+1/3*(-1/2*cos(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d-1/2*(-si
n(b*x+a)/((b*x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+
Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d)+1/48*b^4*(-sin(3*b*x+3
*a)/((b*x+a)*d-a*d+b*c)^3/d+(-3/2*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^2/d-3/
2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d
)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/
d)/d)/d)/d))
```

Maxima [C] time = 4.00839, size = 706, normalized size = 1.71

$$\frac{b^4\left(-2iE_4\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+2iE_4\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(-iE_4\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right)+iE_4\left(-\frac{3ibc+3i(bx+a)d-}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(b^4*(-2*I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*
exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
+ b^4*(-I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*ex
p_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^4*(I*exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) -
 I*exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c
 - a*d)/d) - 2*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d)
- b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_int
```

```
egral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)
 + b^4*(exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_in
tegral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d
))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^
4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^
4)*(b*x + a))*b)
```

Fricas [B] time = 0.81492, size = 1841, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/192*(160*(b*d^3*x + b*c*d^2)*cos(b*x + a)^5 - 224*(b*d^3*x + b*c*d^2)*cos
(b*x + a)^3 - 250*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)
*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 54*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integr
al(3*(b*d*x + b*c)/d) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 64*(b*d^3*x +
b*c*d^2)*cos(b*x + a) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
 b^3*c^3)*cos_integral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d
) - 27*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integ
ral(3*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b
^3*c^3)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 125*((b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(5*(b*d*x
 + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_
integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) - 32*(2*b^2*d^3*x^2 + 4
*b^2*c*d^2*x + 2*b^2*c^2*d + (25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*
d - 2*d^3)*cos(b*x + a)^4 - (21*b^2*d^3*x^2 + 42*b^2*c*d^2*x + 21*b^2*c^2*d
 - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*
x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.97 $\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=143

$$\text{Unintegrable}\left(\csc(a+bx)(c+dx)^m,x\right)+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)$$

```
[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(
((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2
*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Unintegrable[(c + d*x)^m*Cs
c[a + b*x], x]
```

Rubi [A] time = 0.126564, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(
((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2
*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Csc[
a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^m \csc(a+bx)\,dx-\int (c+dx)^m \sin(a+bx)\,dx\\ &= -\left(\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx\right)+\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx)^m \csc(a\\ &= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right.}{2b}\end{aligned}$$

Mathematica [A] time = 6.43278, size = 0, normalized size = 0.

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x], x]
```

Maple [A] time = 0.297, size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)\cot(bx + a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos(bx + a)\cot(bx + a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m \cos(bx + a)\cot(bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a) \cot(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

3.98 $\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=333

$$-\frac{24id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{24id^3(c+dx)\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}-\frac{12d^2(c+dx)^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{12d^2}{}$$

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + (24*d^4*Cos[a + b*x])/b^5 - (
12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 + ((c + d*x)^4*Cos[a + b*x])/b + ((4*I
)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((4*I)*d*(c + d*x)^3*Po
lyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a +
b*x))])/b^3 + (12*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I
)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((24*I)*d^3*(c + d*x)*P
olyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5
 - (24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5 + (24*d^3*(c + d*x)*Sin[a + b*x
])/b^4 - (4*d*(c + d*x)^3*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.284133, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4408, 3296, 2638, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{24id^3(c+dx)\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}-\frac{12d^2(c+dx)^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{12d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + (24*d^4*Cos[a + b*x])/b^5 - (
12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 + ((c + d*x)^4*Cos[a + b*x])/b + ((4*I
)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((4*I)*d*(c + d*x)^3*Po
lyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a +
b*x))])/b^3 + (12*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I
)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((24*I)*d^3*(c + d*x)*P
olyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5
 - (24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5 + (24*d^3*(c + d*x)*Sin[a + b*x
])/b^4 - (4*d*(c + d*x)^3*Sin[a + b*x])/b^2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
```

```
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^4\csc(a+bx)\,dx - \int (c+dx)^4\sin(a+bx)\,dx$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\cos(a+}{b}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b} + \frac{4id(c+dx)^3\text{Li}_2\left(-e^{i(a+b}\right.}{b^2}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} + \frac{(c+dx)^4\cos(a+b}{b}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} + \frac{(c+dx)^4\cos(a+b}{b}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{24d^4\cos(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} +$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{24d^4\cos(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} +$$

Mathematica [B] time = 1.37105, size = 837, normalized size = 2.51

$$\frac{c^4\cos(a+bx)b^4 + d^4x^4\cos(a+bx)b^4 + 4cd^3x^3\cos(a+bx)b^4 + 6c^2d^2x^2\cos(a+bx)b^4 + 4c^3dx\cos(a+bx)b^4 + c^4\log\left(1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (b^4*c^4*Cos[a + b*x] - 12*b^2*c^2*d^2*Cos[a + b*x] + 24*d^4*Cos[a + b*x] +
 4*b^4*c^3*d*x*Cos[a + b*x] - 24*b^2*c*d^3*x*Cos[a + b*x] + 6*b^4*c^2*d^2*x
^2*Cos[a + b*x] - 12*b^2*d^4*x^2*Cos[a + b*x] + 4*b^4*c*d^3*x^3*Cos[a + b*x
] + b^4*d^4*x^4*Cos[a + b*x] + b^4*c^4*Log[1 - E^(I*(a + b*x))] + 4*b^4*c^3
*d*x*Log[1 - E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))]
```

```
+ 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 - E^(I*(a +
b*x))] - b^4*c^4*Log[1 + E^(I*(a + b*x))] - 4*b^4*c^3*d*x*Log[1 + E^(I*(a +
 b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[
1 + E^(I*(a + b*x))] - b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] + (4*I)*b^3*d*(
c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2
, E^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 24*b^2*c
*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*d^4*x^2*PolyLog[3, -E^(I*(a +
b*x))] + 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*b^2*c*d^3*x*PolyLo
g[3, E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, E^(I*(a + b*x))] - (24*I)
*b*c*d^3*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a
+ b*x))] + (24*I)*b*c*d^3*PolyLog[4, E^(I*(a + b*x))] + (24*I)*b*d^4*x*Poly
Log[4, E^(I*(a + b*x))] + 24*d^4*PolyLog[5, -E^(I*(a + b*x))] - 24*d^4*Poly
Log[5, E^(I*(a + b*x))] - 4*b^3*c^3*d*Sin[a + b*x] + 24*b*c*d^3*Sin[a + b*x
] - 12*b^3*c^2*d^2*x*Sin[a + b*x] + 24*b*d^4*x*Sin[a + b*x] - 12*b^3*c*d^3*
x^2*Sin[a + b*x] - 4*b^3*d^4*x^3*Sin[a + b*x])/b^5
```

Maple [B] time = 0.354, size = 1295, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] 1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+8/b^4*c
*d^3*a^3*arctanh(exp(I*(b*x+a)))-12/b^3*c^2*d^2*a^2*arctanh(exp(I*(b*x+a)))
+8/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3-4
/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3+12
*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,exp(
I*(b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-4/b*c*d^3*ln(exp
(I*(b*x+a))+1)*x^3+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+1/b^5*d^4*a^4*l
n(exp(I*(b*x+a))+1)+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-12/b^3*c^2*d^2
*polylog(3,-exp(I*(b*x+a)))-2/b^5*d^4*a^4*arctanh(exp(I*(b*x+a)))-12/b^3*d^
4*polylog(3,-exp(I*(b*x+a)))*x^2+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b*d
^4*ln(exp(I*(b*x+a))+1)*x^4-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+4/b*c^3*
d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a-4/b*c^3*d*ln(ex
p(I*(b*x+a))+1)*x-24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*c^2*d^2*a
^2*ln(exp(I*(b*x+a))+1)-6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*p
olylog(3,exp(I*(b*x+a)))*x-4/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a-24*I/b^4*d^4*
polylog(4,-exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3+24*I
/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a
)))+4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,exp(I
```

```
*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-e
xp(I*(b*x+a)))*x^3+1/2*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4
*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c
*d^3*x-12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2-4*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+
24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))+1/2*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^
2*d^2*x^2+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2+12*I*b^3*c*d
^3*x^2-24*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2+4*I*b^3*c^3*d-24*I*
b*d^4*x+24*d^4-24*I*b*c*d^3)/b^5*exp(I*(b*x+a))+24*d^4*polylog(5,-exp(I*(b*
x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-2/b*c^4*arctanh(exp(I*(b*x+
a)))+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2
```

Maxima [B] time = 2.32893, size = 2076, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 4*a*c^3*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 6*a^2*c^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
a) - 1))/b^2 - 4*a^3*c*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(co
s(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^4 + (48*d^4*polylog(5, -e^(I*b*x + I*a)) - 48*d^4*po
lylog(5, e^(I*b*x + I*a)) - (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4
)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a
)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*a^2*b*c*d^3 - 8*I*a^3*d^4)
*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*I*(b*x + a)^4*d^4
+ (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^
3 + 12*I*a^2*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*
a^2*b*c*d^3 - 8*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) +
 1) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*(a^2 - 2)*d^4
 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 -
2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3
- (a^3 - 6*a)*d^4)*(b*x + a))*cos(b*x + a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c
^2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*
d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*
a^2*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - (8*I*b^3*c^3*d - 24*I*a*b^2*c
^2*d^2 + 24*I*a^2*b*c*d^3 + 8*I*(b*x + a)^3*d^4 - 8*I*a^3*d^4 + (24*I*b*c*d
^3 - 24*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^
2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) - ((b*x + a)^4*d^4 + 4*(b*c*d^3 -
```

```
a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 +
4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(co
s(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((b*x + a)^4*d^4 + 4*
(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*
x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x +
 a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (48*I*b*c*
d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*polylog(4, -e^(I*b*x + I*a)) - (-48*
I*b*c*d^3 - 48*I*(b*x + a)*d^4 + 48*I*a*d^4)*polylog(4, e^(I*b*x + I*a)) -
24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*
d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e
^(I*b*x + I*a)) - 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2
 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*
c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*sin(b*x + a))/b^4)/b
```

Fricas [C] time = 0.849061, size = 3281, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*
x + a)) - 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 2*(b^4*d^4*x^
4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 -
2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a) - (-4*I*b^3*d^
4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(cos(
b*x + a) + I*sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b
^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (-4*I*
b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilo
g(-cos(b*x + a) + I*sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 +
 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a))
+ (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^
3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b
*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^
4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b
```

```
*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 +
4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2
*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a)
+ 1) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x +
a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x +
a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x +
a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x +
 a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x +
a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polyl
og(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^
2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2
*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*
(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 - 2
*b*d^4)*x)*sin(b*x + a))/b^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a), x)
```

3.99 $\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx$

Optimal. Leaf size=254

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+$$

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (6*d^2*(c + d*x)*Cos[a + b*x]
)/b^3 + ((c + d*x)^3*Cos[a + b*x])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(
I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2
- (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d*x)*Pol
yLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^
4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (6*d^3*Sin[a + b*x])/b^4
- (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.19835, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4408, 3296, 2637, 4183, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```


```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (6*d^2*(c + d*x)*Cos[a + b*x]
)/b^3 + ((c + d*x)^3*Cos[a + b*x])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(
I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2
- (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d*x)*Pol
yLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^
4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (6*d^3*Sin[a + b*x])/b^4
- (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```


Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^3\csc(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\,dx$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\cos(a}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{i(a}\right.}{b^2}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(a+b}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(a+b}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(a+b}{b}$$

Mathematica [A] time = 0.982757, size = 330, normalized size = 1.3

$$\frac{3id\left(b^2(c+dx)^2\text{PolyLog}(2,-\cos(a+bx)-i\sin(a+bx)) + 2ibd(c+dx)\text{PolyLog}(3,-\cos(a+bx)-i\sin(a+bx)) - 2d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^2*(
c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)
*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a + b*x
] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I
*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*
x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]) + Cos[b*x]*(b*(c + d
*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)*Sin[
a]) - (3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*(c + d*x)*(-6*d^2 + b^2*(c
 + d*x)^2)*Sin[a])*Sin[b*x])/b^4
```

Maple [B] time = 0.218, size = 847, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] -3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2-6/
b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))+2/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))
+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)
))-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)
))*x+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2
-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x
+a))-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/
b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c
^2*d*ln(1-exp(I*(b*x+a)))*a+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+
b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6
*I*d^3)/b^4*exp(I*(b*x+a))+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-6*I*d^3*polyl
og(4,-exp(I*(b*x+a)))/b^4+6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^
2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-3
*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,-exp(I*(b*x+
a)))*x^2-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+6/b^2*c^2*d*a*arctanh(exp(
I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-1/b^4*d^3*ln(exp(I*(b*x+a))+1)
*a^3-2/b*c^3*arctanh(exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)
))-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a
```

Maxima [B] time = 1.99071, size = 1241, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 3*a*c^2*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 3*a^2*c*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a)
 - 1))/b^2 - a^3*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
+ a) - 1))/b^3 - (12*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 12*I*d^3*polylog(
4, e^(I*b*x + I*a)) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x
 + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x + a))*arctan2
```

```
(sin(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*
I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x
+ a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^3*d^3 - 6*b*c
*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d
^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*
d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x
 + a))*dilog(-e^(I*b*x + I*a)) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x
 + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*dilog(e^
(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x +
a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^
3)*polylog(3, -e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*poly
log(3, e^(I*b*x + I*a)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a
^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))/b^3)/b
```

Fricas [C] time = 0.742391, size = 2280, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*c
os(b*x + a) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(co
s(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2
*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*
c*d^2*x - 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d
^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x +
 a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*
x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
+ 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*
sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^
```

```
2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
+ 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^
3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c
*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*po
lylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
 b^2*c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a), x)
```

3.100 $\int (c+dx)^2 \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=171

$$\frac{2id(c+dx)\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2d^2\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\mathrm{PolyLog}\left(3,e^{i(\ldots}\right.}{b^3}$$

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (2*d^2*Cos[a + b*x])/b^3 + ((
c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))
])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (2*d^2*PolyL
og[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - (2
*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.138333, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4408, 3296, 2638, 4183, 2531, 2282, 6589}

$$\frac{2id(c+dx)\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2d^2\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\mathrm{PolyLog}\left(3,e^{i(\ldots}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```


```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (2*d^2*Cos[a + b*x])/b^3 + ((
c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))
])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (2*d^2*PolyL
og[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - (2
*d*(c + d*x)*Sin[a + b*x])/b^2
```


Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^2 \csc(a+bx)\,dx - \int (c+dx)^2 \sin(a+bx)\,dx$$
$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^2\cos(a+bx)}{b} - \frac{(2d)\int (c+dx)\cos(a+$$
$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{2id(c+dx)\mathrm{Li}_2\left(-e^{i(a+}\right.}{b^2}$$
$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{2id}{}$$
$$= -\frac{2(c+dx)^2 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{2id}{}$$

Mathematica [A] time = 0.874117, size = 221, normalized size = 1.29

$$\frac{2ibd(c+dx)\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right) - 2ibd(c+dx)\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right) - 2d^2\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right) + 2d^2\mathrm{PolyLog}\left(3,e^{i\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - b^2*(c + d*x)^2*Log[1 + E^(I*(a
 + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*(c
 + d*x)*PolyLog[2, E^(I*(a + b*x))] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))] +
2*d^2*PolyLog[3, E^(I*(a + b*x))] + Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Co
s[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c
 + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

Maple [B] time = 0.184, size = 479, normalized size = 2.8

$$\frac{\left(d^2x^2b^2 + 2\,b^2cdx + b^2c^2 + 2\,ibd^2x - 2\,d^2 + 2\,ibcd\right)e^{i(bx+a)}}{2\,b^3} + \frac{\left(d^2x^2b^2 + 2\,b^2cdx + b^2c^2 - 2\,ibd^2x - 2\,d^2 - 2\,ibcd\right)e^{-i(bx+a}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] 1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I
*(b*x+a))+1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)
```

```
/b^3*exp(-I*(b*x+a))+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b^3*d^2*a^2*arct
anh(exp(I*(b*x+a)))-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2/b*c*d*ln(exp(I*(
b*x+a))+1)*x-2/b^2*c*d*ln(exp(I*(b*x+a))+1)*a+2/b*c*d*ln(1-exp(I*(b*x+a)))*
x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-2/b*c^2*arctanh(exp(I*(b*x+a)))+1/b*d^2*
ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-2*I/b^2*c*d*pol
ylog(2,exp(I*(b*x+a)))-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b^3*d^2*ln(exp(I*
(b*x+a))+1)*a^2-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+2*I/b^2*d^2*polylog
(2,-exp(I*(b*x+a)))*x+4/b^2*c*d*a*arctanh(exp(I*(b*x+a)))+2*I/b^2*c*d*polyl
og(2,-exp(I*(b*x+a)))
```

Maxima [B] time = 1.47035, size = 684, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 2*a*c*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
 + a^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b^2 - (4*d^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*
a)) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin
(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^
2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(b*x + a) + (-4*I*b*c*d - 4*I*(b
*x + a)*d^2 + 4*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (4*I*b*c*d + 4*I*(b*x +
a)*d^2 - 4*I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(b
*x + a))/b^2)/b
```

Fricas [C] time = 0.638458, size = 1462, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) + (-2*I*b*d^2*x - 2*I*b*c*d)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
 + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I*sin
(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) + 1/
2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
 a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*
d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(b*d
^2*x + b*c*d)*sin(b*x + a))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\cos(bx+a)\cot(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a), x)
```

3.101 $\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=94

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] (-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)*Cos[a + b*x])/b + (I
*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^
2 - (d*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.06174, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4408, 3296, 2637, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```


```
[Out] (-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)*Cos[a + b*x])/b + (I
*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^
2 - (d*Sin[a + b*x])/b^2
```


Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)\csc(a+bx)\,dx - \int (c+dx)\sin(a+bx)\,dx$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} - \frac{d\int\cos(a+bx)\,dx}{b} - \frac{d\int$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} - \frac{d\sin(a+bx)}{b^2} + \frac{(id)\,\mathrm{Subs}}{}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{id\,\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} - \frac{id\,\mathrm{Li}_2}{}$$

Mathematica [A] time = 0.17497, size = 176, normalized size = 1.87

$$\frac{d\left(i\left(\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)-\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+\right.\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (c*Cos[a + b*x])/b - (c*Log[Cos[(a + b*x)/2]])/b + (c*Log[Sin[(a + b*x)/2]]
)/b + (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) -
 a*Log[Tan[(a + b*x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(
I*(a + b*x))])))/b^2 + (d*Cos[b*x]*(b*x*Cos[a] - Sin[a]))/b^2 - (d*(Cos[a]
+ b*x*Sin[a])*Sin[b*x])/b^2
```

Maple [B] time = 0.164, size = 203, normalized size = 2.2

$$\frac{(dxb+bc+id)\,e^{i(bx+a)}}{2\,b^2}+\frac{(dxb+bc-id)\,e^{-i(bx+a)}}{2\,b^2}-2\,\frac{c\text{Artanh}\left(e^{i(bx+a)}\right)}{b}+\frac{d\ln\left(1-e^{i(bx+a)}\right)x}{b}+\frac{d\ln\left(1-e^{i(bx+a)}\right)a}{b^2}-\frac{id}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] 1/2*(d*x*b+b*c+I*d)/b^2*exp(I*(b*x+a))+1/2*(d*x*b+b*c-I*d)/b^2*exp(-I*(b*x+
a))-2/b*c*arctanh(exp(I*(b*x+a)))+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1
-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(I*(b*x+a)
)+1)*x-1/b^2*d*ln(exp(I*(b*x+a))+1)*a+I*d*polylog(2,-exp(I*(b*x+a)))/b^2+2/
b^2*d*a*arctanh(exp(I*(b*x+a)))
```

Maxima [B] time = 1.6541, size = 269, normalized size = 2.86

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)+\left(2i\,bdx+2i\,bc\right)\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(b*x + a
), cos(b*x + a) + 1) - 2*(b*d*x + b*c)*cos(b*x + a) - 2*I*d*dilog(-e^(I*b*x
 + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 2*d*sin(b*x + a))/b^2
```

Fricas [B] time = 0.607487, size = 787, normalized size = 8.37

$$\frac{2\,(bdx+bc)\cos(bx+a) - i\,d\mathrm{Li}_2\left(\cos(bx+a)+i\,\sin(bx+a)\right) + i\,d\mathrm{Li}_2\left(\cos(bx+a)-i\,\sin(bx+a)\right) - i\,d\mathrm{Li}_2\left(-\cos(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*d*x + b*c)*cos(b*x + a) - I*d*dilog(cos(b*x + a) + I*sin(b*x + a)
) + I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) + I*
sin(b*x + a)) + I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x + b*c)*l
og(cos(b*x + a) + I*sin(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a) - I*
sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
+ 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b
*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + 1) - 2*d*sin(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)*cot(b*x + a), x)
```

3.102 $\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=69

$$\text{Unintegrable}\left(\frac{\csc(a+bx)}{c+dx},x\right)-\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Unintegrable[Csc[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.106625, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx &= \int \frac{\csc(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int \frac{\csc(a+bx)}{c+dx}\\ &= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\csc(a+bx)}{c+dx}\,dx\end{aligned}$$

Mathematica [A] time = 8.49259, size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x), x]
```

Maple [A] time = 0.302, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(iE_1\left(\frac{ibdx+ibc}{d}\right)-iE_1\left(-\frac{ibdx+ibc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+2d\int\frac{\sin(bx+a)}{(dx+c)\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)}\,dx+2d\int\frac{}{(dx+c)\left(\cos(bx+a)^2-\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*((I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*d*integrate(sin(b*x + a)/((d*x + c
)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a
) + c), x) + 2*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x +
c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-
(b*c - a*d)/d))/d
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

3.103 $\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=87

$$\text{Unintegrable}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}$$

```
[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c
 + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Unintegrab
le[Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.130614, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c
 + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int]
[Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\csc(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\int \frac{c}{}\\
&= -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \frac{\csc(a+}{(c+dx}
\end{aligned}$$

Mathematica [A] time = 4.12491, size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.63, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\cot(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)\cot(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c)^2, x)
```

3.104 $\int (c+dx)^m \cot^2(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\cot^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Cot[a + b*x]^2, x]
```

Rubi [A] time = 0.0347689, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\,dx$$

Mathematica [A] time = 1.17278, size = 0, normalized size = 0.

$$\int (c+dx)^m \cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2, x]
```

Maple [A] time = 0.16, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cot (bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cot(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cot (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \cot (bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cot(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \cot^2 (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*cot(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cot (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

3.105 $\int (c+dx)^4 \cot^2(a+bx)\,dx$

Optimal. Leaf size=155

$$-\frac{6id^2(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{6d^3(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^4}+\frac{3id^4\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^5}+\frac{4d(c+dx)^3\log}{b}$$

```
[Out] ((-I)*(c + d*x)^4)/b - (c + d*x)^5/(5*d) - ((c + d*x)^4*Cot[a + b*x])/b + (
4*d*(c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)^2*
PolyLog[2, E^((2*I)*(a + b*x))])/b^3 + (6*d^3*(c + d*x)*PolyLog[3, E^((2*I)
*(a + b*x))])/b^4 + ((3*I)*d^4*PolyLog[4, E^((2*I)*(a + b*x))])/b^5
```

Rubi [A] time = 0.22885, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3717, 2190, 2531, 6609, 2282, 6589, 32}

$$-\frac{6id^2(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{6d^3(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^4}+\frac{3id^4\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^5}+\frac{4d(c+dx)^3\log}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^2,x]
```


```
[Out] ((-I)*(c + d*x)^4)/b - (c + d*x)^5/(5*d) - ((c + d*x)^4*Cot[a + b*x])/b + (
4*d*(c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)^2*
PolyLog[2, E^((2*I)*(a + b*x))])/b^3 + (6*d^3*(c + d*x)*PolyLog[3, E^((2*I)
*(a + b*x))])/b^4 + ((3*I)*d^4*PolyLog[4, E^((2*I)*(a + b*x))])/b^5
```


Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = -\frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{(4d)\int (c+dx)^3 \cot(a+bx)\,dx}{b} - \int (c+dx)^4\,dx$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} - \frac{(8id)\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{(12d^2)}{}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{6id^2(c}{}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{6id^2(c}{}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{6id^2(c}{}$$

$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4 \cot(a+bx)}{b} + \frac{4d(c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{6id^2(c}{}$$

Mathematica [B] time = 6.72704, size = 795, normalized size = 5.13

$$-\frac{e^{ia}\csc(a)\left(b^4 e^{-2ia} x^4 + 2ib^3\left(1-e^{-2ia}\right)\log\left(1-e^{-i(a+bx)}\right)x^3 + 2ib^3\left(1-e^{-2ia}\right)\log\left(1+e^{-i(a+bx)}\right)x^3 - 6e^{-2ia}\left(-1+e^{2ia}\right)\left(b^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2,x]
```

```
[Out] -(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4))/5 - (2*c
*d^3*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))
*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 +
 E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a +
b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I
)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))
]))/E^((2*I)*a)))/b^4 - (d^4*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*
b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^(
(-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x^2
*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x))
```

```
] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))
*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(a
+ b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^5 + (4*c^3*d*
Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^
2*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c^4*Sin[b*x] + 4*c^3*d*x*S
in[b*x] + 6*c^2*d^2*x^2*Sin[b*x] + 4*c*d^3*x^3*Sin[b*x] + d^4*x^4*Sin[b*x])
)/b - (6*c^2*d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi
 + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]]
)*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[T
an[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan
[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 + Si
n[a]^2)])
```

Maple [B] time = 0.213, size = 913, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^2,x)
```

```
[Out] -24*I*d^3/b^3*c*polylog(2,exp(I*(b*x+a)))*x-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*
d^2*x^2+4*c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))-1)+4*d^4/b^2*ln(1-exp(I*(b*x+a))
)*x^3-2*I*d^4/b*x^4-8*d/b^2*c^3*ln(exp(I*(b*x+a)))+4*d/b^2*c^3*ln(exp(I*(b*
x+a))-1)+24*I*d^4/b^5*polylog(4,-exp(I*(b*x+a)))-6*I*d^4/b^5*a^4+4*d/b^2*c^
3*ln(exp(I*(b*x+a))+1)+24*d^4/b^4*polylog(3,exp(I*(b*x+a)))*x+24*d^4/b^4*po
lylog(3,-exp(I*(b*x+a)))*x-4*d^4/b^5*a^3*ln(exp(I*(b*x+a))-1)+24*d^3/b^4*c*
polylog(3,-exp(I*(b*x+a)))+24*d^3/b^4*c*polylog(3,exp(I*(b*x+a)))+8*d^4/b^5
*a^3*ln(exp(I*(b*x+a)))+24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5-1/5*d^4*x^5-
c^4*x-12*d^2/b^3*c^2*a*ln(exp(I*(b*x+a))-1)+12*d^3/b^4*c*a^2*ln(exp(I*(b*x+
a))-1)-24*I*d^2/b^2*c^2*a*x-24*I*d^3/b^3*c*polylog(2,-exp(I*(b*x+a)))*x+12*
d^3/b^2*c*ln(exp(I*(b*x+a))+1)*x^2+12*d^3/b^2*c*ln(1-exp(I*(b*x+a)))*x^2-12
*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a^2-12*I*d^2/b^3*c^2*polylog(2,exp(I*(b*x+a
)))-12*I*d^4/b^3*polylog(2,exp(I*(b*x+a)))*x^2-2*c^2*d^2*x^3-2*c^3*d*x^2-24
*d^3/b^4*c*a^2*ln(exp(I*(b*x+a)))+24*d^2/b^3*c^2*a*ln(exp(I*(b*x+a)))-c*d^3
*x^4+24*I*d^3/b^3*c*a^2*x-12*I*d^2/b^3*c^2*polylog(2,-exp(I*(b*x+a)))-12*I*
d^4/b^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I*d^2/b*c^2*x^2-8*I*d^3/b*c*x^3-8
*I*d^4/b^4*a^3*x+16*I*d^3/b^4*c*a^3-12*I*d^2/b^3*c^2*a^2+4*d^4/b^5*ln(1-exp
(I*(b*x+a)))*a^3+4*d^4/b^2*ln(exp(I*(b*x+a))+1)*x^3+12*d^2/b^2*c^2*ln(1-exp
(I*(b*x+a)))*x+12*d^2/b^3*c^2*ln(1-exp(I*(b*x+a)))*a+12*d^2/b^2*c^2*ln(exp(
I*(b*x+a))+1)*x
```

Maxima [B] time = 3.24618, size = 4359, normalized size = 28.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((b*x + a + 1/tan(b*x + a))*c^4 - 4*(b*x + a + 1/tan(b*x + a))*a*c^3*d/b +
 6*(b*x + a + 1/tan(b*x + a))*a^2*c^2*d^2/b^2 - 4*(b*x + a + 1/tan(b*x + a)
)*a^3*c*d^3/b^3 + (b*x + a + 1/tan(b*x + a))*a^4*d^4/b^4 + 2*((b*x + a)^2*c
os(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*
x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
- (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(co
s(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x
 + 2*a))*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*
a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*
a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2
*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
 + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a*c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) + 6*((b*x + a)^2*cos(2*b*x +
2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) +
(b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)
 + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*
x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^
2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b^3) - 2*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2
- 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
 + 4*(b*x + a)*sin(2*b*x + 2*a))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^4) - (-I*(b*x + a)^5*d^4 + (-5*I*b*c*d^
3 + 5*I*a*d^4)*(b*x + a)^4 + (-10*I*b^2*c^2*d^2 + 20*I*a*b*c*d^3 - 10*I*a^2
*d^4)*(b*x + a)^3 - (20*(b*x + a)^3*d^4 + 60*(b*c*d^3 - a*d^4)*(b*x + a)^2
+ 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - 20*((b*x + a)^3*d^4
+ 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a))*cos(2*b*x + 2*a) - (20*I*(b*x + a)^3*d^4 + (60*I*b*c*d^3 - 60*I
*a*d^4)*(b*x + a)^2 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + 60*I*a^2*d^4)*(
```

```
b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (20*(
b*x + a)^3*d^4 + 60*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 60*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + a^2*d^4)*(b*x + a) - 20*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*
x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2
*a) + (-20*I*(b*x + a)^3*d^4 + (-60*I*b*c*d^3 + 60*I*a*d^4)*(b*x + a)^2 + (
-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a))*sin(2*b*x +
2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*(b*x + a)^5*d^4 + (5*I*
b*c*d^3 - 5*(I*a + 2)*d^4)*(b*x + a)^4 + (10*I*b^2*c^2*d^2 - 20*(I*a + 2)*b
*c*d^3 + (10*I*a^2 + 40*a)*d^4)*(b*x + a)^3 - 60*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a)^2)*cos(2*b*x + 2*a) + (60*b^2*c^2*d^2 - 120*a*b*c*d^3
 + 60*(b*x + a)^2*d^4 + 60*a^2*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a) - 60*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*
(b*x + a)^2*d^4 - 60*I*a^2*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a))*
sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (60*b^2*c^2*d^2 - 120*a*b*c*d^3
 + 60*(b*x + a)^2*d^4 + 60*a^2*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a) - 60*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*
(b*x + a)^2*d^4 - 60*I*a^2*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a))*
sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (10*I*(b*x + a)^3*d^4 + (30*I*b*
c*d^3 - 30*I*a*d^4)*(b*x + a)^2 + (30*I*b^2*c^2*d^2 - 60*I*a*b*c*d^3 + 30*I
*a^2*d^4)*(b*x + a) + (-10*I*(b*x + a)^3*d^4 + (-30*I*b*c*d^3 + 30*I*a*d^4)
*(b*x + a)^2 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2*d^4)*(b*x + a
))*cos(2*b*x + 2*a) + 10*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2
 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (10*I*(b*x + a)^3*
d^4 + (30*I*b*c*d^3 - 30*I*a*d^4)*(b*x + a)^2 + (30*I*b^2*c^2*d^2 - 60*I*a*
b*c*d^3 + 30*I*a^2*d^4)*(b*x + a) + (-10*I*(b*x + a)^3*d^4 + (-30*I*b*c*d^3
 + 30*I*a*d^4)*(b*x + a)^2 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2
*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 10*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^
4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 120*
(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog(4, -e^(I*b*x
+ I*a)) + 120*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog
(4, e^(I*b*x + I*a)) + (120*I*b*c*d^3 + 120*I*(b*x + a)*d^4 - 120*I*a*d^4 +
 (-120*I*b*c*d^3 - 120*I*(b*x + a)*d^4 + 120*I*a*d^4)*cos(2*b*x + 2*a) + 12
0*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x
+ I*a)) + (120*I*b*c*d^3 + 120*I*(b*x + a)*d^4 - 120*I*a*d^4 + (-120*I*b*c*
d^3 - 120*I*(b*x + a)*d^4 + 120*I*a*d^4)*cos(2*b*x + 2*a) + 120*(b*c*d^3 +
(b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - ((b*
x + a)^5*d^4 + (5*b*c*d^3 - (5*a - 10*I)*d^4)*(b*x + a)^4 + (10*b^2*c^2*d^2
 - (20*a - 40*I)*b*c*d^3 + 10*(a^2 - 4*I*a)*d^4)*(b*x + a)^3 - (-60*I*b^2*c
^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-5
*I*b^4*cos(2*b*x + 2*a) + 5*b^4*sin(2*b*x + 2*a) + 5*I*b^4))/b
```

Fricas [C] time = 0.618904, size = 2034, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/10*(10*b^4*d^4*x^4 + 40*b^4*c*d^3*x^3 + 60*b^4*c^2*d^2*x^2 + 40*b^4*c^3*
d*x + 10*b^4*c^4 - 15*I*d^4*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a
))*sin(2*b*x + 2*a) + 15*I*d^4*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a))*sin(2*b*x + 2*a) - (-30*I*b^2*d^4*x^2 - 60*I*b^2*c*d^3*x - 30*I*b^2*c
^2*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - (30
*I*b^2*d^4*x^2 + 60*I*b^2*c*d^3*x + 30*I*b^2*c^2*d^2)*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2
+ 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*
a) + 1/2)*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^
3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(
2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^
2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x +
2*a) + 1)*sin(2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*
d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) -
I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) - 30*(b*d^4*x + b*c*d^3)*polylog(3
, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 30*(b*d^4*x + b
*c*d^3)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a)
+ 10*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b
^4*c^4)*cos(2*b*x + 2*a) + 2*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*b^5*c^2*d^
2*x^3 + 10*b^5*c^3*d*x^2 + 5*b^5*c^4*x)*sin(2*b*x + 2*a))/(b^5*sin(2*b*x +
2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^4 \cot^2 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cot (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^2, x)
```

3.106 $\int (c+dx)^3 \cot^2(a+bx)\,dx$

Optimal. Leaf size=127

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a+bx}{b}$$

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) - ((c + d*x)^3*Cot[a + b*x])/b + (
3*d*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*Po
lyLog[2, E^((2*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^4)
```

Rubi [A] time = 0.197258, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3717, 2190, 2531, 2282, 6589, 32}

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a+bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) - ((c + d*x)^3*Cot[a + b*x])/b + (
3*d*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*Po
lyLog[2, E^((2*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^4)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = -\frac{(c+dx)^3\cot(a+bx)}{b} + \frac{(3d)\int (c+dx)^2\cot(a+bx)\,dx}{b} - \int (c+dx)^3\,dx$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} - \frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{(6d^2)}{}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c}{}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c}{}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c}{}$$

Mathematica [B] time = 6.44499, size = 374, normalized size = 2.94

$$\frac{3cd^2\left(-i\text{PolyLog}\left(2, e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right) - b^2x^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)} + ibx\left(\pi - 2\tan^{-1}(\tan(a))\right) + 2\left(\tan^{-1}(\tan\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2,x]
```

```
[Out] -(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (3*c^2*d*(b*x*Cot[a] -
 Log[Sin[a + b*x]]))/b^2 + (3*c*d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*Log
[1 + E^((-2*I)*b*x)] + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + Arc
Tan[Tan[a]]))] - Pi*Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[T
an[a]]]] - I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcTan
[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2]))/b^3 + (d^3*(I + Cot[a])*(I*b^3*x^3 -
b^3*x^3*Cot[a] + 3*b^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 3*b^2*x^2*Log[1 +
E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*x
*PolyLog[2, E^((-I)*(a + b*x))] + 6*PolyLog[3, -E^((-I)*(a + b*x))] + 6*Pol
yLog[3, E^((-I)*(a + b*x))])*Sin[a])/(b^4*E^(I*a)) + ((c + d*x)^3*Csc[a]*Cs
c[a + b*x]*Sin[b*x])/b
```

Maple [B] time = 0.194, size = 573, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^2,x)
```

```
[Out] 6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-3*
d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/b
^2*ln(1-exp(I*(b*x+a)))*x^2+3*d^3/b^4*a^2*ln(exp(I*(b*x+a))-1)-c*d^2*x^3-1/
4*d^3*x^4-c^3*x-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))
-1)-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))-6*d/b^2*c^2*ln(exp(I*(b*x+a)))+3*d/b^2
*c^2*ln(exp(I*(b*x+a))+1)+3*d/b^2*c^2*ln(exp(I*(b*x+a))-1)-2*I*d^3/b*x^3+4*
I*d^3/b^4*a^3-12*I*d^2/b^2*c*a*x+6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x+6*d^2/b
^2*c*ln(1-exp(I*(b*x+a)))*x+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a-6*I*d^3/b^3*
polylog(2,exp(I*(b*x+a)))*x-6*d^2/b^3*c*a*ln(exp(I*(b*x+a))-1)-6*I*d^3/b^3*
polylog(2,-exp(I*(b*x+a)))*x-3/2*c^2*d*x^2+12*d^2/b^3*c*a*ln(exp(I*(b*x+a))
)-6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))+6*I*d^3/b^3*a^2*x-6*I*d^2/b^3*c*a
^2-6*I*d^2/b*c*x^2-6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))
```

Maxima [B] time = 2.36897, size = 2626, normalized size = 20.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a + 1/tan(b*x + a))*c^3 - 6*(b*x + a + 1/tan(b*x + a))*a*c^2
*d/b + 6*(b*x + a + 1/tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a + 1/tan(b*x
+ a))*a^3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b
*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b
*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x +
 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) +
 (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a
*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*
b^2) + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 -
 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2
```

```
*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
+ 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(-I*(b*x + a)^4*d^3 + (-4*I*b*c*d
^2 + 4*I*a*d^3)*(b*x + a)^3 - (12*(b*x + a)^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b
*x + a) - 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x +
2*a) - (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (12*(b*x + a)^2*d^3
+ 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3
)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*(b*x + a)^2*d^3 + (-24*I*b*c*d^2 + 2
4*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a)
 + 1) + (I*(b*x + a)^4*d^3 + (4*I*b*c*d^2 - 4*(I*a + 2)*d^3)*(b*x + a)^3 -
24*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) + (24*b*c*d^2 + 24*(b*x
+ a)*d^3 - 24*a*d^3 - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a)
 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilo
g(-e^(I*b*x + I*a)) + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d
^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-24*I*b*c*d^2 - 24*I*(b*x +
 a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (6*I*(b*x
+ a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3
+ (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d
^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*
a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^
3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos
(b*x + a) + 1) + (-24*I*d^3*cos(2*b*x + 2*a) + 24*d^3*sin(2*b*x + 2*a) + 24
*I*d^3)*polylog(3, -e^(I*b*x + I*a)) + (-24*I*d^3*cos(2*b*x + 2*a) + 24*d^3
*sin(2*b*x + 2*a) + 24*I*d^3)*polylog(3, e^(I*b*x + I*a)) - ((b*x + a)^4*d^
3 + (4*b*c*d^2 - (4*a - 8*I)*d^3)*(b*x + a)^3 - (-24*I*b*c*d^2 + 24*I*a*d^3
)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(2*b
*x + 2*a) + 4*I*b^3))/b
```

Fricas [C] time = 0.565249, size = 1439, normalized size = 11.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 3*d^3
*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*d^3
```

```
*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - (-6*I
*b*d^3*x - 6*I*b*c*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*
b*x + 2*a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a))*sin(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/
2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*
x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c
*d^2 - a^2*d^3)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*sin(2*b*x +
 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(2*
b*x + 2*a) - I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) + 4*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(2*b*x + 2*a) + (b^4*d^3*x^4 +
4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x)*sin(2*b*x + 2*a))/(b^4*sin
(2*b*x + 2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \cot^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cot(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^2, x)
```

3.107 $\int (c+dx)^2 \cot^2(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{id^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) - ((c + d*x)^2*Cot[a + b*x])/b + (
2*d*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^2 - (I*d^2*PolyLog[2, E^((2*I
)*(a + b*x))])/b^3
```

Rubi [A] time = 0.129615, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3720, 3717, 2190, 2279, 2391, 32}

$$-\frac{id^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) - ((c + d*x)^2*Cot[a + b*x])/b + (
2*d*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^2 - (I*d^2*PolyLog[2, E^((2*I
)*(a + b*x))])/b^3
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^2\cot^2(a+bx)\,dx = -\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{(2d)\int (c+dx)\cot(a+bx)\,dx}{b}-\int (c+dx)^2\,dx$$

$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{\left(2d^2\right)\int \mathrm{l}}{}$$

$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}+\frac{\left(id^2\right)\mathrm{Su}}{}$$

$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\mathrm{Li}_2\left(e\right.}{b^3}$$

Mathematica [B] time = 6.31777, size = 198, normalized size = 2.04

$$\frac{d^2\left(-i\mathrm{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)-b^2x^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)}+ibx\left(\pi-2\tan^{-1}(\tan(a))\right)+2\left(\tan^{-1}(\tan(a)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2,x]
```

```
[Out] -(x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 - (2*c*d*(b*x*Cot[a] - Log[Sin[a + b*x]]
))/b^2 + (d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*Log[1 + E^((-2*I)*b*x)] +
 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - Pi*Lo
g[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - I*PolyLog[2
, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*S
qrt[Sec[a]^2]))/b^3 + ((c + d*x)^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

Maple [B] time = 0.152, size = 297, normalized size = 3.1

$$-\frac{d^2x^3}{3}-cdx^2-c^2x-\frac{2\,id^2x^2}{b}+2\,\frac{cd\ln\left(e^{i(bx+a)}-1\right)}{b^2}-4\,\frac{cd\ln\left(e^{i(bx+a)}\right)}{b^2}+2\,\frac{cd\ln\left(e^{i(bx+a)}+1\right)}{b^2}-\frac{2\,id^2a^2}{b^3}-\frac{2\,i\left(d^2x^2+2\right.}{b\left(e^{2\,i(bx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^2,x)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2-c^2*x-2*I*d^2/b*x^2+2*d/b^2*c*ln(exp(I*(b*x+a))-1)-4*d
/b^2*c*ln(exp(I*(b*x+a)))+2*d/b^2*c*ln(exp(I*(b*x+a))+1)-2*I*d^2/b^3*a^2-2*
I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))-1)-2*I*d^2*polylog(2,exp(I*(b*x
+a)))/b^3+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a
-2*I*d^2/b^3*polylog(2,-exp(I*(b*x+a)))+2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-4*
I*d^2/b^2*a*x-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)+4*d^2/b^3*a*ln(exp(I*(b*x+a)
))
```

Maxima [B] time = 2.2818, size = 872, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-I*b^3*d^2*x^3 - 3*I*b^3*c*d*x^2 - 3*I*b^3*c^2*x - 6*b^2*c^2 - (6*b*d^2*x
+ 6*b*c*d - 6*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (6*I*b*d^2*x + 6*I*b*c*d
)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*b*c*d*cos(
```

```
2*b*x + 2*a) + 6*I*b*c*d*sin(2*b*x + 2*a) - 6*b*c*d)*arctan2(sin(b*x + a),
cos(b*x + a) - 1) - (6*b*d^2*x*cos(2*b*x + 2*a) + 6*I*b*d^2*x*sin(2*b*x + 2
*a) - 6*b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*d^2*x^3
+ (3*I*b^3*c*d - 6*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(2*b*x +
 2*a) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I
*b*x + I*a)) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilo
g(e^(I*b*x + I*a)) + (3*I*b*d^2*x + 3*I*b*c*d + (-3*I*b*d^2*x - 3*I*b*c*d)*
cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3*I*b*d^2*x + 3*I*b*c*d + (-3*I*
b*d^2*x - 3*I*b*c*d)*cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a
))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*d^2*x^3
 + 3*(b^3*c*d + 2*I*b^2*d^2)*x^2 + (3*b^3*c^2 + 12*I*b^2*c*d)*x)*sin(2*b*x
+ 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

Fricas [B] time = 0.543463, size = 950, normalized size = 9.79

$$-\frac{6\,b^2d^2x^2 + 12\,b^2cdx + 6\,b^2c^2 + 3i\,d^2\mathrm{Li}_2\left(\cos\left(2\,bx + 2\,a\right) + i\,\sin\left(2\,bx + 2\,a\right)\right)\sin\left(2\,bx + 2\,a\right) - 3i\,d^2\mathrm{Li}_2\left(\cos\left(2\,bx + 2\,a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^2*d^2*x^2 + 12*b^2*c*d*x + 6*b^2*c^2 + 3*I*d^2*dilog(cos(2*b*x +
2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*I*d^2*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*(b*c*d - a*d^2)*log(-1/2*cos(2
*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b*c*d - a
*d^2)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x +
 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)
*sin(2*b*x + 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a) + 1)*sin(2*b*x + 2*a) + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(2*b*x + 2*a) + 2*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x)*sin(2*b*x + 2
*a))/(b^3*sin(2*b*x + 2*a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cot^2 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \cot (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^2, x)
```

3.108 $\int (c+dx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-cx-\frac{dx^2}{2}$$

```
[Out] -(c*x) - (d*x^2)/2 - ((c + d*x)*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2
```

Rubi [A] time = 0.0261395, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3720, 3475}

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-cx-\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^2,x]
```

```
[Out] -(c*x) - (d*x^2)/2 - ((c + d*x)*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot^2(a+bx)\,dx &= -\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\int \cot(a+bx)\,dx}{b}-\int (c+dx)\,dx\\ &= -cx-\frac{dx^2}{2}-\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}\end{aligned}$$

Mathematica [C] time = 0.450996, size = 82, normalized size = 2.

$$-\frac{c\cot(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(a+bx)\right)}{b}+\frac{d\log(\sin(a+bx))}{b^2}+\frac{dx\csc(a)\sin(bx)\csc(a+bx)}{b}-$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^2,x]
```

```
[Out] -((c*Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b) + (d
*Log[Sin[a + b*x]])/b^2 - (d*x*Csc[a]*(2*Cos[a] + b*x*Sin[a]))/(2*b) + (d*x
*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

Maple [A] time = 0.041, size = 49, normalized size = 1.2

$$-\frac{dx^2}{2}-cx-\frac{d\cot(bx+a)\,x}{b}+\frac{d\ln(\sin(bx+a))}{b^2}-\frac{c\cot(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^2,x)
```

```
[Out] -1/2*d*x^2-c*x-1/b*d*cot(b*x+a)*x+d*ln(sin(b*x+a))/b^2-1/b*c*cot(b*x+a)
```

Maxima [B] time = 1.68273, size = 394, normalized size = 9.61

$$-\frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)c-\frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)ad}{b}+\frac{\left((bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2-2\,(bx+a)^2\cos(2\,bx+2\,a)+(bx+a)^2-\left(\cos(2\,bx+2\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a + 1/tan(b*x + a))*c - 2*(b*x + a + 1/tan(b*x + a))*a*d/b +
 ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x
+ a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos
(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2
```

```
*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x
 + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos
(2*b*x + 2*a) + 1)*b))/b
```

Fricas [B] time = 0.490468, size = 242, normalized size = 5.9

$$-\frac{2\,bdx - d\log\left(-\frac{1}{2}\cos\left(2\,bx + 2\,a\right) + \frac{1}{2}\right)\sin\left(2\,bx + 2\,a\right) + 2\,bc + 2\,(bdx + bc)\cos\left(2\,bx + 2\,a\right) + \left(b^2dx^2 + 2\,b^2cx\right)\sin\left(2\,bx\right.}{2\,b^2\sin\left(2\,bx + 2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x - d*log(-1/2*cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) + 2*b*c
 + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d*x^2 + 2*b^2*c*x)*sin(2*b*x + 2
*a))/(b^2*sin(2*b*x + 2*a))
```

Sympy [A] time = 1.29077, size = 100, normalized size = 2.44

$$\begin{cases} \tilde{\infty}\left(cx + \frac{dx^2}{2}\right) & \text{for } (a = 0 \vee a = -bx) \wedge (a = -bx \vee b = 0) \\ \left(cx + \frac{dx^2}{2}\right)\cot^2(a) & \text{for } b = 0 \\ -cx - \frac{dx^2}{2} - \frac{c}{b\tan(a+bx)} - \frac{dx}{b\tan(a+bx)} - \frac{d\log\left(\tan^2(a+bx)+1\right)}{2b^2} + \frac{d\log(\tan(a+bx))}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*(c*x + d*x**2/2), (Eq(a, 0) | Eq(a, -b*x)) & (Eq(b, 0) | Eq(
a, -b*x))), ((c*x + d*x**2/2)*cot(a)**2, Eq(b, 0)), (-c*x - d*x**2/2 - c/(b
*tan(a + b*x)) - d*x/(b*tan(a + b*x)) - d*log(tan(a + b*x)**2 + 1)/(2*b**2)
 + d*log(tan(a + b*x))/b**2, True))
```

Giac [B] time = 1.60698, size = 1856, normalized size = 45.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*d*x^2*tan(1/2*b*x)*tan(1/2*
a)^2 + 2*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c*x*tan(1/2*b*x)*tan(1/2
*a)^2 - b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d*x^2*tan(1/2*b*x) - b^2*d*
x^2*tan(1/2*a) - b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*x*tan(1/2*b*x) +
 b*d*x*tan(1/2*b*x)^2 - 2*b^2*c*x*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*tan(1/2
*a) - d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(
1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*ta
n(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x
)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a
)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan
(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan
(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^2*tan
(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)
/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x
)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a
)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1
/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2
*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)
^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/
2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*
a)^2))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2 + 4*b*c*tan(1/2*b*x)*
tan(1/2*a) + b*c*tan(1/2*a)^2 - b*d*x + d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*
a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + ta
n(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*
tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^
4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2
*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan
(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 -
 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) +
 tan(1/2*a)^2))*tan(1/2*b*x) + d*log(16*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)
/(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x
)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a
)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1
/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2
*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)
^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/
2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*
a)^2))*tan(1/2*a) - b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*
tan(1/2*a)^2 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

3.109 $\int \frac{\cot^2(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\cot^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Cot[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0355474, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/(c + d*x),x]

[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx = \int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 4.64655, size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/(c + d*x),x]

[Out] Integrate[Cot[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.332, size = 0, normalized size = 0.

$$\int \frac{\left(\cot \left(bx + a\right)\right)^{2}}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cot(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot \left(bx + a\right)^{2}}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{2}\left(a + bx\right)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cot(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2/(d*x + c), x)
```

3.110 $\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\cot^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Cot[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0344833, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/(c + d*x)^2,x]

[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 2.41796, size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/(c + d*x)^2,x]

[Out] Integrate[Cot[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.447, size = 0, normalized size = 0.

$$\int \frac{(\cot(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(cot(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cot(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2/(d*x + c)^2, x)
```

3.111 $\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\csc^3(a+bx)(c+dx)^m,x\right)-\text{Unintegrable}\left(\csc(a+bx)(c+dx)^m,x\right)$$

```
[Out] -Unintegrable[(c + d*x)^m*Csc[a + b*x], x] + Unintegrable[(c + d*x)^m*Csc[a
 + b*x]^3, x]
```

Rubi [A] time = 0.0752131, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -Defer[Int][(c + d*x)^m*Csc[a + b*x], x] + Defer[Int][(c + d*x)^m*Csc[a + b
*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx=-\int (c+dx)^m \csc(a+bx)\,dx+\int (c+dx)^m \csc^3(a+bx)\,dx$$

Mathematica [A] time = 10.2968, size = 0, normalized size = 0.

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x], x]
```

Maple [A] time = 0.18, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cot (bx+a))^2 \csc (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cot (bx+a)^2 \csc (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \cot (bx+a)^2 \csc (bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \cot^2 (a+bx) \csc (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cot (bx+a)^2 \csc (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

3.112 $\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=416

$$\frac{12id^3(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4}-\frac{12id^3(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4}+\frac{12id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}-\frac{12id^3(}{}$$

```
[Out] (-12*d^2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^4*ArcTanh[E
^(I*(a + b*x))])/b - (2*d*(c + d*x)^3*Csc[a + b*x])/b^2 - ((c + d*x)^4*Cot[
a + b*x]*Csc[a + b*x])/(2*b) + ((12*I)*d^3*(c + d*x)*PolyLog[2, -E^(I*(a +
b*x))])/b^4 - ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((12
*I)*d^3*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^4 + ((2*I)*d*(c + d*x)^3*P
olyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^4*PolyLog[3, -E^(I*(a + b*x))])/b^5
 + (6*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (12*d^4*PolyLog[3
, E^(I*(a + b*x))])/b^5 - (6*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b
^3 + ((12*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 - ((12*I)*d^3*
(c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b^4 - (12*d^4*PolyLog[5, -E^(I*(a +
b*x))])/b^5 + (12*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5
```

Rubi [A] time = 0.501959, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4415, 4183, 2531, 6609, 2282, 6589, 4186}

$$\frac{12id^3(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4}-\frac{12id^3(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4}+\frac{12id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}-\frac{12id^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```


```
[Out] (-12*d^2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^4*ArcTanh[E
^(I*(a + b*x))])/b - (2*d*(c + d*x)^3*Csc[a + b*x])/b^2 - ((c + d*x)^4*Cot[
a + b*x]*Csc[a + b*x])/(2*b) + ((12*I)*d^3*(c + d*x)*PolyLog[2, -E^(I*(a +
b*x))])/b^4 - ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((12
*I)*d^3*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^4 + ((2*I)*d*(c + d*x)^3*P
olyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^4*PolyLog[3, -E^(I*(a + b*x))])/b^5
 + (6*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (12*d^4*PolyLog[3
, E^(I*(a + b*x))])/b^5 - (6*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b
^3 + ((12*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 - ((12*I)*d^3*
(c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b^4 - (12*d^4*PolyLog[5, -E^(I*(a +
b*x))])/b^5 + (12*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^4\csc(a+bx)\,dx + \int (c+dx)^4\csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\csc(a+bx)}{b^2} - \frac{(c+dx)^4\cot(a+bx)\,\mathrm{c}}{2b}$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\,\mathrm{cs}}{b^2}$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\,\mathrm{cs}}{b^2}$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\,\mathrm{cs}}{b^2}$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\,\mathrm{cs}}{b^2}$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\,\mathrm{cs}}{b^2}$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\,\mathrm{cs}}{b^2}$$

Mathematica [B] time = 8.43926, size = 966, normalized size = 2.32

$$\frac{-c^4\log\left(1-e^{i(a+bx)}\right)b^4 - d^4x^4\log\left(1-e^{i(a+bx)}\right)b^4 - 4cd^3x^3\log\left(1-e^{i(a+bx)}\right)b^4 - 6c^2d^2x^2\log\left(1-e^{i(a+bx)}\right)b^4 - 4c^3dx\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-(b^4*c^4*Log[1 - E^(I*(a + b*x))]) + 12*b^2*c^2*d^2*Log[1 - E^(I*(a + b*x
))] - 4*b^4*c^3*d*x*Log[1 - E^(I*(a + b*x))] + 24*b^2*c*d^3*x*Log[1 - E^(I*
```

```
(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] + 12*b^2*d^4*x^2*L
og[1 - E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^4*d^
4*x^4*Log[1 - E^(I*(a + b*x))] + b^4*c^4*Log[1 + E^(I*(a + b*x))] - 12*b^2*
c^2*d^2*Log[1 + E^(I*(a + b*x))] + 4*b^4*c^3*d*x*Log[1 + E^(I*(a + b*x))] -
 24*b^2*c*d^3*x*Log[1 + E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(
a + b*x))] - 12*b^2*d^4*x^2*Log[1 + E^(I*(a + b*x))] + 4*b^4*c*d^3*x^3*Log[
1 + E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] - (4*I)*b*d*(c
+ d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] + (4*I)*b*d*
(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] + 12*b^2*c
^2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 24*d^4*PolyLog[3, -E^(I*(a + b*x))] +
 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, -E
^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*d^4*PolyL
og[3, E^(I*(a + b*x))] - 24*b^2*c*d^3*x*PolyLog[3, E^(I*(a + b*x))] - 12*b^
2*d^4*x^2*PolyLog[3, E^(I*(a + b*x))] + (24*I)*b*c*d^3*PolyLog[4, -E^(I*(a
+ b*x))] + (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*c*d^3*Pol
yLog[4, E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*x))] - 24*
d^4*PolyLog[5, -E^(I*(a + b*x))] + 24*d^4*PolyLog[5, E^(I*(a + b*x))])/(2*b
^5) - (Csc[a + b*x]^2*(b*c^4*Cos[a + b*x] + 4*b*c^3*d*x*Cos[a + b*x] + 6*b*
c^2*d^2*x^2*Cos[a + b*x] + 4*b*c*d^3*x^3*Cos[a + b*x] + b*d^4*x^4*Cos[a + b
*x] + 4*c^3*d*Sin[a + b*x] + 12*c^2*d^2*x*Sin[a + b*x] + 12*c*d^3*x^2*Sin[a
 + b*x] + 4*d^4*x^3*Sin[a + b*x]))/(2*b^2)
```

Maple [B] time = 0.381, size = 1673, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] -1/2/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/2/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4-4/
b^4*c*d^3*a^3*arctanh(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*arctanh(exp(I*(b*x+
a)))-4/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))+24/b^4*c*d^3*a*arctanh(exp(I*(b*
x+a)))+2*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+12*I/b^4*c*d^3*polylog(2,-ex
p(I*(b*x+a)))-2*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+12*I/b^4*d^4*polyl
og(4,-exp(I*(b*x+a)))*x-2/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+6*d^4/b^3*ln(1-e
xp(I*(b*x+a)))*x^2-6*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^2-6*d^4/b^3*ln(exp(I*(b
*x+a))+1)*x^2+2/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a^3-2/b^4*c*d^3*ln(1-exp(I*(
b*x+a)))*a^3-12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(3,exp(I*(
b*x+a)))/b^5+2/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-6/b^3*d^4*polylog(3,exp(I*(
b*x+a)))*x^2-1/2/b^5*d^4*a^4*ln(exp(I*(b*x+a))+1)-6/b^3*c^2*d^2*polylog(3,e
xp(I*(b*x+a)))+6/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+1/b^5*d^4*a^4*arcta
```

```
nh(exp(I*(b*x+a)))+6/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-3/b*c^2*d^2*ln(
1-exp(I*(b*x+a)))*x^2+1/2/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+3/b^3*c^2*d^2*a^2*
ln(1-exp(I*(b*x+a)))-2/b*c^3*d*ln(1-exp(I*(b*x+a)))*x-2/b^2*c^3*d*ln(1-exp(
I*(b*x+a)))*a+2/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+12/b^3*c*d^3*polylog(3,-exp(
I*(b*x+a)))*x-3/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))+1)+3/b*c^2*d^2*ln(exp(I*(
b*x+a))+1)*x^2-12/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x+2/b^2*c^3*d*ln(exp(
I*(b*x+a))+1)*a+12*d^3/b^3*c*ln(1-exp(I*(b*x+a)))*x+12*d^3/b^4*c*ln(1-exp(I
*(b*x+a)))*a-12*d^3/b^3*c*ln(exp(I*(b*x+a))+1)*x-12*I*d^4/b^4*polylog(2,exp
(I*(b*x+a)))*x-12*I*d^3/b^4*c*polylog(2,exp(I*(b*x+a)))+1/b^2/(exp(2*I*(b*x
+a))-1)^2*(d^4*x^4*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b*exp(3*I*(b*x+a))+6*c^2*
d^2*x^2*b*exp(3*I*(b*x+a))+d^4*x^4*b*exp(I*(b*x+a))+4*c^3*d*x*b*exp(3*I*(b*
x+a))+4*c*d^3*x^3*b*exp(I*(b*x+a))+12*I*c*d^3*x^2*exp(I*(b*x+a))+c^4*b*exp(
3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(I*(b*x+a))+4*I*c^3*d*exp(I*(b*x+a))+4*c^3*
d*x*b*exp(I*(b*x+a))-12*I*c*d^3*x^2*exp(3*I*(b*x+a))-4*I*c^3*d*exp(3*I*(b*x
+a))+c^4*b*exp(I*(b*x+a))+4*I*d^4*x^3*exp(I*(b*x+a))-4*I*d^4*x^3*exp(3*I*(b
*x+a))-12*I*c^2*d^2*x*exp(3*I*(b*x+a))+12*I*c^2*d^2*x*exp(I*(b*x+a)))+6/b^5
*d^4*ln(exp(I*(b*x+a))+1)*a^2-12/b^5*d^4*a^2*arctanh(exp(I*(b*x+a)))-12/b^3
*c^2*d^2*arctanh(exp(I*(b*x+a)))-12*d^4*polylog(5,-exp(I*(b*x+a)))/b^5+12*d
^4*polylog(5,exp(I*(b*x+a)))/b^5+1/b*c^4*arctanh(exp(I*(b*x+a)))+6*I/b^2*po
lylog(2,exp(I*(b*x+a)))*c^2*d^2*x-6*I/b^2*polylog(2,-exp(I*(b*x+a)))*c^2*d^
2*x+6*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-12/b^4*c*d^3*ln(exp(I*(b*x+
a))+1)*a-12*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+2*I/b^2*d^4*polylog(2,exp
(I*(b*x+a)))*x^3+12*I/b^4*d^4*polylog(2,-exp(I*(b*x+a)))*x+12*I/b^4*c*d^3*p
olylog(4,-exp(I*(b*x+a)))-2*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))-12*I/b^4
*c*d^3*polylog(4,exp(I*(b*x+a)))-6*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x
^2
```

Maxima [B] time = 18.8847, size = 9385, normalized size = 22.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 4*a*c^3*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 6*a^2*c^2*d^2*(2*cos(b*x + a
)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2
 - 4*a^3*c*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1)
 - log(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1
) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^4 + 4*((2*(b*x + a)^4*
```

d^4 - 24*b^2*c^2*d^2 + 48*a*b*c*d^3 - 24*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 4*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d^4 - 24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + (12*I*a^2 - 24*I)*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^4*d^4 + 48*I*b^2*c^2*d^2 - 96*I*a*b*c*d^3 + 48*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*I*a*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 + (-24*I*a^2 + 48*I)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 + (-48*I*a^2 + 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (24*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*a^2*d^4 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(4*b*x + 4*a) - 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(2*b*x + 2*a) + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)*sin(4*b*x + 4*a) + (-48*I*b^2*c^2*d^2 + 96*I*a*b*c*d^3 - 48*I*a^2*d^4)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 4*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + (12*I*a^2 - 24*I)*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^4*d^4 + (-16*I*b*c*d^3 + 16*I*a*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 + (-24*I*a^2 + 48*I)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 + (-48*I*a^2 + 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-4*I*(b*x + a)^4*d^4 - 16*b^3*c^3*d + 48*a*b^2*c^2*d^2 - 48*a^2*b*c*d^3 + 16*a^3*d^4 + (-16*I*b*c*d^3 - 16*(-I*a + 1)*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 - 48*(-I*a + 1)*b*c*d^3 + (-24*I*a^2 + 48*a)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d - 48*(-I*a + 1)*b^2*c^2*d^2 + (-48*I*a^2 + 96*a)*b*c*d^3 + (16*I*a^3 - 48*a^2)*d^4)*(b*x + a))*cos(3*b*x + 3*a) + (-4*I*(b*x + a)^4*d^4 + 16*b^3*c^3*d - 48*a*b^2*c^2*

```
d^2 + 48*a^2*b*c*d^3 - 16*a^3*d^4 + (-16*I*b*c*d^3 - 16*(-I*a - 1)*d^4)*(b*
x + a)^3 + (-24*I*b^2*c^2*d^2 - 48*(-I*a - 1)*b*c*d^3 + (-24*I*a^2 - 48*a)*
d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d - 48*(-I*a - 1)*b^2*c^2*d^2 + (-48*I*a^
2 - 96*a)*b*c*d^3 + (16*I*a^3 + 48*a^2)*d^4)*(b*x + a))*cos(b*x + a) - (8*b
^3*c^3*d - 24*a*b^2*c^2*d^2 + 8*(b*x + a)^3*d^4 + 24*(a^2 - 2)*b*c*d^3 - 8*
(a^3 - 6*a)*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*
b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x
+ a)^3*d^4 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b
*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(4*
b*x + 4*a) - 16*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2
)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*
d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-8*I*b^3*
c^3*d + 24*I*a*b^2*c^2*d^2 - 8*I*(b*x + a)^3*d^4 + (-24*I*a^2 + 48*I)*b*c*d
^3 + (8*I*a^3 - 48*I*a)*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-
24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 + (-24*I*a^2 + 48*I)*d^4)*(b*x + a))*sin(
4*b*x + 4*a) - (16*I*b^3*c^3*d - 48*I*a*b^2*c^2*d^2 + 16*I*(b*x + a)^3*d^4
+ (48*I*a^2 - 96*I)*b*c*d^3 + (-16*I*a^3 + 96*I*a)*d^4 + (48*I*b*c*d^3 - 48
*I*a*d^4)*(b*x + a)^2 + (48*I*b^2*c^2*d^2 - 96*I*a*b*c*d^3 + (48*I*a^2 - 96
*I)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (8*b^3*c^3*
d - 24*a*b^2*c^2*d^2 + 8*(b*x + a)^3*d^4 + 24*(a^2 - 2)*b*c*d^3 - 8*(a^3 -
6*a)*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + (a^2 - 2)*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*
d^4 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)
^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(4*b*x + 4
*a) - 16*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2)*b*c*d
^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2
*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (8*I*b^3*c^3*d -
24*I*a*b^2*c^2*d^2 + 8*I*(b*x + a)^3*d^4 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*
I*a^3 + 48*I*a)*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c
^2*d^2 - 48*I*a*b*c*d^3 + (24*I*a^2 - 48*I)*d^4)*(b*x + a))*sin(4*b*x + 4*a
) + (-16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 - 16*I*(b*x + a)^3*d^4 + (-48*I*a
^2 + 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)
*(b*x + a)^2 + (-48*I*b^2*c^2*d^2 + 96*I*a*b*c*d^3 + (-48*I*a^2 + 96*I)*d^4
)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^4*d^4
 + 12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4 + (-4*I*b*c*d^3 + 4*I*a
*d^4)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 + (-6*I*a^2 + 12*I)*
d^4)*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 + (-12*I*a^2 + 24*I
)*b*c*d^3 + (4*I*a^3 - 24*I*a)*d^4)*(b*x + a) + (-I*(b*x + a)^4*d^4 + 12*I*
b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4 + (-4*I*b*c*d^3 + 4*I*a*d^4)*(b
*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 + (-6*I*a^2 + 12*I)*d^4)*(b*
x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 + (-12*I*a^2 + 24*I)*b*c*d^
3 + (4*I*a^3 - 24*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^4*
d^4 - 24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4 + (8*I*b*c*d^3 - 8*I
*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + (12*I*a^2 - 24*I
)*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + (24*I*a^2 - 48*I
```

)*b*c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4) *(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b* x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d ^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b* c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^ 2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^4*d^4 - 12*I*b ^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + (6*I*a^2 - 12*I)*d^4)*(b*x + a)^2 + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + (12*I*a^2 - 24*I)*b*c*d^3 + (- 4*I*a^3 + 24*I*a)*d^4)*(b*x + a) + (I*(b*x + a)^4*d^4 - 12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6* I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + (6*I*a^2 - 12*I)*d^4)*(b*x + a)^2 + (4*I*b ^3*c^3*d - 12*I*a*b^2*c^2*d^2 + (12*I*a^2 - 24*I)*b*c*d^3 + (-4*I*a^3 + 24* I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^4*d^4 + 24*I*b^2*c^ 2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4 + (-8*I*b*c*d^3 + 8*I*a*d^4)*(b*x + a)^3 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 + (-12*I*a^2 + 24*I)*d^4)*(b*x + a)^2 + (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 + (-24*I*a^2 + 48*I)*b*c*d^3 + (8*I*a^3 - 48*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^4*d^4 - 12 *b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*sin(4 *b*x + 4*a) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d ^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^ 3 - (a^3 - 6*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (48*I*d^4*cos(4*b*x + 4*a) - 96*I*d^4*co s(2*b*x + 2*a) - 48*d^4*sin(4*b*x + 4*a) + 96*d^4*sin(2*b*x + 2*a) + 48*I*d ^4)*polylog(5, -e^(I*b*x + I*a)) + (-48*I*d^4*cos(4*b*x + 4*a) + 96*I*d^4*c os(2*b*x + 2*a) + 48*d^4*sin(4*b*x + 4*a) - 96*d^4*sin(2*b*x + 2*a) - 48*I* d^4)*polylog(5, e^(I*b*x + I*a)) + (48*b*c*d^3 + 48*(b*x + a)*d^4 - 48*a*d^ 4 + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 96*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) + (48*I*b*c*d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*sin(4*b*x + 4*a) + (-96*I*b*c*d^3 - 96*I*(b*x + a)*d^4 + 96* I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(I*b*x + I*a)) - (48*b*c*d^3 + 48* (b*x + a)*d^4 - 48*a*d^4 + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 96*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-48*I*b*c* d^3 - 48*I*(b*x + a)*d^4 + 48*I*a*d^4)*sin(4*b*x + 4*a) - (96*I*b*c*d^3 + 9 6*I*(b*x + a)*d^4 - 96*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 + (-24*I*a^ 2 + 48*I)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 + (-24*I*a^2 + 48*I)*d^4 + (-48*I* b*c*d^3 + 48*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (48*I*b^2*c^2*d^2 - 96*

```
I*a*b*c*d^3 + 48*I*(b*x + a)^2*d^4 + (48*I*a^2 - 96*I)*d^4 + (96*I*b*c*d^3
- 96*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
 (b*x + a)^2*d^4 + (a^2 - 2)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x
 + 4*a) - 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 2)*d^4 +
 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*
a)) + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + (24*I*a^2
 - 48*I)*d^4 + (48*I*b*c*d^3 - 48*I*a*d^4)*(b*x + a) + (24*I*b^2*c^2*d^2 -
48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + (24*I*a^2 - 48*I)*d^4 + (48*I*b*c*d
^3 - 48*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-48*I*b^2*c^2*d^2 + 96*I*a*
b*c*d^3 - 48*I*(b*x + a)^2*d^4 + (-48*I*a^2 + 96*I)*d^4 + (-96*I*b*c*d^3 +
96*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (
b*x + a)^2*d^4 + (a^2 - 2)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x +
 4*a) + 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 2)*d^4 + 2
*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a))
 + (4*(b*x + a)^4*d^4 - 16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 - 48*I*a^2*b*c*
d^3 + 16*I*a^3*d^4 + (16*b*c*d^3 - (16*a + 16*I)*d^4)*(b*x + a)^3 + (24*b^2
*c^2*d^2 - (48*a + 48*I)*b*c*d^3 + 24*(a^2 + 2*I*a)*d^4)*(b*x + a)^2 + (16*
b^3*c^3*d - (48*a + 48*I)*b^2*c^2*d^2 + 48*(a^2 + 2*I*a)*b*c*d^3 - 16*(a^3
+ 3*I*a^2)*d^4)*(b*x + a))*sin(3*b*x + 3*a) + (4*(b*x + a)^4*d^4 + 16*I*b^3
*c^3*d - 48*I*a*b^2*c^2*d^2 + 48*I*a^2*b*c*d^3 - 16*I*a^3*d^4 + (16*b*c*d^3
 - (16*a - 16*I)*d^4)*(b*x + a)^3 + (24*b^2*c^2*d^2 - (48*a - 48*I)*b*c*d^3
 + 24*(a^2 - 2*I*a)*d^4)*(b*x + a)^2 + (16*b^3*c^3*d - (48*a - 48*I)*b^2*c^
2*d^2 + 48*(a^2 - 2*I*a)*b*c*d^3 - 16*(a^3 - 3*I*a^2)*d^4)*(b*x + a))*sin(b
*x + a))/(-4*I*b^4*cos(4*b*x + 4*a) + 8*I*b^4*cos(2*b*x + 2*a) + 4*b^4*sin(
4*b*x + 4*a) - 8*b^4*sin(2*b*x + 2*a) - 4*I*b^4))/b
```

Fricas [C] time = 1.04884, size = 6342, normalized size = 15.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
 b^4*c^4)*cos(b*x + a) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c
^3*d + 24*I*b*c*d^3 + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d
 - 24*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 12*I*(b^
3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (4*I*b^3*d^4
*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 24*I*b*c*d^3 + (-4*I*b^3*d^4*x^
3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 24*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 -
 2*b*d^4)*x)*cos(b*x + a)^2 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x
```

```
 + a) - I*sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*
c^3*d + 24*I*b*c*d^3 + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*
d - 24*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 12*I*(b
^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (4*I*b^3*d
^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 24*I*b*c*d^3 + (-4*I*b^3*d^4*
x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 24*I*b*c*d^3 - 12*I*(b^3*c^2*d^2
 - 2*b*d^4)*x)*cos(b*x + a)^2 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(
b*x + a) - I*sin(b*x + a)) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*
b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*
x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c
^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(
cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c
^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^
4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 +
4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)
*x)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*(
a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^
4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4
- 12*a^2)*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) +
 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*
b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c
^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4)*cos(b*x + a)^2)*log(-1
/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^
3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a
^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
+ 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2
)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*co
s(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x
 + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2
*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*
d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d
^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^
4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b
^2*c*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 24*(d^4*cos(b*x + a)
^2 - d^4)*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*(d^4*cos(b*x + a)^
2 - d^4)*polylog(5, cos(b*x + a) - I*sin(b*x + a)) - 24*(d^4*cos(b*x + a)^2
 - d^4)*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) - 24*(d^4*cos(b*x + a)^2
 - d^4)*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b
*c*d^3 + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, cos(b*x
+ a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3 + (24*I*b*d^4*x + 24
*I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (24
*I*b*d^4*x + 24*I*b*c*d^3 + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2)*
polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3
+ (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (
```

```
b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*x + a)^2)*polylog(
3, cos(b*x + a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c
^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*
x + a)^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d
^2 - 2*d^4)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12
*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*
c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) - I
*sin(b*x + a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c
^3*d)*sin(b*x + a))/(b^5*cos(b*x + a)^2 - b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \cot(bx+a)^2 \csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^2*csc(b*x + a), x)
```

3.113 $\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx)\,dx$

Optimal. Leaf size=308

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{3id(c+d}{}$$

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^3*ArcTanh[E^(I
*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot
[a + b*x]*Csc[a + b*x])/(2*b) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^
4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3
*PolyLog[2, E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(
I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 - (
3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, -E
^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rubi [A] time = 0.344229, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4415, 4183, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{3id(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^3*ArcTanh[E^(I
*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot
[a + b*x]*Csc[a + b*x])/(2*b) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^
4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3
*PolyLog[2, E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(
I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 - (
3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, -E
^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
```

```
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
```

```
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^3\cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^3\csc(a+bx)\,dx + \int (c+dx)^3\csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} - \frac{(c+dx)^3\cot(a+bx}{2b}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc}{2b^2}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc}{2b^2}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc}{2b^2}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc}{2b^2}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc}{2b^2}$$

Mathematica [A] time = 5.05101, size = 528, normalized size = 1.71

$$-\frac{3id\left(b^2(c+dx)^2-2d^2\right)\text{PolyLog}\left(2,-e^{i(a+bx)}\right) - 3id\left(b^2(c+dx)^2-2d^2\right)\text{PolyLog}\left(2,e^{i(a+bx)}\right) - 6bcd^2\text{PolyLog}\left(3,-e^{i(a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] + b^3*c^3*L
og[1 - E^(I*(a + b*x))] - 6*b*c*d^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*c^2*d*
x*Log[1 - E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 - E^(I*(a + b*x))] + 3*b^3*c*d
^2*x^2*Log[1 - E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^
3*c^3*Log[1 + E^(I*(a + b*x))] + 6*b*c*d^2*Log[1 + E^(I*(a + b*x))] - 3*b^3
*c^2*d*x*Log[1 + E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 + E^(I*(a + b*x))] - 3*
b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a + b*x)
)] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] - (3*I
)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] - 6*b*c*d^2*Poly
Log[3, -E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*x))] + 6*b*c*d
^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, E^(I*(a + b*x))] - (6
*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, E^(I*(a + b*x))
])/(2*b^4)
```

Maple [B] time = 0.342, size = 1056, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 3/2/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/2/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2
+3/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))-3/2*I/b^2*d^3*polylog(2,-exp(I*(b*
x+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3/2*I/b^2*c^2*d*polylo
g(2,-exp(I*(b*x+a)))+3/2*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+3*I*d^3*po
lylog(2,-exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4-1/b^4*d
^3*a^3*arctanh(exp(I*(b*x+a)))-3/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+3/b^3*
c*d^2*polylog(3,-exp(I*(b*x+a)))+3/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x-3/b
^3*d^3*polylog(3,exp(I*(b*x+a)))*x+1/2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-3/2/b
*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/2/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/2/
b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b^4*d
^3*ln(exp(I*(b*x+a))+1)*a+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-e
xp(I*(b*x+a)))*a+6/b^4*d^3*a*arctanh(exp(I*(b*x+a)))-6/b^3*d^2*c*arctanh(ex
p(I*(b*x+a)))-3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x-3*I/b^2*polylog(2,-exp(I*(b*
x+a)))*c*d^2*x+3*I/b^2*polylog(2,exp(I*(b*x+a)))*c*d^2*x-1/2/b^4*d^3*ln(1-e
xp(I*(b*x+a)))*a^3+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))
+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))+d^3*x^3*b*exp(
I*(b*x+a))+c^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*
exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a))+c^
```

```
3*b*exp(I*(b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a))+6*
I*c*d^2*x*exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))-3/b^2*c^2*d*a*arctanh(ex
p(I*(b*x+a)))-1/2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/2/b^4*d^3*ln(exp(I*(b*x+
a))+1)*a^3+1/b*c^3*arctanh(exp(I*(b*x+a)))+3/2/b*c^2*d*ln(exp(I*(b*x+a))+1)
*x+3/2/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b
^4-3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

Maxima [B] time = 6.89686, size = 5227, normalized size = 16.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*cos(b*x + a)/
(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 -
 a^3*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1))/b^3 + 4*((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 + 6
*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3
)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(
4*b*x + 4*a) - 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos
(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (6*I*b*c
*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a^2
- 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3 + 24*I*b*c
*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c
^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*
c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 24*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) + (
12*I*b*c*d^2 - 12*I*a*d^3)*sin(4*b*x + 4*a) + (-24*I*b*c*d^2 + 24*I*a*d^3)*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^3*
d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 -
 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b
*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3
+ (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 +
 (6*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3
```

```
+ (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*
d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), -cos(b*x + a) + 1) + (-4*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*
d^2 - 12*a^2*d^3 + (-12*I*b*c*d^2 - 12*(-I*a + 1)*d^3)*(b*x + a)^2 + (-12*I
*b^2*c^2*d - 24*(-I*a + 1)*b*c*d^2 + (-12*I*a^2 + 24*a)*d^3)*(b*x + a))*cos
(3*b*x + 3*a) + (-4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^
2*d^3 + (-12*I*b*c*d^2 - 12*(-I*a - 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d
- 24*(-I*a - 1)*b*c*d^2 + (-12*I*a^2 - 24*a)*d^3)*(b*x + a))*cos(b*x + a) -
 (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*
c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (
a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 12*(b^2*c^
2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x +
a)^2*d^3 + (-6*I*a^2 + 12*I)*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*
sin(4*b*x + 4*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3
+ (12*I*a^2 - 24*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x
+ 2*a))*dilog(-e^(I*b*x + I*a)) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)
^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
a))*cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2
 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*
d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + (6*I*a^2 - 12*I)*d^3 + (12*I*b*c
*d^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*b^2*c^2*d + 24*I*a*
b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 24*I)*d^3 + (-24*I*b*c*d^2 +
24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x
+ a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)
^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a) + (-
I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b
*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (
6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6
*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*
d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c
*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d
^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*
d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^
2 + (3*I*a^2 - 6*I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I
*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c
*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^3
*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 +
 (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*cos(2
*b*x + 2*a) - ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*
(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4*
```

```
b*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (1
2*d^3*cos(4*b*x + 4*a) - 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a
) - 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*polylog(4, -e^(I*b*x + I*a)) - (12*
d^3*cos(4*b*x + 4*a) - 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a)
- 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*polylog(4, e^(I*b*x + I*a)) + (-12*I*
b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)
*d^3 + 12*I*a*d^3)*cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 -
24*I*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b
*x + 4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(
3, -e^(I*b*x + I*a)) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (1
2*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(4*b*x + 4*a) + (-24*I*b*
c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (
b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^
3)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (4*(b*x + a)^3*d^3 - 12*
I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a + 12*I)*d
^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 + 12*(a^2 + 2*I*a)*
d^3)*(b*x + a))*sin(3*b*x + 3*a) + (4*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24
*I*a*b*c*d^2 + 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I)*d^3)*(b*x + a)^2
+ (12*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a)*d^3)*(b*x + a))*
sin(b*x + a))/(-4*I*b^3*cos(4*b*x + 4*a) + 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3
*sin(4*b*x + 4*a) - 8*b^3*sin(2*b*x + 2*a) - 4*I*b^3))/b
```

Fricas [C] time = 0.808976, size = 4091, normalized size = 13.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x +
a) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3 + (3*I*b
^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*
I*b^2*c^2*d - 6*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d
 + 6*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^
2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3 + (3*I*b^2*d^3*x^2 +
6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x +
 a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d
- 6*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*
```

```
cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^
3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d -
2*b*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 -
 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b
*d^3)*x)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2
 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/
2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 -
(a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6
*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)
+ (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6
*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
(a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d
- 2*b*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(
b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*log(-co
s(b*x + a) - I*sin(b*x + a) + 1) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*poly
log(4, cos(b*x + a) + I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*
polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*
d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 -
 6*I*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2
 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x
+ a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(
3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d
^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x
 + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(b*x + a)
)/(b^4*cos(b*x + a)^2 - b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \cot(bx+a)^2 \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^2*csc(b*x + a), x)
```

3.114 $\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx$

Optimal. Leaf size=179

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}$$

```
[Out] ((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3
- (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x])/
(2*b) - (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*P
olyLog[2, E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 -
(d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```

Rubi [A] time = 0.223015, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4415, 4183, 2531, 2282, 6589, 4186, 3770}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```


```
[Out] ((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3
- (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x])/
(2*b) - (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*P
olyLog[2, E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 -
(d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```


Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2\cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^2\csc(a+bx)\,dx + \int (c+dx)^2\csc^3(a+bx)\,dx$$
$$= \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2\cot(a+bx)\csc}{2b}$$
$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a+bx)}{b^2} -$$
$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a+bx)}{b^2} -$$
$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a+bx)}{b^2} -$$
$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a+bx)}{b^2} -$$

Mathematica [B] time = 7.59481, size = 471, normalized size = 2.63

$$\frac{-2ibd(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)+2ibd(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)+2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)-2d^2\text{PolyLog}\left(3,e^{i}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2
]^2)/(8*b) + (-(b^2*c^2*Log[1 - E^(I*(a + b*x))]) + 2*d^2*Log[1 - E^(I*(a +
 b*x))] - 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 - E^(I*(
a + b*x))] + b^2*c^2*Log[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x
))] + 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 + E^(I*(a +
b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (2*I)*b*d*(c +
d*x)*PolyLog[2, E^(I*(a + b*x))] + 2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 2*d
^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/
2 + (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)/2]) -
 d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)
/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2)
```

Maple [B] time = 0.281, size = 546, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*
(b*x+a))+c^2*b*exp(3*I*(b*x+a))+d^2*x^2*b*exp(I*(b*x+a))+2*c*d*x*b*exp(I*(b
*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))+c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x
+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))+1/b*c^2*arctanh(exp(I
*(b*x+a)))+1/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-1/2/b^3*d^2*ln(exp(I*(b*x+
a))+1)*a^2+1/2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+d^2*polylog(3,-exp(I*(b*x+a
)))/b^3+1/b^2*c*d*ln(exp(I*(b*x+a))+1)*a+1/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2
-I/b^2*polylog(2,-exp(I*(b*x+a)))*d^2*x-1/2/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+
I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-2/b^2*c*d*a*arctanh(exp(I*(b*x+a)))+I/b
^2*polylog(2,exp(I*(b*x+a)))*d^2*x-I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-1/b
*c*d*ln(1-exp(I*(b*x+a)))*x-1/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+1/b*c*d*ln(exp
(I*(b*x+a))+1)*x-d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b^3*d^2*arctanh(exp(I*
(b*x+a)))
```

Maxima [B] time = 2.40362, size = 2607, normalized size = 14.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 2*a*c*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(co
s(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + a^2*d^2*(2*cos(b*x + a)/(cos(b
*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 + 4*((2
*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) + (2*I*(b*x + a)
^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(4*b*x + 4*a) + (-
4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(2*b
*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*d^2*cos(4*b*x + 4*a
) - 8*d^2*cos(2*b*x + 2*a) + 4*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x +
 2*a) + 4*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^2*d^2
 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a))*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
)*cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x +
a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*
```

```
x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-4*I*
(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (-8*I*b*c*d - 8*(-I*a + 1)*d^2)*(b*x
+ a))*cos(3*b*x + 3*a) + (-4*I*(b*x + a)^2*d^2 + 8*b*c*d - 8*a*d^2 + (-8*I*
b*c*d - 8*(-I*a - 1)*d^2)*(b*x + a))*cos(b*x + a) - (4*b*c*d + 4*(b*x + a)*
d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*
d^2 + 4*I*a*d^2)*sin(4*b*x + 4*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*
d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (4*b*c*d + 4*(b*x + a)*d^2
 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d
+ (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2
- 4*I*a*d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2
)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^2*d^2 + (-2*I*b*
c*d + 2*I*a*d^2)*(b*x + a) + 2*I*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d +
2*I*a*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (
4*I*b*c*d - 4*I*a*d^2)*(b*x + a) - 4*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)
^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*
b*c*d - 2*I*a*d^2)*(b*x + a) - 2*I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d -
2*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 +
(-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)
^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x +
a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-4*I*d^2*cos(4*b*x + 4*
a) + 8*I*d^2*cos(2*b*x + 2*a) + 4*d^2*sin(4*b*x + 4*a) - 8*d^2*sin(2*b*x +
2*a) - 4*I*d^2)*polylog(3, -e^(I*b*x + I*a)) + (4*I*d^2*cos(4*b*x + 4*a) -
8*I*d^2*cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a)
+ 4*I*d^2)*polylog(3, e^(I*b*x + I*a)) + (4*(b*x + a)^2*d^2 - 8*I*b*c*d + 8
*I*a*d^2 + (8*b*c*d - (8*a + 8*I)*d^2)*(b*x + a))*sin(3*b*x + 3*a) + (4*(b*
x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (8*b*c*d - (8*a - 8*I)*d^2)*(b*x + a
))*sin(b*x + a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*
b^2*sin(4*b*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b
```

Fricas [C] time = 0.688199, size = 2379, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + (-2*I*b*d^2*x -
 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) +
```

```
 I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*co
s(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*
c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin
(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x
+ a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 -
2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 -
2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + (a^2
 - 2)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*
cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)
*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*
d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a
)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(d^2*cos(b*x + a)
^2 - d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2
 - d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 -
 d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 -
d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 4*(b*d^2*x + b*c*d)*sin(b
*x + a))/(b^3*cos(b*x + a)^2 - b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \cot(bx+a)^2 \csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^2*csc(b*x + a), x)
```

3.115 $\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=108

$$-\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+bx)}{2b}$$

```
[Out] ((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b - (d*Csc[a + b*x])/(2*b^2) - ((c + d
*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) - ((I/2)*d*PolyLog[2, -E^(I*(a + b*x))
])/b^2 + ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rubi [A] time = 0.107197, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4415, 4183, 2279, 2391, 4185}

$$-\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```


```
[Out] ((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b - (d*Csc[a + b*x])/(2*b^2) - ((c + d
*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) - ((I/2)*d*PolyLog[2, -E^(I*(a + b*x))
])/b^2 + ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```


Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)\csc(a+bx)\,dx + \int (c+dx)\csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} + $$

$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} - $$

$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} - $$

$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} - $$

Mathematica [B] time = 1.83628, size = 260, normalized size = 2.41

$$-\frac{d\left(i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{2b^2} - \frac{d\tan\left(\frac{1}{2}(a+bx\right.}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -(d*Cot[(a + b*x)/2])/(4*b^2) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*x*Csc[(a
+ b*x)/2]^2)/(8*b) + (c*Log[Cos[(a + b*x)/2]])/(2*b) - (c*Log[Sin[(a + b*x)
/2]])/(2*b) + (a*d*Log[Tan[(a + b*x)/2]])/(2*b^2) - (d*((a + b*x)*(Log[1 -
E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x
))] - PolyLog[2, E^(I*(a + b*x))])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b)
 + (d*x*Sec[(a + b*x)/2]^2)/(8*b) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

Maple [B] time = 0.139, size = 246, normalized size = 2.3

$$\frac{dxb\mathrm{e}^{3\,i(bx+a)}+bc\mathrm{e}^{3\,i(bx+a)}+dxb\mathrm{e}^{i(bx+a)}+bc\mathrm{e}^{i(bx+a)}-id\mathrm{e}^{3\,i(bx+a)}+id\mathrm{e}^{i(bx+a)}}{b^2\left(\mathrm{e}^{2\,i(bx+a)}-1\right)^2}+\frac{c\,\mathrm{Artanh}\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x}{2\,b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d*x*b*exp(3*I*(b*x+a))+b*c*exp(3*I*(b*x+a))+d
*x*b*exp(I*(b*x+a))+b*c*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*x+
a)))+1/b*c*arctanh(exp(I*(b*x+a)))-1/2/b*d*ln(1-exp(I*(b*x+a)))*x-1/2/b^2*d
*ln(1-exp(I*(b*x+a)))*a+1/2*I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/2/b*d*ln(ex
p(I*(b*x+a))+1)*x+1/2/b^2*d*ln(exp(I*(b*x+a))+1)*a-1/2*I*d*polylog(2,-exp(I
*(b*x+a)))/b^2-1/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

Maxima [B] time = 1.90929, size = 1037, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] ((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos(
2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*b
*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(
4*b*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) - 4*I*b*c*
sin(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*
x*cos(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a)
- 4*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a)
 + 1) + (-4*I*b*d*x - 4*I*b*c - 4*d)*cos(3*b*x + 3*a) + (-4*I*b*d*x - 4*I*b
```

```
*c + 4*d)*cos(b*x + a) - (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I
*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*a))
 + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) -
4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c +
(-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a
) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c +
(I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x + 2*a
) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*d*x + b*c - I*
d)*sin(3*b*x + 3*a) + 4*(b*d*x + b*c + I*d)*sin(b*x + a))/(-4*I*b^2*cos(4*b
*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) - 8*b^2*sin(2
*b*x + 2*a) - 4*I*b^2)
```

Fricas [B] time = 0.576427, size = 1191, normalized size = 11.03

$$\frac{2\left(bdx+bc\right)\cos\left(bx+a\right)+\left(i\,d\cos\left(bx+a\right)^{2}-i\,d\right)\mathrm{Li}_{2}\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+\left(-i\,d\cos\left(bx+a\right)^{2}+i\,d\right)\mathrm{Li}_{2}\left(\cos\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d*x + b*c)*cos(b*x + a) + (I*d*cos(b*x + a)^2 - I*d)*dilog(cos(b*
x + a) + I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b
*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)
) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a
) - I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((b*c - a*d)*cos(b*x + a)^2 - b
*c + a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x - (b*d
*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (
b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1) + 2*d*sin(b*x + a))/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c+dx\right)\cot^{2}\left(a+bx\right)\csc\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cot(a + b*x)**2*csc(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \cot (bx + a)^2 \csc (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cot(b*x + a)^2*csc(b*x + a), x)
```

3.116 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\csc^3(a+bx)}{c+dx},x\right)-\text{Unintegrable}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

```
[Out] -Unintegrable[Csc[a + b*x]/(c + d*x), x] + Unintegrable[Csc[a + b*x]^3/(c +
 d*x), x]
```

Rubi [A] time = 0.0810282, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] -Defer[Int][Csc[a + b*x]/(c + d*x), x] + Defer[Int][Csc[a + b*x]^3/(c + d*x
), x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx=-\int \frac{\csc(a+bx)}{c+dx}\,dx+\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 36.7121, size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x), x]
```

Maple [A] time = 2.23, size = 0, normalized size = 0.

$$\int \frac{(\cot (bx + a))^2 \csc (bx + a)}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

```
[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*x
 + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*c)
*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x + b*
c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(b*x
+ a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 -
4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b
*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*si
n(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) -
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*
```

```
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*
a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x +
 a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) + (d*cos(
3*b*x + 3*a) - d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*x + b
*c)*sin(b*x + a))*sin(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b*d*x + b*c
)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (b*d*x + b*c
)*sin(b*x + a))*sin(2*b*x + 2*a) + d*sin(b*x + a))/(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\cot(bx+a)^2\csc(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (bx+a)^2 \csc (bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

3.117 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\csc^3(a+bx)}{(c+dx)^2},x\right)-\text{Unintegrable}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -Unintegrable[Csc[a + b*x]/(c + d*x)^2, x] + Unintegrable[Csc[a + b*x]^3/(c
 + d*x)^2, x]
```

Rubi [A] time = 0.0804008, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -Defer[Int][Csc[a + b*x]/(c + d*x)^2, x] + Defer[Int][Csc[a + b*x]^3/(c + d
*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx=-\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx+\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 42.6071, size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 3.468, size = 0, normalized size = 0.

$$\int \frac{(\cot(bx+a))^2 \csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*b
*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x +
b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x
+ b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*co
s(b*x + a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 +
 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*
a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x
+ 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x
^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*s
```

```
in(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^
2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)
)*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^
2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (2*d*c
os(3*b*x + 3*a) - 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*
x + b*c)*sin(b*x + a))*sin(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a) - (b*d*x
+ b*c)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(2*d*cos(b*x + a) - (b*d*
x + b*c)*sin(b*x + a))*sin(2*b*x + 2*a) + 2*d*sin(b*x + a))/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x +
 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*c
os(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(2*b*x + 2*a))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot\left(bx+a\right)^{2}\csc\left(bx+a\right)}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(bx+a)^2\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c)^2, x)
```

3.118 $\int (c+dx)^{5/2} \cos^2(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=406

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\right.}{144}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rubi [A] time = 0.666837, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\right.}{144}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```


```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{(5d)\int (c+dx)^{3/2}\cos(3}{24b}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{5d(c+dx)^{3/2}\sin(a+b}{8b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3b}{144b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3b}{144b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3b}{144b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3b}{144b^3}$$

Mathematica [C] time = 16.3898, size = 1168, normalized size = 2.88

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\mathrm{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\mathrm{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b}-\frac{\left(2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(3(a+bx))-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c^2*(2*Sqrt[3]*Sqrt
[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*Fres
nelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (
c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d
*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]
```

```
*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d])
+ 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(8*b^
3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - S
qrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (
b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d
*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x])))/(32
*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*S
in[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a
+ b*x)] - Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3
*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c
^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*
(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)*Sin[3*(a + b*x)])))/(28
8*Sqrt[3]*b^5)
```

Maple [A] time = 0.033, size = 476, normalized size = 1.2

$$2\,\frac{1}{d}\left(-1/8\,\frac{d\,(dx+c)^{5/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+5/8\,\frac{d}{b}\left(1/2\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-3/2\,\frac{d}{b}\left(-1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/
24/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2)
*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.533, size = 1874, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3456*sqrt(3)*(80*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(3*((d*x + c)*b
- b*c + a*d)/d)/abs(d) + 720*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(((d*x
 + c)*b - b*c + a*d)/d)/abs(d) - 8*(12*sqrt(3)*(d*x + c)^(5/2)*b^2*d*abs(b)
/abs(d) - 5*sqrt(3)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(3*((d*x + c)*b - b
*c + a*d)/d) - 72*(4*sqrt(3)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(
3)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(((d*x + c)*b - b*c + a*d)/d) - ((5*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5
*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (5*I*sqrt(pi)*cos(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/ab
s(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - (sqrt(3)*(1
35*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 135*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 135*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 135*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(135*I*sq
rt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*
I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 135*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 135*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d^3*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*
b/d)) - (sqrt(3)*(135*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + 135*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 135*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 135*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d)
- sqrt(3)*(-135*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 135*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) + 135*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - 135*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(s
```

```
qrt(d*x + c)*sqrt(-I*b/d)) - ((5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a
*d)/d) - (-5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-3*I*b/d)))*abs(d)/(b^3*d*abs(b))
```

Fricas [A] time = 0.653204, size = 856, normalized size = 2.11

$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2*cos(b
*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x +
a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*
sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.52087, size = 2724, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1728*(12*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*
I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/
d)/b)*c^2 + d^2*((I*sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^
3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(
12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x +
 c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 5*I*s
qrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^2 + 2
7*(I*sqrt(2)*sqrt(pi)*(-4*I*b^2*c^2*d + 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)
*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*
x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(
(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 27*(I*sqrt(2)*sqrt(pi)*(-4*I
*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(
3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*
sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b^3)/d^2 + (I*sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*d - 12*b*c*d^2 + 5
*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3)
```

```
 - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sq
rt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2
 - 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/
d^2) + 12*(I*sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(2)*sqrt(pi)*(2*I*b*c*d - 3
*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sq
rt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*erf(-1/2*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c +
 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c
)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c
)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 18*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(
d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)
/b^2 - 18*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x
 + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 6*I*(2*I*(d*x + c)^
(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)
*b + 3*I*b*c - 3*I*a*d)/d)/b^2)*c)/d
```

3.119 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\mathrm{S}}{8b^{5/2}}$$

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rubi [A] time = 0.5262, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\mathrm{S}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{d\int \sqrt{c+dx}\cos(3a+}{8b}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)}{8b^{5/2}}$$

Mathematica [C] time = 9.26121, size = 676, normalized size = 1.92

$$\frac{c\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{8b}-d\sqrt{\frac{b}{d}}\left(\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)\left(3d\sin\left(a-\frac{bc}{d}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d
]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC
[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6
/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt
[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a
 - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sq
rt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) -
```

```
(Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Co
s[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (
3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] -
Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

Maple [A] time = 0.031, size = 384, normalized size = 1.1

$$2\frac{1}{d}\left(-1/8\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+3/8\,\frac{d}{b}\left(1/2\,\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-1/4\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.52107, size = 1793, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/576*sqrt(3)*(16*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(3*((d*x + c)*b -
b*c + a*d)/d)/abs(d) + 48*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(((d*x + c)
*b - b*c + a*d)/d)/abs(d) - 8*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(3*((d*x
+ c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(((d
*x + c)*b - b*c + a*d)/d)/abs(d) - ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
```

```
, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*
d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^2*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(3*I*b/d)) - (sqrt(3)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) -
sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(I*b/d)) - (sqrt(3)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d)
 - sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(-I*b/d)) - ((I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(p
i)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi
)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(
abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*ab
s(d)/(b^2*d*abs(b))
```

Fricas [A] time = 0.624486, size = 716, normalized size = 2.03

$$-\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*
cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x + c))/
b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.35446, size = 1513, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/288*(2*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
```

```
(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*
(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*
b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)
/b)*c + I*sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(2)*sqrt(pi)*(2*I*b*c*d - 3*d^
2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(
2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b^2) + I*sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*erf(-1/2*sqrt
(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(
3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b
 - 3*I*b*c + 3*I*a*d)/d)/b^2 - 18*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x
 + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^
2 - 18*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x +
c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 6*I*(2*I*(d*x + c)^(3/
2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b
+ 3*I*b*c - 3*I*a*d)/d)/b^2)/d
```

3.120 $\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S}{12b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rubi [A] time = 0.41614, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S}{12b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int \sqrt{c+dx}\sin(a+bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\sin(3a+3bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{d\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}dx}{24b}+\frac{d\int \frac{\cos}{\sqrt{}}}{}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}\right.}{\sqrt{c+}}}{24b}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\left(\int \text{co}\right.}{12b}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{}}{}\right.}{4b^{3/2}}$$

Mathematica [C] time = 6.64042, size = 278, normalized size = 0.91

$$-\frac{-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(3(}{24\sqrt{3}b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (2*Sqrt[3]*Sqrt[b/d]*Sqr
t[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(24*Sqrt[3]*b*Sqrt[b/d])
```

Maple [A] time = 0.03, size = 296, normalized size = 1.

$$2\frac{1}{d}\left(-1/8\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)b}{d}+\frac{ad-bc}{d}\right)+1/16\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(\frac{ad}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.40068, size = 1656, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/288*sqrt(3)*(8*sqrt(3)*sqrt(d*x + c)*d*abs(b)*cos(3*((d*x + c)*b - b*c +
 a*d)/d)/abs(d) + 24*sqrt(3)*sqrt(d*x + c)*d*abs(b)*cos(((d*x + c)*b - b*c
+ a*d)/d)/abs(d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (I*sqrt(pi)
*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)
/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - (sqrt(3)
*(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(3*I*sqrt(pi)*co
s(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs
(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(3)*
```

```
(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(-3*I*sqrt(pi)*co
s(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs
(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((sqrt(p
i)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi
)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(ab
s(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 0.59076, size = 613, normalized size = 2.02

$$\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(\sqrt{2}\cdots\right.}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3)/b
^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.22731, size = 659, normalized size = 2.17

$$-\frac{\frac{\sqrt{6}\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{9\,\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{9\,\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+}{d}\right.}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*(d*
x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)
/d
```

3.121 $\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S}{12b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rubi [A] time = 0.419121, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S}{12b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```


```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int \sqrt{c+dx}\sin(a+bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\sin(3a+3bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{d\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}dx}{24b}+\frac{d\int \frac{\cos}{\sqrt{}}}{8}$$
$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}\right.}{\sqrt{c+}}}{24b}$$
$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\left(\int \text{co}\right.}{12b}$$
$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{}}{}\right.}{4b^{3/2}}$$

Mathematica [C] time = 6.64312, size = 278, normalized size = 0.91

$$-\frac{-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(3(}{24\sqrt{3}b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (2*Sqrt[3]*Sqrt[b/d]*Sqr
t[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(24*Sqrt[3]*b*Sqrt[b/d])
```

Maple [A] time = 0.03, size = 296, normalized size = 1.

$$2\,\frac{1}{d}\left(-1/8\,\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+1/16\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(\frac{ad}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.21505, size = 1656, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/288*sqrt(3)*(8*sqrt(3)*sqrt(d*x + c)*d*abs(b)*cos(3*((d*x + c)*b - b*c +
 a*d)/d)/abs(d) + 24*sqrt(3)*sqrt(d*x + c)*d*abs(b)*cos(((d*x + c)*b - b*c
+ a*d)/d)/abs(d) - ((sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (I*sqrt(pi)
*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)
/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - (sqrt(3)
*(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(3*I*sqrt(pi)*co
s(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs
(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(3)*
```

```
(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(-3*I*sqrt(pi)*co
s(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs
(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((sqrt(p
i)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi
)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(ab
s(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 0.5916, size = 613, normalized size = 2.02

$$\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}S\left(\sqrt{2}\right.}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3)/b
^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.25722, size = 659, normalized size = 2.17

$$-\frac{\frac{\sqrt{6}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{3ibc-3iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} + \frac{9\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} + \frac{9\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+}{d}\right.}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*(d*
x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)
/d
```

3.122 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right.}{8b^{5/2}}$$

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rubi [A] time = 0.52797, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right.}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```


```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx) + \frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\sin(a+bx)\,dx + \frac{1}{4}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{d\int\sqrt{c+dx}\cos(3a -}{8b}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c+dx}\sin(a+b}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c+dx}\sin(a+b}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} + \frac{3d\sqrt{c+dx}\sin(a+b}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right.}{8b^{5/2}}$$

Mathematica [C] time = 9.21484, size = 676, normalized size = 1.92

$$\frac{c\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{8b} - \frac{d\sqrt{\frac{b}{d}}\left(\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)\left(3d\sin\left(a-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d
]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC
[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6
/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt
[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a
 - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sq
rt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) -
```

```
(Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Co
s[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (
3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] -
Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

Maple [A] time = 0.029, size = 384, normalized size = 1.1

$$2\frac{1}{d}\left(-1/8\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+3/8\,\frac{d}{b}\left(1/2\,\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-1/4\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.49685, size = 1793, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/576*sqrt(3)*(16*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(3*((d*x + c)*b -
b*c + a*d)/d)/abs(d) + 48*sqrt(3)*(d*x + c)^(3/2)*b*d*abs(b)*cos(((d*x + c)
*b - b*c + a*d)/d)/abs(d) - 8*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(3*((d*x
+ c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(3)*sqrt(d*x + c)*d^2*abs(b)*sin(((d
*x + c)*b - b*c + a*d)/d)/abs(d) - ((-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
```

```
, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*
d)/d) - (sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^2*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(3*I*b/d)) - (sqrt(3)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) -
sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(I*b/d)) - (sqrt(3)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d)
 - sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(-I*b/d)) - ((I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (sqrt(p
i)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi
)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^2*sqrt(
abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*ab
s(d)/(b^2*d*abs(b))
```

Fricas [A] time = 0.610789, size = 716, normalized size = 2.03

$$-\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*
cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x + c))/
b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.38964, size = 1513, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/288*(2*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
```

```
(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*
(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*
b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)
/b)*c + I*sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(2)*sqrt(pi)*(2*I*b*c*d - 3*d^
2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(
2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b^2) + I*sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*erf(-1/2*sqrt
(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(
3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b
 - 3*I*b*c + 3*I*a*d)/d)/b^2 - 18*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x
 + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^
2 - 18*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x +
c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 6*I*(2*I*(d*x + c)^(3/
2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b
+ 3*I*b*c - 3*I*a*d)/d)/b^2)/d
```

3.123 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=406

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\right.}{144}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rubi [A] time = 0.627123, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\right.}{144}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```


```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int (c+dx)^{5/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx$$
$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{(5d)\int (c+dx)^{3/2}\cos(3}{24b}$$
$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{5d(c+dx)^{3/2}\sin(a+b}{8b^2}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3b}{144b^3}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3b}{144b^3}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3b}{144b^3}$$
$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3b}{144b^3}$$

Mathematica [C] time = 16.2415, size = 1168, normalized size = 2.88

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\mathrm{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\mathrm{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b}-\frac{\left(2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(3(a+bx))-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c^2*(2*Sqrt[3]*Sqrt
[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*Fres
nelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (
c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d
*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]
```

```
*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d])
+ 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(8*b^
3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - S
qrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (
b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d
*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x])))/(32
*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*S
in[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a
+ b*x)] - Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3
*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c
^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*
(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)*Sin[3*(a + b*x)])))/(28
8*Sqrt[3]*b^5)
```

Maple [A] time = 0.029, size = 476, normalized size = 1.2

$$2\,\frac{1}{d}\left(-1/8\,\frac{d\,(dx+c)^{5/2}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+5/8\,\frac{d}{b}\left(1/2\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-3/2\,\frac{d}{b}\left(-1/2\,d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/
24/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2)
*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.42005, size = 1874, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3456*sqrt(3)*(80*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(3*((d*x + c)*b
- b*c + a*d)/d)/abs(d) + 720*sqrt(3)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(((d*x
 + c)*b - b*c + a*d)/d)/abs(d) - 8*(12*sqrt(3)*(d*x + c)^(5/2)*b^2*d*abs(b)
/abs(d) - 5*sqrt(3)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(3*((d*x + c)*b - b
*c + a*d)/d) - 72*(4*sqrt(3)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(
3)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(((d*x + c)*b - b*c + a*d)/d) - ((5*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5
*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a*d)/d) - (5*I*sqrt(pi)*cos(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/ab
s(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - (sqrt(3)*(1
35*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 135*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 135*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 135*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d) - sqrt(3)*(135*I*sq
rt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 135*
I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 135*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 135*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d^3*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*
b/d)) - (sqrt(3)*(135*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + 135*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 135*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 135*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-(b*c - a*d)/d)
- sqrt(3)*(-135*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 135*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) + 135*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - 135*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-(b*c - a*d)/d))*erf(s
```

```
qrt(d*x + c)*sqrt(-I*b/d)) - ((5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2)))))*d^3*sqrt(abs(b)/abs(d))*cos(-3*(b*c - a
*d)/d) - (-5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-3*I*b/d)))*abs(d)/(b^3*d*abs(b))
```

Fricas [A] time = 0.671895, size = 856, normalized size = 2.11

$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2*cos(b
*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x +
a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*
sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.53231, size = 2724, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1728*(12*(sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*
I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/
d)/b)*c^2 + d^2*((I*sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^
3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(
12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x +
 c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 5*I*s
qrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^2 + 2
7*(I*sqrt(2)*sqrt(pi)*(-4*I*b^2*c^2*d + 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)
*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*
x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(
(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 27*(I*sqrt(2)*sqrt(pi)*(-4*I
*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(
3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*
sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b^3)/d^2 + (I*sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*d - 12*b*c*d^2 + 5
*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3)
```

```
 - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sq
rt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2
 - 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/
d^2) + 12*(I*sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(2)*sqrt(pi)*(2*I*b*c*d - 3
*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sq
rt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*erf(-1/2*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c +
 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c
)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c
)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 18*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(
d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)
/b^2 - 18*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x
 + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 6*I*(2*I*(d*x + c)^
(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)
*b + 3*I*b*c - 3*I*a*d)/d)/b^2)*c)/d
```

3.124 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=228

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+4}{2048b^3}$$

```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.398063, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+4}{2048b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```


```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2} - \frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^{7/2}}{28d} - \frac{1}{8}\int (c+dx)^{5/2}\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{7/2}}{28d} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b} + \frac{(5d)\int (c+dx)^{3/2}\sin(4a+4bx)\,dx}{64b}$$
$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b} + \frac{(15d}{}$$
$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3} - \frac{(c}{}$$
$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3} - \frac{(c}{}$$
$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3} - \frac{(c}{}$$
$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{}}{}\right.}{4096b^{7/2}}$$

Mathematica [A] time = 3.65687, size = 206, normalized size = 0.9

$$\frac{\sqrt{\frac{b}{d}}\left(4\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(-7d\sin(4(a+bx))\left(64b^2(c+dx)^2-15d^2\right)-280bd^2(c+dx)\cos(4(a+bx))+512b^3(c+dx)^3\right)-105\sqrt{}\right.}{57344b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
*(c + d*x)^3 - 280*b*d^2*(c + d*x)*Cos[4*(a + b*x)] - 7*d*(-15*d^2 + 64*b^2
*(c + d*x)^2)*Sin[4*(a + b*x)])))/(57344*b^4)
```

Maple [A] time = 0.037, size = 251, normalized size = 1.1

$$2\frac{1}{d}\left(\frac{(dx+c)^{7/2}}{56}-\frac{d\,(dx+c)^{5/2}}{64\,b}\sin\left(4\,\frac{(dx+c)\,b}{d}+4\,\frac{ad-bc}{d}\right)+\frac{5\,d}{64\,b}\left(-1/8\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(4\,\frac{(dx+c)\,b}{d}+4\,\frac{ad-bc}{d}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/8/
b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d
)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(
b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.08834, size = 922, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/229376*(8192*(d*x + c)^(7/2)*b^3*sqrt(abs(b)/abs(d)) - 4480*(d*x + c)^(3/
2)*b*d^2*sqrt(abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((105*I*s
qrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105
*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 105*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 105*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^3*cos(-4*(b*c - a*d)/d) + (105*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(pi)*sin(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*I*sqrt(pi)*sin(-1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-4*(b*c - a*d)/d))*
erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((-105*I*sqrt(pi)*cos(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(pi)*cos(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*sqrt(pi)*sin(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*cos(-4*(b*c - a*d)/d
```

```
) + (105*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 105*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + 105*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) - 105*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))))*d^3*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/
d)) - 112*(64*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d)) - 15*sqrt(d*x + c)*
d^3*sqrt(abs(b)/abs(d)))*sin(4*((d*x + c)*b - b*c + a*d)/d))/(b^3*d*sqrt(ab
s(b)/abs(d)))
```

Fricas [A] time = 0.651637, size = 840, normalized size = 3.68

$$-\frac{105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-16\left(128\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(pi*
d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 -
 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*c
os(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 1
28*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d^3*x^2
+ 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x + a))*sq
rt(d*x + c))/(b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.68562, size = 1467, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/1720320*(2240*(3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) - 12*I*sqrt(d*x +
 c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*
e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2 - d^2*(4096*(15*(d*x +
c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)/d^2 + 105*(sqrt(2
)*sqrt(pi)*(-64*I*b^2*c^2*d + 48*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 4*(64*I*(d*x + c)^(5/2)*b^2*d - 12
8*I*(d*x + c)^(3/2)*b^2*c*d + 64*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(
3/2)*b*d^2 - 48*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d
*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^2 + 105*(sqrt(2)*sqrt(pi)*(64*I*b^
2*c^2*d + 48*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 4*(-64*I*(d*x + c)^(5/2)*b^2*d + 128*I*(d*x + c)^(3
/2)*b^2*c*d - 64*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 48*
sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I*b
*c + 4*I*a*d)/d)/b^3)/d^2) - 112*(1536*(d*x + c)^(5/2) - 2560*(d*x + c)^(3/
2)*c + 15*sqrt(2)*sqrt(pi)*(8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 15*sqrt(2)*sqrt(pi)*(-8*I*b*c*d - 3*d^2)
*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 60*(-8*I
*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4
*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 60*(8*I*(d*x + c)^(3/2)*b*d -
8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b
*c - 4*I*a*d)/d)/b^2)*c)/d
```

3.125 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=200

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}$$

```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.328737, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```


```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2} - \frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)dx$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{1}{8}\int (c+dx)^{3/2}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d)\int \sqrt{c+dx}\sin(4a+4bx)\,dx}{64b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d^2)\int}{}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d^2 c}{}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d\,co}{}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt{d}}\right.}{512b^{5/2}}$$

Mathematica [A] time = 3.15495, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}}\left(15\sqrt{2\pi}d^2\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - 15\sqrt{2\pi}d^2\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) + 4\sqrt{\frac{b}{d}}\sqrt{c+d}\right.}{5120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]] - 15*d^2*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Co
s[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)]))))/(5
120*b^3)
```

Maple [A] time = 0.036, size = 206, normalized size = 1.

$$2\frac{1}{d}\left(1/40\,(dx+c)^{5/2} - \frac{d\,(dx+c)^{3/2}}{64\,b}\sin\left(4\frac{(dx+c)\,b}{d}+4\frac{ad-bc}{d}\right) + \frac{3\,d}{64\,b}\left(-1/8\frac{d\sqrt{dx+c}}{b}\cos\left(4\frac{(dx+c)\,b}{d}+4\frac{ad-bc}{d}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32
/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 1.94746, size = 886, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20480*(1024*(d*x + c)^(5/2)*b^2*sqrt(abs(b)/abs(d)) - 640*(d*x + c)^(3/2)
*b*d*sqrt(abs(b)/abs(d))*sin(4*((d*x + c)*b - b*c + a*d)/d) - 240*sqrt(d*x
+ c)*d^2*sqrt(abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) + ((15*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqr
t(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I
*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 1
5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^2*cos(-4*(b*c - a*d)/d) - (15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-4*(b*c - a*d)/d) - (-15*I*sq
rt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^2*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/(b^2*d*sqrt
(abs(b)/abs(d)))
```

Fricas [A] time = 0.597456, size = 621, normalized size = 3.1

$$\frac{15\,\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)C\left(2\,\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\,\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+4\left(64\,b^3d^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2
+ 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))
/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.45801, size = 805, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(1536*(d*x + c)^(5/2) - 2560*(d*x + c)^(3/2)*c - 40*(3*I*sqrt(2)*sq
rt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
```

```
)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 3*I*s
qrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) - 64*(d*x + c)^(3/2) - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*
b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c -
 4*I*a*d)/d)/b)*c + 15*sqrt(2)*sqrt(pi)*(8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 15*sqrt(2)*sqrt(pi)*(-8*I*b
*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) - 60*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x +
c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 60*(8*I*(d*x + c)
^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x +
 c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.126 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=174

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+$$

```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.270369, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}-\frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^{3/2}}{12d}-\frac{1}{8}\int \sqrt{c+dx}\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{d\int \frac{\sin(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$
$$= \frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{\sqrt{c+dx}}dx}{64b}+$$
$$= \frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Subst}\left(\int \sin\left(\frac{4bx^2}{d}\right)dx,\right.}{32b}$$
$$= \frac{(c+dx)^{3/2}}{12d}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

Mathematica [A] time = 0.885184, size = 161, normalized size = 0.93

$$\frac{3\sqrt{2\pi}d\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+3\sqrt{2\pi}d\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}(8b(c+}{384d^2\left(\frac{b}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Si
n[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(
a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

Maple [A] time = 0.036, size = 159, normalized size = 0.9

$$2\frac{1}{d}\left(1/24\,(dx+c)^{3/2}-\frac{d\sqrt{dx+c}}{64\,b}\sin\left(4\frac{(dx+c)\,b}{d}+4\frac{ad-bc}{d}\right)+\frac{d\sqrt{2}\sqrt{\pi}}{256\,b}\left(\cos\left(4\frac{ad-bc}{d}\right)\text{FresnelS}\left(2\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*
2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC
(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

Maxima [C] time = 1.93622, size = 817, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(128*(d*x + c)^(3/2)*b*sqrt(abs(b)/abs(d)) - 48*sqrt(d*x + c)*d*sqrt
(abs(b)/abs(d))*sin(4*((d*x + c)*b - b*c + a*d)/d) - ((-3*I*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-4*(b*c
- a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))))*d*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d))
 - ((3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))))*d*cos(-4*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(-I*b/d)))/(b*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 0.569092, size = 447, normalized size = 2.57

$$\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+16\left(2b^2dx+\right.}{384\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fre
snel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 16
*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin(b*x
 + a))*sqrt(d*x + c))/(b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.29019, size = 347, normalized size = 1.99

$$-\frac{\frac{3i\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{3i\sqrt{2}\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-64\,(dx+c)^{\frac{3}{2}}-\frac{12i\sqrt{dx+c}\,de^{\left(\frac{4i\,(dx+c)b}{}\right.}}{b}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/768*(3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
```

```
2*d^2) + 1)*b) - 3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) - 12*I*sqrt(d*x + c)*d*e^((
4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(
d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

3.127 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=174

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+}{1}$$

```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.249162, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```



```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int\left(\frac{1}{8}\sqrt{c+dx}-\frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^{3/2}}{12d}-\frac{1}{8}\int\sqrt{c+dx}\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{d\int\frac{\sin(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$
$$= \frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int\frac{\sin\left(\frac{4bc}{d}+4bx\right)}{\sqrt{c+dx}}dx}{64b}+$$
$$= \frac{(c+dx)^{3/2}}{12d}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Subst}\left(\int\sin\left(\frac{4bx^2}{d}\right)dx,\right.}{32b}$$
$$= \frac{(c+dx)^{3/2}}{12d}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$

Mathematica [A] time = 0.136948, size = 161, normalized size = 0.93

$$\frac{3\sqrt{2\pi}d\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+3\sqrt{2\pi}d\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}(8b(c+}{384d^2\left(\frac{b}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Si
n[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(
a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

Maple [A] time = 0.036, size = 159, normalized size = 0.9

$$2\frac{1}{d}\left(1/24\,(dx+c)^{3/2}-\frac{d\sqrt{dx+c}}{64\,b}\sin\left(4\,\frac{(dx+c)\,b}{d}+4\,\frac{ad-bc}{d}\right)+\frac{d\sqrt{2}\sqrt{\pi}}{256\,b}\left(\cos\left(4\,\frac{ad-bc}{d}\right)\text{FresnelS}\left(2\,\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*
2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC
(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.01321, size = 817, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*(128*(d*x + c)^(3/2)*b*sqrt(abs(b)/abs(d)) - 48*sqrt(d*x + c)*d*sqrt
(abs(b)/abs(d))*sin(4*((d*x + c)*b - b*c + a*d)/d) - ((-3*I*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*cos(-4*(b*c
- a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
qrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))))*d*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d))
 - ((3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) - 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))))*d*cos(-4*(b*c - a*d)/d) - (3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(-I*b/d)))/(b*d*sqrt(abs(b)/abs(d)))
```

Fricas [A] time = 0.571745, size = 447, normalized size = 2.57

$$\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+16\left(2b^2dx+\right.}{384\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fre
snel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 16
*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin(b*x
 + a))*sqrt(d*x + c))/(b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.29994, size = 347, normalized size = 1.99

$$-\frac{\frac{3i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{3i\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-64(dx+c)^{\frac{3}{2}}-\frac{12i\sqrt{dx+c}de^{\left(\frac{4i(dx+c)b}{}\right.}}{b}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/768*(3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
```

```
2*d^2) + 1)*b) - 3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) - 12*I*sqrt(d*x + c)*d*e^((
4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(
d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

3.128 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=200

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}-$$

```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.31823, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```


```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2} - \frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right) dx$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{1}{8}\int (c+dx)^{3/2}\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d)\int \sqrt{c+dx}\sin(4a+4bx)\,dx}{64b}$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d^2)\,\int}{}$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d^2\,\mathrm{c}}{}$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d\,\mathrm{co}}{}$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}}{\sqrt{d}}\right.}{512b^{5/2}}$$

Mathematica [A] time = 1.69443, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}}\left(15\sqrt{2\pi}d^2\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)-15\sqrt{2\pi}d^2\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+d}\right.}{5120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]] - 15*d^2*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Co
s[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)]))))/(5
120*b^3)
```

Maple [A] time = 0.036, size = 206, normalized size = 1.

$$2\frac{1}{d}\left(1/40\,(dx+c)^{5/2} - \frac{d\,(dx+c)^{3/2}}{64\,b}\sin\left(4\,\frac{(dx+c)\,b}{d}+4\,\frac{ad-bc}{d}\right) + \frac{3\,d}{64\,b}\left(-1/8\,\frac{d\sqrt{dx+c}}{b}\cos\left(4\,\frac{(dx+c)\,b}{d}+4\,\frac{ad-bc}{d}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32
/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.15975, size = 886, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20480*(1024*(d*x + c)^(5/2)*b^2*sqrt(abs(b)/abs(d)) - 640*(d*x + c)^(3/2)
*b*d*sqrt(abs(b)/abs(d))*sin(4*((d*x + c)*b - b*c + a*d)/d) - 240*sqrt(d*x
+ c)*d^2*sqrt(abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) + ((15*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqr
t(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I
*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 1
5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^2*cos(-4*(b*c - a*d)/d) - (15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + ((15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*cos(-4*(b*c - a*d)/d) - (-15*I*sq
rt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^2*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/(b^2*d*sqrt
(abs(b)/abs(d)))
```

Fricas [A] time = 0.606359, size = 621, normalized size = 3.1

$$\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+4\left(64b^3d^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2
+ 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))
/(b^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.43897, size = 805, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(1536*(d*x + c)^(5/2) - 2560*(d*x + c)^(3/2)*c - 40*(3*I*sqrt(2)*sq
rt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
```

```
)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 3*I*s
qrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) - 64*(d*x + c)^(3/2) - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*
b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c -
 4*I*a*d)/d)/b)*c + 15*sqrt(2)*sqrt(pi)*(8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 15*sqrt(2)*sqrt(pi)*(-8*I*b
*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) - 60*(-8*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x +
c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 60*(8*I*(d*x + c)
^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x +
 c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.129 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=228

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+}{2048b^3}$$

```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rubi [A] time = 0.377683, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+}{2048b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```


```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2} - \frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{1}{8}\int (c+dx)^{5/2}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b} + \frac{(5d)\int (c+dx)^{3/2}\sin(4a+4bx)\,dx}{64b}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b} + \frac{(15d$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3} - \frac{(c$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3} - \frac{(c$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3} - \frac{(c$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{}}{}\right.}{4096b^{7/2}}$$

Mathematica [A] time = 2.7336, size = 206, normalized size = 0.9

$$\frac{\sqrt{\frac{b}{d}}\left(4\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(-7d\sin(4(a+bx))\left(64b^2(c+dx)^2-15d^2\right)-280bd^2(c+dx)\cos(4(a+bx))+512b^3(c+dx)^3\right)-105\sqrt{\right.}{57344b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
*(c + d*x)^3 - 280*b*d^2*(c + d*x)*Cos[4*(a + b*x)] - 7*d*(-15*d^2 + 64*b^2
*(c + d*x)^2)*Sin[4*(a + b*x)])))/(57344*b^4)
```

Maple [A] time = 0.034, size = 251, normalized size = 1.1

$$2\,\frac{1}{d}\left(\frac{(dx+c)^{7/2}}{56}-\frac{d\,(dx+c)^{5/2}}{64\,b}\sin\left(4\,\frac{(dx+c)\,b}{d}+4\,\frac{ad-bc}{d}\right)+\frac{5\,d}{64\,b}\left(-1/8\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(4\,\frac{(dx+c)\,b}{d}+4\,\frac{ad-bc}{d}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/8/
b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d
)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(
b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.13834, size = 922, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/229376*(8192*(d*x + c)^(7/2)*b^3*sqrt(abs(b)/abs(d)) - 4480*(d*x + c)^(3/
2)*b*d^2*sqrt(abs(b)/abs(d))*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((105*I*s
qrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105
*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 105*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 105*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^3*cos(-4*(b*c - a*d)/d) + (105*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(pi)*sin(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*I*sqrt(pi)*sin(-1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sin(-4*(b*c - a*d)/d))*
erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((-105*I*sqrt(pi)*cos(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*I*sqrt(pi)*cos(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 105*sqrt(pi)*sin(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 105*sqrt(pi)*sin(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*cos(-4*(b*c - a*d)/d
```

```
) + (105*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 105*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + 105*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) - 105*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))))*d^3*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/
d)) - 112*(64*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d)) - 15*sqrt(d*x + c)*
d^3*sqrt(abs(b)/abs(d)))*sin(4*((d*x + c)*b - b*c + a*d)/d))/(b^3*d*sqrt(ab
s(b)/abs(d)))
```

Fricas [A] time = 0.648786, size = 840, normalized size = 3.68

$$-\frac{105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-16\,(128}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(pi*
d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 -
 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*c
os(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 1
28*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d^3*x^2
+ 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x + a))*sq
rt(d*x + c))/(b^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.63911, size = 1467, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/1720320*(2240*(3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 3*I*sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) - 12*I*sqrt(d*x +
 c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*
e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2 - d^2*(4096*(15*(d*x +
c)^(7/2) - 42*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2)/d^2 + 105*(sqrt(2
)*sqrt(pi)*(-64*I*b^2*c^2*d + 48*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 4*(64*I*(d*x + c)^(5/2)*b^2*d - 12
8*I*(d*x + c)^(3/2)*b^2*c*d + 64*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(
3/2)*b*d^2 - 48*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d
*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^2 + 105*(sqrt(2)*sqrt(pi)*(64*I*b^
2*c^2*d + 48*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 4*(-64*I*(d*x + c)^(5/2)*b^2*d + 128*I*(d*x + c)^(3
/2)*b^2*c*d - 64*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 48*
sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I*b
*c + 4*I*a*d)/d)/b^3)/d^2) - 112*(1536*(d*x + c)^(5/2) - 2560*(d*x + c)^(3/
2)*c + 15*sqrt(2)*sqrt(pi)*(8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 15*sqrt(2)*sqrt(pi)*(-8*I*b*c*d - 3*d^2)
*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 60*(-8*I
*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4
*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 60*(8*I*(d*x + c)^(3/2)*b*d -
8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b
*c - 4*I*a*d)/d)/b^2)*c)/d
```

3.130 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=615

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\right.}{}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600
*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*
b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos
[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1
600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rubi [A] time = 1.153, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```


```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600
*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*
```

```
b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos
[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1
600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx)-\frac{1}{16}(c+dx)^{5/2}\sin\right.$$

$$=\frac{1}{16}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{5/2}\sin(5a+5bx)\,dx+\frac{1}{8}\int$$

$$=-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+5}{80b}$$

$$=-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+5}{80b}$$

$$=\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3}{576b^3}$$

$$=\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3}{576b^3}$$

$$=\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3}{576b^3}$$

$$=\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3}{576b^3}$$

Mathematica [C] time = 25.8253, size = 3348, normalized size = 5.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c^2*(2*Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fre
snelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*
Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d])
 - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] +
 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] -
3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d
]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c
*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) -
2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*
d*x)*Sin[a + b*x])))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/
d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b
*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Co
s[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c
 + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((
b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((1
2*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) -
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a
- (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[
b/d]*d*Sqrt[c + d*x]*(d*(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)
*Sin[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) + (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*
Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c +
d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*
Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/
(800*Sqrt[5]*b^3) - (d^2*(Sin[5*a]*((c^2*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c
 + d*x]])*Sin[(5*b*c)/d])/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (c^2*Cos[(5*b*c)/d]
*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) -
 (2*c*Cos[(5*b*c)/d]*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c +
d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 +
5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*
(b/d)^(5/2)*d^3) - (2*c*Sin[(5*b*c)/d]*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3
/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi
]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d])
)/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (Sin[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/
```

```
2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10
/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c
 + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3) + (Cos[(5*b*c)/d]*(25*Sqrt[
5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d
)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[
Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin
[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3)) + Cos[5*a]*((c
^2*Cos[(5*b*c)/d]*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]
) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/(5*Sqrt[5]*(
b/d)^(3/2)*d^3) - (c^2*Sin[(5*b*c)/d]*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x
))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (2*c*Sin[(5*b*c)/d]*((-3*(-(Sqrt[5]*S
qrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b
/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*
Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) - (2*c*Cos[(5*b*c)/d]
*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqr
t[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*
Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (
Cos[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))
/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) +
Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b
/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(
7/2)*d^3) - (Sin[(5*b*c)/d]*(25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*
b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c +
d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]])
+ Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqr
t[5]*(b/d)^(7/2)*d^3))))/16
```

Maple [A] time = 0.043, size = 719, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/16/b*d*(1/2/b
*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(
1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(c
os((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-si
n((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-
```

```
1/96/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/
2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(d*x+c)^(1/2)*b/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5/d*(d*x+c)*b+5*(a
*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/100/
b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 3.00513, size = 2943, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1728000*sqrt(5)*sqrt(3)*(720*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*sqrt(
abs(b)/abs(d))*abs(b)*sin(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 2000*sqrt
(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*abs(b)*sin(3*((d*x +
c)*b - b*c + a*d)/d)/abs(d) - 36000*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*s
qrt(abs(b)/abs(d))*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) - 72*(20*
sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d))*abs(b)/abs(d) - 3
*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d))*abs(b)/abs(d))*cos(5
*((d*x + c)*b - b*c + a*d)/d) + 200*(12*sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2
*d*sqrt(abs(b)/abs(d))*abs(b)/abs(d) - 5*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*
sqrt(abs(b)/abs(d))*abs(b)/abs(d))*cos(3*((d*x + c)*b - b*c + a*d)/d) + 360
0*(4*sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d))*abs(b)/abs(d
) - 15*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d))*abs(b)/abs(d))
*cos(((d*x + c)*b - b*c + a*d)/d) - (sqrt(3)*(27*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-5*(b
*c - a*d)/d)/abs(d) - sqrt(3)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-5*(b*c - a*d)/d)/ab
```

```
s(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (sqrt(5)*(125*sqrt(pi)*cos(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*cos(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*
cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(-125*I*sqrt(pi)*cos(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*I*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-3*(b*
c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(5)*sqrt(3)*(67
50*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 6750*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 6750*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + 6750*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^3*abs(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(-67
50*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 6750*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) - 6750*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 6750*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^3*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) + (sqrt(5)*sqrt(3)*(6750*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*I*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-(b*c - a*d)/
d)/abs(d) + sqrt(5)*sqrt(3)*(6750*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*sqrt(pi)*sin(1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-(b*c - a*d)/d
)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (sqrt(5)*(125*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*I*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*I*sqrt(p
i)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(
b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(125*I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*I*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-3*(
b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*(27*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqr
t(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I
*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 2
```

```
7*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^3*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(-27*I*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*s
in(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b^3
*d*sqrt(abs(b)/abs(d))*abs(b))
```

Fricas [A] time = 0.84257, size = 1337, normalized size = 2.17

$$\frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-101250}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d
))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
- 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*s
qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.23538, size = 4084, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/864000*(60*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I
*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt
(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)
/d)/b - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b -
 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*sqrt(d*x
 + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((
-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*sqrt(d*x + c)*d*e^((-5*I*(d
*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c^2 - d^2*(27*(I*sqrt(10)*sqrt(pi)*(20
*I*b^2*c^2*d - 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x
 + c)^(3/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d
^2 + 12*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((-5*I*(d*x + c)*b
 + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^2 + 125*(I*sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*
d + 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^
2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d
*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3
```

```
*I*a*d)/d)/b^3)/d^2 + 6750*(I*sqrt(2)*sqrt(pi)*(-4*I*b^2*c^2*d + 12*b*c*d^2
 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) -
 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*
x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 15
*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 675
0*(I*sqrt(2)*sqrt(pi)*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5
/2)*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*
(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*
e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^2 + 125*(I*sqrt(6)*sqrt(pi)*(-
12*I*b^2*c^2*d - 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d
*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b
*d^2 + 12*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*
b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^2 + 27*(I*sqrt(10)*sqrt(pi)*(20*I*b^2*c^2*
d + 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x + c)^(3
/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*
sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b*
c + 5*I*a*d)/d)/b^3)/d^2) - 12*(9*I*sqrt(10)*sqrt(pi)*(-10*I*b*c*d + 3*d^2)
*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 125*I*
sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b^2) + 2250*I*sqrt(2)*sqrt(pi)*(2*I*b*c*d - 3*d^2)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2250*I*sqrt(2)*s
qrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)*b^2) + 125*I*sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*erf(-1/2*sqrt
(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(10)*sqrt(pi
)*(-10*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 90*I*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*
c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2
- 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*
d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 4500*I*(2*I*(d*x + c
)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b^2 - 4500*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/
```

```
b^2 - 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
 c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 - 90*I*(-10*I*(d*
x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-5*I
*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)*c)/d
```

3.131 $\int (c+dx)^{3/2} \cos^2(a+bx) \sin^3(a+bx)\, dx$

Optimal. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{b}{d}\right.}{}$$

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rubi [A] time = 0.879348, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{b}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
```

```
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx)-\frac{1}{16}(c+dx)^{3/2}\right.$$

$$=\frac{1}{16}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{3/2}\sin(5a+5bx)\,dx+\frac{1}{8}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

$$=-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

Mathematica [C] time = 12.2837, size = 1041, normalized size = 1.95

$$\frac{ce^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+\frac{c\left(2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(5(a+bx))-\sqrt{2\pi}\cos\left(5a-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c*(2*Sqrt[5]*Sqrt[b/
d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fresnel
```

```
C[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (
c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*Fres
nelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]
*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin
[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a
+ b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*
x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*
b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)
/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin
[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

Maple [A] time = 0.042, size = 580, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/16/b*d*(1/2/b
*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1/160/b*d*(d*x+c)^(3/2)*co
s(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d
*x+c)*b+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(
5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*
b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d))))
```

Maxima [C] time = 2.90146, size = 2790, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288000*sqrt(5)*sqrt(3)*(240*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(
b)/abs(d))*abs(b)*cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 400*sqrt(5)*s
qrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*abs(b)*cos(3*((d*x + c)*b -
b*c + a*d)/d)/abs(d) - 2400*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)
/abs(d))*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(5)*sqrt(3
)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*sin(5*((d*x + c)*b - b*c + a
*d)/d)/abs(d) + 200*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*a
bs(b)*sin(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 3600*sqrt(5)*sqrt(3)*sqrt
(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/a
bs(d) - (sqrt(3)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(
9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d^2*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/
d)) - (sqrt(5)*(25*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 25*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 25*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) - 25*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*
(25*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 25*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 25*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 25*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))))*d^2*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) - (sqrt(5)*sqrt(3)*(450*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*sin(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-(b*c - a*d)/d)/ab
s(d) + sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
```

```
rctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*sin(-(b*c - a*d)/d)/abs(d))
*erf(sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(5)*sqrt(3)*(-450*I*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b
)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*sin
(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (sqrt(5)*(-25*I*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25
*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 25*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 25*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)))*d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(25*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*I*sqrt(pi)*sin
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*
sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*
(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(9*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*si
n(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b^2*
d*sqrt(abs(b)/abs(d))*abs(b))
```

Fricas [A] time = 0.780473, size = 1118, normalized size = 2.09

$$\frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-6750\sqrt{2}\pi d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
 b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^
3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.80713, size = 2269, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/144000*(10*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
```

d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I
*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt
(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)
/d)/b - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b -
 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*sqrt(d*x
 + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((
-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*sqrt(d*x + c)*d*e^((-5*I*(d
*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c - 9*I*sqrt(10)*sqrt(pi)*(-10*I*b*c*d
 + 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 125*I*sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2250*I*sqrt(2)*sqrt(pi)*(2*I*b*c*d - 3*
d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2250*I*
sqrt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b^2) - 125*I*sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*erf(
-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I
*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqrt(10
)*sqrt(pi)*(-10*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^2) + 90*I*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d
)/d)/b^2 + 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(
d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 4500*I*(2*I
*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 + 4500*I*(2*I*(d*x + c)^(3/2)*b*d - 2*
I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b^2 + 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + s
qrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 + 90*I*(
-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)
*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d

3.132 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{F}}{80b}$$

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rubi [A] time = 0.670548, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{F}}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
```

```
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\sin(a+bx) + \frac{1}{16}\sqrt{c+dx}\sin(3a+3bx) - \frac{1}{16}\sqrt{c+dx}\sin(5a+$$

$$= \frac{1}{16}\int \sqrt{c+dx}\sin(3a+3bx)\,dx - \frac{1}{16}\int \sqrt{c+dx}\sin(5a+5bx)\,dx + \frac{1}{8}\int \sqrt{c$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} -$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} -$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} -$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} +$$

Mathematica [C] time = 7.40148, size = 432, normalized size = 0.94

$$\frac{\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} + \frac{-\sqrt{2\pi}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqr
t[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/P
i]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt
[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)
/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/
d])
```

Maple [A] time = 0.041, size = 447, normalized size = 1.

$$2\frac{1}{d}\left(-1/16\,\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+1/32\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(\frac{ad-}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)
/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c
)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/16
0/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.84933, size = 2531, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/28800*sqrt(5)*sqrt(3)*(24*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs
(d))*abs(b)*cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 40*sqrt(5)*sqrt(3)*
sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*cos(3*((d*x + c)*b - b*c + a*d)/
d)/abs(d) - 240*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*
cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(3)*(3*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-5*(b
*c - a*d)/d)/abs(d) - sqrt(3)*(3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
```

+ 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (sqrt(5)*(5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(-5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(-30*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(30*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (sqrt(5)*(5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*(3*sqrt(pi)*cos(1/4

```
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(
-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/ab
s(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b*d*sqrt(abs(b)/abs(d))*ab
s(b))
```

Fricas [A] time = 0.712261, size = 952, normalized size = 2.07

$$-\frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-450\sqrt{2}\pi d\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(
-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*s
qrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b
/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*
x + a)^3)*sqrt(d*x + c))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.42523, size = 988, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/14400*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*
d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqr
t(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b
 - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b - 900*
sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*sqrt(d*x + c)
*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((-3*I*
(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*sqrt(d*x + c)*d*e^((-5*I*(d*x +
c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d
```

3.133 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{Fr}\cdots}{80b^{3\cdots}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rubi [A] time = 0.658185, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{Fr}\cdots}{80b^{3\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
```

```
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\sin(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\sin(5a+5\right.$$

$$= \frac{1}{16}\int \sqrt{c+dx}\sin(3a+3bx)\,dx - \frac{1}{16}\int \sqrt{c+dx}\sin(5a+5bx)\,dx + \frac{1}{8}\int \sqrt{c+}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} - \frac{d}{}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} - \Big($$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} - \frac{\mathrm{co}}{}$$

$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} + \frac{\sqrt{}}{}$$

Mathematica [C] time = 7.34617, size = 432, normalized size = 0.94

$$\frac{\sqrt{c+dx}\,e^{-\frac{i(ad+bc)}{d}}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+\frac{-\sqrt{2\pi}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqr
t[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/P
i]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt
[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)
/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/
d])
```

Maple [A] time = 0.042, size = 447, normalized size = 1.

$$2\frac{1}{d}\left(-1/16\,\frac{d\sqrt{dx+c}}{b}\cos\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)+1/32\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(\frac{ad}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)
/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c
)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/16
0/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.75907, size = 2531, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/28800*sqrt(5)*sqrt(3)*(24*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs
(d))*abs(b)*cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 40*sqrt(5)*sqrt(3)*
sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*cos(3*((d*x + c)*b - b*c + a*d)/
d)/abs(d) - 240*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*
cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(3)*(3*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-5*(b
*c - a*d)/d)/abs(d) - sqrt(3)*(3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
```

```
+ 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*
erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (sqrt(5)*(5*sqrt(pi)*cos(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-3*(b*c - a*
d)/d)/abs(d) + sqrt(5)*(-5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sq
rt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-(b*c
- a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(-30*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*cos(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c - a*d)/d)/
abs(d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(
pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b
)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(30*I*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*cos(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c
 - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (sqrt(5)*(5*sqrt(pi)*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*c
os(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi
)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(
pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b
)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(5*I*sqrt(pi)*cos(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/
d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*(3*sqrt(pi)*cos(1/4
```

```
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(
-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/ab
s(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b*d*sqrt(abs(b)/abs(d))*ab
s(b))
```

Fricas [A] time = 0.72081, size = 952, normalized size = 2.07

$$-\frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-450\sqrt{2}\pi d\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(
-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*s
qrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b
/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*
x + a)^3)*sqrt(d*x + c))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.45461, size = 988, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/14400*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*
d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqr
t(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b
 - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b - 900*
sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*sqrt(d*x + c)
*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((-3*I*
(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*sqrt(d*x + c)*d*e^((-5*I*(d*x +
c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d
```

3.134 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\cdots\right.}{\cdots}$$

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rubi [A] time = 0.8014, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
```

```
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx)-\frac{1}{16}(c+dx)^{3/2}\text{s}\right.$$

$$= \frac{1}{16}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{3/2}\sin(5a+5bx)\,dx+\frac{1}{8}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+}{80b}$$

Mathematica [C] time = 12.1833, size = 1041, normalized size = 1.95

$$\frac{ce^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+\frac{c\left(2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(5(a+bx))-\sqrt{2\pi}\cos\left(5a-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c*(2*Sqrt[5]*Sqrt[b/
d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fresnel
```

```
C[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (
c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*Fres
nelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]
*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin
[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a
+ b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*
x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*
b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)
/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin
[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

Maple [A] time = 0.038, size = 580, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/16/b*d*(1/2/b
*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1/160/b*d*(d*x+c)^(3/2)*co
s(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d
*x+c)*b+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(
5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*
b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d))))
```

Maxima [C] time = 2.93574, size = 2790, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288000*sqrt(5)*sqrt(3)*(240*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(
b)/abs(d))*abs(b)*cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 400*sqrt(5)*s
qrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*abs(b)*cos(3*((d*x + c)*b -
b*c + a*d)/d)/abs(d) - 2400*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)
/abs(d))*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(5)*sqrt(3
)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*sin(5*((d*x + c)*b - b*c + a
*d)/d)/abs(d) + 200*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*a
bs(b)*sin(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 3600*sqrt(5)*sqrt(3)*sqrt
(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/a
bs(d) - (sqrt(3)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(
9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d^2*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/
d)) - (sqrt(5)*(25*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 25*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 25*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) - 25*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*
(25*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 25*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 25*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 25*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))))*d^2*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) - (sqrt(5)*sqrt(3)*(450*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*sin(1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-(b*c - a*d)/d)/ab
s(d) + sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
```

```
rctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*sin(-(b*c - a*d)/d)/abs(d))
*erf(sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(5)*sqrt(3)*(-450*I*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b
)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*sin
(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (sqrt(5)*(-25*I*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25
*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 25*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 - 25*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)))*d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(25*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*I*sqrt(pi)*sin
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*
sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*
(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(9*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*si
n(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b^2*
d*sqrt(abs(b)/abs(d))*abs(b))
```

Fricas [A] time = 0.767489, size = 1118, normalized size = 2.09

$$\frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-6750\sqrt{2}\pi}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
 b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^
3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.81472, size = 2269, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/144000*(10*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
```

```
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I
*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt
(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)
/d)/b - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b -
 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*sqrt(d*x
 + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((
-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*sqrt(d*x + c)*d*e^((-5*I*(d
*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c - 9*I*sqrt(10)*sqrt(pi)*(-10*I*b*c*d
 + 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 125*I*sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2250*I*sqrt(2)*sqrt(pi)*(2*I*b*c*d - 3*
d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2250*I*
sqrt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b^2) - 125*I*sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*erf(
-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I
*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqrt(10
)*sqrt(pi)*(-10*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^2) + 90*I*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d
)/d)/b^2 + 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(
d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 4500*I*(2*I
*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I
*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 + 4500*I*(2*I*(d*x + c)^(3/2)*b*d - 2*
I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I
*a*d)/d)/b^2 + 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + s
qrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 + 90*I*(
-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)
*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d
```

3.135 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=615

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a\right.}{}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600
*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*
b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos
[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1
600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rubi [A] time = 0.952804, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```



```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600
*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*
```

```
b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos
[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1
600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx)-\frac{1}{16}(c+dx)^{5/2}\mathrm{s}\right.$$

$$= \frac{1}{16}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{5/2}\sin(5a+5bx)\,dx+\frac{1}{8}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+}{80b}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+}{80b}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\cos(3a+}{576b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\cos(3a+}{576b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\cos(3a+}{576b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}\cos(3a+}{576b^3}$$

Mathematica [C] time = 25.3137, size = 3348, normalized size = 5.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c^2*(2*Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fre
snelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*
Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d])
 - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] +
 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] -
3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d
]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c
*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) -
2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*
d*x)*Sin[a + b*x])))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/
d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b
*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Co
s[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c
 + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((
b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((1
2*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) -
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a
- (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[
b/d]*d*Sqrt[c + d*x]*(d*(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)
*Sin[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) + (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*
Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c +
d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*
Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/
(800*Sqrt[5]*b^3) - (d^2*(Sin[5*a]*((c^2*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c
 + d*x]])*Sin[(5*b*c)/d])/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (c^2*Cos[(5*b*c)/d]
*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) -
 (2*c*Cos[(5*b*c)/d]*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c +
d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 +
5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*
(b/d)^(5/2)*d^3) - (2*c*Sin[(5*b*c)/d]*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3
/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi
]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d])
)/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (Sin[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/
```

```
2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10
/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c
 + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3) + (Cos[(5*b*c)/d]*(25*Sqrt[
5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d
)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[
Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin
[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3)) + Cos[5*a]*((c
^2*Cos[(5*b*c)/d]*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]
) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/(5*Sqrt[5]*(
b/d)^(3/2)*d^3) - (c^2*Sin[(5*b*c)/d]*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x
))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (2*c*Sin[(5*b*c)/d]*((-3*(-(Sqrt[5]*S
qrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b
/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*
Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) - (2*c*Cos[(5*b*c)/d]
*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqr
t[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*
Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (
Cos[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))
/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) +
Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b
/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(
7/2)*d^3) - (Sin[(5*b*c)/d]*(25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*
b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c +
d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]])
+ Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqr
t[5]*(b/d)^(7/2)*d^3))))/16
```

Maple [A] time = 0.043, size = 719, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/16/b*d*(1/2/b
*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(
1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(c
os((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-si
n((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-
```

```
1/96/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/
2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(d*x+c)^(1/2)*b/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5/d*(d*x+c)*b+5*(a
*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/100/
b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.95017, size = 2943, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1728000*sqrt(5)*sqrt(3)*(720*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*sqrt(
abs(b)/abs(d))*abs(b)*sin(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 2000*sqrt
(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*abs(b)*sin(3*((d*x +
c)*b - b*c + a*d)/d)/abs(d) - 36000*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*s
qrt(abs(b)/abs(d))*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) - 72*(20*
sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d))*abs(b)/abs(d) - 3
*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d))*abs(b)/abs(d))*cos(5
*((d*x + c)*b - b*c + a*d)/d) + 200*(12*sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2
*d*sqrt(abs(b)/abs(d))*abs(b)/abs(d) - 5*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*
sqrt(abs(b)/abs(d))*abs(b)/abs(d))*cos(3*((d*x + c)*b - b*c + a*d)/d) + 360
0*(4*sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d))*abs(b)/abs(d
) - 15*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d))*abs(b)/abs(d))
*cos(((d*x + c)*b - b*c + a*d)/d) - (sqrt(3)*(27*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-5*(b
*c - a*d)/d)/abs(d) - sqrt(3)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-5*(b*c - a*d)/d)/ab
```

s(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (sqrt(5)*(125*sqrt(pi)*cos(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*cos(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*I*sqrt(pi)*sin(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*
cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(-125*I*sqrt(pi)*cos(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*I*sqrt(pi)*cos(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*sqrt(pi)*sin(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-3*(b*
c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(5)*sqrt(3)*(67
50*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
6750*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 6750*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + 6750*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^3*abs(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(-67
50*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 6750*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) - 6750*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 6750*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^3*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) + (sqrt(5)*sqrt(3)*(6750*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*I*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-(b*c - a*d)/
d)/abs(d) + sqrt(5)*sqrt(3)*(6750*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*sqrt(pi)*sin(1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-(b*c - a*d)/d
)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (sqrt(5)*(125*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*I*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*I*sqrt(p
i)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(
b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(125*I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*I*sqrt(pi)*cos(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 125*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 125*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-3*(
b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*(27*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqr
t(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I
*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 2

```
7*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^3*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(-27*I*sqrt(pi)*cos(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*s
in(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b^3
*d*sqrt(abs(b)/abs(d))*abs(b))
```

Fricas [A] time = 0.832086, size = 1337, normalized size = 2.17

$$81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-101250\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d
))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
- 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*s
qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.21758, size = 4084, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/864000*(60*(9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*d^2*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I
*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*sqrt(6)*sqrt
(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
9*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b) + 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)
/d)/b - 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b -
 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*sqrt(d*x
 + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b - 150*sqrt(d*x + c)*d*e^((
-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*sqrt(d*x + c)*d*e^((-5*I*(d
*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c^2 - d^2*(27*(I*sqrt(10)*sqrt(pi)*(20
*I*b^2*c^2*d - 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x
 + c)^(3/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d
^2 + 12*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((-5*I*(d*x + c)*b
 + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^2 + 125*(I*sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*
d + 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^
2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d
*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3
```

*I*a*d)/d)/b^3)/d^2 + 6750*(I*sqrt(2)*sqrt(pi)*(-4*I*b^2*c^2*d + 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^2 + 6750*(I*sqrt(2)*sqrt(pi)*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^2 + 125*(I*sqrt(6)*sqrt(pi)*(-12*I*b^2*c^2*d - 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^2 + 27*(I*sqrt(10)*sqrt(pi)*(20*I*b^2*c^2*d + 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x + c)^(3/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^2) - 12*(9*I*sqrt(10)*sqrt(pi)*(-10*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 125*I*sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2250*I*sqrt(2)*sqrt(pi)*(2*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2250*I*sqrt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 125*I*sqrt(6)*sqrt(pi)*(2*I*b*c*d + d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(10)*sqrt(pi)*(-10*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 90*I*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2 - 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 4500*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 - 4500*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/

```
b^2 - 750*I*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
 c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 - 90*I*(-10*I*(d*
x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-5*I
*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)*c)/d
```

3.136 $\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=273

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{b}$$

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) - (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rubi [A] time = 0.288119, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3308, 2181}

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) - (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^m \sin(2a+2bx)+\frac{1}{8}(c+dx)^m \sin(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int (c+dx)^m \sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^m \sin(2a+2bx)\,dx$$
$$= \frac{1}{16}i\int e^{-i(4a+4bx)}(c+dx)^m\,dx-\frac{1}{16}i\int e^{i(4a+4bx)}(c+dx)^m\,dx+\frac{1}{8}i\int e^{-i(2a+2bx)}$$
$$= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-4-m}e^{-2i\left(a-\frac{bc}{d}\right)}}{}$$

Mathematica [A] time = 0.247976, size = 245, normalized size = 0.9

$$-\frac{4^{-m-3}e^{-\frac{4i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(2^{m+2}e^{2i\left(a+\frac{3bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^m \text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right)+2^{m+2}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((4^(-3 - m)*(c + d*x)^m*(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c + d
*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d] + 2^(2 + m)*E^((2*I)*(a + (3
*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d] + E^
((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] + E^((
(8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])
```

```
)/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m))
```

Maple [F] time = 0.266, size = 0, normalized size = 0.

$$\int (dx + c)^m (\cos (bx + a))^3 \sin (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \cos (bx + a)^3 \sin (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

Fricas [A] time = 0.54418, size = 487, normalized size = 1.78

$$-\frac{e^{\left(-\frac{dm \log\left(\frac{4ib}{d}\right) - 4ibc + 4iad}{d}\right)}\Gamma\left(m+1, \frac{4ibdx + 4ibc}{d}\right) + 4e^{\left(-\frac{dm \log\left(\frac{2ib}{d}\right) - 2ibc + 2iad}{d}\right)}\Gamma\left(m+1, \frac{2ibdx + 2ibc}{d}\right) + 4e^{\left(-\frac{dm \log\left(-\frac{2ib}{d}\right) + 2ibc - 2iad}{d}\right)}\Gamma\left(m+1\right.}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/64*(e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m + 1, (4*I*b*d*
x + 4*I*b*c)/d) + 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
 1, (2*I*b*d*x + 2*I*b*c)/d) + 4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d
```

```
)/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*
b*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

3.137 $\int (c+dx)^4 \cos^3(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=260

$$\frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^3(c+dx)\sin(a+bx)\cos^3(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)}{64b^4}$$

```
[Out] (-45*c*d^3*x)/(64*b^3) - (45*d^4*x^2)/(128*b^3) + (3*(c + d*x)^4)/(32*b) -
(45*d^4*Cos[a + b*x]^2)/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[a + b*x]^2)/(16*
b^3) - (3*d^4*Cos[a + b*x]^4)/(128*b^5) + (3*d^2*(c + d*x)^2*Cos[a + b*x]^4
)/(16*b^3) - ((c + d*x)^4*Cos[a + b*x]^4)/(4*b) - (45*d^3*(c + d*x)*Cos[a +
 b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/
(8*b^2) - (3*d^3*(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(32*b^4) + (d*(c +
d*x)^3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b^2)
```

Rubi [A] time = 0.234001, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4405, 3311, 32, 3310}

$$\frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^3(c+dx)\sin(a+bx)\cos^3(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)}{64b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```



```
[Out] (-45*c*d^3*x)/(64*b^3) - (45*d^4*x^2)/(128*b^3) + (3*(c + d*x)^4)/(32*b) -
(45*d^4*Cos[a + b*x]^2)/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[a + b*x]^2)/(16*
b^3) - (3*d^4*Cos[a + b*x]^4)/(128*b^5) + (3*d^2*(c + d*x)^2*Cos[a + b*x]^4
)/(16*b^3) - ((c + d*x)^4*Cos[a + b*x]^4)/(4*b) - (45*d^3*(c + d*x)*Cos[a +
 b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/
(8*b^2) - (3*d^3*(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(32*b^4) + (d*(c +
d*x)^3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b^2)
```


Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)^4\cos^4(a+bx)}{4b}+\frac{d\int (c+dx)^3\cos^4(a+bx)\,dx}{b}\\
&= \frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}-\frac{(c+dx)^4\cos^4(a+bx)}{4b}+\frac{d(c+dx)^3\cos^3(a+bx)}{4b^2}\\
&= \frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^4\cos^4(a+bx)}{128b^5}+\frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}-\\
&= \frac{3(c+dx)^4}{32b}-\frac{45d^4\cos^2(a+bx)}{128b^5}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^4\cos^4(a+b}{128b^5}\\
&= -\frac{45cd^3x}{64b^3}-\frac{45d^4x^2}{128b^3}+\frac{3(c+dx)^4}{32b}-\frac{45d^4\cos^2(a+bx)}{128b^5}+\frac{9d^2(c+dx)^2\cos^2(a-}{16b^3}\end{aligned}$$

Mathematica [A] time = 1.8831, size = 158, normalized size = 0.61

$$-\frac{64\cos(2(a+bx))\left(-6b^2d^2(c+dx)^2+2b^4(c+dx)^4+3d^4\right)+\cos(4(a+bx))\left(-24b^2d^2(c+dx)^2+32b^4(c+dx)^4+3d^4\right)}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -(64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
 (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*x)] - 8
*b*d*(c + d*x)*(16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c + d*x)
^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/(1024*b^5)
```

Maple [B] time = 0.049, size = 1150, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/4*(b*x+a)^4*cos(b*x+a)^4+(b*x+a)^3*(1/4*(cos(b*x+a)^3+3/
2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/16*(b*x+a)^2*cos(b*x+a)^4-3/8*(b*
x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+45/128*(b
*x+a)^2-3/128*cos(b*x+a)^4-9/128*cos(b*x+a)^2+9/16*(b*x+a)^2*cos(b*x+a)^2-9
/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*sin(b*x+a)^2-9/32
*(b*x+a)^4)-4/b^4*a*d^4*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(co
s(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a
)^4-3/128*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32
*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)+4/b^3*c*d^
3*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^3+3/2*cos(b*x
+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*cos(b*x+a)^2-
9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)+6/b^4*a^2*d^4*(-1/4*(b*x+a)^2*co
s(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*
x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a)^2)-12/b^3*a*c*d^3
*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a)
)*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a
)^2)+6/b^2*c^2*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a
)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a
)^4+3/32*cos(b*x+a)^2)-4/b^4*a^3*d^4*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b
*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+12/b^3*a^2*c*d^3*(-1/4*
(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x
+3/32*a)-12/b^2*a*c^2*d^2*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2
*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+4/b*c^3*d*(-1/4*(b*x+a)*cos(b*x+a)
^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-1/4/b^4*a
^4*d^4*cos(b*x+a)^4+1/b^3*a^3*c*d^3*cos(b*x+a)^4-3/2/b^2*a^2*c^2*d^2*cos(b*
x+a)^4+1/b*a*c^3*d*cos(b*x+a)^4-1/4*c^4*cos(b*x+a)^4)
```

Maxima [B] time = 1.394, size = 1305, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(256*c^4*cos(b*x + a)^4 - 1024*a*c^3*d*cos(b*x + a)^4/b + 1536*a^2*
c^2*d^2*cos(b*x + a)^4/b^2 - 1024*a^3*c*d^3*cos(b*x + a)^4/b^3 + 256*a^4*d^
4*cos(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(
2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x
+ a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) -
8*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(
b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^2*c*d^
3/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) -
sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1
)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*
sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x
 + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4
*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 +
24*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
 + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a^2
*d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x
 + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x +
 4*a) - 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x +
 a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*co
s(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2
 - 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*
cos(4*b*x + 4*a) + 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a)
- 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) - 128*(2*(b*x + a)^3 - 3
*b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

Fricas [A] time = 0.536091, size = 802, normalized size = 3.08

$$\frac{12\,b^4d^4x^4 + 48\,b^4cd^3x^3 - \left(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 32\,b^4c^4 - 24\,b^2c^2d^2 + 3\,d^4 + 24\left(8\,b^4c^2d^2 - b^2d^4\right)x^2 + 16\left(8\,b^4c^3d -\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(12*b^4*d^4*x^4 + 48*b^4*c*d^3*x^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*
x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^
2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 9*(8*b^4*c^2*d^2 - 5
*b^2*d^4)*x^2 + 9*(8*b^2*d^4*x^2 + 16*b^2*c*d^3*x + 8*b^2*c^2*d^2 - 5*d^4)*
cos(b*x + a)^2 + 6*(8*b^4*c^3*d - 15*b^2*c*d^3)*x + 2*(2*(8*b^3*d^4*x^3 + 2
4*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*co
s(b*x + a)^3 + 3*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 15*b*c*d
^3 + 3*(8*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

Sympy [A] time = 21.9182, size = 976, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**4/(4*b) + c**4*sin(a + b*x)**2*cos(a + b*x)**
2/(2*b) + 3*c**3*d*x*sin(a + b*x)**4/(8*b) + 3*c**3*d*x*sin(a + b*x)**2*cos
(a + b*x)**2/(4*b) - 5*c**3*d*x*cos(a + b*x)**4/(8*b) + 9*c**2*d**2*x**2*si
n(a + b*x)**4/(16*b) + 9*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(8*
b) - 15*c**2*d**2*x**2*cos(a + b*x)**4/(16*b) + 3*c*d**3*x**3*sin(a + b*x)*
*4/(8*b) + 3*c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 5*c*d**3*x
**3*cos(a + b*x)**4/(8*b) + 3*d**4*x**4*sin(a + b*x)**4/(32*b) + 3*d**4*x**
4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d**4*x**4*cos(a + b*x)**4/(32*
b) + 3*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 5*c**3*d*sin(a + b*x)
*cos(a + b*x)**3/(8*b**2) + 9*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(8*b
**2) + 15*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 9*c*d**3*x**2
*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*c*d**3*x**2*sin(a + b*x)*cos(a
+ b*x)**3/(8*b**2) + 3*d**4*x**3*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 5*
d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) - 3*c**2*d**2*sin(a + b*x)*
*4/(4*b**3) - 15*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**3) - 45*c
*d**3*x*sin(a + b*x)**4/(64*b**3) - 9*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)
**2/(32*b**3) + 51*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 45*d**4*x**2*sin(a
+ b*x)**4/(128*b**3) - 9*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**3
) + 51*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 45*c*d**3*sin(a + b*x)**3*cos
(a + b*x)/(64*b**4) - 51*c*d**3*sin(a + b*x)*cos(a + b*x)**3/(64*b**4) - 45
*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*d**4*x*sin(a + b*x)*cos
(a + b*x)**3/(64*b**4) + 3*d**4*sin(a + b*x)**4/(8*b**5) + 51*d**4*sin(a +
b*x)**2*cos(a + b*x)**2/(128*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2
*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**3, True))
```

Giac [A] time = 1.10955, size = 487, normalized size = 1.87

$$-\frac{\left(32\,b^4d^4x^4+128\,b^4cd^3x^3+192\,b^4c^2d^2x^2+128\,b^4c^3dx+32\,b^4c^4-24\,b^2d^4x^2-48\,b^2cd^3x-24\,b^2c^2d^2+3\,d^4\right)\cos\left(4\,bx\right.}{1024\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4
*c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 +
3*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^
4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x
- 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 + 1/256*(8*b^3*d^4*x^3 + 24*b
^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(
4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

3.138 $\int (c+dx)^3 \cos^3(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=196

$$\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a+bx)\cos}{32b^2}$$

```
[Out] (-45*d^3*x)/(256*b^3) + (3*(c + d*x)^3)/(32*b) + (9*d^2*(c + d*x)*Cos[a + b
*x]^2)/(32*b^3) + (3*d^2*(c + d*x)*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^3*
Cos[a + b*x]^4)/(4*b) - (45*d^3*Cos[a + b*x]*Sin[a + b*x])/(256*b^4) + (9*d
*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) - (3*d^3*Cos[a + b*x]^3*Si
n[a + b*x])/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x])/(16*b
^2)
```

Rubi [A] time = 0.160841, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4405, 3311, 32, 2635, 8}

$$\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a+bx)\cos}{32b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```


```
[Out] (-45*d^3*x)/(256*b^3) + (3*(c + d*x)^3)/(32*b) + (9*d^2*(c + d*x)*Cos[a + b
*x]^2)/(32*b^3) + (3*d^2*(c + d*x)*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^3*
Cos[a + b*x]^4)/(4*b) - (45*d^3*Cos[a + b*x]*Sin[a + b*x])/(256*b^4) + (9*d
*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) - (3*d^3*Cos[a + b*x]^3*Si
n[a + b*x])/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x])/(16*b
^2)
```


Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^3\cos^4(a+bx)}{4b}+\frac{(3d)\int (c+dx)^2\cos^4(a+bx)\,dx}{4b}$$

$$= \frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}+\frac{3d(c+dx)^2\cos^3(a+bx)}{16b^2}$$

$$= \frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$= \frac{3(c+dx)^3}{32b}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^3}{}$$

$$= -\frac{45d^3x}{256b^3}+\frac{3(c+dx)^3}{32b}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}$$

Mathematica [A] time = 1.01191, size = 135, normalized size = 0.69

$$\frac{-64b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)-4b(c+dx)\cos(4(a+bx))\left(8b^2(c+dx)^2-3d^2\right)+6d\sin(2(a+bx))\left(\right.}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 4*b*(c + d
*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + 6*d*(16*(-d^2 + 2*b^2*(
c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)]
)/(1024*b^4)
```

Maple [B] time = 0.021, size = 594, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128
*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*
cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)-3/b^3*a*d^3*(-1/4*(
b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*
x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a)^2)+3/b
^2*c*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*co
s(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*c
os(b*x+a)^2)+3/b^3*a^2*d^3*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/
2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-6/b^2*a*c*d^2*(-1/4*(b*x+a)*cos(b
*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+3/b*
c^2*d*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x
+a)+3/32*b*x+3/32*a)+1/4/b^3*a^3*d^3*cos(b*x+a)^4-3/4/b^2*a^2*c*d^2*cos(b*x
+a)^4+3/4/b*a*c^2*d*cos(b*x+a)^4-1/4*c^3*cos(b*x+a)^4)
```

Maxima [B] time = 1.2684, size = 741, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(256*c^3*cos(b*x + a)^4 - 768*a*c^2*d*cos(b*x + a)^4/b + 768*a^2*c*
d^2*cos(b*x + a)^4/b^2 - 256*a^3*d^3*cos(b*x + a)^4/b^3 + 24*(4*(b*x + a)*c
```

```
os(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(
2*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos
(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24*(4*
(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*
a) - 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*b*x +
4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*
a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2 - 1)*cos
(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4
*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b*x + a)^3
- 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*
x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2 - 1)*
sin(2*b*x + 2*a))*d^3/b^3)/b
```

Fricas [A] time = 0.506452, size = 518, normalized size = 2.64

$$\frac{24\,b^3d^3x^3 + 72\,b^3cd^2x^2 - 8\left(8\,b^3d^3x^3 + 24\,b^3cd^2x^2 + 8\,b^3c^3 - 3\,bcd^2 + 3\left(8\,b^3c^2d - bd^3\right)x\right)\cos\left(bx + a\right)^4 + 72\left(bd^3x + bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/256*(24*b^3*d^3*x^3 + 72*b^3*c*d^2*x^2 - 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*
x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 + 7
2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 9*(8*b^3*c^2*d - 5*b*d^3)*x + 3*(2*(
8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 + 3*(8*b
^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 5*d^3)*cos(b*x + a))*sin(b*x +
a))/b^4
```

Sympy [A] time = 11.6901, size = 634, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**4/(4*b) + c**3*sin(a + b*x)**2*cos(a + b*x)**
2/(2*b) + 9*c**2*d*x*sin(a + b*x)**4/(32*b) + 9*c**2*d*x*sin(a + b*x)**2*co
```

```
s(a + b*x)**2/(16*b) - 15*c**2*d*x*cos(a + b*x)**4/(32*b) + 9*c*d**2*x**2*s
in(a + b*x)**4/(32*b) + 9*c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b
) - 15*c*d**2*x**2*cos(a + b*x)**4/(32*b) + 3*d**3*x**3*sin(a + b*x)**4/(32
*b) + 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d**3*x**3*cos(
a + b*x)**4/(32*b) + 9*c**2*d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 15*c
**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) + 9*c*d**2*x*sin(a + b*x)**3*c
os(a + b*x)/(16*b**2) + 15*c*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**2)
+ 9*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 15*d**3*x**2*sin(a +
 b*x)*cos(a + b*x)**3/(32*b**2) - 3*c*d**2*sin(a + b*x)**4/(8*b**3) - 15*c*
d**2*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) - 45*d**3*x*sin(a + b*x)**4/
(256*b**3) - 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**3) + 51*d**3*
x*cos(a + b*x)**4/(256*b**3) - 45*d**3*sin(a + b*x)**3*cos(a + b*x)/(256*b*
*4) - 51*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b, 0)), ((c**3*x
+ 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a)**3, True))
```

Giac [A] time = 1.14536, size = 325, normalized size = 1.66

$$-\frac{\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^4}-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b
*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^
2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)
/b^4 + 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*
x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin
(2*b*x + 2*a)/b^4
```

3.139 $\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{d(c+dx)\sin(a+bx)\cos^3(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}+\frac{d^2\cos^4(a+bx)}{32b^3}+\frac{3d^2\cos^2(a+bx)}{32b^3}-\frac{(c+dx}{}$$

```
[Out] (3*c*d*x)/(16*b) + (3*d^2*x^2)/(32*b) + (3*d^2*Cos[a + b*x]^2)/(32*b^3) + (
d^2*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^2*Cos[a + b*x]^4)/(4*b) + (3*d*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/(16*b^2) + (d*(c + d*x)*Cos[a + b*x]^3*S
in[a + b*x])/(8*b^2)
```

Rubi [A] time = 0.086674, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4405, 3310}

$$\frac{d(c+dx)\sin(a+bx)\cos^3(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}+\frac{d^2\cos^4(a+bx)}{32b^3}+\frac{3d^2\cos^2(a+bx)}{32b^3}-\frac{(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*c*d*x)/(16*b) + (3*d^2*x^2)/(32*b) + (3*d^2*Cos[a + b*x]^2)/(32*b^3) + (
d^2*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^2*Cos[a + b*x]^4)/(4*b) + (3*d*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/(16*b^2) + (d*(c + d*x)*Cos[a + b*x]^3*S
in[a + b*x])/(8*b^2)
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)^2\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{d\int (c+dx)\cos^4(a+bx)\,dx}{2b}$$
$$=\frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{d(c+dx)\cos^3(a+bx)\sin(a+bx)}{8b^2}$$
$$=\frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{3d(c+dx)\cos(}{1}$$
$$=\frac{3cdx}{16b}+\frac{3d^2x^2}{32b}+\frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

Mathematica [A] time = 0.486177, size = 89, normalized size = 0.66

$$\frac{-16\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+\cos(4(a+bx))\left(d^2-8b^2(c+dx)^2\right)+4bd(c+dx)(8\sin(2(a+bx))+\sin(4(a+b}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (d^2 - 8*b^2*(c + d*x)^2
)*Cos[4*(a + b*x)] + 4*b*d*(c + d*x)*(8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)]
))/(256*b^3)
```

Maple [B] time = 0.019, size = 260, normalized size = 1.9

$$\frac{1}{b}\left(\frac{d^2}{b^2}\left(-\frac{(bx+a)^2(\cos(bx+a))^4}{4}+\frac{bx+a}{2}\left(\frac{\sin(bx+a)}{4}\left((\cos(bx+a))^3+\frac{3\cos(bx+a)}{2}\right)+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{3(bx+a)^2}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+
3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+
3/32*cos(b*x+a)^2)-2/b^2*a*d^2*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+2/b*c*d*(-1/4*(b*x+a)*cos(b*x
+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-1/4*d^
```

```
2/b^2*a^2*cos(b*x+a)^4+1/2*c*d/b*a*cos(b*x+a)^4-1/4*c^2*cos(b*x+a)^4)
```

Maxima [B] time = 1.15485, size = 355, normalized size = 2.65

$$-\frac{64\,c^2\cos\left(bx+a\right)^4-\frac{128\,acd\cos(bx+a)^4}{b}+\frac{64\,a^2d^2\cos(bx+a)^4}{b^2}+\frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a)}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*(64*c^2*cos(b*x + a)^4 - 128*a*c*d*cos(b*x + a)^4/b + 64*a^2*d^2*cos
(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x
+ 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(
4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b
*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*s
in(2*b*x + 2*a))*d^2/b^2)/b
```

Fricas [A] time = 0.49275, size = 294, normalized size = 2.19

$$\frac{3\,b^2d^2x^2+6\,b^2cdx-\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(bx+a\right)^4+3\,d^2\cos\left(bx+a\right)^2+2\left(2\left(bd^2x+bcd\right)\cos\left(bx+\right.\right.}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*d^2*x^2 + 6*b^2*c*d*x - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c
^2 - d^2)*cos(b*x + a)^4 + 3*d^2*cos(b*x + a)^2 + 2*(2*(b*d^2*x + b*c*d)*co
s(b*x + a)^3 + 3*(b*d^2*x + b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [A] time = 13.5359, size = 350, normalized size = 2.61

$$\begin{cases}\frac{c^2\sin^4(a+bx)}{4b}+\frac{c^2\sin^2(a+bx)\cos^2(a+bx)}{2b}+\frac{3cdx\sin^4(a+bx)}{16b}+\frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b}-\frac{5cdx\cos^4(a+bx)}{16b}+\frac{3d^2x^2\sin^4(a+bx)}{32b}+\frac{3d^2x^2\sin^2}{} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin\left(a\right)\cos^3\left(a\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**4/(4*b) + c**2*sin(a + b*x)**2*cos(a + b*x)**
2/(2*b) + 3*c*d*x*sin(a + b*x)**4/(16*b) + 3*c*d*x*sin(a + b*x)**2*cos(a +
b*x)**2/(8*b) - 5*c*d*x*cos(a + b*x)**4/(16*b) + 3*d**2*x**2*sin(a + b*x)**
4/(32*b) + 3*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d**2*x**2
*cos(a + b*x)**4/(32*b) + 3*c*d*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 5*
c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) + 3*d**2*x*sin(a + b*x)**3*cos(a
 + b*x)/(16*b**2) + 5*d**2*x*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) - d**2*
sin(a + b*x)**4/(8*b**3) - 5*d**2*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3)
, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a)**3, True))
```

Giac [A] time = 1.10493, size = 196, normalized size = 1.46

$$-\frac{\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^3}-\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(2\,bx+2\,a\right)}{16\,b^3}+\frac{\left(bd^2x+bcd\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^
3 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b
^3 + 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*si
n(2*b*x + 2*a)/b^3
```

3.140 $\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=72

$$\frac{d\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3dx}{32b}$$

```
[Out] (3*d*x)/(32*b) - ((c + d*x)*Cos[a + b*x]^4)/(4*b) + (3*d*Cos[a + b*x]*Sin[a
 + b*x])/(32*b^2) + (d*Cos[a + b*x]^3*Sin[a + b*x])/(16*b^2)
```

Rubi [A] time = 0.0470773, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4405, 2635, 8}

$$\frac{d\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{3d\sin(a+bx)\cos(a+bx)}{32b^2}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3dx}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*d*x)/(32*b) - ((c + d*x)*Cos[a + b*x]^4)/(4*b) + (3*d*Cos[a + b*x]*Sin[a
 + b*x])/(32*b^2) + (d*Cos[a + b*x]^3*Sin[a + b*x])/(16*b^2)
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)\cos^4(a+bx)}{4b} + \frac{d\int \cos^4(a+bx)\,dx}{4b}$$
$$= -\frac{(c+dx)\cos^4(a+bx)}{4b} + \frac{d\cos^3(a+bx)\sin(a+bx)}{16b^2} + \frac{(3d)\int \cos^2(a+bx)\,dx}{16b}$$
$$= -\frac{(c+dx)\cos^4(a+bx)}{4b} + \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos^3(a+bx)\sin(a+b}{16b^2}$$
$$= \frac{3dx}{32b} - \frac{(c+dx)\cos^4(a+bx)}{4b} + \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos^3(a+bx)\sin(}{16b^2}$$

Mathematica [A] time = 0.151772, size = 75, normalized size = 1.04

$$\frac{d(\sin(2(a+bx)) - 2bx\cos(2(a+bx)))}{16b^2} + \frac{d(\sin(4(a+bx)) - 4bx\cos(4(a+bx)))}{128b^2} - \frac{c\cos^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -(c*Cos[a + b*x]^4)/(4*b) + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)])
)/(16*b^2) + (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

Maple [A] time = 0.02, size = 85, normalized size = 1.2

$$\frac{1}{b}\left(\frac{d}{b}\left(-\frac{(bx+a)(\cos(bx+a))^4}{4} + \frac{\sin(bx+a)}{16}\left((\cos(bx+a))^3 + \frac{3\cos(bx+a)}{2}\right) + \frac{3bx}{32} + \frac{3a}{32}\right) + \frac{ad(\cos(bx+a))^4}{4b} - \frac{(c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(d/b*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(
b*x+a)+3/32*b*x+3/32*a)+1/4/b*d*a*cos(b*x+a)^4-1/4*cos(b*x+a)^4*c)
```

Maxima [A] time = 1.11205, size = 124, normalized size = 1.72

$$-\frac{32\,c\cos\left(bx+a\right)^4-\frac{32\,ad\cos(bx+a)^4}{b}+\frac{(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a))d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/128*(32*c*cos(b*x + a)^4 - 32*a*d*cos(b*x + a)^4/b + (4*(b*x + a)*cos(4*
b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x
 + 2*a))*d/b)/b
```

Fricas [A] time = 0.48835, size = 147, normalized size = 2.04

$$-\frac{8\,(bdx+bc)\cos\left(bx+a\right)^4-3\,bdx-\left(2\,d\cos\left(bx+a\right)^3+3\,d\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 - 3*b*d*x - (2*d*cos(b*x + a)^3 + 3*d
*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A] time = 2.81213, size = 160, normalized size = 2.22

$$\begin{cases}\frac{c\sin^4(a+bx)}{4b}+\frac{c\sin^2(a+bx)\cos^2(a+bx)}{2b}+\frac{3dx\sin^4(a+bx)}{32b}+\frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b}-\frac{5dx\cos^4(a+bx)}{32b}+\frac{3d\sin^3(a+bx)\cos(a+bx)}{32b^2}+\frac{5d\sin(\ldots}{} \\ \left(cx+\frac{dx^2}{2}\right)\sin(a)\cos^3(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((c*sin(a + b*x)**4/(4*b) + c*sin(a + b*x)**2*cos(a + b*x)**2/(2*b
) + 3*d*x*sin(a + b*x)**4/(32*b) + 3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(1
6*b) - 5*d*x*cos(a + b*x)**4/(32*b) + 3*d*sin(a + b*x)**3*cos(a + b*x)/(32*
```

```
b**2) + 5*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2), Ne(b, 0)), ((c*x + d*x*
*2/2)*sin(a)*cos(a)**3, True))
```

Giac [A] time = 1.11548, size = 101, normalized size = 1.4

$$-\frac{(bdx + bc)\cos(4bx + 4a)}{32b^2} - \frac{(bdx + bc)\cos(2bx + 2a)}{8b^2} + \frac{d\sin(4bx + 4a)}{128b^2} + \frac{d\sin(2bx + 2a)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*
a)/b^2 + 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

3.141 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=129

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}+$$

```
[Out] (CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral[
(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*c)/d + 2*b*x])/(4*d) + (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*
c)/d + 4*b*x])/(8*d)
```

Rubi [A] time = 0.211813, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral[
(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*c)/d + 2*b*x])/(4*d) + (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*
c)/d + 4*b*x])/(8*d)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)}+\frac{\sin(4a+4bx)}{8(c+dx)}\right)dx \\
&= \frac{1}{8}\int \frac{\sin(4a+4bx)}{c+dx}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{c+dx}\,dx \\
&= \frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx+\frac{1}{4}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{8}\,\mathrm{s} \\
&= \frac{\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d}+\frac{\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}\right.}{4d}
\end{aligned}
$$

Mathematica [A] time = 0.344294, size = 110, normalized size = 0.85

$$
\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)+\cos\left(4a\right.}{8d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 2*CosIntegral[(2*b*(
c + d*x))/d]*Sin[2*a - (2*b*c)/d] + 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b
*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

Maple [A] time = 0.021, size = 178, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b}{8}\left(2\,\frac{1}{d}\mathrm{Si}\left(2\,bx+2\,a+2\,\frac{-ad+bc}{d}\right)\cos\left(2\,\frac{-ad+bc}{d}\right)-2\,\frac{1}{d}\mathrm{Ci}\left(2\,bx+2\,a+2\,\frac{-ad+bc}{d}\right)\sin\left(2\,\frac{-ad+bc}{d}\right)\right)+\frac{b}{32}\left(4\,\frac{1}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x)
```

```
[Out] 1/b*(1/8*b*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x
+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)+1/32*b*(4*Si(4*b*x+4*a+4*(-a*d+
b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*
c)/d)/d))
```

Maxima [C] time = 1.4869, size = 370, normalized size = 2.87

$$-\frac{b\left(2i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_1\left(-\frac{4i\,bc+4i\,(\cdots}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*(b*(2*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) -
2*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*
c - a*d)/d) + b*(I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/
d) - I*exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*
(b*c - a*d)/d) + 2*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*
d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
*(b*c - a*d)/d) + b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d
)/d) + exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*
(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 0.481013, size = 421, normalized size = 3.26

$$\frac{2\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+\left(\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+2\,\cos\left(-\frac{4\,(bc-a\cdots}{d}\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) + (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-
4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) + 2*cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x), x)
```

Giac [C] time = 1.75453, size = 8162, normalized size = 63.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2
*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_in
tegral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b
*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*ta
n(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integra
l(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_
part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b
```

```
*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(
2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2
*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^
2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*ta
n(2*b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^
2*tan(2*b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*ta
n(2*b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2 + 8*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan
(2*b*c/d)^2*tan(b*c/d) - 8*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*
a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 16*sin_integral(2*(b*d*x + b*c)/d)*
tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_integra
l(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b
*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x +
 b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2
 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(b
*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*t
an(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_inte
gral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*sin_integr
al(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 2*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 4*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 + 4*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 2*
real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2 -
 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)
^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c
/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b
```

```
*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^
2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2
*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*
tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(2*a)^2*tan(a)*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c
/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_inte
gral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d
)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(
b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^
2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)
^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2 + 2*ima
g_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2 + imag_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*sin_integral(4*(b*d*x +
 b*c)/d)*tan(2*a)^2*tan(a)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2
*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*ta
n(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*
tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*
c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 +
 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - 2*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - imag_p
art(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*sin_integ
ral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 4*sin_integral(2*(b*d*x
+ b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/
d))*tan(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^
2*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2
*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 - 4*s
in_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 + 8*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) - 8*imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) + 16*sin_integral(2*(
b*d*x + b*c)/d)*tan(2*a)^2*tan(a)*tan(b*c/d) + 8*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 8*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 16*sin_integral(2*(b*
d*x + b*c)/d)*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
```

```
(2*a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c
/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c/d)^2 + imag_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^
2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 +
4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(b*c/d)
^2 - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(
b*c/d)^2 + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)*tan(b*c/
d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2
 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 +
imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*s
in_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral
(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(
2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a) + 4*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(2*a)^2*tan(a) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*a)*tan(a)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^
2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*real_
part(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 4*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)*tan(2*b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2
*a)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*t
an(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)
+ 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 4*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(co
s_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 + 2*real_part(cos_integr
al(-4*b*x - 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*
c/d)*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 2
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2 - 2*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(2*a)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/
d))*tan(2*a)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 4*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(2*a)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(a)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + 2*imag_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - imag_part(cos_integral(-4*b*
x - 4*b*c/d))*tan(a)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2 - 4*sin
```

```
_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*a)*tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b
*c/d) - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2 - 2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/
d))*tan(2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 + 4*s
in_integral(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 + 8*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 8*imag_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)*tan(b*c/d) + 16*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/
d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(b*c/d)^2 - 2*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(b
*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(b*c/d)^2 - 4*sin_integral(2
*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))
*tan(2*a) + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 4*real_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 4*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/
d) - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) - 4*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d)) + 2*ima
g_part(cos_integral(2*b*x + 2*b*c/d)) - 2*imag_part(cos_integral(-2*b*x - 2
*b*c/d)) - imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*sin_integral(4*(b*
d*x + b*c)/d) + 4*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(2*a)^2*tan(a)^2*t
an(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + d*tan(2
*a)^2*tan(a)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*
tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2 + d*tan(2*a)^2
*tan(2*b*c/d)^2 + d*tan(a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2*tan(b*c/d)^2 + d
*tan(a)^2*tan(b*c/d)^2 + d*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2 + d*t
an(a)^2 + d*tan(2*b*c/d)^2 + d*tan(b*c/d)^2 + d)
```

3.142 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=179

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}$$

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) - Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rubi [A] time = 0.268476, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```



```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) - Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2}+\frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx \\
&= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx \\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}+\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{2d}+\frac{b\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{2d} \\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}+\frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d}+\frac{\left(b\cos\left(2a-\right.\right.}{} \\
&= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin}{8}
\end{aligned}$$

Mathematica [A] time = 1.67309, size = 151, normalized size = 0.84

$$-\frac{-4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-4b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -(-4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*a -
(4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (2*d*Sin[2*(a + b*x)])/(c + d*x
) + (d*Sin[4*(a + b*x)])/(c + d*x) + 4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(
2*b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]
)/(8*d^2)
```

Maple [A] time = 0.023, size = 256, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^2}{8}\left(-2\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)d}+2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\sin\left(2\frac{-ad+bc}{d}\right)+2\frac{1}{d}\text{Ci}\left(2bx+2a+2\frac{-ad}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/8*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b*x+2*a+2*(
-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a
*d+b*c)/d)/d)/d)+1/32*b^2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d+4*(4*Si(
4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)
/d)*cos(4*(-a*d+b*c)/d)/d)/d))
```

Maxima [C] time = 1.74053, size = 406, normalized size = 2.27

$$-\frac{b^2\left(2iE_2\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)-2iE_2\left(-\frac{2ibc+2i(bx+a)d-2iad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^2\left(iE_2\left(\frac{4ibc+4i(bx+a)d-4iad}{d}\right)-iE_2\left(-\frac{4ibc+4}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(b^2*(2*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
- 2*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) + 2*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 0.557949, size = 597, normalized size = 3.34

$$-\frac{4\,d\cos\left(bx+a\right)^{3}\sin\left(bx+a\right)+2\left(bdx+bc\right)\sin\left(-\frac{4\left(bc-ad\right)}{d}\right)\operatorname{Si}\left(\frac{4\left(bdx+bc\right)}{d}\right)+2\left(bdx+bc\right)\sin\left(-\frac{2\left(bc-ad\right)}{d}\right)\operatorname{Si}\left(\frac{2\left(bdx+bc\right)}{d}\right)-\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*cos(b*x + a)^3*sin(b*x + a) + 2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/
d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*
sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*
c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/
d) - ((b*d*x + b*c)*cos_integral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_int
egral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+bx\right)\cos^{3}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3 \sin(bx+a)}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3*sin(b*x + a)/(d*x + c)^2, x)
```

3.143 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=231

$$-\frac{b^2 \sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2 \sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2 \cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2b\right.}{2d^3}$$

```
[Out] -(b*Cos[2*a + 2*b*x])/(4*d^2*(c + d*x)) - (b*Cos[4*a + 4*b*x])/(4*d^2*(c +
d*x)) - (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (b^
2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a +
2*b*x]/(8*d*(c + d*x)^2) - Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos[2
*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) - (b^2*Cos[4*a - (4
*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Rubi [A] time = 0.32919, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2 \sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2 \sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2 \cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2b\right.}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```


```
[Out] -(b*Cos[2*a + 2*b*x])/(4*d^2*(c + d*x)) - (b*Cos[4*a + 4*b*x])/(4*d^2*(c +
d*x)) - (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (b^
2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a +
2*b*x]/(8*d*(c + d*x)^2) - Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos[2
*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) - (b^2*Cos[4*a - (4
*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3}+\frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx\\
&= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx\\
&= -\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}+\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{4d}+\frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{4d}\\
&= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{b^2\int \frac{\sin(2a+2b}{c+dx}}{2d^2}\\
&= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{\left(b^2\cos\left(4a-\right.\right.}{}\\
&= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{b^2\,\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3}-\frac{b^2\,\mathrm{Ci}\left(\frac{2bc}{d}+\right.}{}
\end{aligned}
$$

Mathematica [A] time = 3.79172, size = 197, normalized size = 0.85

$$-\frac{16b^2\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+8b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+8b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 8*b^2*CosInt
egral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (2*d*(2*b*(c + d*x)*Cos[2*(
a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + (d*(4*b*(c + d*x)*Cos[4*(a +
 b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 + 8*b^2*Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*(c + d*x))/d] + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(
c + d*x))/d])/(16*d^3)
```

Maple [A] time = 0.023, size = 329, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^3}{8}\left(-\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)^2d}+\frac{1}{d}\left(-2\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)d}-2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\cos\left(2\frac{-ad+b}{d}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/8*b^3*(-sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d+(-2*cos(2*b*x+2*a)/((
b*x+a)*d-a*d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d
-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)+1/32*b^3*(-2*s
in(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)^2/d+2*(-4*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b
*c)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*
a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] time = 2.2536, size = 454, normalized size = 1.97

$$-\frac{b^3\left(2iE_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2iE_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(iE_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-iE_3\left(-\frac{4i\,bc+4i\,(\cdots}{\cdots}\right.\right.}{16\left(b^2c^{\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(b^3*(2*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
- 2*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) + 2*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A] time = 0.624906, size = 918, normalized size = 3.97

$$-\frac{2\,d^2\cos\left(bx+a\right)^3\sin\left(bx+a\right)+8\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4-6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+4\left(b^2d^2x^2+2\,b^2cdx+b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cos(b*x + a)^3*sin(b*x + a) + 8*(b*d^2*x + b*c*d)*cos(b*x + a)^
4 - 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*
c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)
/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d
))*sin(-2*(b*c - a*d)/d) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d
^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.144 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{3d^4}$$

```
[Out] -(b*Cos[2*a + 2*b*x])/(12*d^2*(c + d*x)^2) - (b*Cos[4*a + 4*b*x])/(12*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) - (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
- Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c +
d*x)) - Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) + (b^2*Sin[4*a + 4*b*x])/(3*d^3
*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) + (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rubi [A] time = 0.451177, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```


```
[Out] -(b*Cos[2*a + 2*b*x])/(12*d^2*(c + d*x)^2) - (b*Cos[4*a + 4*b*x])/(12*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) - (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
- Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c +
d*x)) - Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) + (b^2*Sin[4*a + 4*b*x])/(3*d^3
*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) + (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4} + \frac{\sin(4a+4bx)}{8(c+dx)^4}\right) dx$$
$$= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^4}\,dx + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$
$$= -\frac{\sin(2a+2bx)}{12d(c+dx)^3} - \frac{\sin(4a+4bx)}{24d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{6d} + \frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$
$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} - \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} - \frac{\sin(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\int \frac{\sin(2a+2b}{(c+dx)^2}}{6d^2}$$
$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} - \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} - \frac{\sin(4a+4}{24d(c+d}$$
$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} - \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} - \frac{\sin(4a+4}{24d(c+d}$$
$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} - \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4} - \frac{4b^3\cos\left(4a\right.}{}$$

Mathematica [A] time = 2.54981, size = 316, normalized size = 1.1

$$-\frac{8b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)\right)+32b^3(c+dx)^3\left(\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -(2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*
a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^2)*Si
n[4*a]) - 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a]
)*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x)*Sin
[4*a])*Sin[4*b*x] + 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*
b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]) + 32
*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - Sin
[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

Maple [A] time = 0.024, size = 404, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^4}{8}\left(-\frac{2\sin(2bx+2a)}{3((bx+a)d-ad+bc)^3d}+\frac{2}{3d}\left(-\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)^2d}-\frac{1}{d}\left(-2\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)d}+2\frac{1}{d}\left(2\frac{1}{d}\mathrm{Si}\left(2bx\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/8*b^4*(-2/3*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^3/d+2/3*(-cos(2*b*x+2
*a)/((b*x+a)*d-a*d+b*c)^2/d-(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*S
i(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*
c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)+1/32*b^4*(-4/3*sin(4*b*x+4*a)/((b*x+a
)*d-a*d+b*c)^3/d+4/3*(-2*cos(4*b*x+4*a)/((b*x+a)*d-a*d+b*c)^2/d-2*(-4*sin(4
*b*x+4*a)/((b*x+a)*d-a*d+b*c)/d+4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a
*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)/d)
)
```

Maxima [C] time = 2.91, size = 521, normalized size = 1.82

$$-\frac{b^4\left(2iE_4\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)-2iE_4\left(-\frac{2ibc+2i(bx+a)d-2iad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^4\left(iE_4\left(\frac{4ibc+4i(bx+a)d-4iad}{d}\right)-iE_4\left(-\frac{4ibc+4i(\cdots}{\cdots}\right.\right.}{16\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(b^4*(2*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
- 2*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) + 2*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] time = 0.682596, size = 1256, normalized size = 4.38

$$-\frac{4\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^4-3\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2-8\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x
+ a)^2 - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-4
*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*
d*x + b*c)/d) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) +
4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(4
*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^
3)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d) - 2*((8*b^2*d^3*
x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - 3*(b^2*d^3*x^2 +
 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*
x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.145 $\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=419

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{32b}$$

```
[Out] ((-I/16)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]
)/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/16)*(c + d*x)^m*Gamma[1 + m, (I*b*(c +
 d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/32)*3^(-1 -
m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]
)/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m
, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
 + ((I/32)*5^(-1 - m)*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5
*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*5^(-1 - m)*(c +
 d*x)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d])/(b*E^((5*I)*(a - (b*c)/d))*((I
*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.434733, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3307, 2181}

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]
)/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/16)*(c + d*x)^m*Gamma[1 + m, (I*b*(c +
 d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/32)*3^(-1 -
m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]
)/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m
, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
 + ((I/32)*5^(-1 - m)*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5
*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*5^(-1 - m)*(c +
 d*x)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d])/(b*E^((5*I)*(a - (b*c)/d))*((I
*b*(c + d*x))/d)^m)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^m\cos(a+bx) - \frac{1}{16}(c+dx)^m\cos(3a+3bx) - \frac{1}{16}(c+dx)^m\cos(5$$

$$= -\left(\frac{1}{16}\int (c+dx)^m\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^m\cos(5a+5bx)\,dx + \frac{1}{8}$$

$$= -\left(\frac{1}{32}\int e^{-i(3a+3bx)}(c+dx)^m\,dx\right) - \frac{1}{32}\int e^{i(3a+3bx)}(c+dx)^m\,dx - \frac{1}{32}\int e^{-i(5a+$$

$$= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(}{}\right.}{}$$

Mathematica [A] time = 0.605986, size = 409, normalized size = 0.98

$$-\frac{i3^{-m-1}e^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(m+1,\frac{3ib(c+dx)}{d}\right)-e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^m\text{Gamma}\left(m+1,-\right.\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(((-
I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d])/
((I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d)) - ((I/32)*3^(-1 - m)*(c +
 d*x)^m*(-(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*
x))/d]) + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*
(c + d*x))/d]))/(b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m) - (
(I/32)*5^(-1 - m)*(c + d*x)^m*(-(E^((10*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1
 + m, ((-5*I)*b*(c + d*x))/d]) + E^(((10*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^
m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*I)*(b*c + a*d))/d)*((b^2*
(c + d*x)^2)/d^2)^m)
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cos(bx+a))^3 (\sin(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

Fricas [A] time = 0.577308, size = 726, normalized size = 1.73

$$-3ie^{\left(-\frac{dm\log\left(\frac{5ib}{d}\right)-5ibc+5iad}{d}\right)}\Gamma\left(m+1,\frac{5ibdx+5ibc}{d}\right)-5ie^{\left(-\frac{dm\log\left(\frac{3ib}{d}\right)-3ibc+3iad}{d}\right)}\Gamma\left(m+1,\frac{3ibdx+3ibc}{d}\right)+30ie^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(-3*I*e^(-(d*m*log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, (5*I
*b*d*x + 5*I*b*c)/d) - 5*I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*ga
mma(m + 1, (3*I*b*d*x + 3*I*b*c)/d) + 30*I*e^(-(d*m*log(I*b/d) - I*b*c + I*
a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*c)/d) - 30*I*e^(-(d*m*log(-I*b/d) + I*b
*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) + 5*I*e^(-(d*m*log(-3*I*b
/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d) + 3*I*e^
(-(d*m*log(-5*I*b/d) + 5*I*b*c - 5*I*a*d)/d)*gamma(m + 1, (-5*I*b*d*x - 5*I
*b*c)/d))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

3.146 $\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\,dx$

Optimal. Leaf size=330

$$-\frac{3d^2(c+dx)^2\sin(a+bx)}{2b^3}+\frac{d^2(c+dx)^2\sin(3a+3bx)}{36b^3}+\frac{3d^2(c+dx)^2\sin(5a+5bx)}{500b^3}-\frac{3d^3(c+dx)\cos(a+bx)}{b^4}+\frac{d^3(c\cdots}{}$$

```
[Out] (-3*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (d*(c + d*x)^3*Cos[a + b*x])/(2*b^2)
+ (d^3*(c + d*x)*Cos[3*a + 3*b*x])/(54*b^4) - (d*(c + d*x)^3*Cos[3*a + 3*b*
x])/(36*b^2) + (3*d^3*(c + d*x)*Cos[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)
^3*Cos[5*a + 5*b*x])/(100*b^2) + (3*d^4*Sin[a + b*x])/b^5 - (3*d^2*(c + d*x
)^2*Sin[a + b*x])/(2*b^3) + ((c + d*x)^4*Sin[a + b*x])/(8*b) - (d^4*Sin[3*a
 + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*Sin[3*a + 3*b*x])/(36*b^3) - ((c +
d*x)^4*Sin[3*a + 3*b*x])/(48*b) - (3*d^4*Sin[5*a + 5*b*x])/(6250*b^5) + (3*
d^2*(c + d*x)^2*Sin[5*a + 5*b*x])/(500*b^3) - ((c + d*x)^4*Sin[5*a + 5*b*x]
)/(80*b)
```

Rubi [A] time = 0.368317, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$-\frac{3d^2(c+dx)^2\sin(a+bx)}{2b^3}+\frac{d^2(c+dx)^2\sin(3a+3bx)}{36b^3}+\frac{3d^2(c+dx)^2\sin(5a+5bx)}{500b^3}-\frac{3d^3(c+dx)\cos(a+bx)}{b^4}+\frac{d^3(c\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```


```
[Out] (-3*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (d*(c + d*x)^3*Cos[a + b*x])/(2*b^2)
+ (d^3*(c + d*x)*Cos[3*a + 3*b*x])/(54*b^4) - (d*(c + d*x)^3*Cos[3*a + 3*b*
x])/(36*b^2) + (3*d^3*(c + d*x)*Cos[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)
^3*Cos[5*a + 5*b*x])/(100*b^2) + (3*d^4*Sin[a + b*x])/b^5 - (3*d^2*(c + d*x
)^2*Sin[a + b*x])/(2*b^3) + ((c + d*x)^4*Sin[a + b*x])/(8*b) - (d^4*Sin[3*a
 + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*Sin[3*a + 3*b*x])/(36*b^3) - ((c +
d*x)^4*Sin[3*a + 3*b*x])/(48*b) - (3*d^4*Sin[5*a + 5*b*x])/(6250*b^5) + (3*
d^2*(c + d*x)^2*Sin[5*a + 5*b*x])/(500*b^3) - ((c + d*x)^4*Sin[5*a + 5*b*x]
)/(80*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^4\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4\cos(a+bx) - \frac{1}{16}(c+dx)^4\cos(3a+3bx) - \frac{1}{16}(c+dx)^4\cos(5a$$

$$= -\left(\frac{1}{16}\int (c+dx)^4\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^4\cos(5a+5bx)\,dx + \frac{1}{8}\int$$

$$= \frac{(c+dx)^4\sin(a+bx)}{8b} - \frac{(c+dx)^4\sin(3a+3bx)}{48b} - \frac{(c+dx)^4\sin(5a+5bx)}{80b} + \frac{d}{}$$

$$= \frac{d(c+dx)^3\cos(a+bx)}{2b^2} - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} - \frac{d(c+dx)^3\cos(5a+5bx)}{100b^2}$$

$$= \frac{d(c+dx)^3\cos(a+bx)}{2b^2} - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} - \frac{d(c+dx)^3\cos(5a+5bx)}{100b^2}$$

$$= -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} + \frac{d^3(c+dx)\cos(3a+3bx}{54b^4}$$

$$= -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} + \frac{d^3(c+dx)\cos(3a+3bx}{54b^4}$$

Mathematica [A] time = 3.75681, size = 563, normalized size = 1.71

$$-\frac{-3037500b^2c^2d^2\left(\left(b^2x^2-2\right)\sin(a+bx)+2bx\cos(a+bx)\right)+56250b^2c^2d^2\left(\left(9b^2x^2-2\right)\sin(3(a+bx))+6bx\cos(3(a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -(-506250*b^4*c^4*Sin[a + b*x] - 2025000*b^3*c^3*d*(Cos[a + b*x] + b*x*Sin[
a + b*x]) - 2025000*b*c*d^3*(3*(-2 + b^2*x^2)*Cos[a + b*x] + b*x*(-6 + b^2*
x^2)*Sin[a + b*x]) - 3037500*b^2*c^2*d^2*(2*b*x*Cos[a + b*x] + (-2 + b^2*x^
2)*Sin[a + b*x]) - 506250*d^4*(4*b*x*(-6 + b^2*x^2)*Cos[a + b*x] + (24 - 12
*b^2*x^2 + b^4*x^4)*Sin[a + b*x]) + 84375*b^4*c^4*Sin[3*(a + b*x)] + 112500
*b^3*c^3*d*(Cos[3*(a + b*x)] + 3*b*x*Sin[3*(a + b*x)]) + 37500*b*c*d^3*((-2
 + 9*b^2*x^2)*Cos[3*(a + b*x)] + 3*b*x*(-2 + 3*b^2*x^2)*Sin[3*(a + b*x)]) +
 56250*b^2*c^2*d^2*(6*b*x*Cos[3*(a + b*x)] + (-2 + 9*b^2*x^2)*Sin[3*(a + b*
x)]) + 3125*d^4*(12*b*x*(-2 + 3*b^2*x^2)*Cos[3*(a + b*x)] + (8 - 36*b^2*x^2
 + 27*b^4*x^4)*Sin[3*(a + b*x)]) + 50625*b^4*c^4*Sin[5*(a + b*x)] + 40500*b
^3*c^3*d*(Cos[5*(a + b*x)] + 5*b*x*Sin[5*(a + b*x)]) + 1620*b*c*d^3*((-6 +
75*b^2*x^2)*Cos[5*(a + b*x)] + 5*b*x*(-6 + 25*b^2*x^2)*Sin[5*(a + b*x)]) +
12150*b^2*c^2*d^2*(10*b*x*Cos[5*(a + b*x)] + (-2 + 25*b^2*x^2)*Sin[5*(a + b
*x)]) + 81*d^4*(20*b*x*(-6 + 25*b^2*x^2)*Cos[5*(a + b*x)] + (24 - 300*b^2*x
^2 + 625*b^4*x^4)*Sin[5*(a + b*x)]))/(4050000*b^5)
```

Maple [B] time = 0.052, size = 1842, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/3*(b*x+a)^4*(2+cos(b*x+a)^2)*sin(b*x+a)+8/15*(b*x+a)^3*co
s(b*x+a)-8/5*(b*x+a)^2*sin(b*x+a)+3424/1125*sin(b*x+a)-3424/1125*(b*x+a)*co
s(b*x+a)+4/45*(b*x+a)^3*cos(b*x+a)^3-4/45*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*
x+a)+88/3375*(b*x+a)*cos(b*x+a)^3-88/10125*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*
(b*x+a)^4*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-4/25*(b*x+a)^3*cos
(b*x+a)^5+12/125*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+2
4/625*(b*x+a)*cos(b*x+a)^5-24/3125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(
b*x+a))-4/b^4*a*d^4*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*(b*x+a)^
2*cos(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a)^2*cos(
b*x+a)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a)^3-1/5*
(b*x+a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+a)^2*cos
(b*x+a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+6/62
5*cos(b*x+a)^5)+4/b^3*c*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*
(b*x+a)^2*cos(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a
)^2*cos(b*x+a)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a
)^3-1/5*(b*x+a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+
a)^2*cos(b*x+a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x
+a)+6/625*cos(b*x+a)^5)+6/b^4*a^2*d^4*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b
```

```
*x+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/1
35*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+
a)^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(
b*x+a)^2)*sin(b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*
x+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/13
5*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a
)^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b
*x+a)^2)*sin(b*x+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+
a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*
(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^
2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x
+a)^2)*sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1
/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+
a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*(2+cos(b*
x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b
*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)-12/b^2*a*c^2*d^2*(1
/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/
5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)
+4/b*c^3*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*
cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*
cos(b*x+a)^5)+1/b^4*a^4*d^4*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a
)^2)*sin(b*x+a))-4/b^3*a^3*c*d^3*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(
b*x+a)^2)*sin(b*x+a))+6/b^2*a^2*c^2*d^2*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*
(2+cos(b*x+a)^2)*sin(b*x+a))-4/b*a*c^3*d*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15
*(2+cos(b*x+a)^2)*sin(b*x+a))+c^4*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos
(b*x+a)^2)*sin(b*x+a)))
```

Maxima [B] time = 1.48933, size = 1808, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4050000*(270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^4 - 1080000*(3*s
in(b*x + a)^5 - 5*sin(b*x + a)^3)*a*c^3*d/b + 1620000*(3*sin(b*x + a)^5 - 5
*sin(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*sin(b*x + a)^5 - 5*sin(b*x +
a)^3)*a^3*c*d^3/b^3 + 270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^4*d^4/
b^4 + 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) -
 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 45
0*cos(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
```

```
a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*
cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x + a)*si
n(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a)
 + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*c*d^3/b
^3 - 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) -
450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450
*cos(b*x + a))*a^3*d^4/b^4 + 450*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x
 + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2
- 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b
*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 900*(270*(b*x + a)*cos(5*b*x + 5
*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(2
5*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3
*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 + 450*(270*(b*x + a)
*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*
x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)
*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 + 60*(
81*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*
x + 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*
b*x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x
+ 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 - 60*(81
*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*x
+ 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*b*
x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x +
3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*a*d^4/b^4 + (1620*(2
5*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) + 37500*(3*(b*x + a)^3 - 2*b*
x - 2*a)*cos(3*b*x + 3*a) - 2025000*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a
) + 81*(625*(b*x + a)^4 - 300*(b*x + a)^2 + 24)*sin(5*b*x + 5*a) + 3125*(27
*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*sin(3*b*x + 3*a) - 506250*((b*x + a)^4 -
 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

Fricas [A] time = 0.568267, size = 1245, normalized size = 3.77

$$-\frac{1620\left(25\,b^3d^4x^3+75\,b^3cd^3x^2+25\,b^3c^3d-6\,bcd^3+3\left(25\,b^3c^2d^2-2\,bd^4\right)x\right)\cos\left(bx+a\right)^5-300\left(75\,b^3d^4x^3+225\,b^3cd^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/253125*(1620*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d
^3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^5 - 300*(75*b^3*d^4*x^3 +
 225*b^3*c*d^3*x^2 + 75*b^3*c^3*d + 22*b*c*d^3 + (225*b^3*c^2*d^2 + 22*b*d^
```

```
4)*x)*cos(b*x + a)^3 - 1800*(75*b^3*d^4*x^3 + 225*b^3*c*d^3*x^2 + 75*b^3*c^
3*d - 428*b*c*d^3 + (225*b^3*c^2*d^2 - 428*b*d^4)*x)*cos(b*x + a) - (33750*
b^4*d^4*x^4 + 135000*b^4*c*d^3*x^3 + 33750*b^4*c^4 - 385200*b^2*c^2*d^2 - 8
1*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*c^2*d^2 + 2
4*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d - 6*b^2*c*
d^3)*x)*cos(b*x + a)^4 + 760816*d^4 + 900*(225*b^4*c^2*d^2 - 428*b^2*d^4)*x
^2 + (16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*x^3 + 16875*b^4*c^4 + 9900*b^2*c^
2*d^2 - 4792*d^4 + 450*(225*b^4*c^2*d^2 + 22*b^2*d^4)*x^2 + 900*(75*b^4*c^3
*d + 22*b^2*c*d^3)*x)*cos(b*x + a)^2 + 1800*(75*b^4*c^3*d - 428*b^2*c*d^3)*
x)*sin(b*x + a))/b^5
```

Sympy [A] time = 48.0104, size = 1098, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**4*sin(a + b*x)**5/(15*b) + c**4*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 8*c**3*d*x*sin(a + b*x)**5/(15*b) + 4*c**3*d*x*sin(a + b*x)**3
*cos(a + b*x)**2/(3*b) + 4*c**2*d**2*x**2*sin(a + b*x)**5/(5*b) + 2*c**2*d*
*2*x**2*sin(a + b*x)**3*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)**5/(
15*b) + 4*c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*d**4*x**4*s
in(a + b*x)**5/(15*b) + d**4*x**4*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 8
*c**3*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*c**3*d*sin(a + b*x)**2*
cos(a + b*x)**3/(45*b**2) + 104*c**3*d*cos(a + b*x)**5/(225*b**2) + 8*c**2*
d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c**2*d**2*x*sin(a + b*x)*
*2*cos(a + b*x)**3/(15*b**2) + 104*c**2*d**2*x*cos(a + b*x)**5/(75*b**2) +
8*c*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c*d**3*x**2*sin(a
+ b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c*d**3*x**2*cos(a + b*x)**5/(75*b
**2) + 8*d**4*x**3*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*d**4*x**3*si
n(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 104*d**4*x**3*cos(a + b*x)**5/(22
5*b**2) - 1712*c**2*d**2*sin(a + b*x)**5/(1125*b**3) - 676*c**2*d**2*sin(a
+ b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*c**2*d**2*sin(a + b*x)*cos(a + b
*x)**4/(75*b**3) - 3424*c*d**3*x*sin(a + b*x)**5/(1125*b**3) - 1352*c*d**3*
x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 208*c*d**3*x*sin(a + b*x)*co
s(a + b*x)**4/(75*b**3) - 1712*d**4*x**2*sin(a + b*x)**5/(1125*b**3) - 676*
d**4*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*d**4*x**2*sin(a
+ b*x)*cos(a + b*x)**4/(75*b**3) - 3424*c*d**3*sin(a + b*x)**4*cos(a + b*x)
/(1125*b**4) - 20456*c*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 5
0272*c*d**3*cos(a + b*x)**5/(16875*b**4) - 3424*d**4*x*sin(a + b*x)**4*cos(
```

```
a + b*x)/(1125*b**4) - 20456*d**4*x*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b
**4) - 50272*d**4*x*cos(a + b*x)**5/(16875*b**4) + 760816*d**4*sin(a + b*x)
**5/(253125*b**5) + 303368*d**4*sin(a + b*x)**3*cos(a + b*x)**2/(50625*b**5
) + 50272*d**4*sin(a + b*x)*cos(a + b*x)**4/(16875*b**5), Ne(b, 0)), ((c**4
*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**
2*cos(a)**3, True))
```

Giac [A] time = 1.14666, size = 717, normalized size = 2.17

$$-\frac{\left(25\,b^{3}d^{4}x^{3}+75\,b^{3}cd^{3}x^{2}+75\,b^{3}c^{2}d^{2}x+25\,b^{3}c^{3}d-6\,bd^{4}x-6\,bcd^{3}\right)\cos\left(5\,bx+5\,a\right)}{2500\,b^{5}}-\frac{\left(3\,b^{3}d^{4}x^{3}+9\,b^{3}cd^{3}x^{2}+9\,b^{3}c^{2}d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c^3*
d - 6*b*d^4*x - 6*b*c*d^3)*cos(5*b*x + 5*a)/b^5 - 1/108*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*cos(
3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b
^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1/50000*(625*b^4*d^4*x
^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500*b^4*c^3*d*x + 625*b^4
*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^2*d^2 + 24*d^4)*sin(5*
b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d
^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 3
6*b^2*c^2*d^2 + 8*d^4)*sin(3*b*x + 3*a)/b^5 + 1/8*(b^4*d^4*x^4 + 4*b^4*c*d^
3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b
^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

3.147 $\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx) \, dx$

Optimal. Leaf size=259

$$-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}+\frac{3d^2(c+dx)\sin(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+dx)^2}{}$$

```
[Out] (-3*d^3*Cos[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x])/(8*b^2) + (d
^3*Cos[3*a + 3*b*x])/(216*b^4) - (d*(c + d*x)^2*Cos[3*a + 3*b*x])/(48*b^2)
+ (3*d^3*Cos[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x)^2*Cos[5*a + 5*b*x])/
(400*b^2) - (3*d^2*(c + d*x)*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Sin[a + b
*x])/(8*b) + (d^2*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Sin[3
*a + 3*b*x])/(48*b) + (3*d^2*(c + d*x)*Sin[5*a + 5*b*x])/(1000*b^3) - ((c +
 d*x)^3*Sin[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.272605, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}+\frac{3d^2(c+dx)\sin(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+dx)^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (-3*d^3*Cos[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x])/(8*b^2) + (d
^3*Cos[3*a + 3*b*x])/(216*b^4) - (d*(c + d*x)^2*Cos[3*a + 3*b*x])/(48*b^2)
+ (3*d^3*Cos[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x)^2*Cos[5*a + 5*b*x])/
(400*b^2) - (3*d^2*(c + d*x)*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Sin[a + b
*x])/(8*b) + (d^2*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Sin[3
*a + 3*b*x])/(48*b) + (3*d^2*(c + d*x)*Sin[5*a + 5*b*x])/(1000*b^3) - ((c +
 d*x)^3*Sin[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^3\cos(a+bx) - \frac{1}{16}(c+dx)^3\cos(3a+3bx) - \frac{1}{16}(c+dx)^3\cos(5a$$

$$= -\left(\frac{1}{16}\int (c+dx)^3\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^3\cos(5a+5bx)\,dx + \frac{1}{8}$$

$$= \frac{(c+dx)^3\sin(a+bx)}{8b} - \frac{(c+dx)^3\sin(3a+3bx)}{48b} - \frac{(c+dx)^3\sin(5a+5bx)}{80b} +$$

$$= \frac{3d(c+dx)^2\cos(a+bx)}{8b^2} - \frac{d(c+dx)^2\cos(3a+3bx)}{48b^2} - \frac{3d(c+dx)^2\cos(5a+}{400b^2}$$

$$= \frac{3d(c+dx)^2\cos(a+bx)}{8b^2} - \frac{d(c+dx)^2\cos(3a+3bx)}{48b^2} - \frac{3d(c+dx)^2\cos(5a+}{400b^2}$$

$$= -\frac{3d^3\cos(a+bx)}{4b^4} + \frac{3d(c+dx)^2\cos(a+bx)}{8b^2} + \frac{d^3\cos(3a+3bx)}{216b^4} - \frac{d(c+dx)}{}$$

Mathematica [A] time = 2.2165, size = 195, normalized size = 0.75

$$-\frac{30b(c+dx)\sin(a+bx)\left(8\cos(2(a+bx))\left(75b^2(c+dx)^2-38d^2\right)+9\cos(4(a+bx))\left(25b^2(c+dx)^2-6d^2\right)-825b^2c^2-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -(-101250*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 625*d*(-2*d^2 + 9*b^2
*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*d*(-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*(
a + b*x)] + 30*b*(c + d*x)*(-825*b^2*c^2 + 6598*d^2 - 1650*b^2*c*d*x - 825*
b^2*d^2*x^2 + 8*(-38*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 9*(-6*d^2
 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/(270000*b^4)
```

Maple [B] time = 0.021, size = 1016, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*(b*x+a)^2*cos
(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a)^2*cos(b*x+a
)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a)^3-1/5*(b*x+
a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+a)^2*cos(b*x+
a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+6/625*cos
(b*x+a)^5)-3/b^3*a*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(
b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a
)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a
)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b
*x+a))+3/b^2*c*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+
a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)
*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/
25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a
))+3/b^3*a^2*d^3*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3
+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)
-1/25*cos(b*x+a)^5)-6/b^2*a*c*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+
1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x
+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)+3/b*c^2*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2
)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^
4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)-1/b^3*a^3*d^3*(-1/5*sin(b
*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b^2*a^2*c*d^2*(-1/5*
sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))-3/b*a*c^2*d*(-1/5
*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+c^3*(-1/5*sin(b*
x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a)))
```

Maxima [B] time = 1.26809, size = 1034, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/270000*(18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^3 - 54000*(3*sin(b
*x + a)^5 - 5*sin(b*x + a)^3)*a*c^2*d/b + 54000*(3*sin(b*x + a)^5 - 5*sin(b
*x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^3*
d^3/b^3 + 225*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a
) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) -
 450*cos(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
 a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25
*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a)*sin(5
*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) +
9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*d^3/b^3 +
15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500
*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9
*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*c
*d^2/b^2 - 15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3
*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*
a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(
b*x + a))*a*d^3/b^3 + (81*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b
*x + a)^2 - 2)*cos(3*b*x + 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 1
35*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 -
2*b*x - 2*a)*sin(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x +
 a))*d^3/b^3)/b
```

Fricas [A] time = 0.539504, size = 798, normalized size = 3.08

$$-\frac{81\left(25\,b^{2}d^{3}x^{2}+50\,b^{2}cd^{2}x+25\,b^{2}c^{2}d-2\,d^{3}\right)\cos\left(bx+a\right)^{5}-5\left(225\,b^{2}d^{3}x^{2}+450\,b^{2}cd^{2}x+225\,b^{2}c^{2}d+22\,d^{3}\right)\cos\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16875*(81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b
*x + a)^5 - 5*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d + 22*d^3)*
cos(b*x + a)^3 - 30*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d - 42
8*d^3)*cos(b*x + a) - 15*(150*b^3*d^3*x^3 + 450*b^3*c*d^2*x^2 + 150*b^3*c^3
 - 9*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 + 3*(25*b^
3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^4 - 856*b*c*d^2 + (75*b^3*d^3*x^3 + 225*
b^3*c*d^2*x^2 + 75*b^3*c^3 + 22*b*c*d^2 + (225*b^3*c^2*d + 22*b*d^3)*x)*cos
(b*x + a)^2 + 2*(225*b^3*c^2*d - 428*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [A] time = 53.5758, size = 690, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**3*sin(a + b*x)**5/(15*b) + c**3*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 2*c**2*d*x*sin(a + b*x)**5/(5*b) + c**2*d*x*sin(a + b*x)**3*co
s(a + b*x)**2/b + 2*c*d**2*x**2*sin(a + b*x)**5/(5*b) + c*d**2*x**2*sin(a +
 b*x)**3*cos(a + b*x)**2/b + 2*d**3*x**3*sin(a + b*x)**5/(15*b) + d**3*x**3
*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*c**2*d*sin(a + b*x)**4*cos(a + b
*x)/(5*b**2) + 13*c**2*d*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*c**
2*d*cos(a + b*x)**5/(75*b**2) + 4*c*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*
b**2) + 26*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 52*c*d**2*x
*cos(a + b*x)**5/(75*b**2) + 2*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b*
*2) + 13*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*d**3*x**2
*cos(a + b*x)**5/(75*b**2) - 856*c*d**2*sin(a + b*x)**5/(1125*b**3) - 338*c
*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*c*d**2*sin(a + b*x)*c
os(a + b*x)**4/(75*b**3) - 856*d**3*x*sin(a + b*x)**5/(1125*b**3) - 338*d**
3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*d**3*x*sin(a + b*x)*cos
(a + b*x)**4/(75*b**3) - 856*d**3*sin(a + b*x)**4*cos(a + b*x)/(1125*b**4)
- 5114*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 12568*d**3*cos(a
+ b*x)**5/(16875*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4)*sin(a)**2*cos(a)**3, True))
```

Giac [A] time = 1.11776, size = 474, normalized size = 1.83

$$-\frac{3\left(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3\right)\cos\left(5\,bx+5\,a\right)}{10000\,b^4}-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(3\,bx+3\,a\right)}{432\,b^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(5*b*x
 + 5*a)/b^4 - 1/432*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*
cos(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3
)*cos(b*x + a)/b^4 - 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2
*d*x + 25*b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(5*b*x + 5*a)/b^4 - 1/144*(3*
b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b
```

```
*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

3.148 $\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\,dx$

Optimal. Leaf size=184

$$\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}-\frac{d^2\sin(a+bx)}{4b^3}+\frac{d^2\sin(3a+3bx)}{216b^3}+\frac{d^2\sin}{1}$$

```
[Out] (d*(c + d*x)*Cos[a + b*x])/(4*b^2) - (d*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^2
) - (d*(c + d*x)*Cos[5*a + 5*b*x])/(200*b^2) - (d^2*Sin[a + b*x])/(4*b^3) +
 ((c + d*x)^2*Sin[a + b*x])/(8*b) + (d^2*Sin[3*a + 3*b*x])/(216*b^3) - ((c
+ d*x)^2*Sin[3*a + 3*b*x])/(48*b) + (d^2*Sin[5*a + 5*b*x])/(1000*b^3) - ((c
 + d*x)^2*Sin[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.189954, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}-\frac{d^2\sin(a+bx)}{4b^3}+\frac{d^2\sin(3a+3bx)}{216b^3}+\frac{d^2\sin}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```


```
[Out] (d*(c + d*x)*Cos[a + b*x])/(4*b^2) - (d*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^2
) - (d*(c + d*x)*Cos[5*a + 5*b*x])/(200*b^2) - (d^2*Sin[a + b*x])/(4*b^3) +
 ((c + d*x)^2*Sin[a + b*x])/(8*b) + (d^2*Sin[3*a + 3*b*x])/(216*b^3) - ((c
+ d*x)^2*Sin[3*a + 3*b*x])/(48*b) + (d^2*Sin[5*a + 5*b*x])/(1000*b^3) - ((c
 + d*x)^2*Sin[5*a + 5*b*x])/(80*b)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^2\cos^3(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{8}(c+dx)^2\cos(a+bx)-\frac{1}{16}(c+dx)^2\cos(3a+3bx)-\frac{1}{16}(c+dx)^2\cos(5\right.\\
&= -\left(\frac{1}{16}\int (c+dx)^2\cos(3a+3bx)\,dx\right)-\frac{1}{16}\int (c+dx)^2\cos(5a+5bx)\,dx+\frac{1}{8}\\
&= \frac{(c+dx)^2\sin(a+bx)}{8b}-\frac{(c+dx)^2\sin(3a+3bx)}{48b}-\frac{(c+dx)^2\sin(5a+5bx)}{80b}+\\
&= \frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}\\
&= \frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}
\end{aligned}
$$

Mathematica [A] time = 1.01095, size = 252, normalized size = 1.37

$$
-\frac{-6750b^2c^2\sin(a+bx)+1125b^2c^2\sin(3(a+bx))+675b^2c^2\sin(5(a+bx))-13500b^2cdx\sin(a+bx)+2250b^2cdx\sin(}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -(-13500*b*d*(c + d*x)*Cos[a + b*x] + 750*b*d*(c + d*x)*Cos[3*(a + b*x)] +
270*b*c*d*Cos[5*(a + b*x)] + 270*b*d^2*x*Cos[5*(a + b*x)] - 6750*b^2*c^2*Si
n[a + b*x] + 13500*d^2*Sin[a + b*x] - 13500*b^2*c*d*x*Sin[a + b*x] - 6750*b
^2*d^2*x^2*Sin[a + b*x] + 1125*b^2*c^2*Sin[3*(a + b*x)] - 250*d^2*Sin[3*(a
+ b*x)] + 2250*b^2*c*d*x*Sin[3*(a + b*x)] + 1125*b^2*d^2*x^2*Sin[3*(a + b*x
)] + 675*b^2*c^2*Sin[5*(a + b*x)] - 54*d^2*Sin[5*(a + b*x)] + 1350*b^2*c*d*
x*Sin[5*(a + b*x)] + 675*b^2*d^2*x^2*Sin[5*(a + b*x)])/(54000*b^3)
```

Maple [B] time = 0.02, size = 484, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+a)+4
/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)*sin
(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/25*(
b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))-2
/b^2*a*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*
cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*
cos(b*x+a)^5)+2/b*c*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x
+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b
*x+a)-1/25*cos(b*x+a)^5)+d^2/b^2*a^2*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+
cos(b*x+a)^2)*sin(b*x+a))-2*c*d/b*a*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+c
os(b*x+a)^2)*sin(b*x+a))+c^2*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+
a)^2)*sin(b*x+a)))
```

Maxima [B] time = 1.17677, size = 506, normalized size = 2.75

$$-\frac{3600\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)c^2-\frac{7200\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)acd}{b}+\frac{3600\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)a^2d^2}{b^2}+\frac{30\,(45\,(bx+a)\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/54000*(3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^2 - 7200*(3*sin(b*x
+ a)^5 - 5*sin(b*x + a)^3)*a*c*d/b + 3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a
)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b
*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x
+ 3*a) - 450*cos(b*x + a))*c*d/b - 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(
b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a)
 + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*d^2/b^2 + (270*(b*x + a)*cos(5
*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a)
 + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3
*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

Fricas [A] time = 0.499786, size = 470, normalized size = 2.55

$$-\frac{270\left(bd^2x+bcd\right)\cos(bx+a)^5-150\left(bd^2x+bcd\right)\cos(bx+a)^3-900\left(bd^2x+bcd\right)\cos(bx+a)-\left(450\,b^2d^2x^2+900\,b^2c\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3375*(270*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 150*(b*d^2*x + b*c*d)*cos(b
*x + a)^3 - 900*(b*d^2*x + b*c*d)*cos(b*x + a) - (450*b^2*d^2*x^2 + 900*b^2
*c*d*x - 27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x +
a)^4 + 450*b^2*c^2 + (225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 + 22*d^
2)*cos(b*x + a)^2 - 856*d^2)*sin(b*x + a))/b^3
```

Sympy [A] time = 11.5424, size = 382, normalized size = 2.08

$$\begin{cases} \frac{2c^2\sin^5(a+bx)}{15b} + \frac{c^2\sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{4cdx\sin^5(a+bx)}{15b} + \frac{2cdx\sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{2d^2x^2\sin^5(a+bx)}{15b} + \frac{d^2x^2\sin^3(a+bx)\cos^2(a+bx)}{3b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**2*sin(a + b*x)**5/(15*b) + c**2*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 4*c*d*x*sin(a + b*x)**5/(15*b) + 2*c*d*x*sin(a + b*x)**3*cos(a
 + b*x)**2/(3*b) + 2*d**2*x**2*sin(a + b*x)**5/(15*b) + d**2*x**2*sin(a + b
*x)**3*cos(a + b*x)**2/(3*b) + 4*c*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2)
 + 26*c*d*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*c*d*cos(a + b*x)**
5/(225*b**2) + 4*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 26*d**2*x*
sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*d**2*x*cos(a + b*x)**5/(225*
b**2) - 856*d**2*sin(a + b*x)**5/(3375*b**3) - 338*d**2*sin(a + b*x)**3*cos
(a + b*x)**2/(675*b**3) - 52*d**2*sin(a + b*x)*cos(a + b*x)**4/(225*b**3),
Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a)**3, True))
```

Giac [A] time = 1.11354, size = 282, normalized size = 1.53

$$-\frac{\left(bd^2x + bcd\right)\cos(5\,bx + 5\,a)}{200\,b^3} - \frac{\left(bd^2x + bcd\right)\cos(3\,bx + 3\,a)}{72\,b^3} + \frac{\left(bd^2x + bcd\right)\cos(bx + a)}{4\,b^3} - \frac{\left(25\,b^2d^2x^2 + 50\,b^2cdx + 2\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/200*(b*d^2*x + b*c*d)*cos(5*b*x + 5*a)/b^3 - 1/72*(b*d^2*x + b*c*d)*cos(
3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 - 1/2000*(25*b^2*
d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*sin(5*b*x + 5*a)/b^3 - 1/432*(
9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*b*x + 3*a)/b^3 + 1/
8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/b^3
```

3.149 $\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+}{80b}$$

```
[Out] (d*Cos[a + b*x])/(8*b^2) - (d*Cos[3*a + 3*b*x])/(144*b^2) - (d*Cos[5*a + 5*
b*x])/(400*b^2) + ((c + d*x)*Sin[a + b*x])/(8*b) - ((c + d*x)*Sin[3*a + 3*b
*x])/(48*b) - ((c + d*x)*Sin[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.0939269, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2638}

$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (d*Cos[a + b*x])/(8*b^2) - (d*Cos[3*a + 3*b*x])/(144*b^2) - (d*Cos[5*a + 5*
b*x])/(400*b^2) + ((c + d*x)*Sin[a + b*x])/(8*b) - ((c + d*x)*Sin[3*a + 3*b
*x])/(48*b) - ((c + d*x)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)\cos(a+bx) - \frac{1}{16}(c+dx)\cos(3a+3bx) - \frac{1}{16}(c+dx)\cos(5a+5b$$
$$= -\left(\frac{1}{16}\int (c+dx)\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)\cos(5a+5bx)\,dx + \frac{1}{8}\int (c$$
$$= \frac{(c+dx)\sin(a+bx)}{8b} - \frac{(c+dx)\sin(3a+3bx)}{48b} - \frac{(c+dx)\sin(5a+5bx)}{80b} + \frac{d\int \mathrm{si}}{}$$
$$= \frac{d\cos(a+bx)}{8b^2} - \frac{d\cos(3a+3bx)}{144b^2} - \frac{d\cos(5a+5bx)}{400b^2} + \frac{(c+dx)\sin(a+bx)}{8b} - \frac{(c}{}$$

Mathematica [A] time = 0.365338, size = 110, normalized size = 1.01

$$-\frac{-450bc\sin(a+bx) + 75bc\sin(3(a+bx)) + 45bc\sin(5(a+bx)) - 450bdx\sin(a+bx) + 75bdx\sin(3(a+bx)) + 45bdx\,\mathrm{si}}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -(-450*d*Cos[a + b*x] + 25*d*Cos[3*(a + b*x)] + 9*d*Cos[5*(a + b*x)] - 450*
b*c*Sin[a + b*x] - 450*b*d*x*Sin[a + b*x] + 75*b*c*Sin[3*(a + b*x)] + 75*b*
d*x*Sin[3*(a + b*x)] + 45*b*c*Sin[5*(a + b*x)] + 45*b*d*x*Sin[5*(a + b*x)])
/(3600*b^2)
```

Maple [A] time = 0.023, size = 175, normalized size = 1.6

$$\frac{1}{b}\left(\frac{d}{b}\left(\frac{(bx+a)\left(2+(\cos(bx+a))^2\right)\sin(bx+a)}{3} + \frac{(\cos(bx+a))^3}{45} + \frac{2\cos(bx+a)}{15} - \frac{(bx+a)\sin(bx+a)}{5}\left(\frac{8}{3} + (\cos(bx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(d/b*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*co
s(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*co
```

```
s(b*x+a)^5)-1/b*d*a*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin
(b*x+a))+c*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a)))
```

Maxima [A] time = 1.05287, size = 188, normalized size = 1.72

$$-\frac{240\left(3\,\sin\left(bx+a\right)^{5}-5\,\sin\left(bx+a\right)^{3}\right)c-\frac{240\left(3\,\sin(bx+a)^{5}-5\,\sin(bx+a)^{3}\right)ad}{b}+\frac{(45\,(bx+a)\sin(5\,bx+5\,a)+75\,(bx+a)\sin(3\,bx+3\,a)-450\,(bx+a}{}}{3600\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3600*(240*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c - 240*(3*sin(b*x + a)^
5 - 5*sin(b*x + a)^3)*a*d/b + (45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)
*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*co
s(3*b*x + 3*a) - 450*cos(b*x + a))*d/b)/b
```

Fricas [A] time = 0.488231, size = 235, normalized size = 2.16

$$-\frac{9\,d\cos\left(bx+a\right)^{5}-5\,d\cos\left(bx+a\right)^{3}-30\,d\cos\left(bx+a\right)+15\left(3\,(bdx+bc)\cos\left(bx+a\right)^{4}-2\,bdx-(bdx+bc)\cos\left(bx+\right.\right.}{225\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/225*(9*d*cos(b*x + a)^5 - 5*d*cos(b*x + a)^3 - 30*d*cos(b*x + a) + 15*(3
*(b*d*x + b*c)*cos(b*x + a)^4 - 2*b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 - 2*
b*c)*sin(b*x + a))/b^2
```

Sympy [A] time = 5.46708, size = 163, normalized size = 1.5

$$\begin{cases}\frac{2c\sin^{5}(a+bx)}{15b}+\frac{c\sin^{3}(a+bx)\cos^{2}(a+bx)}{3b}+\frac{2dx\sin^{5}(a+bx)}{15b}+\frac{dx\sin^{3}(a+bx)\cos^{2}(a+bx)}{3b}+\frac{2d\sin^{4}(a+bx)\cos(a+bx)}{15b^{2}}+\frac{13d\sin^{2}(a+bx)\cos^{3}(a+bx)}{45b^{2}} \\ \left(cx+\frac{dx^{2}}{2}\right)\sin^{2}(a)\cos^{3}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c*sin(a + b*x)**5/(15*b) + c*sin(a + b*x)**3*cos(a + b*x)**2/(
3*b) + 2*d*x*sin(a + b*x)**5/(15*b) + d*x*sin(a + b*x)**3*cos(a + b*x)**2/(
3*b) + 2*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 13*d*sin(a + b*x)**2*co
s(a + b*x)**3/(45*b**2) + 26*d*cos(a + b*x)**5/(225*b**2), Ne(b, 0)), ((c*x
 + d*x**2/2)*sin(a)**2*cos(a)**3, True))
```

Giac [A] time = 1.11593, size = 143, normalized size = 1.31

$$-\frac{d\cos(5\,bx+5\,a)}{400\,b^2}-\frac{d\cos(3\,bx+3\,a)}{144\,b^2}+\frac{d\cos(bx+a)}{8\,b^2}-\frac{(bdx+bc)\sin(5\,bx+5\,a)}{80\,b^2}-\frac{(bdx+bc)\sin(3\,bx+3\,a)}{48\,b^2}+\frac{(bdx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/400*d*cos(5*b*x + 5*a)/b^2 - 1/144*d*cos(3*b*x + 3*a)/b^2 + 1/8*d*cos(b*
x + a)/b^2 - 1/80*(b*d*x + b*c)*sin(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*s
in(3*b*x + 3*a)/b^2 + 1/8*(b*d*x + b*c)*sin(b*x + a)/b^2
```

3.150 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=185

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5b\right.}{16d}$$

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*CosIntegral
[(5*b*c)/d + 5*b*x])/(16*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])
/(8*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) + (Si
n[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rubi [A] time = 0.279852, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5b\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*CosIntegral
[(5*b*c)/d + 5*b*x])/(16*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])
/(8*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) + (Si
n[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{\cos(a+bx)}{8(c+dx)}-\frac{\cos(3a+3bx)}{16(c+dx)}-\frac{\cos(5a+5bx)}{16(c+dx)}\right)dx\\ &= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{c+dx}\,dx\right)-\frac{1}{16}\int \frac{\cos(5a+5bx)}{c+dx}\,dx+\frac{1}{8}\int \frac{\cos(a+bx)}{c+dx}\,dx\\ &= -\left(\frac{1}{16}\cos\left(5a-\frac{5bc}{d}\right)\int \frac{\cos\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,dx\right)-\frac{1}{16}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,d\\ &= \frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5bc}{d}+5\right.}{16d}\end{aligned}$$

Mathematica [A] time = 0.520874, size = 154, normalized size = 0.83

$$\frac{2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)-}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosInte
gral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x))/
d] - 2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a - (3*b*c)/d]*Sin
Integral[(3*b*(c + d*x))/d] + Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*
```

```
x))/d])/(16*d)
```

Maple [A] time = 0.023, size = 252, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b}{8}\left(\frac{1}{d}\mathrm{Si}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)+\frac{1}{d}\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)\right)-\frac{b}{80}\left(5\frac{1}{d}\mathrm{Si}\left(5bx+5a+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/b*(1/8*b*(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/
d)*cos((-a*d+b*c)/d)/d)-1/80*b*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+
b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)-1/48*b*(3*S
i(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*
c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

Maxima [C] time = 1.62652, size = 551, normalized size = 2.98

$$-\frac{2\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_1\left(-\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/32*(2*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp
_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1,
 -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b*(exp_
integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(1,
-(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b*(-2*I*
exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*exp_integral_e(1
, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(I*exp_integ
ral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral_e(1, -(3
*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b*(I*exp_in
tegral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*exp_integral_e(1,
-(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 0.49701, size = 612, normalized size = 3.31

$$\frac{2\left(\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)+\operatorname{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)-\left(\operatorname{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\right.\right.}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/32*(2*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*co
s(-(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d
*x + b*c)/d))*cos(-3*(b*c - a*d)/d) - (cos_integral(5*(b*d*x + b*c)/d) + co
s_integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) + 2*sin(-5*(b*c - a*d
)/d)*sin_integral(5*(b*d*x + b*c)/d) + 2*sin(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) - 4*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/
d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.151 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=257

$$\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}(}{8d^2}$$

```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)) + Cos[3*a + 3*b*x]/(16*d*(c + d*x)) + Cos[5*a
 + 5*b*x]/(16*d*(c + d*x)) + (5*b*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a -
(5*b*c)/d])/(16*d^2) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c
)/d])/(16*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^2) -
(b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Cos[5*a - (5*b*c)/
d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rubi [A] time = 0.344944, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}(}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)) + Cos[3*a + 3*b*x]/(16*d*(c + d*x)) + Cos[5*a
 + 5*b*x]/(16*d*(c + d*x)) + (5*b*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a -
(5*b*c)/d])/(16*d^2) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c
)/d])/(16*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^2) -
(b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Cos[5*a - (5*b*c)/
d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)^2} - \frac{\cos(3a+3bx)}{16(c+dx)^2} - \frac{\cos(5a+5bx)}{16(c+dx)^2}\right) dx$$

$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^2}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} - \frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{8d} + \frac{(3b)\int \frac{\sin(3a+3bx)}{c+dx}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{\left(5b\cos\left(5a-\frac{5bc}{d}\right)\right)\int \frac{\sin\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,dx}{16d}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{5b\,\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d^2} + $$

Mathematica [A] time = 2.13917, size = 212, normalized size = 0.82

$$-2\left(b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d\cos(a+bx)}{c+dx}\right)+5b\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] ((d*Cos[3*(a + b*x)])/(c + d*x) + (d*Cos[5*(a + b*x)])/(c + d*x) + 5*b*CosI
ntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + 3*b*CosIntegral[(3*b*(c +
 d*x))/d]*Sin[3*a - (3*b*c)/d] - 2*((d*Cos[a + b*x])/(c + d*x) + b*CosInteg
ral[b*(c/d + x)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d +
 x)]) + 3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Cos[5
*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

Maple [A] time = 0.026, size = 367, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^2}{8}\left(-\frac{\cos(bx+a)}{((bx+a)d-ad+bc)d}-\frac{1}{d}\left(\frac{1}{d}\text{Si}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)-\frac{1}{d}\text{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+}{d}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/8*b^2*(-cos(b*x+a)/((b*x+a)*d-a*d+b*c)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos
((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/80*b^2*(-
5*cos(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*co
s(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d
)-1/48*b^2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d-3*(3*Si(3*b*x+3*a+3*(-a
*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d
+b*c)/d)/d)/d))
```

Maxima [C] time = 1.96772, size = 593, normalized size = 2.31

$$-\frac{1073741824\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-536870912\,b^2\left(E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/17179869184*(1073741824*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d -
I*a*d)/d) + exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*
c - a*d)/d) - 536870912*b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d -
 3*I*a*d)/d) + exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*cos(-3*(b*c - a*d)/d) - 536870912*b^2*(exp_integral_e(2, (5*I*b*c + 5*I*(b
*x + a)*d - 5*I*a*d)/d) + exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5
*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^2*(-1073741824*I*exp_integral_e(2, (I
*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(2, -(I*b*c +
 I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(536870912*I*exp_inte
gral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integr
al_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) +
b^2*(536870912*I*exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d)
 - 536870912*I*exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))
*sin(-5*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 0.63162, size = 867, normalized size = 3.37

$$\frac{32\,d\cos\left(bx+a\right)^{5}-32\,d\cos\left(bx+a\right)^{3}+10\left(bdx+bc\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right)+6\left(bdx+bc\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bd}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*d*cos(b*x + a)^5 - 32*d*cos(b*x + a)^3 + 10*(b*d*x + b*c)*cos(-5*(
b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 6*(b*d*x + b*c)*cos(-3*(b*c
 - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b*d*x + b*c)*cos(-(b*c - a*
d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b*d*x + b*c)*cos_integral((b*d*x
+ b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/
d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_i
ntegral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 5*((b*d*x + b*c)*cos_i
ntegral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-5*(b*d*x + b*c)/d)
)*sin(-5*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^3\sin(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3*sin(b*x + a)^2/(d*x + c)^2, x)
```

3.152 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=338

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegr}}{32d^3}$$

```
[Out] -Cos[a + b*x]/(16*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(32*d*(c + d*x)^2) + Co
s[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)
/d + b*x])/(16*d^3) + (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3
*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*c)/d + 5*b*
x])/(32*d^3) + (b*Sin[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x])
/(32*d^2*(c + d*x)) - (5*b*Sin[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (b^2*Sin[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Sin[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) - (25*b^2*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rubi [A] time = 0.4378, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegr}}{32d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```


```
[Out] -Cos[a + b*x]/(16*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(32*d*(c + d*x)^2) + Co
s[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)
/d + b*x])/(16*d^3) + (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3
*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*c)/d + 5*b*
x])/(32*d^3) + (b*Sin[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x])
/(32*d^2*(c + d*x)) - (5*b*Sin[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (b^2*Sin[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Sin[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) - (25*b^2*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int\left(\frac{\cos(a+bx)}{8(c+dx)^3} - \frac{\cos(3a+3bx)}{16(c+dx)^3} - \frac{\cos(5a+5bx)}{16(c+dx)^3}\right)dx$$

$$= -\left(\frac{1}{16}\int\frac{\cos(3a+3bx)}{(c+dx)^3}\,dx\right) - \frac{1}{16}\int\frac{\cos(5a+5bx)}{(c+dx)^3}\,dx + \frac{1}{8}\int\frac{\cos(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} - \frac{b\int\frac{\sin(a+bx)}{(c+dx)^2}\,dx}{16d} + \frac{(3b)\int\frac{\sin(3a+3bx)}{(c+dx)^2}}{32d}$$

$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} + \frac{b\sin(a+bx)}{16d^2(c+dx)} - \frac{3b\sin(3a+3bx)}{32d^2(c+dx)}$$

$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} + \frac{b\sin(a+bx)}{16d^2(c+dx)} - \frac{3b\sin(3a+3bx)}{32d^2(c+dx)}$$

$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{16d^3} + \frac{9b^2}{}$$

Mathematica [A] time = 3.29578, size = 283, normalized size = 0.84

$$\frac{-2b^2\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) + 9b^2\cos\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) + 25b^2\cos\left(5a-\frac{5bc}{d}\right)\mathrm{CosIntegr}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] ((d^2*Cos[3*(a + b*x)])/(c + d*x)^2 + (d^2*Cos[5*(a + b*x)])/(c + d*x)^2 -
2*b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 9*b^2*Cos[3*a - (3*b*c)/d
]*CosIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[
(5*b*(c + d*x))/d] + (2*d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Sin[a + b*x]))/(
c + d*x)^2 - (3*b*d*Sin[3*(a + b*x)])/(c + d*x) - (5*b*d*Sin[5*(a + b*x)])/
(c + d*x) + 2*b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b^2*Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] - 25*b^2*Sin[5*a - (5*b*c)/d]*
SinIntegral[(5*b*(c + d*x))/d])/(32*d^3)
```

Maple [A] time = 0.026, size = 473, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^3}{8}\left(-\frac{\cos(bx+a)}{2\left((bx+a)d-ad+bc\right)^2 d} - \frac{1}{2d}\left(-\frac{\sin(bx+a)}{\left((bx+a)d-ad+bc\right)d} + \frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right) + \frac{1}{d}\mathrm{C}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/8*b^3*(-1/2*cos(b*x+a)/((b*x+a)*d-a*d+b*c)^2/d-1/2*(-sin(b*x+a)/((b*
x+a)*d-a*d+b*c)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*
d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/80*b^3*(-5/2*cos(5*b*x+5*a)/((b*x+a)
*d-a*d+b*c)^2/d-5/2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)/d+5*(5*Si(5*b*x+
5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*co
s(5*(-a*d+b*c)/d)/d)/d)/d)-1/48*b^3*(-3/2*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c
)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d+3*(3*Si(3*b*x+3*a+3*(-a*
d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+
b*c)/d)/d)/d)/d))
```

Maxima [C] time = 2.60331, size = 640, normalized size = 1.89

$$-\frac{1073741824\, b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-536870912\, b^3\left(E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/17179869184*(1073741824*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d -
I*a*d)/d) + exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*
c - a*d)/d) - 536870912*b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d -
 3*I*a*d)/d) + exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*cos(-3*(b*c - a*d)/d) - 536870912*b^3*(exp_integral_e(3, (5*I*b*c + 5*I*(b
*x + a)*d - 5*I*a*d)/d) + exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5
*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^3*(-1073741824*I*exp_integral_e(3, (I
*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(3, -(I*b*c +
 I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(536870912*I*exp_inte
gral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integr
al_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) +
b^3*(536870912*I*exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d)
 - 536870912*I*exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))
*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A] time = 0.723896, size = 1312, normalized size = 3.88

$$32\,d^2\cos\left(bx+a\right)^5-32\,d^2\cos\left(bx+a\right)^3-50\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right)-18\left(b^2d^2x^2+2\,b^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(32*d^2*cos(b*x + a)^5 - 32*d^2*cos(b*x + a)^3 - 50*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) -
 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c -
a*d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) +
 25*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*c
os(-5*(b*c - a*d)/d) - 32*(5*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 3*(b*d^2*x
+ b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin^2\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.153 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=413

$$-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4}-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosInte}}{48d^4}$$

```
[Out] -Cos[a + b*x]/(24*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(48*d^3*(c + d*x)) +
Cos[3*a + 3*b*x]/(48*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(32*d^3*(c +
 d*x)) + Cos[5*a + 5*b*x]/(48*d*(c + d*x)^3) - (25*b^2*Cos[5*a + 5*b*x])/(9
6*d^3*(c + d*x)) - (125*b^3*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c
)/d])/(96*d^4) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d]
)/(32*d^4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(48*d^4) + (
b*Sin[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(32*d^2*(c + d*
x)^2) - (5*b*Sin[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) + (b^3*Cos[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rubi [A] time = 0.538389, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4}-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosInte}}{48d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -Cos[a + b*x]/(24*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(48*d^3*(c + d*x)) +
Cos[3*a + 3*b*x]/(48*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(32*d^3*(c +
 d*x)) + Cos[5*a + 5*b*x]/(48*d*(c + d*x)^3) - (25*b^2*Cos[5*a + 5*b*x])/(9
6*d^3*(c + d*x)) - (125*b^3*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c
)/d])/(96*d^4) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d]
)/(32*d^4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(48*d^4) + (
b*Sin[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(32*d^2*(c + d*
x)^2) - (5*b*Sin[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) + (b^3*Cos[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Cos[5*a - (5*b*c)/d]*SinIntegr
```

```
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)^4} - \frac{\cos(3a+3bx)}{16(c+dx)^4} - \frac{\cos(5a+5bx)}{16(c+dx)^4}\right) dx$$
$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^4}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^4}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$$
$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{24d} + \frac{b\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx}{16d}$$
$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} + \frac{b\sin(a+bx)}{48d^2(c+dx)^2} - \frac{b\sin(3a+3bx)}{32d^2(c+dx)^2}$$
$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos(5a+5bx)}{48d(c+dx)^3}$$
$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos(5a+5bx)}{48d(c+dx)^3}$$
$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos(5a+5bx)}{48d(c+dx)^3}$$

Mathematica [A] time = 3.47955, size = 451, normalized size = 1.09

$$-\frac{-2\left(b^3(c+dx)^3\left(\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)+d\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -(d*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin
[3*a]) + d*Cos[5*b*x]*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d*(c +
d*x)*Sin[5*a]) + d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)
*Sin[3*a])*Sin[3*b*x] + d*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2*(c +
 d*x)^2)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*C
os[a] + b*d*(c + d*x)*Sin[a]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c
+ d*x)^2)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[
a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) + 27*b^3*(c + d*
x)^3*(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a - (3*b*
c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(CosIntegral[(5
*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + Cos[5*a - (5*b*c)/d]*SinIntegral[(5
*b*(c + d*x))/d]))/(96*d^4*(c + d*x)^3)
```

Maple [A] time = 0.024, size = 583, normalized size = 1.4

$$\frac{1}{b}\left(\frac{b^4}{8}\left(-\frac{\cos(bx+a)}{3\left((bx+a)d-ad+bc\right)^3 d}-\frac{1}{3d}\left(-\frac{\sin(bx+a)}{2\left((bx+a)d-ad+bc\right)^2 d}+\frac{1}{2d}\left(-\frac{\cos(bx+a)}{\left((bx+a)d-ad+bc\right)d}-\frac{1}{d}\left(\frac{1}{d}\mathrm{Si}\left(bx+\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/8*b^4*(-1/3*cos(b*x+a)/((b*x+a)*d-a*d+b*c)^3/d-1/3*(-1/2*sin(b*x+a)/
((b*x+a)*d-a*d+b*c)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-a*d+b*c)/d-(Si(b*x+a+(-
a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)
/d)/d)/d)-1/80*b^4*(-5/3*cos(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)^3/d-5/3*(-5/2*s
in(5*b*x+5*a)/((b*x+a)*d-a*d+b*c)^2/d+5/2*(-5*cos(5*b*x+5*a)/((b*x+a)*d-a*d
+b*c)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+
5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)/d)-1/48*b^4*(-cos(3*b*x+3*
a)/((b*x+a)*d-a*d+b*c)^3/d-(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)^2/d+3/2
*(-3*cos(3*b*x+3*a)/((b*x+a)*d-a*d+b*c)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)
*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d
)/d)/d)/d))
```

Maxima [C] time = 3.97539, size = 707, normalized size = 1.71

$$-\frac{1073741824\,b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-536870912\,b^4\left(E_4\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/17179869184*(1073741824*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d -
I*a*d)/d) + exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*
c - a*d)/d) - 536870912*b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d -
 3*I*a*d)/d) + exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*cos(-3*(b*c - a*d)/d) - 536870912*b^4*(exp_integral_e(4, (5*I*b*c + 5*I*(b
*x + a)*d - 5*I*a*d)/d) + exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5
*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^4*(-1073741824*I*exp_integral_e(4, (I
*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(4, -(I*b*c +
 I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(536870912*I*exp_inte
```

```
gral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integr
al_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) +
b^4*(536870912*I*exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d)
 - 536870912*I*exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))
*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] time = 0.844789, size = 1814, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/192*(32*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b*x
 + a)^5 - 32*(29*b^2*d^3*x^2 + 58*b^2*c*d^2*x + 29*b^2*c^2*d - 2*d^3)*cos(b
*x + a)^3 + 250*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*c
os(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 54*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 192*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a) + 32*(5*(b*d^3*x + b*c*d^2)*cos(
b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x + a) - 2*((b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral((b*d*x + b
*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_inte
gral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c)/d) + (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-3*(b*d*x +
b*c)/d))*sin(-3*(b*c - a*d)/d) + 125*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos_integral(5*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-5*(b*d*x + b*c)/d))*sin
(-5*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.154 $\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=285

$$-\frac{3\,2^{-m-7}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamm}}{b}$$

```
[Out] (-3*2^(-7 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(
c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - (3*2^(-7 - m)*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x)
)/d)^m) + (2^(-7 - m)*3^(-1 - m)*E^((6*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[
1 + m, ((-6*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-7 - m)*
3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*b*(c + d*x))/d])/(b*E^((6*I)*(a
- (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.316992, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3308, 2181}

$$-\frac{3\,2^{-m-7}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamm}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```


```
[Out] (-3*2^(-7 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(
c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - (3*2^(-7 - m)*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x)
)/d)^m) + (2^(-7 - m)*3^(-1 - m)*E^((6*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[
1 + m, ((-6*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-7 - m)*
3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*b*(c + d*x))/d])/(b*E^((6*I)*(a
- (b*c)/d))*((I*b*(c + d*x))/d)^m)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^m \sin(2a+2bx) - \frac{1}{32}(c+dx)^m \sin(6a+6bx)\right) dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^m \sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^m \sin(2a+2bx)\,dx$$
$$= -\left(\frac{1}{64}i\int e^{-i(6a+6bx)}(c+dx)^m\,dx\right) + \frac{1}{64}i\int e^{i(6a+6bx)}(c+dx)^m\,dx + \frac{3}{64}i\int e^{-$$
$$= -\frac{3\,2^{-7-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{3\,2^{-7-m}e^{-2i}}{}$$

Mathematica [A] time = 3.41225, size = 255, normalized size = 0.89

$$2^{-m-7}3^{-m-1}e^{-\frac{6i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{m+2}e^{4ia+\frac{8ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m \text{Gamma}\left(m+1,\frac{2ib(c+dx)}{d}\right) - 3^{m+2}e^{4i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (2^(-7 - m)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((4*I)*(2*a + (b*c)/d))*(
(I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 3^(2 + m)*E^((
4*I)*a + ((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c +
 d*x))/d] + E^((12*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-6*I)*b*(c +
d*x))/d] + E^(((12*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((6*I)*
```

```
b*(c + d*x))/d]))/(b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cos (bx+a))^3 (\sin (bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos (bx+a)^3 \sin (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

Fricas [A] time = 0.556983, size = 487, normalized size = 1.71

$$\frac{e^{\left(-\frac{dm\log\left(\frac{6ib}{d}\right)-6ibc+6iad}{d}\right)}\Gamma\left(m+1, \frac{6ibdx+6ibc}{d}\right) - 9e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1, \frac{2ibdx+2ibc}{d}\right) - 9e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(m+1, \right.}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(e^(-(d*m*log(6*I*b/d) - 6*I*b*c + 6*I*a*d)/d)*gamma(m + 1, (6*I*b*d*
x + 6*I*b*c)/d) - 9*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
 1, (2*I*b*d*x + 2*I*b*c)/d) - 9*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d
```

```
)/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-6*I*b/d) + 6*I*
b*c - 6*I*a*d)/d)*gamma(m + 1, (-6*I*b*d*x - 6*I*b*c)/d))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

3.155 $\int (c+dx)^4 \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=233

$$-\frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4}+\frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4}+\frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{d^2(c+dx)^2\cos(6a+6bx)}{576b^3}+\frac{3d(\cdots}{\cdots}$$

```
[Out] (-9*d^4*Cos[2*a + 2*b*x])/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[2*a + 2*b*x])/
(64*b^3) - (3*(c + d*x)^4*Cos[2*a + 2*b*x])/(64*b) + (d^4*Cos[6*a + 6*b*x])
/(10368*b^5) - (d^2*(c + d*x)^2*Cos[6*a + 6*b*x])/(576*b^3) + ((c + d*x)^4*
Cos[6*a + 6*b*x])/(192*b) - (9*d^3*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^4) + (
3*d*(c + d*x)^3*Sin[2*a + 2*b*x])/(32*b^2) + (d^3*(c + d*x)*Sin[6*a + 6*b*x
])/(1728*b^4) - (d*(c + d*x)^3*Sin[6*a + 6*b*x])/(288*b^2)
```

Rubi [A] time = 0.265973, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$-\frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4}+\frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4}+\frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{d^2(c+dx)^2\cos(6a+6bx)}{576b^3}+\frac{3d(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-9*d^4*Cos[2*a + 2*b*x])/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[2*a + 2*b*x])/
(64*b^3) - (3*(c + d*x)^4*Cos[2*a + 2*b*x])/(64*b) + (d^4*Cos[6*a + 6*b*x])
/(10368*b^5) - (d^2*(c + d*x)^2*Cos[6*a + 6*b*x])/(576*b^3) + ((c + d*x)^4*
Cos[6*a + 6*b*x])/(192*b) - (9*d^3*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^4) + (
3*d*(c + d*x)^3*Sin[2*a + 2*b*x])/(32*b^2) + (d^3*(c + d*x)*Sin[6*a + 6*b*x
])/(1728*b^4) - (d*(c + d*x)^3*Sin[6*a + 6*b*x])/(288*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos^3(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{32}(c+dx)^4\sin(2a+2bx)-\frac{1}{32}(c+dx)^4\sin(6a+6bx)\right)dx\\ &= -\left(\frac{1}{32}\int (c+dx)^4\sin(6a+6bx)\,dx\right)+\frac{3}{32}\int (c+dx)^4\sin(2a+2bx)\,dx\\ &= -\frac{3(c+dx)^4\cos(2a+2bx)}{64b}+\frac{(c+dx)^4\cos(6a+6bx)}{192b}-\frac{d\int (c+dx)^3\cos(6a\\ &= -\frac{3(c+dx)^4\cos(2a+2bx)}{64b}+\frac{(c+dx)^4\cos(6a+6bx)}{192b}+\frac{3d(c+dx)^3\sin(2a+\\ &= \frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{3(c+dx)^4\cos(2a+2bx)}{64b}-\frac{d^2(c+dx)^2\cos(6a\\ &= \frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{3(c+dx)^4\cos(2a+2bx)}{64b}-\frac{d^2(c+dx)^2\cos(6a\\ &= -\frac{9d^4\cos(2a+2bx)}{128b^5}+\frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{3(c+dx)^4\cos(2a+2bx)}{64b}\end{aligned}$$

Mathematica [A] time = 1.5526, size = 153, normalized size = 0.66

$$\frac{-243\cos(2(a+bx))\left(-6b^2d^2(c+dx)^2+2b^4(c+dx)^4+3d^4\right)+\cos(6(a+bx))\left(-18b^2d^2(c+dx)^2+54b^4(c+dx)^4+d^4\right)}{10368b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-243*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)]
+ (d^4 - 18*b^2*d^2*(c + d*x)^2 + 54*b^4*(c + d*x)^4)*Cos[6*(a + b*x)] - 12
*b*d*(c + d*x)*(121*d^2 - 78*b^2*(c + d*x)^2 + (-d^2 + 6*b^2*(c + d*x)^2)*C
os[4*(a + b*x)])*Sin[2*(a + b*x)])/(10368*b^5)
```

Maple [B] time = 0.074, size = 2061, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/4*(b*x+a)^4*sin(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/
2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/12*(b*x+a)^2*sin(b*x+a)^4+1/6*(b*
x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/9*(b*x
+a)^2+1/216*sin(b*x+a)^4+5/36*sin(b*x+a)^2+1/4*(b*x+a)^2*cos(b*x+a)^2-1/2*(
b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/8*(b*x+a)^4-1/6*(b*x+a)^
4*sin(b*x+a)^6+2/3*(b*x+a)^3*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(
b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/18*(b*x+a)^2*sin(b*x+a)^6-1/9*(b*x+a)
*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+
5/16*a)-1/324*sin(b*x+a)^6)-4/b^4*a*d^4*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*
x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(
b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-
1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1
/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^
3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/
216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+4/b^3*c*d^3
*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+
a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+
3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*co
s(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2
*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+
5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8
*sin(b*x+a))*cos(b*x+a))+6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x
+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x
+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b
*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16
*b*x+5/16*a)+1/108*sin(b*x+a)^6)-12/b^3*a*c*d^3*(1/4*(b*x+a)^2*sin(b*x+a)^4
-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+
1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)
^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*
x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+6/b^2*c^2*d^2*(1/4*(b*x+a)^2*sin(
b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x
+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*si
n(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a)
)*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)-4/b^4*a^3*d^4*(1/4*(b*x+a
)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*
a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x
```

```
+a))*cos(b*x+a))+12/b^3*a^2*c*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a
)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/3
6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-12/b^2*a*c^2*
d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)
-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)
^3+15/8*sin(b*x+a))*cos(b*x+a))+4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(s
in(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+
a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+1/b^4
*a^4*d^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b^3*a^3*c*d^3
*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+6/b^2*a^2*c^2*d^2*(-1/6
*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b*a*c^3*d*(-1/6*sin(b*x+a)^
2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*
cos(b*x+a)^4))
```

Maxima [B] time = 1.44756, size = 1395, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/10368*(864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^4 - 3456*(2*sin(b*x +
 a)^6 - 3*sin(b*x + a)^4)*a*c^3*d/b + 5184*(2*sin(b*x + a)^6 - 3*sin(b*x +
a)^4)*a^2*c^2*d^2/b^2 - 3456*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*c*d^
3/b^3 + 864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^4*d^4/b^4 - 36*(6*(b*x
+ a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) +
27*sin(2*b*x + 2*a))*c^3*d/b + 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x
+ a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a*c^2*d^2/b
^2 - 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - si
n(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*c*d^3/b^3 + 36*(6*(b*x + a)*cos(6
*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b
*x + 2*a))*a^3*d^4/b^4 - 18*((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*
(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x
 + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 + 36*((18*(b*x + a)^2 - 1)*cos(6*b*x +
6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*
a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 - 18*((18*(b*x + a)^2 - 1)
*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*s
in(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/b^4 - 6*(6*(6*(b*
x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*co
s(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2
 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 + 6*(6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*
```

```
x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x +
a)^2 - 1)*sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^
4/b^4 - ((54*(b*x + a)^4 - 18*(b*x + a)^2 + 1)*cos(6*b*x + 6*a) - 243*(2*(b
*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 6*(6*(b*x + a)^3 - b*x -
a)*sin(6*b*x + 6*a) + 486*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*d
^4/b^4)/b
```

Fricas [B] time = 0.562245, size = 1153, normalized size = 4.95

$$\frac{27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 2\left(54\,b^4d^4x^4 + 216\,b^4cd^3x^3 + 54\,b^4c^4 - 18\,b^2c^2d^2 + d^4 + 18\left(18\,b^4c^2d^2 - b^2d^4\right)x^2 + 36\left(6\,b^4c^3d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 2*(54*b^4*d^4*x^4 + 216*b^4*c*d
^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2 - b^2*d^4)*
x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^6 - 3*(54*b^4*d^4*x^4 +
216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2
- b^2*d^4)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^4 + 18*(9*b^4
*c^2*d^2 - 5*b^2*d^4)*x^2 + 18*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*
d^2 - 5*d^4)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d - 5*b^2*c*d^3)*x - 12*((6*b^3
*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b*c*d^3 + (18*b^3*c^2*d^2 - b*d
^4)*x)*cos(b*x + a)^5 - (6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b
*c*d^3 + (18*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - 3*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 3*b^3*c^3*d - 5*b*c*d^3 + (9*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(
b*x + a))*sin(b*x + a))/b^5
```

Sympy [A] time = 46.2417, size = 1334, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**4*cos(a + b*x)*
*6/(12*b) + c**3*d*x*sin(a + b*x)**6/(6*b) + c**3*d*x*sin(a + b*x)**4*cos(a
```

```
 + b*x)**2/(2*b) - c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c**3*d*
x*cos(a + b*x)**6/(6*b) + c**2*d**2*x**2*sin(a + b*x)**6/(4*b) + 3*c**2*d**
2*x**2*sin(a + b*x)**4*cos(a + b*x)**2/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x
)**2*cos(a + b*x)**4/(4*b) - c**2*d**2*x**2*cos(a + b*x)**6/(4*b) + c*d**3*
x**3*sin(a + b*x)**6/(6*b) + c*d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(2
*b) - c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c*d**3*x**3*cos(a
 + b*x)**6/(6*b) + d**4*x**4*sin(a + b*x)**6/(24*b) + d**4*x**4*sin(a + b*x
)**4*cos(a + b*x)**2/(8*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**4/(8*b
) - d**4*x**4*cos(a + b*x)**6/(24*b) + c**3*d*sin(a + b*x)**5*cos(a + b*x)/
(6*b**2) + 4*c**3*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c**3*d*sin(a
 + b*x)*cos(a + b*x)**5/(6*b**2) + c**2*d**2*x*sin(a + b*x)**5*cos(a + b*x)
/(2*b**2) + 4*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**2) + c**2*d
**2*x*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + c*d**3*x**2*sin(a + b*x)**5*c
os(a + b*x)/(2*b**2) + 4*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**
2) + c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + d**4*x**3*sin(a +
b*x)**5*cos(a + b*x)/(6*b**2) + 4*d**4*x**3*sin(a + b*x)**3*cos(a + b*x)**3
/(9*b**2) + d**4*x**3*sin(a + b*x)*cos(a + b*x)**5/(6*b**2) - c**2*d**2*sin
(a + b*x)**6/(12*b**3) + c**2*d**2*sin(a + b*x)**2*cos(a + b*x)**4/(3*b**3)
 + 7*c**2*d**2*cos(a + b*x)**6/(36*b**3) - 5*c*d**3*x*sin(a + b*x)**6/(18*b
**3) - c*d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(3*b**3) + c*d**3*x*sin(a +
 b*x)**2*cos(a + b*x)**4/(3*b**3) + 5*c*d**3*x*cos(a + b*x)**6/(18*b**3) -
5*d**4*x**2*sin(a + b*x)**6/(36*b**3) - d**4*x**2*sin(a + b*x)**4*cos(a + b
*x)**2/(6*b**3) + d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(6*b**3) + 5*d*
*4*x**2*cos(a + b*x)**6/(36*b**3) - 5*c*d**3*sin(a + b*x)**5*cos(a + b*x)/(
18*b**4) - 31*c*d**3*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*c*d**3*s
in(a + b*x)*cos(a + b*x)**5/(18*b**4) - 5*d**4*x*sin(a + b*x)**5*cos(a + b*
x)/(18*b**4) - 31*d**4*x*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*d**4
*x*sin(a + b*x)*cos(a + b*x)**5/(18*b**4) + 5*d**4*sin(a + b*x)**6/(108*b**
5) - 31*d**4*sin(a + b*x)**2*cos(a + b*x)**4/(216*b**5) - 61*d**4*cos(a + b
*x)**6/(648*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 +
 c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a)**3, True))
```

Giac [A] time = 1.13064, size = 485, normalized size = 2.08

$$\frac{\left(54\,b^4d^4x^4+216\,b^4cd^3x^3+324\,b^4c^2d^2x^2+216\,b^4c^3dx+54\,b^4c^4-18\,b^2d^4x^2-36\,b^2cd^3x-18\,b^2c^2d^2+d^4\right)\cos\left(6\,bx+6\right.}{10368\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/10368*(54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 324*b^4*c^2*d^2*x^2 + 216*b^4
*c^3*d*x + 54*b^4*c^4 - 18*b^2*d^4*x^2 - 36*b^2*c*d^3*x - 18*b^2*c^2*d^2 +
```

```
d^4)*cos(6*b*x + 6*a)/b^5 - 3/128*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x -
 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/1728*(6*b^3*d^4*x^3 + 18*b
^3*c*d^3*x^2 + 18*b^3*c^2*d^2*x + 6*b^3*c^3*d - b*d^4*x - b*c*d^3)*sin(6*b*
x + 6*a)/b^5 + 3/64*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*
b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

3.156 $\int (c+dx)^3 \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=181

$$\frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3}+\frac{9d(c+dx)^2\sin(2a+2bx)}{128b^2}-\frac{d(c+dx)^2\sin(6a+6bx)}{384b^2}-\frac{9d^3\text{si}}{}$$

```
[Out] (9*d^2*(c + d*x)*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^3*Cos[2*a + 2*b
*x])/(64*b) - (d^2*(c + d*x)*Cos[6*a + 6*b*x])/(1152*b^3) + ((c + d*x)^3*Co
s[6*a + 6*b*x])/(192*b) - (9*d^3*Sin[2*a + 2*b*x])/(256*b^4) + (9*d*(c + d*
x)^2*Sin[2*a + 2*b*x])/(128*b^2) + (d^3*Sin[6*a + 6*b*x])/(6912*b^4) - (d*(
c + d*x)^2*Sin[6*a + 6*b*x])/(384*b^2)
```

Rubi [A] time = 0.218944, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$\frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3}+\frac{9d(c+dx)^2\sin(2a+2bx)}{128b^2}-\frac{d(c+dx)^2\sin(6a+6bx)}{384b^2}-\frac{9d^3\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```



```
[Out] (9*d^2*(c + d*x)*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^3*Cos[2*a + 2*b
*x])/(64*b) - (d^2*(c + d*x)*Cos[6*a + 6*b*x])/(1152*b^3) + ((c + d*x)^3*Co
s[6*a + 6*b*x])/(192*b) - (9*d^3*Sin[2*a + 2*b*x])/(256*b^4) + (9*d*(c + d*
x)^2*Sin[2*a + 2*b*x])/(128*b^2) + (d^3*Sin[6*a + 6*b*x])/(6912*b^4) - (d*(
c + d*x)^2*Sin[6*a + 6*b*x])/(384*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^3\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^3\sin(2a+2bx)-\frac{1}{32}(c+dx)^3\sin(6a+6bx)\right)dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^3\sin(6a+6bx)\,dx\right)+\frac{3}{32}\int (c+dx)^3\sin(2a+2bx)\,dx$$
$$= -\frac{3(c+dx)^3\cos(2a+2bx)}{64b}+\frac{(c+dx)^3\cos(6a+6bx)}{192b}-\frac{d\int (c+dx)^2\cos(6a+}{64b}$$
$$= -\frac{3(c+dx)^3\cos(2a+2bx)}{64b}+\frac{(c+dx)^3\cos(6a+6bx)}{192b}+\frac{9d(c+dx)^2\sin(2a+}{128b^2}$$
$$= \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{3(c+dx)^3\cos(2a+2bx)}{64b}-\frac{d^2(c+dx)\cos(6a+}{1152b^3}$$
$$= \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{3(c+dx)^3\cos(2a+2bx)}{64b}-\frac{d^2(c+dx)\cos(6a+}{1152b^3}$$

Mathematica [A] time = 2.38983, size = 132, normalized size = 0.73

$$\frac{-324b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+12b(c+dx)\cos(6(a+bx))\left(6b^2(c+dx)^2-d^2\right)-4d\sin(2(a+bx))\left(c\right.}{13824b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-324*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 12*b*(c +
 d*x)*(-d^2 + 6*b^2*(c + d*x)^2)*Cos[6*(a + b*x)] - 4*d*(121*d^2 - 234*b^2*
(c + d*x)^2 + (-d^2 + 18*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[2*(a + b*x)
])/(13824*b^4)
```

Maple [B] time = 0.027, size = 1100, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*
(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*
x+a)^2-1/16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6
+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*
x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin
(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-3/b^3*a*d^3*(1/4*(b*x+a)^2*sin(b*x+a
)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*
a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x
+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos
(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+3/b^2*c*d^2*(1/4*(b*x+a)^2*sin
(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*
x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*s
in(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a
))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+3/b^3*a^2*d^3*(1/4*(b*x+
a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24
*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*
x+a))*cos(b*x+a))-6/b^2*a*c*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*
(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+3/b*c^2*d*(1/4*
(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x
-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*s
in(b*x+a))*cos(b*x+a))-1/b^3*a^3*d^3*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*c
os(b*x+a)^4)+3/b^2*a^2*c*d^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a
)^4)-3/b*a*c^2*d*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^3*(-1
/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))
```

Maxima [B] time = 1.33376, size = 813, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6912*(576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^3 - 1728*(2*sin(b*x +
a)^6 - 3*sin(b*x + a)^4)*a*c^2*d/b + 1728*(2*sin(b*x + a)^6 - 3*sin(b*x + a
)^4)*a^2*c*d^2/b^2 - 576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*d^3/b^3
- 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*
b*x + 6*a) + 27*sin(2*b*x + 2*a))*c^2*d/b + 36*(6*(b*x + a)*cos(6*b*x + 6*a
```

```
) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))
*a*c*d^2/b^2 - 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x +
2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*d^3/b^3 - 6*((18*(b*x +
a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b
*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 + 6*((
18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2
*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/
b^3 - (6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 -
3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) + 243
*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

Fricas [B] time = 0.533301, size = 747, normalized size = 4.13

$$\frac{9\,b^3d^3x^3+27\,b^3cd^2x^2+6\left(6\,b^3d^3x^3+18\,b^3cd^2x^2+6\,b^3c^3-bcd^2+\left(18\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^6-9\left(6\,b^3d^3x^3+18\,b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 6*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x
^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^6 - 9*(6*
b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^
3)*x)*cos(b*x + a)^4 + 27*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 3*(9*b^3*c^2
*d - 5*b*d^3)*x - ((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*c
os(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*cos(
b*x + a)^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 5*d^3)*cos(b
*x + a))*sin(b*x + a))/b^4
```

Sympy [A] time = 60.5033, size = 867, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**6/(12*b) + c**3*sin(a + b*x)**4*cos(a + b*x)*
*2/(4*b) + c**2*d*x*sin(a + b*x)**6/(8*b) + 3*c**2*d*x*sin(a + b*x)**4*cos(
```

```
a + b*x)**2/(8*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - c**2
*d*x*cos(a + b*x)**6/(8*b) + c*d**2*x**2*sin(a + b*x)**6/(8*b) + 3*c*d**2*x
**2*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - 3*c*d**2*x**2*sin(a + b*x)**2*c
os(a + b*x)**4/(8*b) - c*d**2*x**2*cos(a + b*x)**6/(8*b) + d**3*x**3*sin(a
+ b*x)**6/(24*b) + d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - d**3*x
**3*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d**3*x**3*cos(a + b*x)**6/(24*b
) + c**2*d*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + c**2*d*sin(a + b*x)**3*c
os(a + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**5/(8*b**2) + c*
d**2*x*sin(a + b*x)**5*cos(a + b*x)/(4*b**2) + 2*c*d**2*x*sin(a + b*x)**3*c
os(a + b*x)**3/(3*b**2) + c*d**2*x*sin(a + b*x)*cos(a + b*x)**5/(4*b**2) +
d**3*x**2*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + d**3*x**2*sin(a + b*x)**3
*cos(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(8*b**2)
 - 5*c*d**2*sin(a + b*x)**6/(36*b**3) - 7*c*d**2*sin(a + b*x)**4*cos(a + b*
x)**2/(24*b**3) - c*d**2*sin(a + b*x)**2*cos(a + b*x)**4/(8*b**3) - 5*d**3*
x*sin(a + b*x)**6/(72*b**3) - d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(12*b*
*3) + d**3*x*sin(a + b*x)**2*cos(a + b*x)**4/(12*b**3) + 5*d**3*x*cos(a + b
*x)**6/(72*b**3) - 5*d**3*sin(a + b*x)**5*cos(a + b*x)/(72*b**4) - 31*d**3*
sin(a + b*x)**3*cos(a + b*x)**3/(216*b**4) - 5*d**3*sin(a + b*x)*cos(a + b*
x)**5/(72*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3
*x**4/4)*sin(a)**3*cos(a)**3, True))
```

Giac [A] time = 1.13246, size = 325, normalized size = 1.8

$$\frac{\left(6\,b^3d^3x^3+18\,b^3cd^2x^2+18\,b^3c^2dx+6\,b^3c^3-bd^3x-bcd^2\right)\cos\left(6\,bx+6\,a\right)}{1152\,b^4}-\frac{3\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c\right.}{128\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/1152*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*b^3*c^2*d*x + 6*b^3*c^3 - b*d
^3*x - b*c*d^2)*cos(6*b*x + 6*a)/b^4 - 3/128*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x
^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^
4 - 1/6912*(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*sin(6*b*x
 + 6*a)/b^4 + 9/256*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin
(2*b*x + 2*a)/b^4
```

3.157 $\int (c+dx)^2 \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=129

$$\frac{3d(c+dx)\sin(2a+2bx)}{64b^2}-\frac{d(c+dx)\sin(6a+6bx)}{576b^2}+\frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{d^2\cos(6a+6bx)}{3456b^3}-\frac{3(c+dx)^2\cos(2a+2bx)}{64b}$$

```
[Out] (3*d^2*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*b
) - (d^2*Cos[6*a + 6*b*x])/(3456*b^3) + ((c + d*x)^2*Cos[6*a + 6*b*x])/(192
*b) + (3*d*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^2) - (d*(c + d*x)*Sin[6*a + 6*
b*x])/(576*b^2)
```

Rubi [A] time = 0.143795, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$\frac{3d(c+dx)\sin(2a+2bx)}{64b^2}-\frac{d(c+dx)\sin(6a+6bx)}{576b^2}+\frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{d^2\cos(6a+6bx)}{3456b^3}-\frac{3(c+dx)^2\cos(2a+2bx)}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```


```
[Out] (3*d^2*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*b
) - (d^2*Cos[6*a + 6*b*x])/(3456*b^3) + ((c + d*x)^2*Cos[6*a + 6*b*x])/(192
*b) + (3*d*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^2) - (d*(c + d*x)*Sin[6*a + 6*
b*x])/(576*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^2\sin(2a+2bx) - \frac{1}{32}(c+dx)^2\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^2\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^2\sin(2a+2bx)\,dx$$

$$= -\frac{3(c+dx)^2\cos(2a+2bx)}{64b} + \frac{(c+dx)^2\cos(6a+6bx)}{192b} - \frac{d\int (c+dx)\cos(6a+}{96b}$$

$$= -\frac{3(c+dx)^2\cos(2a+2bx)}{64b} + \frac{(c+dx)^2\cos(6a+6bx)}{192b} + \frac{3d(c+dx)\sin(2a+}{64b^2}$$

$$= \frac{3d^2\cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^2\cos(2a+2bx)}{64b} - \frac{d^2\cos(6a+6bx)}{3456b^3} + \frac{(c+dx}{}$$

Mathematica [A] time = 0.544526, size = 91, normalized size = 0.71

$$\frac{-81\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+\cos(6(a+bx))\left(18b^2(c+dx)^2-d^2\right)-6bd(c+dx)(\sin(6(a+bx))-27\sin(2(a}{3456b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-81*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 18*b^2*(c + d*x)
^2)*Cos[6*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[2*(a + b*x)] + Sin[6*(a + b
*x)]))/(3456*b^3)
```

Maple [B] time = 0.024, size = 498, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+
3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-
1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^
5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b
*x+a)^6)-2/b^2*a*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b
*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)
^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+2/b*c*d*(1/4*(b*x+a)*sin(b
*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(
b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*co
s(b*x+a))+d^2/b^2*a^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-2*
c*d/b*a*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^2*(-1/6*sin(b*
x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))
```

Maxima [B] time = 1.23834, size = 409, normalized size = 3.17

$$-\frac{288\left(2\,\sin\left(bx+a\right)^{6}-3\,\sin\left(bx+a\right)^{4}\right)c^{2}-\frac{576\left(2\,\sin(bx+a)^{6}-3\,\sin(bx+a)^{4}\right)acd}{b}+\frac{288\left(2\,\sin(bx+a)^{6}-3\,\sin(bx+a)^{4}\right)a^{2}d^{2}}{b^{2}}-\frac{6\,(6\,(bx+a)\cos(6}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456*(288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^2 - 576*(2*sin(b*x + a
)^6 - 3*sin(b*x + a)^4)*a*c*d/b + 288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)
*a^2*d^2/b^2 - 6*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2
*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*c*d/b + 6*(6*(b*x + a)*cos(6*
b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*
x + 2*a))*a*d^2/b^2 - ((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*
sin(2*b*x + 2*a))*d^2/b^2)/b
```

Fricas [A] time = 0.503551, size = 447, normalized size = 3.47

$$\frac{2\left(18\,b^{2}d^{2}x^{2}+36\,b^{2}cdx+18\,b^{2}c^{2}-d^{2}\right)\cos\left(bx+a\right)^{6}+9\,b^{2}d^{2}x^{2}+18\,b^{2}cdx-3\left(18\,b^{2}d^{2}x^{2}+36\,b^{2}cdx+18\,b^{2}c^{2}-d^{2}\right)\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(2*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(b*x + a)^6
+ 9*b^2*d^2*x^2 + 18*b^2*c*d*x - 3*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*
c^2 - d^2)*cos(b*x + a)^4 + 9*d^2*cos(b*x + a)^2 - 6*(2*(b*d^2*x + b*c*d)*c
os(b*x + a)^5 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 3*(b*d^2*x + b*c*d)*co
s(b*x + a))*sin(b*x + a))/b^3
```

Sympy [A] time = 22.629, size = 471, normalized size = 3.65

$$\begin{cases} \frac{c^2 \sin^6(a+bx)}{12b} + \frac{c^2 \sin^4(a+bx)\cos^2(a+bx)}{4b} + \frac{cdx \sin^6(a+bx)}{12b} + \frac{cdx \sin^4(a+bx)\cos^2(a+bx)}{4b} - \frac{cdx \sin^2(a+bx)\cos^4(a+bx)}{4b} - \frac{cdx \cos^6(a+bx)}{12b} + \frac{d^2x^2 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**6/(12*b) + c**2*sin(a + b*x)**4*cos(a + b*x)*
*2/(4*b) + c*d*x*sin(a + b*x)**6/(12*b) + c*d*x*sin(a + b*x)**4*cos(a + b*x
)**2/(4*b) - c*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c*d*x*cos(a + b*
x)**6/(12*b) + d**2*x**2*sin(a + b*x)**6/(24*b) + d**2*x**2*sin(a + b*x)**4
*cos(a + b*x)**2/(8*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) -
d**2*x**2*cos(a + b*x)**6/(24*b) + c*d*sin(a + b*x)**5*cos(a + b*x)/(12*b**
2) + 2*c*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c*d*sin(a + b*x)*cos(
a + b*x)**5/(12*b**2) + d**2*x*sin(a + b*x)**5*cos(a + b*x)/(12*b**2) + 2*d
**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + d**2*x*sin(a + b*x)*cos(a
+ b*x)**5/(12*b**2) - 5*d**2*sin(a + b*x)**6/(108*b**3) - 7*d**2*sin(a + b*
x)**4*cos(a + b*x)**2/(72*b**3) - d**2*sin(a + b*x)**2*cos(a + b*x)**4/(24*
b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**3, T
rue))
```

Giac [A] time = 1.14459, size = 196, normalized size = 1.52

$$\frac{\left(18\,b^2d^2x^2 + 36\,b^2cdx + 18\,b^2c^2 - d^2\right)\cos\left(6\,bx + 6\,a\right)}{3456\,b^3} - \frac{3\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(2\,bx + 2\,a\right)}{128\,b^3} - \frac{\left(bd^2x + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3456*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(6*b*x + 6*a)/
b^3 - 3/128*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a
)/b^3 - 1/576*(b*d^2*x + b*c*d)*sin(6*b*x + 6*a)/b^3 + 3/64*(b*d^2*x + b*c*
d)*sin(2*b*x + 2*a)/b^3
```

3.158 $\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=77

$$\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin(6a+6bx)}{1152b^2}-\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}$$

```
[Out] (-3*(c + d*x)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)*Cos[6*a + 6*b*x])/(192*
b) + (3*d*Sin[2*a + 2*b*x])/(128*b^2) - (d*Sin[6*a + 6*b*x])/(1152*b^2)
```

Rubi [A] time = 0.0744242, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2637}

$$\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin(6a+6bx)}{1152b^2}-\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```



```
[Out] (-3*(c + d*x)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)*Cos[6*a + 6*b*x])/(192*
b) + (3*d*Sin[2*a + 2*b*x])/(128*b^2) - (d*Sin[6*a + 6*b*x])/(1152*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{32}(c+dx)\sin(2a+2bx)-\frac{1}{32}(c+dx)\sin(6a+6bx)\right)dx \\
&= -\left(\frac{1}{32}\int (c+dx)\sin(6a+6bx)\,dx\right)+\frac{3}{32}\int (c+dx)\sin(2a+2bx)\,dx \\
&= -\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}-\frac{d\int \cos(6a+6bx)\,dx}{192b}+ \\
&= -\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}+\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin
\end{aligned}
$$

Mathematica [A] time = 0.222521, size = 63, normalized size = 0.82

$$
\frac{-54b(c+dx)\cos(2(a+bx))+6b(c+dx)\cos(6(a+bx))+d(27\sin(2(a+bx))-\sin(6(a+bx)))}{1152b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*b*(c + d*x)*Cos[2*(a + b*x)] + 6*b*(c + d*x)*Cos[6*(a + b*x)] + d*(27*
Sin[2*(a + b*x)] - Sin[6*(a + b*x)]))/(1152*b^2)
```

Maple [B] time = 0.022, size = 176, normalized size = 2.3

$$
\frac{1}{b}\left(\frac{d}{b}\left(\frac{(bx+a)(\sin(bx+a))^4}{4}+\frac{\cos(bx+a)}{16}\left((\sin(bx+a))^3+\frac{3\sin(bx+a)}{2}\right)-\frac{bx}{24}-\frac{a}{24}-\frac{(bx+a)(\sin(bx+a))^6}{6}-\frac{\mathrm{co}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(d/b*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b
*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b
*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-1/b*d*a*(-1/6*sin(b*x+a)^2*cos(b*x+a)^
4-1/12*cos(b*x+a)^4)+c*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))
```

Maxima [A] time = 1.17966, size = 161, normalized size = 2.09

$$-\frac{96\left(2\,\sin\left(bx+a\right)^{6}-3\,\sin\left(bx+a\right)^{4}\right)c-\frac{96\left(2\,\sin(bx+a)^{6}-3\,\sin(bx+a)^{4}\right)ad}{b}-\frac{(6\,(bx+a)\cos(6\,bx+6\,a)-54\,(bx+a)\cos(2\,bx+2\,a)-\sin(6\,bx+6\,a)}{b}}{1152\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1152*(96*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c - 96*(2*sin(b*x + a)^6
- 3*sin(b*x + a)^4)*a*d/b - (6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*co
s(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*d/b)/b
```

Fricas [A] time = 0.486378, size = 221, normalized size = 2.87

$$\frac{12\,(bdx+bc)\cos\left(bx+a\right)^{6}-18\,(bdx+bc)\cos\left(bx+a\right)^{4}+3\,bdx-\left(2\,d\cos\left(bx+a\right)^{5}-2\,d\cos\left(bx+a\right)^{3}-3\,d\cos\left(bx+a\right.\right.}{72\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72*(12*(b*d*x + b*c)*cos(b*x + a)^6 - 18*(b*d*x + b*c)*cos(b*x + a)^4 + 3
*b*d*x - (2*d*cos(b*x + a)^5 - 2*d*cos(b*x + a)^3 - 3*d*cos(b*x + a))*sin(b
*x + a))/b^2
```

Sympy [A] time = 11.1778, size = 201, normalized size = 2.61

$$\begin{cases}\frac{c\sin^{6}(a+bx)}{12b}+\frac{c\sin^{4}(a+bx)\cos^{2}(a+bx)}{4b}+\frac{dx\sin^{6}(a+bx)}{24b}+\frac{dx\sin^{4}(a+bx)\cos^{2}(a+bx)}{8b}-\frac{dx\sin^{2}(a+bx)\cos^{4}(a+bx)}{8b}-\frac{dx\cos^{6}(a+bx)}{24b}+\frac{d\sin^{5}(a+b}{2} \\ \left(cx+\frac{dx^{2}}{2}\right)\sin^{3}(a)\cos^{3}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**6/(12*b) + c*sin(a + b*x)**4*cos(a + b*x)**2/(4*
b) + d*x*sin(a + b*x)**6/(24*b) + d*x*sin(a + b*x)**4*cos(a + b*x)**2/(8*b)
```

```
 - d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d*x*cos(a + b*x)**6/(24*b) +
 d*sin(a + b*x)**5*cos(a + b*x)/(24*b**2) + d*sin(a + b*x)**3*cos(a + b*x)*
*3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**5/(24*b**2), Ne(b, 0)), ((c*x +
d*x**2/2)*sin(a)**3*cos(a)**3, True))
```

Giac [A] time = 1.12175, size = 101, normalized size = 1.31

$$\frac{(bdx+bc)\cos(6\,bx+6\,a)}{192\,b^2}-\frac{3\,(bdx+bc)\cos(2\,bx+2\,a)}{64\,b^2}-\frac{d\sin(6\,bx+6\,a)}{1152\,b^2}+\frac{3\,d\sin(2\,bx+2\,a)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/192*(b*d*x + b*c)*cos(6*b*x + 6*a)/b^2 - 3/64*(b*d*x + b*c)*cos(2*b*x + 2
*a)/b^2 - 1/1152*d*sin(6*b*x + 6*a)/b^2 + 3/128*d*sin(2*b*x + 2*a)/b^2
```

3.159 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=129

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{32d}+\frac{3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{32d}$$

```
[Out] -(CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(32*d) + (3*CosInteg
ral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(32*d) + (3*Cos[2*a - (2*b*c)/
d]*SinIntegral[(2*b*c)/d + 2*b*x])/(32*d) - (Cos[6*a - (6*b*c)/d]*SinIntegr
al[(6*b*c)/d + 6*b*x])/(32*d)
```

Rubi [A] time = 0.246071, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{32d}+\frac{3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(32*d) + (3*CosInteg
ral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(32*d) + (3*Cos[2*a - (2*b*c)/
d]*SinIntegral[(2*b*c)/d + 2*b*x])/(32*d) - (Cos[6*a - (6*b*c)/d]*SinIntegr
al[(6*b*c)/d + 6*b*x])/(32*d)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)} - \frac{\sin(6a+6bx)}{32(c+dx)}\right)dx\\ &= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{c+dx}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\\ &= -\left(\frac{1}{32}\cos\left(6a-\frac{6bc}{d}\right)\int \frac{\sin\left(\frac{6bc}{d}+6bx\right)}{c+dx}\,dx\right) + \frac{1}{32}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2b\right.}{c+dx}\\ &= -\frac{\text{Ci}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{32d} + \frac{3\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{32d} + \frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}}{32d}\end{aligned}$$

Mathematica [A] time = 0.309157, size = 110, normalized size = 0.85

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right) - 3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(6\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 3*CosIntegral[(2*b*
(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*
b*(c + d*x))/d] + Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])/(32*
d)
```

Maple [A] time = 0.024, size = 178, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b}{192}\left(6\frac{1}{d}\mathrm{Si}\left(6bx+6a+6\frac{-ad+bc}{d}\right)\cos\left(6\frac{-ad+bc}{d}\right)-6\frac{1}{d}\mathrm{Ci}\left(6bx+6a+6\frac{-ad+bc}{d}\right)\sin\left(6\frac{-ad+bc}{d}\right)\right)+\frac{3b}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(-1/192*b*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d-6*Ci(6*
b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)+3/64*b*(2*Si(2*b*x+2*a+2*(-a
*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d
+b*c)/d)/d))
```

Maxima [C] time = 1.5355, size = 370, normalized size = 2.87

$$\frac{b\left(-3i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E_1\left(-\frac{6i\,bc+6i\,(}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/64*(b*(-3*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
3*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*
c - a*d)/d) + b*(I*exp_integral_e(1, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/
d) - I*exp_integral_e(1, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos(-6*
(b*c - a*d)/d) - 3*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*
d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
*(b*c - a*d)/d) + b*(exp_integral_e(1, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d
)/d) + exp_integral_e(1, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*sin(-6*
(b*c - a*d)/d))/(b*d)
```

Fricas [A] time = 0.490778, size = 421, normalized size = 3.26

$$\frac{3\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-\left(\mathrm{Ci}\left(\frac{6\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{6\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-2\cos\left(-\frac{6\,(bc-ad}{d}\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) - (cos_integral(6*(b*d*x + b*c)/d) + cos_integral(-
6*(b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d) - 2*cos(-6*(b*c - a*d)/d)*sin_int
egral(6*(b*d*x + b*c)/d) + 6*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x), x)
```

Giac [C] time = 1.78309, size = 8162, normalized size = 63.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/64*(imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b
*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^
2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_i
ntegral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2
+ 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(
b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/
d)^2*tan(b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*t
an(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integr
```

al(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 2*real
_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(
b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan
(3*b*c/d)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(6*b*x
+ 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos
_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2
+ imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)
^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b
*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*t
an(3*b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)
^2*tan(3*b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*t
an(3*b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)^2 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*t
an(3*b*c/d)^2*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/
d)*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(
2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 3*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_inte
gral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(6
*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*
x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 4*imag_part(co
s_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 +
 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d
)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2*ta
n(b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*
c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^
2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*
c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_i
ntegral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*sin_int
egral(6*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(c
os_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d) + 2*real_par
t(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d) - 6*real
_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2 - 6*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2 -
 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^
2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c

/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2 + 4*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d) - 4*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d) + 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(a)^2*tan(3*b*c/d) + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(3*b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(3*b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2*tan(3*b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)*tan(b*c/d) - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(2* b*x + 2*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d)*tan(b*c/d)^2 - 4*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/d)*tan(b*c/d)^2 + 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(3*b*c/d)*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a) + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2 + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2 + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d) + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d) - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2*tan(3*b*c/d) - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d) - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(3*b*c/d)^2 - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(3*b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(b*c/d) + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(b*c/d) - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d) + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(b*c/d)^2 + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*c/d)*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2 + 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 3

```
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - imag_part(cos_integra
l(-6*b*x - 6*b*c/d))*tan(a)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(a)^2
+ 6*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d) - 4*imag_part(cos_integral(-6*b*x - 6*
b*c/d))*tan(3*a)*tan(3*b*c/d) + 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*
tan(3*b*c/d) - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*c/d)^2 - 3*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*c/d)^2 + 3*imag_part(cos_i
ntegral(-2*b*x - 2*b*c/d))*tan(3*b*c/d)^2 + imag_part(cos_integral(-6*b*x -
 6*b*c/d))*tan(3*b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*c/d)^
2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*b*c/d)^2 - 12*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(a)*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/d)*tan(a)
*tan(b*c/d) + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(b*c/d)^2 + 3*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 3*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/
d))*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 6*sin_i
ntegral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x +
6*b*c/d))*tan(3*a) + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a) -
 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) - 6*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*t
an(3*b*c/d) - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*c/d) + 6*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 6*real_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(6*b*x + 6*b*c/d)
) - 3*imag_part(cos_integral(2*b*x + 2*b*c/d)) + 3*imag_part(cos_integral(-
2*b*x - 2*b*c/d)) - imag_part(cos_integral(-6*b*x - 6*b*c/d)) + 2*sin_integ
ral(6*(b*d*x + b*c)/d) - 6*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(3*a)^2*t
an(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + d*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2
+ d*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + d*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c/
d)^2 + d*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + d*tan(3*a)^2*tan(a)^2 + d*t
an(3*a)^2*tan(3*b*c/d)^2 + d*tan(a)^2*tan(3*b*c/d)^2 + d*tan(3*a)^2*tan(b*c
/d)^2 + d*tan(a)^2*tan(b*c/d)^2 + d*tan(3*b*c/d)^2*tan(b*c/d)^2 + d*tan(3*a
)^2 + d*tan(a)^2 + d*tan(3*b*c/d)^2 + d*tan(b*c/d)^2 + d)
```

3.160 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=179

$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2\right.}{16d^2}$$

```
[Out] (3*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) - (3*b*C
os[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(16*d^2) - (3*Sin[2*a +
 2*b*x])/(32*d*(c + d*x)) + Sin[6*a + 6*b*x]/(32*d*(c + d*x)) - (3*b*Sin[2*
a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) + (3*b*Sin[6*a - (6
*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^2)
```

Rubi [A] time = 0.297098, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2\right.}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```


```
[Out] (3*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) - (3*b*C
os[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(16*d^2) - (3*Sin[2*a +
 2*b*x])/(32*d*(c + d*x)) + Sin[6*a + 6*b*x]/(32*d*(c + d*x)) - (3*b*Sin[2*
a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) + (3*b*Sin[6*a - (6
*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^2}-\frac{\sin(6a+6bx)}{32(c+dx)^2}\right)dx\\
&= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^2}\,dx\right)+\frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\\
&= -\frac{3\sin(2a+2bx)}{32d(c+dx)}+\frac{\sin(6a+6bx)}{32d(c+dx)}+\frac{(3b)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{16d}-\frac{(3b)\int \frac{\cos(6a+6bx)}{c+dx}\,dx}{16d}\\
&= -\frac{3\sin(2a+2bx)}{32d(c+dx)}+\frac{\sin(6a+6bx)}{32d(c+dx)}-\frac{\left(3b\cos\left(6a-\frac{6bc}{d}\right)\right)\int \frac{\cos\left(\frac{6bc}{d}+6bx\right)}{c+dx}\,dx}{16d}+\frac{\left(3b\cos\left(2\right.\right.}{}\\
&= \frac{3b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\mathrm{Ci}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3\sin(2a+2bx)}{32d(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.9808, size = 189, normalized size = 1.06

$$6b(c+dx)\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-6b(c+dx)\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right)-6b(c+dx)\sin\left(2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (6*b*(c + d*x)*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 6*b*(c
 + d*x)*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] - 3*d*Cos[2*b*x
]*Sin[2*a] + d*Cos[6*b*x]*Sin[6*a] - 3*d*Cos[2*a]*Sin[2*b*x] + d*Cos[6*a]*S
in[6*b*x] - 6*b*(c + d*x)*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/
d] + 6*b*(c + d*x)*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])/(32
*d^2*(c + d*x))
```

Maple [A] time = 0.028, size = 256, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b^2}{192}\left(-6\,\frac{\sin(6\,bx+6\,a)}{((bx+a)\,d-ad+bc)\,d}+6\,\frac{1}{d}\left(6\,\frac{1}{d}\text{Si}\left(6\,bx+6\,a+6\,\frac{-ad+bc}{d}\right)\sin\left(6\,\frac{-ad+bc}{d}\right)+6\,\frac{1}{d}\text{Ci}\left(6\,bx+6\,a+6\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/192*b^2*(-6*sin(6*b*x+6*a)/((b*x+a)*d-a*d+b*c)/d+6*(6*Si(6*b*x+6*a+
6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*
(-a*d+b*c)/d)/d)/d)+3/64*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*
Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b
*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

Maxima [C] time = 1.76057, size = 406, normalized size = 2.27

$$b^2\left(-3i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E_2\left(-\frac{6i\,bc+6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(b^2*(-3*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 3*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a
*d)/d) - I*exp_integral_e(2, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos
(-6*(b*c - a*d)/d) - 3*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (6*I*b*c + 6*I*(b*x + a)*d -
 6*I*a*d)/d) + exp_integral_e(2, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))
*sin(-6*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Fricas [A] time = 0.596735, size = 632, normalized size = 3.53

$$\frac{6\,(bdx+bc)\sin\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{6\,(bdx+bc)}{d}\right)-6\,(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(6*(b*d*x + b*c)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d)
 - 6*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) +
3*((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integr
al(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - 3*((b*d*x + b*c)*cos_integr
al(6*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-6*(b*d*x + b*c)/d))*cos
(-6*(b*c - a*d)/d) + 32*(d*cos(b*x + a)^5 - d*cos(b*x + a)^3)*sin(b*x + a))
/(d^3*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx + a)^3 \sin (bx + a)^3}{(dx + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3*sin(b*x + a)^3/(d*x + c)^2, x)
```

3.161 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=235

$$\frac{9b^2\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^3}-\frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+\right.}{16d^3}$$

```
[Out] (-3*b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)) + (3*b*Cos[6*a + 6*b*x])/(32*d^2
*(c + d*x)) + (9*b^2*CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(
16*d^3) - (3*b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(16*d
^3) - (3*Sin[2*a + 2*b*x])/(64*d*(c + d*x)^2) + Sin[6*a + 6*b*x]/(64*d*(c +
 d*x)^2) - (3*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*
d^3) + (9*b^2*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^3)
```

Rubi [A] time = 0.353223, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{9b^2\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^3}-\frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+\right.}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (-3*b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)) + (3*b*Cos[6*a + 6*b*x])/(32*d^2
*(c + d*x)) + (9*b^2*CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(
16*d^3) - (3*b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(16*d
^3) - (3*Sin[2*a + 2*b*x])/(64*d*(c + d*x)^2) + Sin[6*a + 6*b*x]/(64*d*(c +
 d*x)^2) - (3*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*
d^3) + (9*b^2*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^3)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^3} - \frac{\sin(6a+6bx)}{32(c+dx)^3}\right) dx$$

$$= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^3}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} + \frac{(3b)\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{32d} - \frac{(3b)\int \frac{\cos(6a+6bx)}{(c+dx)^2}\,dx}{32d}$$

$$= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} - \frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} - \frac{(3b^2)\int}{}$$

$$= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} - \frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} + \frac{(9b^2\,\mathrm{co}}{}$$

$$= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} + \frac{9b^2\,\mathrm{Ci}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{16d^3} - \frac{3b^2\,\mathrm{Ci}\left(\right.}{}$$

Mathematica [A] time = 1.09066, size = 239, normalized size = 1.02

$$\frac{6b^2(c+dx)^2\left(6\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right)-2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c}{\ldots}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (-3*d*Cos[2*b*x]*(2*b*(c + d*x)*Cos[2*a] + d*Sin[2*a]) + d*Cos[6*b*x]*(6*b*
(c + d*x)*Cos[6*a] + d*Sin[6*a]) + 3*d*(-(d*Cos[2*a]) + 2*b*(c + d*x)*Sin[2
*a])*Sin[2*b*x] + d*(d*Cos[6*a] - 6*b*(c + d*x)*Sin[6*a])*Sin[6*b*x] + 6*b^
2*(c + d*x)^2*(6*CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 2*Co
sIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*
SinIntegral[(2*b*(c + d*x))/d] + 6*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c
 + d*x))/d]))/(64*d^3*(c + d*x)^2)
```

Maple [A] time = 0.028, size = 329, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b^3}{192}\left(-3\frac{\sin(6bx+6a)}{((bx+a)d-ad+bc)^2d}+3\frac{1}{d}\left(-6\frac{\cos(6bx+6a)}{((bx+a)d-ad+bc)d}-6\frac{1}{d}\left(6\frac{1}{d}\text{Si}\left(6bx+6a+6\frac{-ad+bc}{d}\right)\cos\left(6\frac{-}{}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/192*b^3*(-3*sin(6*b*x+6*a)/((b*x+a)*d-a*d+b*c)^2/d+3*(-6*cos(6*b*x+
6*a)/((b*x+a)*d-a*d+b*c)/d-6*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*
c)/d)/d-6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)/d)/d)+3/64*b^
3*(-sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-a*
d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x
+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

Maxima [C] time = 2.3008, size = 454, normalized size = 1.93

$$\frac{b^3\left(-3iE_3\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)+3iE_3\left(-\frac{2ibc+2i(bx+a)d-2iad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(iE_3\left(\frac{6ibc+6i(bx+a)d-6iad}{d}\right)-iE_3\left(-\frac{6ibc+6i\ldots}{}\right.\right.}{64\left(b^2c^2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(b^3*(-3*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 3*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a
*d)/d) - I*exp_integral_e(3, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos
(-6*(b*c - a*d)/d) - 3*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (6*I*b*c + 6*I*(b*x + a)*d -
 6*I*a*d)/d) + exp_integral_e(3, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))
*sin(-6*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Fricas [A] time = 0.705712, size = 1010, normalized size = 4.3

$$\frac{96\left(bd^2x + bcd\right)\cos\left(bx + a\right)^6 - 144\left(bd^2x + bcd\right)\cos\left(bx + a\right)^4 + 48\left(bd^2x + bcd\right)\cos\left(bx + a\right)^2 + 18\left(b^2d^2x^2 + 2\,b^2cdx + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(96*(b*d^2*x + b*c*d)*cos(b*x + a)^6 - 144*(b*d^2*x + b*c*d)*cos(b*x +
 a)^4 + 48*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d) - 6*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*
x + b*c)/d) + 16*(d^2*cos(b*x + a)^5 - d^2*cos(b*x + a)^3)*sin(b*x + a) - 3
*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*sin(
-2*(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(6
*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-6*(
b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.162 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4}+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{8d^4}$$

```
[Out] -(b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)^2) + (b*Cos[6*a + 6*b*x])/(32*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(8*
d^4) + (9*b^3*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(8*d^4)
- Sin[2*a + 2*b*x]/(32*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(16*d^3*(c +
 d*x)) + Sin[6*a + 6*b*x]/(96*d*(c + d*x)^3) - (3*b^2*Sin[6*a + 6*b*x])/(16
*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])
/(8*d^4) - (9*b^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(8*d
^4)
```

Rubi [A] time = 0.419222, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4}+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```


```
[Out] -(b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)^2) + (b*Cos[6*a + 6*b*x])/(32*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(8*
d^4) + (9*b^3*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(8*d^4)
- Sin[2*a + 2*b*x]/(32*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(16*d^3*(c +
 d*x)) + Sin[6*a + 6*b*x]/(96*d*(c + d*x)^3) - (3*b^2*Sin[6*a + 6*b*x])/(16
*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])
/(8*d^4) - (9*b^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(8*d
^4)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^4} - \frac{\sin(6a+6bx)}{32(c+dx)^4}\right)dx$$

$$= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^4}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{\sin(6a+6bx)}{96d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}dx}{16d} - \frac{b\int \frac{\cos(6a+6bx)}{(c+dx)^3}dx}{16d}$$

$$= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{\sin(6a+6bx)}{96d(c+dx)^3} - \frac{b^2\int \frac{\sin(2a+}{(c+dx}}{16d^2}$$

$$= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{16d^3(c+dx)} + \frac{\sin(6a+}{96d(c+}$$

$$= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{16d^3(c+dx)} + \frac{\sin(6a+}{96d(c+}$$

$$= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{8d^4} + \frac{9b^3\cos\left(6\right.}{}$$

Mathematica [A] time = 5.13101, size = 554, normalized size = 1.93

$$\frac{12b^3c^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + 36b^3c^2dx\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) - 108b^3c^3\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6b(c+dx)}{d}\right) - 324b^3c^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-3*b*c*d^2*Cos[2*(a + b*x)] - 3*b*d^3*x*Cos[2*(a + b*x)] + 3*b*c*d^2*Cos[6
*(a + b*x)] + 3*b*d^3*x*Cos[6*(a + b*x)] - 12*b^3*(c + d*x)^3*Cos[2*a - (2*
b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + 108*b^3*(c + d*x)^3*Cos[6*a - (6*b
*c)/d]*CosIntegral[(6*b*(c + d*x))/d] + 6*b^2*c^2*d*Sin[2*(a + b*x)] - 3*d^
3*Sin[2*(a + b*x)] + 12*b^2*c*d^2*x*Sin[2*(a + b*x)] + 6*b^2*d^3*x^2*Sin[2*
(a + b*x)] - 18*b^2*c^2*d*Sin[6*(a + b*x)] + d^3*Sin[6*(a + b*x)] - 36*b^2*
c*d^2*x*Sin[6*(a + b*x)] - 18*b^2*d^3*x^2*Sin[6*(a + b*x)] + 12*b^3*c^3*Sin
[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c^2*d*x*Sin[2*a -
 (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c*d^2*x^2*Sin[2*a - (2*
b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 12*b^3*d^3*x^3*Sin[2*a - (2*b*c)/d
]*SinIntegral[(2*b*(c + d*x))/d] - 108*b^3*c^3*Sin[6*a - (6*b*c)/d]*SinInte
gral[(6*b*(c + d*x))/d] - 324*b^3*c^2*d*x*Sin[6*a - (6*b*c)/d]*SinIntegral[
```

```
(6*b*(c + d*x))/d] - 324*b^3*c*d^2*x^2*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*
b*(c + d*x))/d] - 108*b^3*d^3*x^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c
+ d*x))/d])/(96*d^4*(c + d*x)^3)
```

Maple [A] time = 0.028, size = 404, normalized size = 1.4

$$\frac{1}{b}\left(-\frac{b^4}{192}\left(-2\frac{\sin(6bx+6a)}{((bx+a)d-ad+bc)^3 d}+2\frac{1}{d}\left(-3\frac{\cos(6bx+6a)}{((bx+a)d-ad+bc)^2 d}-3\frac{1}{d}\left(-6\frac{\sin(6bx+6a)}{((bx+a)d-ad+bc)d}+6\frac{1}{d}\left(6\frac{1}{d}\mathrm{Si}\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x)
```

```
[Out] 1/b*(-1/192*b^4*(-2*sin(6*b*x+6*a)/((b*x+a)*d-a*d+b*c)^3/d+2*(-3*cos(6*b*x+
6*a)/((b*x+a)*d-a*d+b*c)^2/d-3*(-6*sin(6*b*x+6*a)/((b*x+a)*d-a*d+b*c)/d+6*(
6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*b*x+6*a+6*(-a*d
+b*c)/d)*cos(6*(-a*d+b*c)/d)/d)/d)/d)/d)+3/64*b^4*(-2/3*sin(2*b*x+2*a)/((b*
x+a)*d-a*d+b*c)^3/d+2/3*(-cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d-(-2*sin(2*
b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*
d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d))
```

Maxima [C] time = 3.22064, size = 521, normalized size = 1.82

$$\frac{b^4\left(-3iE_4\left(\frac{2ibc+2i(bx+a)d-2iad}{d}\right)+3iE_4\left(-\frac{2ibc+2i(bx+a)d-2iad}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^4\left(iE_4\left(\frac{6ibc+6i(bx+a)d-6iad}{d}\right)-iE_4\left(-\frac{6ibc+6i}{\cdots}\right.\right.}{64\left(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(b^4*(-3*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 3*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a
*d)/d) - I*exp_integral_e(4, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos
(-6*(b*c - a*d)/d) - 3*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (6*I*b*c + 6*I*(b*x + a)*d -
 6*I*a*d)/d) + exp_integral_e(4, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))
*sin(-6*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
```

```
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Fricas [B] time = 0.761782, size = 1419, normalized size = 4.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*(b*d^3*x + b*c*d^2)*cos(b*x + a)^6 - 72*(b*d^3*x + b*c*d^2)*cos(b*
x + a)^4 + 24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 - 54*(b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(
b*d*x + b*c)/d) + 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^
3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 3*((b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d
) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(
-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(6*(b*d*x + b*c)/d) + (b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-6*(b*d*x + b
*c)/d))*cos(-6*(b*c - a*d)/d) - 16*((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b
^2*c^2*d - d^3)*cos(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*
c^2*d - d^3)*cos(b*x + a)^3 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*c
os(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.163 $\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=151

$$\text{Unintegrable}(\cot(a+bx)(c+dx)^m,x)+\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-3}e^{-}\cdots}{}$$

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Unintegrable[(c + d*x)^m*Cot[a + b*x], x]
```

Rubi [A] time = 0.172095, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Defer[Int][(c + d*x)^m*Cot[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\cos^2(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^m\cot(a+bx)\,dx-\int (c+dx)^m\cos(a+bx)\sin(a+bx)\,dx\\ &= \int (c+dx)^m\cot(a+bx)\,dx-\int \frac{1}{2}(c+dx)^m\sin(2a+2bx)\,dx\\ &= -\left(\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)\,dx\right)+\int (c+dx)^m\cot(a+bx)\,dx\\ &= -\left(\frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx\right)+\frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx+\int (c+dx)^m\cdots\\ &= \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(\cdots}{}\end{aligned}$$

Mathematica [A] time = 7.75228, size = 0, normalized size = 0.

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x], x]
```

Maple [A] time = 0.192, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cos(bx+a))^2 \cot(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^2 \cot(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \cos(bx+a)^2 \cot(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)^2 \cot(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

3.164 $\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=307

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{3d^4\text{PolyL}}{}$$

```
[Out] (-3*c*d^3*x)/(2*b^3) - (3*d^4*x^2)/(4*b^3) + (c + d*x)^4/(4*b) - ((I/5)*(c
+ d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c +
d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3,
 E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a +
b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) + (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) - (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 - (3*d^4*Sin[a + b*x]^2)/(4*b^5) + (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) - ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.339897, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4408, 4404, 3311, 32, 3310, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{3d^4\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (-3*c*d^3*x)/(2*b^3) - (3*d^4*x^2)/(4*b^3) + (c + d*x)^4/(4*b) - ((I/5)*(c
+ d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c +
d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3,
 E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a +
b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) + (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) - (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 - (3*d^4*Sin[a + b*x]^2)/(4*b^5) + (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) - ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
```

```
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^4\cot(a+bx)\,dx - \int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}}\,dx + \frac{(2d)\int (c+$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)^3\cos(a+bx)\sin(a+bx}{b^2}$$
$$= \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+}\right.}{b^2}$$
$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id}{}$$
$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id}{}$$
$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id}{}$$
$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id}{}$$

Mathematica [B] time = 6.52244, size = 2828, normalized size = 9.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] -((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*L
og[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)
*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E
^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a +
 b*x))]))/E^((2*I)*a)))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a)
 + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3
*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))
*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(
a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^(
(2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^
((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^4 -
(d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a))
*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 +
```

E^((-I)*(a + b*x))] - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, -E^((-I)*(a + b*x))] - (3*I)*b^2*x^2*PolyLog[3, -E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, -E^((-I)*(a + b*x))] + (6*I)*PolyLog[5, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, E^((-I)*(a + b*x))] - (3*I)*b^2*x^2*PolyLog[3, E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, E^((-I)*(a + b*x))] + (6*I)*PolyLog[5, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(10*b^5) + (c^4*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(160*b^5) - ((I/160)*Sin[2*a + 2*b*x])/b^5)*(80*b^5*c^4*x*Cos[a + 2*b*x] + 160*b^5*c^3*d*x^2*Cos[a + 2*b*x] + 160*b^5*c^2*d^2*x^3*Cos[a + 2*b*x] + 80*b^5*c*d^3*x^4*Cos[a + 2*b*x] + 16*b^5*d^4*x^5*Cos[a + 2*b*x] + 80*b^5*c^4*x*Cos[3*a + 2*b*x] + 160*b^5*c^3*d*x^2*Cos[3*a + 2*b*x] + 160*b^5*c^2*d^2*x^3*Cos[3*a + 2*b*x] + 80*b^5*c*d^3*x^4*Cos[3*a + 2*b*x] + 16*b^5*d^4*x^5*Cos[3*a + 2*b*x] + (10*I)*b^4*c^4*Cos[3*a + 4*b*x] - 20*b^3*c^3*d*Cos[3*a + 4*b*x] - (30*I)*b^2*c^2*d^2*Cos[3*a + 4*b*x] + 30*b*c*d^3*Cos[3*a + 4*b*x] + (15*I)*d^4*Cos[3*a + 4*b*x] + (40*I)*b^4*c^3*d*x*Cos[3*a + 4*b*x] - 60*b^3*c^2*d^2*x*Cos[3*a + 4*b*x] - (60*I)*b^2*c*d^3*x*Cos[3*a + 4*b*x] + 30*b*d^4*x*Cos[3*a + 4*b*x] + (60*I)*b^4*c^2*d^2*x^2*Cos[3*a + 4*b*x] - 60*b^3*c*d^3*x^2*Cos[3*a + 4*b*x] - (30*I)*b^2*d^4*x^2*Cos[3*a + 4*b*x] + (40*I)*b^4*c*d^3*x^3*Cos[3*a + 4*b*x] - 20*b^3*d^4*x^3*Cos[3*a + 4*b*x] + (10*I)*b^4*d^4*x^4*Cos[3*a + 4*b*x] - (10*I)*b^4*c^4*Cos[5*a + 4*b*x] + 20*b^3*c^3*d*Cos[5*a + 4*b*x] + (30*I)*b^2*c^2*d^2*Cos[5*a + 4*b*x] - 30*b*c*d^3*Cos[5*a + 4*b*x] - (15*I)*d^4*Cos[5*a + 4*b*x] - (40*I)*b^4*c^3*d*x*Cos[5*a + 4*b*x] + 60*b^3*c^2*d^2*x*Cos[5*a + 4*b*x] + (60*I)*b^2*c*d^3*x*Cos[5*a + 4*b*x] - 30*b*d^4*x*Cos[5*a + 4*b*x] - (60*I)*b^4*c^2*d^2*x^2*Cos[5*a + 4*b*x] + 60*b^3*c*d^3*x^2*Cos[5*a + 4*b*x] + (30*I)*b^2*d^4*x^2*Cos[5*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3*Cos[5*a + 4*b*x] + 20*b^3*d^4*x^3*Cos[5*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Cos[5*a + 4*b*x] + 20*b^4*c^4*Sin[a] - (40*I)*b^3*c^3*d*Sin[a] - 60*b^2*c^2*d^2*Sin[a] + (60*I)*b*c*d^3*Sin[a] + 30*d^4*Sin[a] + 80*b^4*c^3*d*x*Sin[a] - (120*I)*b^3*c^2*d^2*x*Sin[a] - 120*b^2*c*d^3*x*Sin[a] + (60*I)*b*d^4*x*Sin[a] + 120*b^4*c^2*d^2*x^2*Sin[a] - (120*I)*b^3*c*d^3*x^2*Sin[a] - 60*b^2*d^4*x^2*Sin[a] + 80*b^4*c*d^3*x^3*Sin[a] - (40*I)*b^3*d^4*x^3*Sin[a] + 20*b^4*d^4*x^4*Sin[a] + (80*I)*b^5*c^4*x*Sin[a + 2*b*x] + (160*I)*b^5*c^3*d*x^2*Sin[a + 2*b*x] + (160*I)*b^5*c^2*d^2*x^3*Sin[a + 2*b*x] + (80*I)*b^5*c*d^3*x^4*Sin[a + 2*b*x] + (16*I)*b^5*d^4*x^5*Sin[a + 2*b*x] + (80*I)*b^5*c^4*x*Sin[3*a + 2*b*x] + (160*I)*b^5*c^3*d*x^2*Sin[3*a + 2*b*x] + (160*I)*b^5*c^2*d^2*x^3*Sin[3*a + 2*b*x] + (80*I)*b^5*c*d^3*x^4*Sin[3*a + 2*b*x] + (16*I)*b^5*d^4*x^5*Sin[3*a + 2*b*x] - 10*b^4*c^4*Sin[3*a + 4*b*x] - (20*I)*b^3*c^3*d*Sin[3*a + 4*b*x] + 30*b^2*c^2*d^2*Sin[3*a + 4*b*x] + (30*I)*b*c*d^3*Sin[3*a + 4*b*x] - 15*d^4*Sin[3*a + 4*b*x] - 40*b^4*c^3*d*x*Sin[3*a + 4*b*x] - (60*I)*b^3*c^2*d^2*x*Sin[3*a + 4*b*x] + 60*b^2*c*d^3*x*Sin[3*a + 4*b*x] + (30*I)*b*d^4*x*Sin[3*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Sin[3*a + 4*b*x] - (60*I)*b^3*c*d^3*x^2*Sin[3*a + 4*b*x] + 30*b^2*d^4*x^2*Sin[3*a + 4*b*x] - 40*b^4*c*d^3*x^3*Sin[3*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Sin[3*a + 4*b*x] - 10*b^4*d^4*x^4*Sin[3*a + 4*b*x] + 10*b^4*c^4*Sin[5*a + 4*b*x] + (20*I)*b^3*c^3*d*Sin[5*a +

```
 4*b*x] - 30*b^2*c^2*d^2*Sin[5*a + 4*b*x] - (30*I)*b*c*d^3*Sin[5*a + 4*b*x]
 + 15*d^4*Sin[5*a + 4*b*x] + 40*b^4*c^3*d*x*Sin[5*a + 4*b*x] + (60*I)*b^3*c
^2*d^2*x*Sin[5*a + 4*b*x] - 60*b^2*c*d^3*x*Sin[5*a + 4*b*x] - (30*I)*b*d^4*
x*Sin[5*a + 4*b*x] + 60*b^4*c^2*d^2*x^2*Sin[5*a + 4*b*x] + (60*I)*b^3*c*d^3
*x^2*Sin[5*a + 4*b*x] - 30*b^2*d^4*x^2*Sin[5*a + 4*b*x] + 40*b^4*c*d^3*x^3*
Sin[5*a + 4*b*x] + (20*I)*b^3*d^4*x^3*Sin[5*a + 4*b*x] + 10*b^4*d^4*x^4*Sin
[5*a + 4*b*x]) - (2*c^3*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I
*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTa
n[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2
*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
 + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[
a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.49, size = 1492, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] 1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+I*c^4*x
+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/16*(2*d^4*x^4*b^4+4*I*b^3*d^4*x^3+8*b
^4*c*d^3*x^3+12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2+12*I*b^3*c^2*d^2*x+8*b^4
*c^3*d*x+4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2-6*I*b*d^4*x-12*b^2*c*d^3*x-6
*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp(2*I*(b*x+a))+4/b^4*c*d^3*ln(1-exp(I
*(b*x+a)))*a^3+8/5*I/b^5*d^4*a^5-I*c*d^3*x^4-2*I*c^2*d^2*x^3-2*I*c^3*d*x^2+
4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2
+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,-exp(I*(
b*x+a)))+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+6/b*c^2*d^2*ln(1-exp(I*(
b*x+a)))*x^2+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-6/b^3*c^2*d^2*a^2*ln(1-exp(I*
(b*x+a)))+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))
*a+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))
*x+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a
)))*x+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-1
/5*I*d^4*x^5+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*ln(exp(I*(b
*x+a))-1)-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b
*x+a))-1)+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a)
)-1)-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,-exp(I
*(b*x+a)))+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+2*I/b^4*d^4*a^4*x+8*I/b
^3*c^2*d^2*a^3-4*I/b^2*c^3*d*a^2-6*I/b^4*c*d^3*a^4-4*I/b^2*d^4*polylog(2,ex
p(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*p
```

```
olylog(4,-exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+24*I/b
^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-2
4*d^4*polylog(5,exp(I*(b*x+a)))/b^5-8*I/b*c^3*d*a*x+12*I/b^2*c^2*d^2*a^2*x-
12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,ex
p(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^4*ln(ex
p(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-2/b*c^4*ln(exp(I*(b*x+a)))+1/1
6*(2*d^4*x^4*b^4-4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3-12*I*b^3*c*d^3*x^2+12*b^4*
c^2*d^2*x^2-12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x-4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*
d^4*x^2+6*I*b*d^4*x-12*b^2*c*d^3*x+6*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp
(-2*I*(b*x+a))-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-8*I/b^3*c*d^3*a
^3*x
```

Maxima [B] time = 2.63538, size = 2207, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/40*(20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^4 - 80*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c^3*d/b + 120*(sin(b*x + a)^2 - log(sin(b*x + a)^2)
)*a^2*c^2*d^2/b^2 - 80*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*c*d^3/b^3
 + 20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^4*d^4/b^4 - (-8*I*(b*x + a)^
5*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^4 - 960*d^4*polylog(5, -e^(I
*b*x + I*a)) - 960*d^4*polylog(5, e^(I*b*x + I*a)) + (-80*I*b^2*c^2*d^2 + 1
60*I*a*b*c*d^3 - 80*I*a^2*d^4)*(b*x + a)^3 + (-80*I*b^3*c^3*d + 240*I*a*b^2
*c^2*d^2 - 240*I*a^2*b*c*d^3 + 80*I*a^3*d^4)*(b*x + a)^2 + (40*I*(b*x + a)^
4*d^4 + (160*I*b*c*d^3 - 160*I*a*d^4)*(b*x + a)^3 + (240*I*b^2*c^2*d^2 - 48
0*I*a*b*c*d^3 + 240*I*a^2*d^4)*(b*x + a)^2 + (160*I*b^3*c^3*d - 480*I*a*b^2
*c^2*d^2 + 480*I*a^2*b*c*d^3 - 160*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x +
a), cos(b*x + a) + 1) + (-40*I*(b*x + a)^4*d^4 + (-160*I*b*c*d^3 + 160*I*a*
d^4)*(b*x + a)^3 + (-240*I*b^2*c^2*d^2 + 480*I*a*b*c*d^3 - 240*I*a^2*d^4)*(
b*x + a)^2 + (-160*I*b^3*c^3*d + 480*I*a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 +
160*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 5*(2*(
b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 3*(2*a^2 - 1)*d^4 + 8*(b*c*
d^3 - a*d^4)*(b*x + a)^3 + 6*(2*b^2*c^2*d^2 - 4*a*b*c*d^3 + (2*a^2 - 1)*d^4
)*(b*x + a)^2 + 4*(2*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 3*(2*a^2 - 1)*b*c*d^3 -
(2*a^3 - 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-160*I*b^3*c^3*d + 480*I*
a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 - 160*I*(b*x + a)^3*d^4 + 160*I*a^3*d^4 +
 (-480*I*b*c*d^3 + 480*I*a*d^4)*(b*x + a)^2 + (-480*I*b^2*c^2*d^2 + 960*I*a
*b*c*d^3 - 480*I*a^2*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (-160*I*b^3*
```

```
c^3*d + 480*I*a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 - 160*I*(b*x + a)^3*d^4 + 1
60*I*a^3*d^4 + (-480*I*b*c*d^3 + 480*I*a*d^4)*(b*x + a)^2 + (-480*I*b^2*c^2
*d^2 + 960*I*a*b*c*d^3 - 480*I*a^2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) +
 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*
b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b
*x + a) + 1) + 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*cos(b*x + a) + 1) + (960*I*b*c*d^3 + 960*I*(b*x + a)*d^4 - 960*I*a
*d^4)*polylog(4, -e^(I*b*x + I*a)) + (960*I*b*c*d^3 + 960*I*(b*x + a)*d^4 -
 960*I*a*d^4)*polylog(4, e^(I*b*x + I*a)) + 480*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^
(I*b*x + I*a)) + 480*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4
 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e^(I*b*x + I*a)) - 10*(2*b^3*c
^3*d - 6*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(2*a^2 - 1)*b*c*d^3 - (2*a^3
 - 3*a)*d^4 + 6*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(2*b^2*c^2*d^2 - 4*a*b*c*
d^3 + (2*a^2 - 1)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/b^4)/b
```

Fricas [C] time = 0.922434, size = 3460, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 48*d^4*polylog(5, cos(b*x + a) + I*si
n(b*x + a)) + 48*d^4*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 48*d^4*pol
ylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 48*d^4*polylog(5, -cos(b*x + a) -
 I*sin(b*x + a)) + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b^4*d^4*x^4 + 8*b^4
*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4
)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(2*b^3*d^4*x^3
+ 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*
cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)*x - (-8*I*b^3*d^4
*x^3 - 24*I*b^3*c*d^3*x^2 - 24*I*b^3*c^2*d^2*x - 8*I*b^3*c^3*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) - (8*I*b^3*d^4*x^3 + 24*I*b^3*c*d^3*x^2 + 24*I*b^
3*c^2*d^2*x + 8*I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (8*I*b^
3*d^4*x^3 + 24*I*b^3*c*d^3*x^2 + 24*I*b^3*c^2*d^2*x + 8*I*b^3*c^3*d)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) - (-8*I*b^3*d^4*x^3 - 24*I*b^3*c*d^3*x^2 -
24*I*b^3*c^2*d^2*x - 8*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4
```

```
*c^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3
*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*si
n(b*x + a) + 1) - 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/
2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x
^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2
+ 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^4
*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^
3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*si
n(b*x + a) + 1) - (48*I*b*d^4*x + 48*I*b*c*d^3)*polylog(4, cos(b*x + a) + I
*sin(b*x + a)) - (-48*I*b*d^4*x - 48*I*b*c*d^3)*polylog(4, cos(b*x + a) - I
*sin(b*x + a)) - (-48*I*b*d^4*x - 48*I*b*c*d^3)*polylog(4, -cos(b*x + a) +
I*sin(b*x + a)) - (48*I*b*d^4*x + 48*I*b*c*d^3)*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3,
 cos(b*x + a) + I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2
*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c
*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 24*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x
 + a)))/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \cos(bx+a)^2 \cot(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)^2*cot(b*x + a), x)
```

3.165 $\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=246

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\sin^2(a}{4b^3}$$

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) - ((I/4)*(c + d*x)^4)/d + ((c + d*x)
^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^
((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])
/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x
])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin[a
+ b*x]^2)/(2*b)
```

Rubi [A] time = 0.278089, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4408, 4404, 3311, 32, 2635, 8, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\sin^2(a}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```


```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) - ((I/4)*(c + d*x)^4)/d + ((c + d*x)
^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^
((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])
/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x
])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin[a
+ b*x]^2)/(2*b)
```


Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^3\cot(a+bx)\,dx - \int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx + \frac{(3d)\int(c+$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)\sin(a+b}{4b^2}$$
$$= \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2\left(e^{2i(a+}\right.}{2b^2}$$
$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2}{2}$$
$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2}{2}$$
$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2}{2}$$

Mathematica [B] time = 6.40242, size = 1918, normalized size = 7.8

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] -(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*
a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[
1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a
 + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((
2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*
x))]))/E^((2*I)*a)))/(2*b^3) - (d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) +
 (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(
1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(
b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a
+ b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2
*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((
-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(4*b^4)
+ (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a
]))/(b*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)
*Sin[2*a + 2*b*x])/b^4)*(32*b^4*c^3*x*Cos[a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos
[a + 2*b*x] + 32*b^4*c*d^2*x^3*Cos[a + 2*b*x] + 8*b^4*d^3*x^4*Cos[a + 2*b*x
] + 32*b^4*c^3*x*Cos[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos[3*a + 2*b*x] + 32*
```

```
b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + 8*b^4*d^3*x^4*Cos[3*a + 2*b*x] + (4*I)*b^3
*c^3*Cos[3*a + 4*b*x] - 6*b^2*c^2*d*Cos[3*a + 4*b*x] - (6*I)*b*c*d^2*Cos[3*
a + 4*b*x] + 3*d^3*Cos[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*Cos[3*a + 4*b*x] -
 12*b^2*c*d^2*x*Cos[3*a + 4*b*x] - (6*I)*b*d^3*x*Cos[3*a + 4*b*x] + (12*I)*
b^3*c*d^2*x^2*Cos[3*a + 4*b*x] - 6*b^2*d^3*x^2*Cos[3*a + 4*b*x] + (4*I)*b^3
*d^3*x^3*Cos[3*a + 4*b*x] - (4*I)*b^3*c^3*Cos[5*a + 4*b*x] + 6*b^2*c^2*d*Co
s[5*a + 4*b*x] + (6*I)*b*c*d^2*Cos[5*a + 4*b*x] - 3*d^3*Cos[5*a + 4*b*x] -
(12*I)*b^3*c^2*d*x*Cos[5*a + 4*b*x] + 12*b^2*c*d^2*x*Cos[5*a + 4*b*x] + (6*
I)*b*d^3*x*Cos[5*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*Cos[5*a + 4*b*x] + 6*b^2
*d^3*x^2*Cos[5*a + 4*b*x] - (4*I)*b^3*d^3*x^3*Cos[5*a + 4*b*x] + 8*b^3*c^3*
Sin[a] - (12*I)*b^2*c^2*d*Sin[a] - 12*b*c*d^2*Sin[a] + (6*I)*d^3*Sin[a] + 2
4*b^3*c^2*d*x*Sin[a] - (24*I)*b^2*c*d^2*x*Sin[a] - 12*b*d^3*x*Sin[a] + 24*b
^3*c*d^2*x^2*Sin[a] - (12*I)*b^2*d^3*x^2*Sin[a] + 8*b^3*d^3*x^3*Sin[a] + (3
2*I)*b^4*c^3*x*Sin[a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Sin[a + 2*b*x] + (32*I
)*b^4*c*d^2*x^3*Sin[a + 2*b*x] + (8*I)*b^4*d^3*x^4*Sin[a + 2*b*x] + (32*I)*
b^4*c^3*x*Sin[3*a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] + (32*I)
*b^4*c*d^2*x^3*Sin[3*a + 2*b*x] + (8*I)*b^4*d^3*x^4*Sin[3*a + 2*b*x] - 4*b^
3*c^3*Sin[3*a + 4*b*x] - (6*I)*b^2*c^2*d*Sin[3*a + 4*b*x] + 6*b*c*d^2*Sin[3
*a + 4*b*x] + (3*I)*d^3*Sin[3*a + 4*b*x] - 12*b^3*c^2*d*x*Sin[3*a + 4*b*x]
- (12*I)*b^2*c*d^2*x*Sin[3*a + 4*b*x] + 6*b*d^3*x*Sin[3*a + 4*b*x] - 12*b^3
*c*d^2*x^2*Sin[3*a + 4*b*x] - (6*I)*b^2*d^3*x^2*Sin[3*a + 4*b*x] - 4*b^3*d^
3*x^3*Sin[3*a + 4*b*x] + 4*b^3*c^3*Sin[5*a + 4*b*x] + (6*I)*b^2*c^2*d*Sin[5
*a + 4*b*x] - 6*b*c*d^2*Sin[5*a + 4*b*x] - (3*I)*d^3*Sin[5*a + 4*b*x] + 12*
b^3*c^2*d*x*Sin[5*a + 4*b*x] + (12*I)*b^2*c*d^2*x*Sin[5*a + 4*b*x] - 6*b*d^
3*x*Sin[5*a + 4*b*x] + 12*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] + (6*I)*b^2*d^3*x^
2*Sin[5*a + 4*b*x] + 4*b^3*d^3*x^3*Sin[5*a + 4*b*x]) - (3*c^2*d*Csc[a]*Sec[
a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Lo
g[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + Ar
cTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[
Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1
+ Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.438, size = 1001, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] 3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3*I/b
^2*c^2*d*a^2+4*I/b^3*c*d^2*a^3-2*I/b^3*d^3*a^3*x-3*I/b^2*d^3*polylog(2,exp(
```

```
I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*c^2*d*po
lylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+6*I*d^3*po
lylog(4,exp(I*(b*x+a)))/b^4+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1
-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-6/b^3*c*d^2*a^2*ln(ex
p(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b^2*c^2*d*a*ln(exp(I*(
b*x+a)))-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x
+a)))+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b
^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+I*c^3*x-I*c*d^2*x^3-3/2*I*c^2*d*x^2+6/b^3*d
^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3
*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+2/b
^4*d^3*a^3*ln(exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)-3/2*I/b^4*
d^3*a^4+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-6*I/b*c^2*d*a*x+6*I/b^2*c*d^
2*a^2*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-
exp(I*(b*x+a)))*x+1/32*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2+12*I
*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d
^2*b)/b^4*exp(2*I*(b*x+a))-2/b*c^3*ln(exp(I*(b*x+a)))+1/b*c^3*ln(exp(I*(b*x
+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)-1/4*I*d^3*x^4+1/32*(4*d^3*x^3*b^3-6*I*
b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+
4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b^4*exp(-2*I*(b*x+a))
```

Maxima [B] time = 2.01961, size = 1305, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^3 - 24*(sin(b*x + a)^2 -
log(sin(b*x + a)^2))*a*c^2*d/b + 24*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*
a^2*c*d^2/b^2 - 8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*d^3/b^3 - (-4*
I*(b*x + a)^4*d^3 + (-16*I*b*c*d^2 + 16*I*a*d^3)*(b*x + a)^3 + 96*I*d^3*pol
ylog(4, -e^(I*b*x + I*a)) + 96*I*d^3*polylog(4, e^(I*b*x + I*a)) + (-24*I*b
^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3)*(b*x + a)^2 + (16*I*(b*x + a)^3*d
^3 + (48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a)^2 + (48*I*b^2*c^2*d - 96*I*a*b*c
*d^2 + 48*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (
-16*I*(b*x + a)^3*d^3 + (-48*I*b*c*d^2 + 48*I*a*d^3)*(b*x + a)^2 + (-48*I*b
^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a),
-cos(b*x + a) + 1) + 2*(2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b
*x + a))*cos(2*b*x + 2*a) + (-48*I*b^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*(b*x +
 a)^2*d^3 - 48*I*a^2*d^3 + (-96*I*b*c*d^2 + 96*I*a*d^3)*(b*x + a))*dilog(-e
```

```
^(I*b*x + I*a)) + (-48*I*b^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*(b*x + a)^2*d^3
- 48*I*a^2*d^3 + (-96*I*b*c*d^2 + 96*I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I
*a)) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*cos(b*x + a) + 1) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) + 96*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*pol
ylog(3, -e^(I*b*x + I*a)) + 96*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3,
 e^(I*b*x + I*a)) - 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a
^2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))/b^3)/b
```

Fricas [C] time = 0.825092, size = 2437, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, cos(b*x + a) +
I*sin(b*x + a)) + 24*I*d^3*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 24*I
*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 24*I*d^3*polylog(4, -cos(
b*x + a) - I*sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*
c^2*d - b*d^3)*x - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*
dilog(cos(b*x + a) + I*sin(b*x + a)) - (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x
 + 12*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (12*I*b^2*d^3*x^2
 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a))
 - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*dilog(-cos(b*x +
 a) - I*sin(b*x + a)) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) -
 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*d^3*x
^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*
d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*
x + a) - I*sin(b*x + a) + 1) - 24*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x +
a) + I*sin(b*x + a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*s
in(b*x + a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x
```

```
+ a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/
b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \cos(bx+a)^2 \cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)^2*cot(b*x + a), x)
```

3.166 $\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=181

$$-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c-$$

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 + (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^2
*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.226928, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4408, 4404, 3310, 3717, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 + (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^2
*Sin[a + b*x]^2)/(2*b)
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^2\cot(a+bx)\,dx - \int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx + \frac{d\int (c+dx)$$
$$= -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2}$$
$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx}\right.}{b^2}$$
$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx}\right.}{b^2}$$
$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx}\right.}{b^2}$$

Mathematica [B] time = 2.98839, size = 564, normalized size = 3.12

$$\frac{-48ibcd\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+96ibd^2x\text{PolyLog}\left(2,-e^{-i(a+bx)}\right)+96ibd^2x\text{PolyLog}\left(2,e^{-i(a+bx)}\right)+96d^2\text{PolyL}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] ((48*I)*b^2*c*d*Pi*x + (16*I)*b^3*d^2*x^3 - (96*I)*b^2*c*d*x*ArcTan[Tan[a]]
 + 48*b^3*c*d*x^2*Cot[a] - 6*b*c*d*Cos[a + 2*b*x]*Csc[a] - 6*b*d^2*x*Cos[a
+ 2*b*x]*Csc[a] + 6*b*c*d*Cos[3*a + 2*b*x]*Csc[a] + 6*b*d^2*x*Cos[3*a + 2*b
*x]*Csc[a] + 48*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 48*b^2*d^2*x^2*Log[1 - E
^((-I)*(a + b*x))] + 48*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 96*b^2*c*
d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 96*b*c*d*ArcTan[Tan[a]]*Log
[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 48*b*c*d*Pi*Log[Cos[b*x]] + 48*b^2
*c^2*Log[Sin[a + b*x]] - 96*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a
]]]] + (96*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (96*I)*b*d^2*x*Poly
Log[2, E^((-I)*(a + b*x))] - (48*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + ArcTan
[Tan[a]]))] + 96*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 96*d^2*PolyLog[3, E^
((-I)*(a + b*x))] - 48*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^
2] - 6*b^2*c^2*Csc[a]*Sin[a + 2*b*x] + 3*d^2*Csc[a]*Sin[a + 2*b*x] - 12*b^2
```

```
*c*d*x*Csc[a]*Sin[a + 2*b*x] - 6*b^2*d^2*x^2*Csc[a]*Sin[a + 2*b*x] + 6*b^2*
c^2*Csc[a]*Sin[3*a + 2*b*x] - 3*d^2*Csc[a]*Sin[3*a + 2*b*x] + 12*b^2*c*d*x*
Csc[a]*Sin[3*a + 2*b*x] + 6*b^2*d^2*x^2*Csc[a]*Sin[3*a + 2*b*x])/(48*b^3)
```

Maple [B] time = 0.47, size = 590, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] 1/16*(2*d^2*x^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-d^2)/b^3*ex
p(-2*I*(b*x+a))-4*I/b*c*d*a*x-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln
(exp(I*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-2*I/b^2*c*d*polylog
(2,exp(I*(b*x+a)))+I*c^2*x+2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b*c^2*ln(exp(I*
(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))+1)+1/b*c^2*ln(exp(I*(b*x+a))-1)-1/3*I*d
^2*x^3+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))
/b^3+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-2*I/b^
2*d^2*polylog(2,exp(I*(b*x+a)))*x+1/16*(2*d^2*x^2*b^2+2*I*b*d^2*x+4*b^2*c*d
*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*I*(b*x+a))-2/b^3*d^2*a^2*ln(exp(I*(b*
x+a)))+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+4/3*I/b^3*d^2*a^3-I*c*d*x^2-2/b^2
*c*d*a*ln(exp(I*(b*x+a))-1)+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-2*I/b^2*c*d*a^2-
2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+
a)))+2*I/b^2*d^2*a^2*x
```

Maxima [B] time = 1.67788, size = 705, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*(12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^2 - 24*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c*d/b + 12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a
^2*d^2/b^2 - (-8*I*(b*x + a)^3*d^2 + (-24*I*b*c*d + 24*I*a*d^2)*(b*x + a)^2
 + 48*d^2*polylog(3, -e^(I*b*x + I*a)) + 48*d^2*polylog(3, e^(I*b*x + I*a))
 + (24*I*(b*x + a)^2*d^2 + (48*I*b*c*d - 48*I*a*d^2)*(b*x + a))*arctan2(sin
```

```
(b*x + a), cos(b*x + a) + 1) + (-24*I*(b*x + a)^2*d^2 + (-48*I*b*c*d + 48*I
*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 3*(2*(b*x + a
)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (-48*I*b*c*
d - 48*I*(b*x + a)*d^2 + 48*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (-48*I*b*c*d
 - 48*I*(b*x + a)*d^2 + 48*I*a*d^2)*dilog(e^(I*b*x + I*a)) + 12*((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
2*cos(b*x + a) + 1) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 6*(b*c*d + (b*x +
 a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

Fricas [C] time = 0.689083, size = 1538, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 - 4*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 4
*d^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x
+ a) + I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) +
 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (-4*I*b*d^2*x - 4*I*b*c*d)
*dilog(cos(b*x + a) + I*sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(cos
(b*x + a) - I*sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(-cos(b*x + a)
 + I*sin(b*x + a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*dilog(-cos(b*x + a) - I*sin
(b*x + a)) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*s
in(b*x + a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
 - I*sin(b*x + a) + 1) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x +
a) + 1))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \cos^2 (a + bx) \cot (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\cos(bx+a)^2\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)^2*cot(b*x + a), x)
```

3.167 $\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=114

$$-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}-\frac{i(c+d}{2d}$$

```
[Out] (d*x)/(4*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Sin
[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.127923, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4408, 4404, 2635, 8, 3717, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}-\frac{i(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```


```
[Out] (d*x)/(4*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Sin
[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```


Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)\cot(a+bx)\,dx - \int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^2}{2d} - \frac{(c+dx)\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx + \frac{d\int \sin^2(a+b}{2b}$$
$$= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{(c+dx)}{}$$
$$= \frac{dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{(c+}{}$$
$$= \frac{dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} - \frac{d\cos(a+bx)}{4b^2}$$

Mathematica [A] time = 0.352627, size = 131, normalized size = 1.15

$$\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2} - \frac{d\sin(2(a+bx))}{8b^2} - \frac{ad\log(\sin(a+bx))}{b^2} - \frac{c\sin^2(}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (c*Log[Sin[a + b*x]])/b - (a*d*Log[Sin[a + b
*x]])/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2
 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 - (c*Sin[a + b*x]^2)/(2*b) - (d*S
in[2*(a + b*x)])/(8*b^2)
```

Maple [B] time = 0.383, size = 271, normalized size = 2.4

$$icx - \frac{i}{2}dx^2 + \frac{(2\,dxb+id+2\,bc)\,e^{2\,i(bx+a)}}{16\,b^2} + \frac{(2\,dxb-id+2\,bc)\,e^{-2\,i(bx+a)}}{16\,b^2} + \frac{c\ln\left(e^{i(bx+a)}-1\right)}{b} - 2\,\frac{c\ln\left(e^{i(bx+a)}\right)}{b} + \frac{c\ln\left(e^{i(b}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] I*c*x-1/2*I*d*x^2+1/16*(2*d*x*b+I*d+2*b*c)/b^2*exp(2*I*(b*x+a))+1/16*(2*d*x
*b-I*d+2*b*c)/b^2*exp(-2*I*(b*x+a))+1/b*c*ln(exp(I*(b*x+a))-1)-2/b*c*ln(exp
(I*(b*x+a)))+1/b*c*ln(exp(I*(b*x+a))+1)-I/b^2*d*a^2-2*I/b*d*a*x-I*d*polylog
```

```
(2,-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*
x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-1/b
^2*d*a*ln(exp(I*(b*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 1.53569, size = 300, normalized size = 2.63

$$\frac{-4i\,b^2dx^2 - 8i\,b^2cx - 8i\,bdx\arctan\left(\sin\left(bx+a\right), -\cos\left(bx+a\right)+1\right) + 8i\,bc\arctan\left(\sin\left(bx+a\right), \cos\left(bx+a\right)-1\right) + (8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*I*b*d*x*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) + 8*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (8*I*b*d*x
+ 8*I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) - 8*I*d*dilog(-e^(I*b*x + I*a)) - 8*I*d*dilog(e^(I*b*x + I*a)) +
 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
+ 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
 - d*sin(2*b*x + 2*a))/b^2
```

Fricas [B] time = 0.601803, size = 844, normalized size = 7.4

$$-\frac{bdx - 2\left(bdx+bc\right)\cos\left(bx+a\right)^2 + d\cos\left(bx+a\right)\sin\left(bx+a\right) + 2i\,d\mathrm{Li}_2\left(\cos\left(bx+a\right) + i\sin\left(bx+a\right)\right) - 2i\,d\mathrm{Li}_2\left(\cos\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
+ 2*I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - 2*I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 2*I*d*dilog(
-cos(b*x + a) - I*sin(b*x + a)) - 2*(b*d*x + b*c)*log(cos(b*x + a) + I*sin(
b*x + a) + 1) - 2*(b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 2*
(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b*c - a*
d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b*d*x + a*d)*log(
-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*d*x + a*d)*log(-cos(b*x + a) - I
*sin(b*x + a) + 1))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)**2*cot(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\cos(bx+a)^2\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)^2*cot(b*x + a), x)
```

3.168 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=81

$$\text{Unintegrable}\left(\frac{\cot(a+bx)}{c+dx},x\right)-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Unintegrable[Cot[a + b*x]
/(c + d*x), x]
```

Rubi [A] time = 0.144313, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Cot[a + b*x]/(
c + d*x), x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx &= \int \frac{\cot(a+bx)}{c+dx}\,dx - \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx \\
&= \int \frac{\cot(a+bx)}{c+dx}\,dx - \int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx \\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\right) + \int \frac{\cot(a+bx)}{c+dx}\,dx \\
&= -\left(\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\right) - \frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx + \\
&= -\frac{\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} - \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d} + \int \frac{\cot(a+bx)}{c+dx}\,dx
\end{aligned}
$$

Mathematica [A] time = 0.794424, size = 0, normalized size = 0.

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x), x]
```

Maple [A] time = 0.355, size = 0, normalized size = 0.

$$\int \frac{(\cos(bx+a))^2\cot(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(-iE_1\left(\frac{2ibdx+2ibc}{d}\right)+iE_1\left(-\frac{2ibdx+2ibc}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+4d\int\frac{\sin(bx+a)}{(dx+c)\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)}\,dx-4d\int\frac{}{(dx+c)(}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*((-I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + I*exp_integral_e(1,
-(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*d*integrate(sin(b*x +
a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)
*cos(b*x + a) + c), x) - 4*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)
^2 + (d*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) - (
exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(1, -(2*I*b*d*x
+ 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))/d
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)^2\cot(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)^2 \cot(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

3.169 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=101

$$\text{Unintegrable}\left(\frac{\cot(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a}{2d(c}$$

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Unintegrable[Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.1703, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx - \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$$
$$= \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx$$
$$= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\right) + \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d} + \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,d}{d}$$
$$= -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} + \frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2} + \int$$

Mathematica [A] time = 2.49899, size = 0, normalized size = 0.

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.63, size = 0, normalized size = 0.

$$\int \frac{(\cos(bx+a))^2\cot(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(bx+a\right)^{2}\cot\left(bx+a\right)}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos^{2}\left(a+bx\right)\cot\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(bx+a\right)^{2}\cot\left(bx+a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c)^2, x)
```

3.170 $\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=153

$$\text{CannotIntegrate}\left(\cot(a+bx)\csc(a+bx)(c+dx)^m,x\right)+\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-$$

```
[Out] CannotIntegrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x] + ((I/2)*E^(I*(a
- (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(b*(((-I)*b*(c
+ d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(b*E^(I
*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.239148, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*
x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x
)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx=-\int (c+dx)^m\cos(a+bx)\,dx+\int (c+dx)^m\cot(a+bx)\csc(a+bx)\,dx$$
$$=-\left(\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx\right)-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx)^m\cot(a$$
$$=\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c\right.}{}}{}$$

Mathematica [A] time = 3.96008, size = 0, normalized size = 0.

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2, x]
```

Maple [A] time = 0.164, size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)(\cot(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\cot(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \cos(bx+a)\cot(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cos(bx+a)\cot(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

3.171 $\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{12id^2}{}$$

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 + (24*d^3*(c + d*x)*Cos[a +
 b*x])/b^4 - (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5 - (24*d^4*Sin[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*
Sin[a + b*x])/b^3 - ((c + d*x)^4*Sin[a + b*x])/b
```

Rubi [A] time = 0.292642, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4408, 3296, 2637, 4410, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{12id^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```


```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 + (24*d^3*(c + d*x)*Cos[a +
 b*x])/b^4 - (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5 - (24*d^4*Sin[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*
Sin[a + b*x])/b^3 - ((c + d*x)^4*Sin[a + b*x])/b
```


Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
```

```
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)^4\cos(a+bx)\,dx + \int (c+dx)^4\cot(a+bx)\csc(a+bx)\,dx$$

$$= -\frac{(c+dx)^4\csc(a+bx)}{b} - \frac{(c+dx)^4\sin(a+bx)}{b} + \frac{(4d)\int (c+dx)^3\csc(a+bx)\,d}{b}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{(c+dx)^4\csc(a+b}{b}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{(c+dx)^4\csc(a+b}{b}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d(c+dx)^3\cos(a}{b^2}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d(c+dx)^3\cos(a}{b^2}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d(c+dx)^3\cos(a}{b^2}$$

Mathematica [B] time = 1.72215, size = 798, normalized size = 2.67

$$\frac{\csc(a+bx)\left(-3c^4b^4 - 3d^4x^4b^4 - 12cd^3x^3b^4 - 18c^2d^2x^2b^4 - 12c^3dxb^4 + c^4\cos(2(a+bx))b^4 + d^4x^4\cos(2(a+bx))b^4 + 4cd\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]*(-3*b^4*c^4 + 12*b^2*c^2*d^2 - 24*d^4 - 12*b^4*c^3*d*x + 24*b
^2*c*d^3*x - 18*b^4*c^2*d^2*x^2 + 12*b^2*d^4*x^2 - 12*b^4*c*d^3*x^3 - 3*b^4
*d^4*x^4 + b^4*c^4*Cos[2*(a + b*x)] - 12*b^2*c^2*d^2*Cos[2*(a + b*x)] + 24*
d^4*Cos[2*(a + b*x)] + 4*b^4*c^3*d*x*Cos[2*(a + b*x)] - 24*b^2*c*d^3*x*Cos[
2*(a + b*x)] + 6*b^4*c^2*d^2*x^2*Cos[2*(a + b*x)] - 12*b^2*d^4*x^2*Cos[2*(a
 + b*x)] + 4*b^4*c*d^3*x^3*Cos[2*(a + b*x)] + b^4*d^4*x^4*Cos[2*(a + b*x)]
- 16*b^3*c^3*d*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 48*b^3
*c^2*d^2*x*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 48*b^3*c*d
^3*x^2*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 16*b^3*d^4*x^3
*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + (24*I)*b^2*d^2*(c +
d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] - (24*I)*b^2
*d^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 4
8*b*c*d^3*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] - 48*b*d^
4*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] + 48*b*c*d^3*Po
lyLog[3, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + 48*b*d^4*x*PolyLog[3
, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - (48*I)*d^4*PolyLog[4, -Cos[
a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] + (48*I)*d^4*PolyLog[4, Cos[a + b*x
] + I*Sin[a + b*x]]*Sin[a + b*x] - 4*b^3*c^3*d*Sin[2*(a + b*x)] + 24*b*c*d^
3*Sin[2*(a + b*x)] - 12*b^3*c^2*d^2*x*Sin[2*(a + b*x)] + 24*b*d^4*x*Sin[2*(
a + b*x)] - 12*b^3*c*d^3*x^2*Sin[2*(a + b*x)] - 4*b^3*d^4*x^3*Sin[2*(a + b*
x)]))/(2*b^5)
```

Maple [B] time = 0.209, size = 1056, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] -4*d^4/b^5*ln(exp(I*(b*x+a))+1)*a^3-24*d^3/b^4*c*a^2*arctanh(exp(I*(b*x+a))
)+24*d^2/b^3*c^2*a*arctanh(exp(I*(b*x+a)))-24*I*d^3/b^3*c*polylog(2,exp(I*(
b*x+a)))*x+8*d^4/b^5*a^3*arctanh(exp(I*(b*x+a)))-8*d/b^2*c^3*arctanh(exp(I*
(b*x+a)))+4*d^4/b^2*ln(1-exp(I*(b*x+a)))*x^3+24*d^4/b^4*polylog(3,exp(I*(b*
x+a)))*x-24*d^4/b^4*polylog(3,-exp(I*(b*x+a)))*x-24*d^3/b^4*c*polylog(3,-ex
p(I*(b*x+a)))+24*d^3/b^4*c*polylog(3,exp(I*(b*x+a)))+12*d^3/b^4*c*a^2*ln(ex
p(I*(b*x+a))+1)-12*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)*a+12*I*d^2/b^3*c^2*poly
log(2,-exp(I*(b*x+a)))+12*I*d^4/b^3*polylog(2,-exp(I*(b*x+a)))*x^2+24*I*d^4
*polylog(4,exp(I*(b*x+a)))/b^5-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3
*d*x+c^4)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-1/2*I*(d^4*x^4*b^4+4*b^4*c*
d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*
x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2-4*I
```

```
*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))-12*d^3/b^2
*c*ln(exp(I*(b*x+a))+1)*x^2+12*d^3/b^2*c*ln(1-exp(I*(b*x+a)))*x^2-12*d^3/b^
4*c*ln(1-exp(I*(b*x+a)))*a^2-12*I*d^2/b^3*c^2*polylog(2,exp(I*(b*x+a)))-12*
I*d^4/b^3*polylog(2,exp(I*(b*x+a)))*x^2+24*I*d^3/b^3*c*polylog(2,-exp(I*(b*
x+a)))*x-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+1/2*I*(d^4*x^4*b^4+4*b^4*c
*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4
*x^2+12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2+4*
I*b^3*c^3*d-24*I*b*d^4*x+24*d^4-24*I*b*c*d^3)/b^5*exp(I*(b*x+a))+4*d^4/b^5*
ln(1-exp(I*(b*x+a)))*a^3-4*d^4/b^2*ln(exp(I*(b*x+a))+1)*x^3+12*d^2/b^2*c^2*
ln(1-exp(I*(b*x+a)))*x+12*d^2/b^3*c^2*ln(1-exp(I*(b*x+a)))*a-12*d^2/b^2*c^2
*ln(exp(I*(b*x+a))+1)*x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 0.88748, size = 3009, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 12*b^2*c^2*d^2 - 12*I*d^4*p
olylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4,
 cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*polylog(4, -cos(b*x
 + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -cos(b*x + a) -
I*sin(b*x + a))*sin(b*x + a) + 24*d^4 + 12*(b^4*c^2*d^2 - b^2*d^4)*x^2 - (b
^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c
^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4
*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 -
2*b*d^4)*x)*cos(b*x + a)*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*
x - 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6
```

```
*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) - I
*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^
2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b^2*d^
4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin(b*x
 + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x +
b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b^3*d^4
*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a) - I*
sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c
*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x +
 a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*co
s(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^
3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*
log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b
^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)
*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x + b*c*d
^3)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^4*x +
b*c*d^3)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4
*x + b*c*d^3)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*
(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a)
 + 8*(b^4*c^3*d - 3*b^2*c*d^3)*x)/(b^5*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \cos(bx + a) \cot(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a)^2, x)
```

3.172 $\int (c+dx)^3 \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=216

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}\left(3,\right.}{b^4}$$

```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 + (6*d^3*Cos[a + b*x])/b^4
- (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I
)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 +
 (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*Sin[a + b*x])/b
^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rubi [A] time = 0.222293, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4408, 3296, 2638, 4410, 4183, 2531, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}\left(3,\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 + (6*d^3*Cos[a + b*x])/b^4
- (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I
)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 +
 (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*Sin[a + b*x])/b
^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)^3\cos(a+bx)\,dx + \int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx$$
$$= -\frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b} + \frac{(3d)\int (c+dx)^2\csc(a+bx)}{b}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} - \frac{(c+dx)^3\csc(a+}{b}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} - \frac{(c+dx)^3\csc(a+}{b}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} -$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} -$$

Mathematica [B] time = 1.46379, size = 539, normalized size = 2.5

$$\frac{\csc(a+bx)\left(12ibd^2(c+dx)\sin(a+bx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right) - 12ibd^2(c+dx)\sin(a+bx)\text{PolyLog}\left(2,e^{i(a+bx)}\right) - 12d^3\text{ s}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]*(-3*b^3*c^3 + 6*b*c*d^2 - 9*b^3*c^2*d*x + 6*b*d^3*x - 9*b^3*c
*d^2*x^2 - 3*b^3*d^3*x^3 + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a +
b*x)] + 3*b^3*c^2*d*x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3
*c*d^2*x^2*Cos[2*(a + b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] + 6*b^2*c^2*d*Lo
g[1 - E^(I*(a + b*x))]*Sin[a + b*x] + 12*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x)
)]*Sin[a + b*x] + 6*b^2*d^3*x^2*Log[1 - E^(I*(a + b*x))]*Sin[a + b*x] - 6*b
^2*c^2*d*Log[1 + E^(I*(a + b*x))]*Sin[a + b*x] - 12*b^2*c*d^2*x*Log[1 + E^(
I*(a + b*x))]*Sin[a + b*x] - 6*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))]*Sin[a +
 b*x] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))]*Sin[a + b*x] -
(12*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))]*Sin[a + b*x] - 12*d^3*Po
lyLog[3, -E^(I*(a + b*x))]*Sin[a + b*x] + 12*d^3*PolyLog[3, E^(I*(a + b*x))
]*Sin[a + b*x] - 3*b^2*c^2*d*Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a + b*x)] - 6*
b^2*c*d^2*x*Sin[2*(a + b*x)] - 3*b^2*d^3*x^2*Sin[2*(a + b*x)]))/(2*b^4)
```

Maple [B] time = 0.189, size = 649, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*
b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))
-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))
-1)-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*
x^2-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)
))*x-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/b^4*ln(exp(I*(b*x+a))+1)*a^2+
6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))-6*d^3/b^4*a^2*arctanh(exp(I*(b*x+a
)))-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-6*d^2/b^3*c*ln(exp(I*(b*x+a))+1)*a-1
/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b
*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))
-6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))+6*d^3*polylog(3,exp(I*(b*x+a)))/b^
4-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x+6
*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a+12*d^2/b^3*c*a*arctanh(exp(I*(b*x+a)))-6*
d/b^2*c^2*arctanh(exp(I*(b*x+a)))
```

Maxima [B] time = 6.75717, size = 14876, normalized size = 68.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^3*(1/sin(b*x + a) + sin(b*x + a)) - 6*a*c^2*d*(1/sin(b*x + a) + s
in(b*x + a))/b + 6*a^2*c*d^2*(1/sin(b*x + a) + sin(b*x + a))/b^2 - 2*a^3*d^
3*(1/sin(b*x + a) + sin(b*x + a))/b^3 - 3*(((b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a)
+ a - sin(2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2
*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + c
os(b*x + a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*co
s(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)
^2 + (8*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*
```

```
a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) -
 sin(b*x + a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a
)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x
 + a)*cos(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) +
 cos(b*x + a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2
 + cos(b*x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2
)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3
*a) + 2*(3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a
)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x +
a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + c
os(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x
+ a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*
a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)
^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
 + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x +
 a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) +
 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)
^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin
(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*
(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a
)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*
a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*si
n(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x
+ 2*a) + a - sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2
*a)^2 + (b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x +
 a)*sin(b*x + a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2
*b*x + 2*a) - (b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x +
a))*sin(2*b*x + 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x
+ a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*
(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(
2*b*x + 2*a) + a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a
)*cos(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)
^2 + b*x + a)*sin(b*x + a) - cos(b*x + a))*c^2*d/(((cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x +
a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + c
os(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x
+ a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*
a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)
```

```
^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
 + 3*a) + sin(b*x + a)^2)*b) + 6*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x +
 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin
(2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x
+ a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x +
a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x +
 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(
b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(
2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x
+ a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b
*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos
(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*
x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b
*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(
3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x
 + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
 a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos
(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(
b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*
x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) +
 sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
+ ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x +
3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*
x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2
*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin
(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(
b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a
) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) +
a - sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (
b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b
*x + a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*
a) - (b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2
*b*x + 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 +
13*(b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)
*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2
*a) + a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x
 + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x
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+ a)*sin(b*x + a) - cos(b*x + a))*a*c*d^2/(((cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
 + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x
+ a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*c
os(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + co
s(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(
b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a)
 + sin(b*x + a)^2)*b^2) - 3*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*
x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*
cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*c
os(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)
^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x +
 a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x
 + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))
*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x +
a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x
+ a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^
2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a
)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x +
2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*
x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a)
 + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
 + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2
)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*
x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x +
 a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a
) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(
b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((c
os(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x
+ 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*
x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^
2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2
*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*c
os(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x
+ a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x +
 a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + s
in(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) + a - s
in(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (b*x +

a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) - (b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2*b*x + 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) - cos(b*x + a))*a^2*d^3/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b^3) - 2*(-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*(I*a - 1)*d^3 + (-3*I*b*c*d^2 - 3*(-I*a + 1)*d^3)*(b*x + a)^2 + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 - 6*(-I*a - 1)*d^3 + (3*I*b*c*d^2 - 3*(I*a + 1)*d^3)*(b*x + a)^2 - (6*b*c*d^2 - (6*a - 6*I)*d^3)*(b*x + a))*cos(3*b*x + 3*a)^2 + (6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2)*cos(b*x + a)^2 + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*(I*a + 1)*d^3 + (-3*I*b*c*d^2 - 3*(-I*a - 1)*d^3)*(b*x + a)^2 + (6*b*c*d^2 - (6*a - 6*I)*d^3)*(b*x + a))*sin(3*b*x + 3*a)^2 - 12*((b*x + a)^3*d^3 - 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(b*x + a)*sin(b*x + a) + (-6*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2)*sin(b*x + a)^2 - (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b*x + a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x

+ a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x
+ a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a)
+ (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(b*x +
a) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*
d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^
3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*
a) - (6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-
6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))
*sin(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin
(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + ((-7*I*(b*x + a)^3*d^
3 + 18*I*b*c*d^2 - 6*(3*I*a + 1)*d^3 + (-21*I*b*c*d^2 - 3*(-7*I*a - 1)*d^3)
*(b*x + a)^2 + (6*b*c*d^2 - (6*a - 18*I)*d^3)*(b*x + a))*cos(b*x + a) + (7*
(b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I
)*d^3)*(b*x + a)^2 + (6*I*b*c*d^2 - 6*(I*a + 3)*d^3)*(b*x + a))*sin(b*x + a
))*cos(3*b*x + 3*a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 - 6*(-I*a + 1)*d^3 +
(3*I*b*c*d^2 - 3*(I*a - 1)*d^3)*(b*x + a)^2 + (6*b*c*d^2 - (6*a + 6*I)*d^3
)*(b*x + a))*cos(2*b*x + 2*a) + ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I
*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(2*b*x + 2*a)
- 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) +
((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(b*x + a) + 12*(b*c*
d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*cos(2*b*x + 2*a) + (12*I*b*c*d^2
+ 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(b*x + a) + (12*b*c*d^2 + 12*(b*x +
a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) +
(-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3*
b*x + 3*a) + (12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(b*x + a) + (12*I*b*c
*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(b*x + a))*sin(2*b*x + 2*a) - 12
*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*dilog(-e^(I*b*x + I*a)) +
((12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b
*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 -
a*d^3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((12*I*b*c*d^2 + 12*I*(b*x + a)
*d^3 - 12*I*a*d^3)*cos(b*x + a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(
b*x + a))*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d
^3)*cos(b*x + a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2
+ (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (12*I*b*c*d^2 + 12*I*(b*x + a)*
d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (12*(b*c*d^2 + (b*x
+ a)*d^3 - a*d^3)*cos(b*x + a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I
*a*d^3)*sin(b*x + a))*sin(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^
3)*sin(b*x + a))*dilog(e^(I*b*x + I*a)) + ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2
- a*d^3)*(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*co
s(2*b*x + 2*a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x +
a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 -
a*d^3)*(b*x + a))*cos(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I
*a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*
(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c
*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a

```
*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) + ((3*I*(b*x + a)^2*d^3 + (
6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(
b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) + (-3*I*(b*x + a
)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((3*(b*x + a)^2*d^3 + 6*(b*c
*d^2 - a*d^3)*(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*cos(2*b*x + 2*a) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x
 + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2
+ 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (-
6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 -
6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 -
a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - ((-3*I*(b*x + a)^2*d
^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(b*x + a) + 3*((b*x + a)^2*d^
3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) - (3*I*(b
*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (12*d^3*cos(b*x + a) +
12*I*d^3*sin(b*x + a) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) -
 d^3)*cos(3*b*x + 3*a) - 12*(d^3*cos(b*x + a) + I*d^3*sin(b*x + a))*cos(2*b
*x + 2*a) - (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^
3)*sin(3*b*x + 3*a) - (12*I*d^3*cos(b*x + a) - 12*d^3*sin(b*x + a))*sin(2*b
*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) + (12*d^3*cos(b*x + a) + 12*I*d^3*s
in(b*x + a) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3)*cos(
3*b*x + 3*a) - 12*(d^3*cos(b*x + a) + I*d^3*sin(b*x + a))*cos(2*b*x + 2*a)
+ (12*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*sin(3*b*
x + 3*a) + (-12*I*d^3*cos(b*x + a) + 12*d^3*sin(b*x + a))*sin(2*b*x + 2*a))
*polylog(3, e^(I*b*x + I*a)) - ((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + (12*a - 1
2*I)*d^3 + (6*b*c*d^2 - (6*a - 6*I)*d^3)*(b*x + a)^2 - (-12*I*b*c*d^2 - 12*
(-I*a - 1)*d^3)*(b*x + a))*cos(3*b*x + 3*a) - (7*(b*x + a)^3*d^3 - 18*b*c*d
^2 + (18*a - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*
b*c*d^2 - 6*(I*a + 3)*d^3)*(b*x + a))*cos(b*x + a) - (7*I*(b*x + a)^3*d^3 -
 18*I*b*c*d^2 - 6*(-3*I*a - 1)*d^3 + (21*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*
x + a)^2 - (6*b*c*d^2 - (6*a - 18*I)*d^3)*(b*x + a))*sin(b*x + a))*sin(3*b*
x + 3*a) - ((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a + 6*I)*d^3 + (3*b*c*d^2 - (3
*a + 3*I)*d^3)*(b*x + a)^2 - (6*I*b*c*d^2 - 6*(I*a - 1)*d^3)*(b*x + a))*sin
(2*b*x + 2*a))/(2*b^3*cos(b*x + a) + 2*I*b^3*sin(b*x + a) + (2*b^3*cos(2*b*
x + 2*a) + 2*I*b^3*sin(2*b*x + 2*a) - 2*b^3)*cos(3*b*x + 3*a) - (2*b^3*cos(
b*x + a) + 2*I*b^3*sin(b*x + a))*cos(2*b*x + 2*a) - (-2*I*b^3*cos(2*b*x + 2
*a) + 2*b^3*sin(2*b*x + 2*a) + 2*I*b^3)*sin(3*b*x + 3*a) + 2*(-I*b^3*cos(b*
x + a) + b^3*sin(b*x + a))*sin(2*b*x + 2*a)))/b
```

Fricas [C] time = 0.725879, size = 2018, normalized size = 9.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 4*b^3*c^3 - 6*d^3*polylog(3, cos(b
*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*
sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a
))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x
+ a) - 12*b*c*d^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2
+ 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*
d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I
*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x
 - 6*I*b*c*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b
*d^3*x + 6*I*b*c*d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) +
3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x +
a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(
b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a
^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1)*sin(b*x + a) + 12*(b^3*c^2*d - b*d^3)*x)/(b^4*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \cos (bx + a) \cot (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a)^2, x)
```

3.173 $\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\, dx$

Optimal. Leaf size=139

$$\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{2d^2\text{si}}{}$$

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (2*d*(c + d*x)*Cos[a + b*x]
)/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x
))])/b^3 - ((2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 + (2*d^2*Sin[a + b*x
])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```

Rubi [A] time = 0.14899, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4408, 3296, 2637, 4410, 4183, 2279, 2391}

$$\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{2d^2\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (2*d*(c + d*x)*Cos[a + b*x]
)/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x
))])/b^3 - ((2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 + (2*d^2*Sin[a + b*x
])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2 \cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\,dx + \int (c+dx)^2\cot(a+bx)\csc(a+bx)\,dx$$

$$= -\frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b} + \frac{(2d)\int(c+dx)\csc(a+bx)\,d}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\csc(a+b}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\csc(a+b}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\csc(a+b}{b}$$

Mathematica [B] time = 3.95586, size = 310, normalized size = 2.23

$$-\frac{-4d^2\left(2\tan^{-1}(\tan(a))\tanh^{-1}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right) + \frac{\sec(a)\left(i\text{PolyLog}\left(2,-e^{i\left(\tan^{-1}(\tan(a))+bx\right)}\right)-i\text{PolyLog}\left(2,e^{i\left(\tan^{-1}(\tan(a))+bx\right)}\right)\right.}{\sqrt{s}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -(8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + 2*b^2*(c + d*x)^2*Csc[a]
- 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*x +
ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x
+ ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*Poly
Log[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + 2*Cos[b*x]*
(2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - b^2*(c + d*x
)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] + b^2*(c + d*x)^2*Sec[a/2]*Sec[(
a + b*x)/2]*Sin[(b*x)/2] + 2*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c
+ d*x)*Sin[a])*Sin[b*x])/(2*b^3)
```

Maple [B] time = 0.16, size = 332, normalized size = 2.4

$$\frac{\frac{i}{2}\left(d^2x^2b^2+2\,b^2cdx+b^2c^2+2\,ibd^2x-2\,d^2+2\,ibcd\right)e^{i(bx+a)}}{b^3} - \frac{\frac{i}{2}\left(d^2x^2b^2+2\,b^2cdx+b^2c^2-2\,ibd^2x-2\,d^2-2\,ibcd\right)e^{-i(}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp
(I*(b*x+a))-1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*
c*d)/b^3*exp(-I*(b*x+a))-2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp(2*
I*(b*x+a))-1)-4*d/b^2*c*arctanh(exp(I*(b*x+a)))+2*d^2/b^2*ln(1-exp(I*(b*x+a
)))*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^
3-2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*d^2/b^3*ln(exp(I*(b*x+a))+1)*a+2*I*d^2
*polylog(2,-exp(I*(b*x+a)))/b^3+4*d^2/b^3*a*arctanh(exp(I*(b*x+a)))
```

Maxima [B] time = 5.39042, size = 4435, normalized size = 31.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (b^2*d^2*x^2*(-I*cos(a) + sin(a)) + b^2*c^2*(-I*cos(a) + sin(a)) - b*c*d*(2
*cos(a) + 2*I*sin(a)) - 2*d^2*(-I*cos(a) + sin(a)) - (2*b^2*c*d*(I*cos(a) -
 sin(a)) + b*d^2*(2*cos(a) + 2*I*sin(a)))*x - ((4*b*d^2*x*(-I*cos(a) + sin(
a)) + 4*b*c*d*(-I*cos(a) + sin(a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*cos(2*b*x +
 3*a) - 4*(b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) - ((4*I*b*d^
2*x + 4*I*b*c*d)*cos(b*x + a) - 4*(b*d^2*x + b*c*d)*sin(b*x + a))*cos(2*b*x
 + 3*a) + 4*(b*d^2*x*(I*cos(a) - sin(a)) + b*c*d*(I*cos(a) - sin(a)))*cos(b
*x + a) + (b*d^2*x*(4*cos(a) + 4*I*sin(a)) + b*c*d*(4*cos(a) + 4*I*sin(a))
- 4*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) - (4*I*b*d^2*x + 4*I*b*c*d)*sin(2*b*
x + 3*a))*sin(3*b*x + 3*a) + (4*(b*d^2*x + b*c*d)*cos(b*x + a) - (-4*I*b*d^
2*x - 4*I*b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) - (b*d^2*x*(4*cos(a) + 4*I*
sin(a)) + b*c*d*(4*cos(a) + 4*I*sin(a)))*sin(b*x + a))*arctan2(sin(b*x + a)
, cos(b*x + a) + 1) - (4*b*c*d*(-I*cos(a) + sin(a))*cos(b*x + a) + b*c*d*(4
*cos(a) + 4*I*sin(a))*sin(b*x + a) + (4*b*c*d*(I*cos(a) - sin(a)) - 4*I*b*c
*d*cos(2*b*x + 3*a) + 4*b*c*d*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(I*b*c
*d*cos(b*x + a) - b*c*d*sin(b*x + a))*cos(2*b*x + 3*a) - (b*c*d*(4*cos(a) +
 4*I*sin(a)) - 4*b*c*d*cos(2*b*x + 3*a) - 4*I*b*c*d*sin(2*b*x + 3*a))*sin(3
*b*x + 3*a) - (4*b*c*d*cos(b*x + a) + 4*I*b*c*d*sin(b*x + a))*sin(2*b*x + 3
*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (4*b*d^2*x*(I*cos(a) - sin(a
))*cos(b*x + a) - b*d^2*x*(4*cos(a) + 4*I*sin(a))*sin(b*x + a) + 4*(b*d^2*x
*(-I*cos(a) + sin(a)) + I*b*d^2*x*cos(2*b*x + 3*a) - b*d^2*x*sin(2*b*x + 3*
a))*cos(3*b*x + 3*a) + 4*(-I*b*d^2*x*cos(b*x + a) + b*d^2*x*sin(b*x + a))*c
os(2*b*x + 3*a) + (b*d^2*x*(4*cos(a) + 4*I*sin(a)) - 4*b*d^2*x*cos(2*b*x +
3*a) - 4*I*b*d^2*x*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + (4*b*d^2*x*cos(b*x
```

```
+ a) + 4*I*b*d^2*x*sin(b*x + a))*sin(2*b*x + 3*a))*arctan2(sin(b*x + a), -c
os(b*x + a) + 1) + ((I*b^2*d^2*x^2 + I*b^2*c^2 - 2*b*c*d - 2*I*d^2 + (2*I*b
^2*c*d - 2*b*d^2)*x)*cos(3*b*x + 3*a) + (-I*b^2*d^2*x^2 - I*b^2*c^2 + 2*b*c
*d + 2*I*d^2 + (-2*I*b^2*c*d + 2*b*d^2)*x)*cos(b*x + a) - (b^2*d^2*x^2 + b^
2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*sin(3*b*x + 3*a) + (b^
2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*sin(b*x
+ a))*cos(3*b*x + 4*a) + ((-6*I*b^2*d^2*x^2 - 12*I*b^2*c*d*x - 6*I*b^2*c^2
+ 4*I*d^2)*cos(b*x + 2*a) + 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*
d^2)*sin(b*x + 2*a))*cos(3*b*x + 3*a) + (I*b^2*d^2*x^2 + I*b^2*c^2 + 2*b*c*
d - 2*I*d^2 + (2*I*b^2*c*d + 2*b*d^2)*x)*cos(2*b*x + 3*a) + ((6*I*b^2*d^2*x
^2 + 12*I*b^2*c*d*x + 6*I*b^2*c^2 - 4*I*d^2)*cos(b*x + a) - 2*(3*b^2*d^2*x^
2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(b*x + a))*cos(b*x + 2*a) - (4*d^2*
(-I*cos(a) + sin(a))*cos(b*x + a) + d^2*(4*cos(a) + 4*I*sin(a))*sin(b*x + a
) + (4*d^2*(I*cos(a) - sin(a)) - 4*I*d^2*cos(2*b*x + 3*a) + 4*d^2*sin(2*b*x
 + 3*a))*cos(3*b*x + 3*a) - (-4*I*d^2*cos(b*x + a) + 4*d^2*sin(b*x + a))*co
s(2*b*x + 3*a) - (d^2*(4*cos(a) + 4*I*sin(a)) - 4*d^2*cos(2*b*x + 3*a) - 4*
I*d^2*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - 4*(d^2*cos(b*x + a) + I*d^2*sin(
b*x + a))*sin(2*b*x + 3*a))*dilog(-e^(I*b*x + I*a)) - (4*d^2*(I*cos(a) - si
n(a))*cos(b*x + a) - d^2*(4*cos(a) + 4*I*sin(a))*sin(b*x + a) + (4*d^2*(-I*
cos(a) + sin(a)) + 4*I*d^2*cos(2*b*x + 3*a) - 4*d^2*sin(2*b*x + 3*a))*cos(3
*b*x + 3*a) - (4*I*d^2*cos(b*x + a) - 4*d^2*sin(b*x + a))*cos(2*b*x + 3*a)
+ (d^2*(4*cos(a) + 4*I*sin(a)) - 4*d^2*cos(2*b*x + 3*a) - 4*I*d^2*sin(2*b*x
 + 3*a))*sin(3*b*x + 3*a) + 4*(d^2*cos(b*x + a) + I*d^2*sin(b*x + a))*sin(2
*b*x + 3*a))*dilog(e^(I*b*x + I*a)) + ((b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b
*c*d*(2*cos(a) + 2*I*sin(a)) - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) + (-2*I
*b*d^2*x - 2*I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (2*(b*d^2*x + b*
c*d)*cos(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a
) - (b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(2*cos(a) + 2*I*sin(a)))*cos(b
*x + a) - (2*b*d^2*x*(-I*cos(a) + sin(a)) + 2*b*c*d*(-I*cos(a) + sin(a)) -
(-2*I*b*d^2*x - 2*I*b*c*d)*cos(2*b*x + 3*a) - 2*(b*d^2*x + b*c*d)*sin(2*b*x
 + 3*a))*sin(3*b*x + 3*a) + ((2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a) - 2*(b*
d^2*x + b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) - 2*(b*d^2*x*(I*cos(a) - sin(
a)) + b*c*d*(I*cos(a) - sin(a)))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x
 + a)^2 + 2*cos(b*x + a) + 1) - ((b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(
2*cos(a) + 2*I*sin(a)) - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) - (2*I*b*d^2*
x + 2*I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (2*(b*d^2*x + b*c*d)*co
s(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a) - (b
*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(2*cos(a) + 2*I*sin(a)))*cos(b*x + a
) + (2*b*d^2*x*(I*cos(a) - sin(a)) + 2*b*c*d*(I*cos(a) - sin(a)) - (2*I*b*d
^2*x + 2*I*b*c*d)*cos(2*b*x + 3*a) + 2*(b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*
sin(3*b*x + 3*a) - ((-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a) + 2*(b*d^2*x +
b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) + 2*(b*d^2*x*(-I*cos(a) + sin(a)) + b
*c*d*(-I*cos(a) + sin(a)))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 - 2*cos(b*x + a) + 1) - ((b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(
b^2*c*d + I*b*d^2)*x)*cos(3*b*x + 3*a) - (b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d
```

```
 - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*cos(b*x + a) - (-I*b^2*d^2*x^2 - I*b^2*
c^2 + 2*b*c*d + 2*I*d^2 + (-2*I*b^2*c*d + 2*b*d^2)*x)*sin(3*b*x + 3*a) - (I
*b^2*d^2*x^2 + I*b^2*c^2 - 2*b*c*d - 2*I*d^2 + (2*I*b^2*c*d - 2*b*d^2)*x)*s
in(b*x + a))*sin(3*b*x + 4*a) + (2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2
 - 2*d^2)*cos(b*x + 2*a) + (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x + 6*I*b^2*c^2
- 4*I*d^2)*sin(b*x + 2*a))*sin(3*b*x + 3*a) - (b^2*d^2*x^2 + b^2*c^2 - 2*I*
b*c*d - 2*d^2 + 2*(b^2*c*d - I*b*d^2)*x)*sin(2*b*x + 3*a) - (2*(3*b^2*d^2*x
^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(b*x + a) - (-6*I*b^2*d^2*x^2 - 12
*I*b^2*c*d*x - 6*I*b^2*c^2 + 4*I*d^2)*sin(b*x + a))*sin(b*x + 2*a))/(b^3*(2
*cos(a) + 2*I*sin(a))*cos(b*x + a) + 2*b^3*(I*cos(a) - sin(a))*sin(b*x + a)
 - (b^3*(2*cos(a) + 2*I*sin(a)) - 2*b^3*cos(2*b*x + 3*a) - 2*I*b^3*sin(2*b*
x + 3*a))*cos(3*b*x + 3*a) - (2*b^3*cos(b*x + a) + 2*I*b^3*sin(b*x + a))*co
s(2*b*x + 3*a) + (2*b^3*(-I*cos(a) + sin(a)) + 2*I*b^3*cos(2*b*x + 3*a) - 2
*b^3*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + 2*(-I*b^3*cos(b*x + a) + b^3*sin(
b*x + a))*sin(2*b*x + 3*a))
```

Fricas [B] time = 0.61068, size = 1172, normalized size = 8.43

$$-\frac{2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+i\,d^2\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-i\,d^2\mathrm{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*si
n(b*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b
*x + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*
dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*s
in(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(
b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
 + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*s
in(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) +
1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) +
1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
*sin(b*x + a) - 2*d^2)/(b^3*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2\cos(a+bx)\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\cos(bx+a)\cot(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a)^2, x)
```

3.174 $\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{d\cos(a+bx)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - (d*Cos[a + b*x])/b^2 - ((c + d*x)*Csc[a
+ b*x])/b - ((c + d*x)*Sin[a + b*x])/b
```

Rubi [A] time = 0.0631534, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4408, 3296, 2638, 4410, 3770}

$$-\frac{d\cos(a+bx)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - (d*Cos[a + b*x])/b^2 - ((c + d*x)*Csc[a
+ b*x])/b - ((c + d*x)*Sin[a + b*x])/b
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)\cos(a+bx)\,dx + \int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$$
$$= -\frac{(c+dx)\csc(a+bx)}{b} - \frac{(c+dx)\sin(a+bx)}{b} + \frac{d\int \csc(a+bx)\,dx}{b} + \frac{d\int \sin(a$$
$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\csc(a+bx)}{b} - \frac{(c+dx)\sin$$

Mathematica [A] time = 0.706704, size = 104, normalized size = 1.79

$$-\frac{2bc\sin(a+bx)+2bc\csc(a+bx)+2bdx\sin(a+bx)+2d\cos(a+bx)+bdx\tan\left(\frac{1}{2}(a+bx)\right)+bdx\cot\left(\frac{1}{2}(a+bx)\right)-}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -(2*d*Cos[a + b*x] + b*d*x*Cot[(a + b*x)/2] + 2*b*c*Csc[a + b*x] + 2*d*Log[
Cos[(a + b*x)/2]] - 2*d*Log[Sin[(a + b*x)/2]] + 2*b*c*Sin[a + b*x] + 2*b*d*
x*Sin[a + b*x] + b*d*x*Tan[(a + b*x)/2])/(2*b^2)
```

Maple [C] time = 0.119, size = 124, normalized size = 2.1

$$\frac{\frac{i}{2}(dxb+bc+id)\,e^{i(bx+a)}}{b^2} - \frac{\frac{i}{2}(dxb+bc-id)\,e^{-i(bx+a)}}{b^2} - \frac{2\,ie^{i(bx+a)}(dx+c)}{b\left(e^{2\,i(bx+a)}-1\right)} - \frac{d\ln\left(e^{i(bx+a)}+1\right)}{b^2} + \frac{d\ln\left(e^{i(bx+a)}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d*x*b+b*c+I*d)/b^2*exp(I*(b*x+a))-1/2*I*(d*x*b+b*c-I*d)/b^2*exp(-I*(
b*x+a))-2*I*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))-1)-d/b^2*ln(exp(I*(b
*x+a))+1)+d/b^2*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 1.18379, size = 2849, normalized size = 49.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*(1/sin(b*x + a) + sin(b*x + a)) - 2*a*d*(1/sin(b*x + a) + sin(b*x
 + a))/b - (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x +
 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*sin(3*b
*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*cos(b*x + a)*sin(
2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) +
 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)^2 - ((b*x + a)*s
in(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*cos(2*b*x + 2
*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(
3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)
 - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) +
 cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a)
 - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^
2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a)^2 + (13*(b*x +
a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) + sin(2*b*x
+ 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*sin(b*x +
a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) + cos(b*x + a))*
cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
+ 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*
x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)
+ 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a
)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2
- 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos
(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*
b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x +
 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((cos(2*b*x + 2*a)^2
```

```
 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b
*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)
^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a
) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos
(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x
 + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x +
 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(
3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*
cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (b*x + a)*cos(b*x + a)^
2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + b*x + 2*((
(b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) - (b*x + a)*cos(b*x
 + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2*b*x + 2*a) - sin(b*x
+ a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x +
 a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x
+ a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*sin(3*b*x + 3
*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x + a)*sin(b*x + a)^2)*
sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) - co
s(b*x + a))*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x +
2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2
- 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x
 + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x
+ 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a
)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b))/b
```

Fricas [A] time = 0.521035, size = 269, normalized size = 4.64

$$-\frac{4\,bdx - 2\,(bdx + bc)\cos{(bx + a)}^{2} + 2\,d\cos{(bx + a)}\sin{(bx + a)} + d\log\left(\frac{1}{2}\,\cos{(bx + a)} + \frac{1}{2}\right)\sin{(bx + a)} - d\log\left(-\frac{1}{2}\,c\right.}{2\,b^{2}\sin{(bx + a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
 a) + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos(b*x + a)
+ 1/2)*sin(b*x + a) + 4*b*c)/(b^2*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x)**2, x)
```

Giac [B] time = 2.22117, size = 2655, normalized size = 45.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
6*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3 -
 d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3
*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)
*tan(1/2*a) + 1))*tan(1/2*b*x)^4*tan(1/2*a)^3 + d*log(4*(tan(1/2*a)^2 + 1)/
(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2
 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)
^4*tan(1/2*a)^3 + 6*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - d*log(4*(tan(1/2*a)
^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/
2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))*ta
n(1/2*b*x)^3*tan(1/2*a)^4 + d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*
tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 +
2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*
b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*d*t
an(1/2*b*x)^4*tan(1/2*a)^3 + 6*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/
2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan(1/2*b*x)^4 - 8*b*d*x*tan(1/2*b*x)^3*tan(1
/2*a) - d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2
*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1
/2*b*x)*tan(1/2*a) + 1))*tan(1/2*b*x)^4*tan(1/2*a) + d*log(4*(tan(1/2*a)^2
+ 1)/(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2
*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2
*b*x)^4*tan(1/2*a) - 12*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 8*b*d*x*tan(1/2
```

```
*b*x)*tan(1/2*a)^3 + b*d*x*tan(1/2*a)^4 - d*log(4*(tan(1/2*a)^2 + 1)/(tan(1
/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1
/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))*tan(1/2*b*x)*tan
(1/2*a)^4 + d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*t
an(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*t
an(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4 -
 8*b*c*tan(1/2*b*x)^3*tan(1/2*a) + 2*d*tan(1/2*b*x)^4*tan(1/2*a) - 12*b*c*t
an(1/2*b*x)^2*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 8*b*c*tan(1
/2*b*x)*tan(1/2*a)^3 + 12*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + b*c*tan(1/2*a)^4
+ 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 6*b*d*x*tan(1/2*b*x)^2 + d*log(4*(tan(1/2
*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan
(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))
*tan(1/2*b*x)^3 - d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*tan(1/2*b*
x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*
b*x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x)^3 + 8*b*d*x*tan(1/2*b*x)*tan(
1/2*a) + 6*b*d*x*tan(1/2*a)^2 + d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*
tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 +
tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))*tan(1/2*a)^3 - d*log(4*(ta
n(1/2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x
)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^
2))*tan(1/2*a)^3 + 6*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2*b*x)^3 + 8*b*c*tan(1/
2*b*x)*tan(1/2*a) - 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 6*b*c*tan(1/2*a)^2 - 1
2*d*tan(1/2*b*x)*tan(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + d*log(4*(tan(1/2
*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan
(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))
*tan(1/2*b*x) - d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)
^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*
x)*tan(1/2*a) + tan(1/2*a)^2))*tan(1/2*b*x) + d*log(4*(tan(1/2*a)^2 + 1)/(t
an(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*t
an(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1))*tan(1/2*a) -
 d*log(4*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) +
 tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) +
 tan(1/2*a)^2))*tan(1/2*a) + b*c + 2*d*tan(1/2*b*x) + 2*d*tan(1/2*a))/(b^2*
tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^2*tan(1/2
*b*x)^4*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2*a)^4 - b^2*tan(1/2*b*x)^3 - b
^2*tan(1/2*a)^3 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

3.175 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=74

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)-\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x] - (Cos[a - (b*c)/
d]*CosIntegral[(b*c)/d + b*x])/d + (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d +
b*x])/d
```

Rubi [A] time = 0.213677, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x
), x]
```

Rubi steps

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx=-\int \frac{\cos(a+bx)}{c+dx}\,dx+\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

$$=-\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)+\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int \frac{\cot(a+b}{c}$$

$$=-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 3.70494, size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x), x]
```

Maple [A] time = 0.346, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)(\cot(bx+a))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(a+bx)}\cot^2{(a+bx)}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(bx+a)}\cot{(bx+a)}^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

3.176 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=92

$$\text{CannotIntegrate}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2},x\right)+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+$$

```
[Out] CannotIntegrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x] + Cos[a + b*x]/
(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d^2 + (b*Co
s[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```

Rubi [A] time = 0.256355, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][(C
ot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d}+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\int\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \frac{\cot(a+b}{(c}
\end{aligned}$$

Mathematica [A] time = 4.00792, size = 0, normalized size = 0.

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A] time = 0.492, size = 0, normalized size = 0.

$$\int \frac{\cos(bx+a)(\cot(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(bx+a)\cot(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (a + bx) \cot^2 (a + bx)}{(c + dx)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (bx + a) \cot (bx + a)^2}{(dx + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c)^2, x)
```

3.177 $\int (c+dx)^m \cot^3(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\cot^3(a+bx)(c+dx)^m, x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Cot[a + b*x]^3, x]
```

Rubi [A] time = 0.0356774, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \cot^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^3,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (c+dx)^m \cot^3(a+bx)\,dx$$

Mathematica [A] time = 5.53587, size = 0, normalized size = 0.

$$\int (c+dx)^m \cot^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^3,x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^3, x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int (dx+c)^m (\cot (bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cot(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cot (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \cot (bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cot(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \cot (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

3.178 $\int (c+dx)^4 \cot^3(a+bx)\,dx$

Optimal. Leaf size=302

$$-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}-\frac{6id^3(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}+\frac{2id(c+$$

```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (c + d*x)^4/(2*b) + ((I/5)*(c + d*x)^5)/d - (2
*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x)^4*Cot[a + b*x]^2)/(2*b) + (6*
d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^4*Log[1 - E^
((2*I)*(a + b*x))])/b - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^4
*PolyLog[3, E^((2*I)*(a + b*x))])/b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, E^((2
*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))]
)/b^4 + (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rubi [A] time = 0.462334, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3717, 2190, 2531, 2282, 6589, 32, 6609}

$$-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}-\frac{6id^3(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}+\frac{2id(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^3,x]
```


```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (c + d*x)^4/(2*b) + ((I/5)*(c + d*x)^5)/d - (2
*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x)^4*Cot[a + b*x]^2)/(2*b) + (6*
d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^4*Log[1 - E^
((2*I)*(a + b*x))])/b - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^4
*PolyLog[3, E^((2*I)*(a + b*x))])/b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, E^((2
*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))]
)/b^4 + (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```


Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
```

```
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cot^3(a+bx)\,dx = -\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{(2d)\int (c+dx)^3\cot^2(a+bx)\,dx}{b}-\int (c+dx)^4\cot(a+bx)\,dx$$

$$= \frac{i(c+dx)^5}{5d}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}}\,dx$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{(c+dx)^4}{2b}+\frac{i(c+dx)^5}{5d}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{(c+dx)^4}{2b}+\frac{i(c+dx)^5}{5d}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{(c+dx)^4}{2b}+\frac{i(c+dx)^5}{5d}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{(c+dx)^4}{2b}+\frac{i(c+dx)^5}{5d}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{(c+dx)^4}{2b}+\frac{i(c+dx)^5}{5d}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2}-\frac{(c+dx)^4}{2b}+\frac{i(c+dx)^5}{5d}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

Mathematica [B] time = 7.10651, size = 1534, normalized size = 5.08

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^3,x]
```

```
[Out] -(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Cot[a])/5
 - ((c + d*x)^4*Csc[a + b*x]^2)/(2*b) + (c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3
)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))
```

```
] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 +
 E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*
(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(
a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^3 - (d^4*E^
(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Lo
g[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I
)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))]
- I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(
b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^(
(2*I)*a)))/b^5 + (c*d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(
1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I
)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x^2*Poly
Log[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x))] - 2
*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b^2
*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(a + b*x
))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^4 + (d^4*E^(I*a)*C
sc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 - E
^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 + E^((-I)*(a +
b*x))] - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, -E^((-I)*(a + b*x))] -
(3*I)*b^2*x^2*PolyLog[3, -E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, -E^((-I)*(
a + b*x))] + (6*I)*PolyLog[5, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (20*(-1
+ E^((2*I)*a))*(b^3*x^3*PolyLog[2, E^((-I)*(a + b*x))] - (3*I)*b^2*x^2*Poly
Log[3, E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, E^((-I)*(a + b*x))] + (6*I)*P
olyLog[5, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(10*b^5) - (c^4*Csc[a]*(-(b*x
*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + S
in[a]^2)) + (6*c^2*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]
*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*(c
^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b
*x]))/b^2 + (2*c^3*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*
(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan
[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcT
an[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + Ar
cTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2
+ Sin[a]^2)]) - (6*c*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*
b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan
[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*
ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
+ ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos[a
]^2 + Sin[a]^2)])
```

Maple [B] time = 0.403, size = 1868, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^3,x)
```

```
[Out] -1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+1/5*I*
d^4*x^5+I*c*d^3*x^4-4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+6*d^4/b^5*a^2*ln(exp
(I*(b*x+a))-1)-12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))-12*d^2/b^3*c^2*ln(exp(I*(b
*x+a)))+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))-
1)+6*d^4/b^3*ln(1-exp(I*(b*x+a)))*x^2-6*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^2+6*
d^4/b^3*ln(exp(I*(b*x+a))+1)*x^2-4*I*d^4/b^2*x^3+8*I*d^4/b^5*a^3-4/b^4*c*d^
3*ln(1-exp(I*(b*x+a)))*a^3-I*c^4*x+12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+12
*d^4*polylog(3,exp(I*(b*x+a)))/b^5-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-12*I/
b^2*a^2*c^2*d^2*x+12*I/b^2*polylog(2,exp(I*(b*x+a)))*c^2*d^2*x+12*I/b^2*pol
ylog(2,-exp(I*(b*x+a)))*c^2*d^2*x+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))
*x^2+8*I/b^3*c*d^3*a^3*x+12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+2*(b*
d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp
(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c
^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b
*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))+6*I*c*d^3*x^2+6*I*c^2*d^2*x+2
*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2-12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x
^2-12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-12/b^3*c^2*d^2*polylog(3,-exp(I
*(b*x+a)))-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-8/5*I/b^5*d^4*a^5-6/b*
c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+6/b^3*c^2
*d^2*a^2*ln(1-exp(I*(b*x+a)))-4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x-4/b^2*c^3*d*
ln(1-exp(I*(b*x+a)))*a-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x-24/b^3*c*d^3*polylo
g(3,-exp(I*(b*x+a)))*x-6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2-24/b^3*c*d^3*po
lylog(3,exp(I*(b*x+a)))*x+12*d^3/b^3*c*ln(1-exp(I*(b*x+a)))*x+12*d^3/b^4*c*
ln(1-exp(I*(b*x+a)))*a+12*d^3/b^3*c*ln(exp(I*(b*x+a))+1)*x-12*I*d^4/b^4*pol
ylog(2,exp(I*(b*x+a)))*x-12*I*d^4/b^4*polylog(2,-exp(I*(b*x+a)))*x-12*I*d^3
/b^4*c*polylog(2,exp(I*(b*x+a)))-12*I*d^3/b^4*c*polylog(2,-exp(I*(b*x+a)))+
12*I*d^4/b^4*a^2*x-12*I*d^3/b^4*c*a^2-12*I*d^3/b^2*c*x^2-12*d^3/b^4*c*a*ln(
exp(I*(b*x+a))-1)+24*d^3/b^4*c*a*ln(exp(I*(b*x+a)))+8*I/b*a*c^3*d*x-2*I/b^4
*d^4*a^4*x+4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-
exp(I*(b*x+a)))*x^3-24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-24*I/b^4*d^4*p
olylog(4,-exp(I*(b*x+a)))*x+4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3-24*I/
b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-8*I/b^3*a^3*c^2*d^2-24*I/b^4*c*d^3*poly
log(4,-exp(I*(b*x+a)))+4*I/b^2*a^2*c^3*d+6*I/b^4*c*d^3*a^4+4*I/b^2*c^3*d*po
lylog(2,exp(I*(b*x+a)))+2*I*c^2*d^2*x^3+2*I*c^3*d*x^2-1/b^5*d^4*a^4*ln(exp(
I*(b*x+a))-1)+2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-8/b^2*c^3*d*a*ln(exp(I*(b*x+
a)))-6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)+12/b^3*c^2*d^2*a^2*ln(exp(I*(b*
x+a)))+4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a
)))+4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^
5+24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-1/b*c^4*ln(exp(I*(b*x+a))+1)-1/b*c^4
*ln(exp(I*(b*x+a))-1)+2/b*c^4*ln(exp(I*(b*x+a)))-24*I*d^3/b^3*c*a*x
```

Maxima [B] time = 19.4093, size = 9600, normalized size = 31.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 4*a*c^3*d*(1/sin(b*x +
 a)^2 + log(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/sin(b*x + a)^2 + log(sin(
b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3
 + a^4*d^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^4 - 2*(2*(b*x + a)^5*
d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4 + 10*
(b*c*d^3 - a*d^4)*(b*x + a)^4 + 20*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b
*x + a)^3 + 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x
 + a)^2 - (10*(b*x + a)^4*d^4 - 60*b^2*c^2*d^2 + 120*a*b*c*d^3 - 60*a^2*d^4
 + 40*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2
- 1)*d^4)*(b*x + a)^2 + 40*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d
^3 - (a^3 - 3*a)*d^4)*(b*x + a) + 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*
a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 -
2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 20*((
b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*
d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^
2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + (10*I*(b*x + a)^4*d^4 - 60*I*b^2*c^2*d^2 + 1
20*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)*(b*x + a)^3 + (
60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2 - 60*I)*d^4)*(b*x + a)^2 + (
40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*a^2 - 120*I)*b*c*d^3 + (-40*I
*a^3 + 120*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-20*I*(b*x + a)^4*d^4 +
 120*I*b^2*c^2*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4 + (-80*I*b*c*d^3 + 80*
I*a*d^4)*(b*x + a)^3 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2
+ 120*I)*d^4)*(b*x + a)^2 + (-80*I*b^3*c^3*d + 240*I*a*b^2*c^2*d^2 + (-240*
I*a^2 + 240*I)*b*c*d^3 + (80*I*a^3 - 240*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2
*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (60*b^2*c^2*d^2 - 120*a*b*c*
d^3 + 60*a^2*d^4 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(4*b*x + 4*a
) - 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(2*b*x + 2*a) - (-60*I*b^2
*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*sin(4*b*x + 4*a) - (120*I*b^2*c^
2*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4)*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), cos(b*x + a) - 1) + (10*(b*x + a)^4*d^4 + 40*(b*c*d^3 - a*d^4)*(b*x +
 a)^3 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 40*(b^
```

3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a
) + 10*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2
- 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2
 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 20*
((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a
^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-10*I*(b*
x + a)^4*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^3 + (-60*I*b^2*c^2*d^
2 + 120*I*a*b*c*d^3 + (-60*I*a^2 + 60*I)*d^4)*(b*x + a)^2 + (-40*I*b^3*c^3*
d + 120*I*a*b^2*c^2*d^2 + (-120*I*a^2 + 120*I)*b*c*d^3 + (40*I*a^3 - 120*I*
a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (20*I*(b*x + a)^4*d^4 + (80*I*b*c*d^3
 - 80*I*a*d^4)*(b*x + a)^3 + (120*I*b^2*c^2*d^2 - 240*I*a*b*c*d^3 + (120*I*
a^2 - 120*I)*d^4)*(b*x + a)^2 + (80*I*b^3*c^3*d - 240*I*a*b^2*c^2*d^2 + (24
0*I*a^2 - 240*I)*b*c*d^3 + (-80*I*a^3 + 240*I*a)*d^4)*(b*x + a))*sin(2*b*x
+ 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*((b*x + a)^5*d^4 + 5*(
b*c*d^3 - a*d^4)*(b*x + a)^4 + 10*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^
4)*(b*x + a)^3 + 10*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a
^3 - 6*a)*d^4)*(b*x + a)^2 - 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x
+ a))*cos(4*b*x + 4*a) - (4*(b*x + a)^5*d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*
d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4 + (20*b*c*d^3 - (20*a - 20*I)*d^4)*(b*x
+ a)^4 + (40*b^2*c^2*d^2 - (80*a - 80*I)*b*c*d^3 + 40*(a^2 - 2*I*a - 1)*d^4
)*(b*x + a)^3 + (40*b^3*c^3*d - (120*a - 120*I)*b^2*c^2*d^2 + 120*(a^2 - 2*
I*a - 1)*b*c*d^3 - (40*a^3 - 120*I*a^2 - 120*a)*d^4)*(b*x + a)^2 + (80*I*b^
3*c^3*d - 120*(2*I*a + 1)*b^2*c^2*d^2 + (240*I*a^2 + 240*a)*b*c*d^3 + (-80*
I*a^3 - 120*a^2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (40*b^3*c^3*d - 120*a*b
^2*c^2*d^2 + 40*(b*x + a)^3*d^4 + 120*(a^2 - 1)*b*c*d^3 - 40*(a^3 - 3*a)*d^
4 + 120*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a
^2 - 1)*d^4)*(b*x + a) + 40*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4
+ 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(4*b*x + 4*a)
- 80*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 1)*b*c*d^3 -
 (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-40*I*b^3*c^3*d + 12
0*I*a*b^2*c^2*d^2 - 40*I*(b*x + a)^3*d^4 + (-120*I*a^2 + 120*I)*b*c*d^3 + (
40*I*a^3 - 120*I*a)*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a)^2 + (-12
0*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2 + 120*I)*d^4)*(b*x + a))*si
n(4*b*x + 4*a) - (80*I*b^3*c^3*d - 240*I*a*b^2*c^2*d^2 + 80*I*(b*x + a)^3*d
^4 + (240*I*a^2 - 240*I)*b*c*d^3 + (-80*I*a^3 + 240*I*a)*d^4 + (240*I*b*c*d
^3 - 240*I*a*d^4)*(b*x + a)^2 + (240*I*b^2*c^2*d^2 - 480*I*a*b*c*d^3 + (240
*I*a^2 - 240*I)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) +
 (40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 40*(b*x + a)^3*d^4 + 120*(a^2 - 1)*b*c
d^3 - 40(a^3 - 3*a)*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 120*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a) + 40*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^
3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x

+ a))*cos(4*b*x + 4*a) - 80*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4
+ 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2
+ 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a)
- (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 40*I*(b*x + a)^3*d^4 + (-120*I*
a^2 + 120*I)*b*c*d^3 + (40*I*a^3 - 120*I*a)*d^4 + (-120*I*b*c*d^3 + 120*I*a
d^4)(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2 + 1
20*I)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (80*I*b^3*c^3*d - 240*I*a*b^2*c^2*
d^2 + 80*I*(b*x + a)^3*d^4 + (240*I*a^2 - 240*I)*b*c*d^3 + (-80*I*a^3 + 240
*I*a)*d^4 + (240*I*b*c*d^3 - 240*I*a*d^4)*(b*x + a)^2 + (240*I*b^2*c^2*d^2
- 480*I*a*b*c*d^3 + (240*I*a^2 - 240*I)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*d
ilog(e^(I*b*x + I*a)) - (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*c^2*d^2 - 60*I*a*b
*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x + a)^3 + (-30*I*b
^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*x + a)^2 + (-20*I*
b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d^3 + (20*I*a^3 - 6
0*I*a)*d^4)*(b*x + a) + (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*c^2*d^2 - 60*I*a*b
*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x + a)^3 + (-30*I*b
^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*x + a)^2 + (-20*I*
b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d^3 + (20*I*a^3 - 6
0*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (10*I*(b*x + a)^4*d^4 - 60*I*b^2*
c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)*(b*x
+ a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2 - 60*I)*d^4)*(b*x
+ a)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*a^2 - 120*I)*b*c*d
^3 + (-40*I*a^3 + 120*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 5*((b*x + a)^
4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x
+ a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a
))*sin(4*b*x + 4*a) - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 -
6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)
*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*
c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x
+ a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*
x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d
^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a) + (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*
c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x
+ a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*
x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d
^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (10*I*(b*x + a)
^4*d^4 - 60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3
- 40*I*a*d^4)*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2
- 60*I)*d^4)*(b*x + a)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*
a^2 - 120*I)*b*c*d^3 + (-40*I*a^3 + 120*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*
a) + 5*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c
*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(

b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 -
3*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2
+ 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*
d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2
d^2 + 3(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*
log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (240*I*d^4*cos(
4*b*x + 4*a) - 480*I*d^4*cos(2*b*x + 2*a) - 240*d^4*sin(4*b*x + 4*a) + 480*
d^4*sin(2*b*x + 2*a) + 240*I*d^4)*polylog(5, -e^(I*b*x + I*a)) - (240*I*d^4
*cos(4*b*x + 4*a) - 480*I*d^4*cos(2*b*x + 2*a) - 240*d^4*sin(4*b*x + 4*a) +
480*d^4*sin(2*b*x + 2*a) + 240*I*d^4)*polylog(5, e^(I*b*x + I*a)) - (240*b
*c*d^3 + 240*(b*x + a)*d^4 - 240*a*d^4 + 240*(b*c*d^3 + (b*x + a)*d^4 - a*d
^4)*cos(4*b*x + 4*a) - 480*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*
a) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*sin(4*b*x + 4*a) +
(-480*I*b*c*d^3 - 480*I*(b*x + a)*d^4 + 480*I*a*d^4)*sin(2*b*x + 2*a))*pol
ylog(4, -e^(I*b*x + I*a)) - (240*b*c*d^3 + 240*(b*x + a)*d^4 - 240*a*d^4 +
240*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 480*(b*c*d^3 + (b*
x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4
- 240*I*a*d^4)*sin(4*b*x + 4*a) + (-480*I*b*c*d^3 - 480*I*(b*x + a)*d^4 +
480*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) - (-120*I*b^2*c^
2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120*I)*d^4
+ (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a) + (-120*I*b^2*c^2*d^2 + 240*I*a*
b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120*I)*d^4 + (-240*I*b*c*d^
3 + 240*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (240*I*b^2*c^2*d^2 - 480*I*a
*b*c*d^3 + 240*I*(b*x + a)^2*d^4 + (240*I*a^2 - 240*I)*d^4 + (480*I*b*c*d^3
- 480*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + (b*x + a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*
b*x + 4*a) - 240*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 1)*d
^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x
+ I*a)) - (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (
-120*I*a^2 + 120*I)*d^4 + (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a) + (-120*
I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120
*I)*d^4 + (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (240
*I*b^2*c^2*d^2 - 480*I*a*b*c*d^3 + 240*I*(b*x + a)^2*d^4 + (240*I*a^2 - 240
*I)*d^4 + (480*I*b*c*d^3 - 480*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 120*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 -
a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 240*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x
+ a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a
))*polylog(3, e^(I*b*x + I*a)) - (-2*I*(b*x + a)^5*d^4 + (-10*I*b*c*d^3 + 1
0*I*a*d^4)*(b*x + a)^4 + (-20*I*b^2*c^2*d^2 + 40*I*a*b*c*d^3 + (-20*I*a^2 +
40*I)*d^4)*(b*x + a)^3 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^
2 + 120*I)*b*c*d^3 + (20*I*a^3 - 120*I*a)*d^4)*(b*x + a)^2 + (120*I*b^2*c^2
*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (4*I*
(b*x + a)^5*d^4 + 40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + 120*I*a^2*b*c*d^3
- 40*I*a^3*d^4 + (20*I*b*c*d^3 - 20*(I*a + 1)*d^4)*(b*x + a)^4 + (40*I*b^2*
c^2*d^2 - 80*(I*a + 1)*b*c*d^3 + (40*I*a^2 + 80*a - 40*I)*d^4)*(b*x + a)^3

```
+ (40*I*b^3*c^3*d - 120*(I*a + 1)*b^2*c^2*d^2 + (120*I*a^2 + 240*a - 120*I)
*b*c*d^3 + (-40*I*a^3 - 120*a^2 + 120*I*a)*d^4)*(b*x + a)^2 - (80*b^3*c^3*d
 - (240*a - 120*I)*b^2*c^2*d^2 + 240*(a^2 - I*a)*b*c*d^3 - 40*(2*a^3 - 3*I*
a^2)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/(-10*I*b^4*cos(4*b*x + 4*a) + 20*I*b
^4*cos(2*b*x + 2*a) + 10*b^4*sin(4*b*x + 4*a) - 20*b^4*sin(2*b*x + 2*a) - 1
0*I*b^4))/b
```

Fricas [C] time = 0.707253, size = 3915, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^4*d^4*x^4 + 16*b^4*c*d^3*x^3 + 24*b^4*c^2*d^2*x^2 + 16*b^4*c^3*d*x
 + 4*b^4*c^4 + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 12*
I*b*c*d^3 - 12*I*(b^3*c^2*d^2 - b*d^4)*x + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^
3*x^2 + 4*I*b^3*c^3*d - 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*
b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (4*I*b^3*d^4*x^3
 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 -
b*d^4)*x + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 12*I*b*
c*d^3 - 12*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a)) + 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^
2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*
d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*co
s(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) +
 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d
^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2
 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a))*log(-1/2*co
s(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^4*d^4*x^4 + 4*b^4*c*d
^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 -
 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x
 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(
a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*
(b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*si
n(2*b*x + 2*a) + 1) + 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*
a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^
2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x - (b^4*d^4*x^4 + 4*b^4*c*d
^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 -
 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x
)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 3*(d^
```

```
4*cos(2*b*x + 2*a) - d^4)*polylog(5, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))
 + 3*(d^4*cos(2*b*x + 2*a) - d^4)*polylog(5, cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a)) + (6*I*b*d^4*x + 6*I*b*c*d^3 + (-6*I*b*d^4*x - 6*I*b*c*d^3)*cos(2*
b*x + 2*a))*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (-6*I*b*d^4
*x - 6*I*b*c*d^3 + (6*I*b*d^4*x + 6*I*b*c*d^3)*cos(2*b*x + 2*a))*polylog(4,
 cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
b^2*c^2*d^2 - d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4)*cos(2
*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 6*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b
^2*c^2*d^2 - d^4)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d
)*sin(2*b*x + 2*a))/(b^5*cos(2*b*x + 2*a) - b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^4 \cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \cot(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^3, x)
```

3.179 $\int (c+dx)^3 \cot^3(a+bx)\,dx$

Optimal. Leaf size=256

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{PolyLog}\left(4,\right.}{4b^4}$$

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) + ((I/4)*(c + d*x)^4)/d
- (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]^2)/(2*
b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^3*Log[
1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3
*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*Po
lyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rubi [A] time = 0.368376, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3720, 3717, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{PolyLog}\left(4,\right.}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^3,x]
```



```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) + ((I/4)*(c + d*x)^4)/d
- (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]^2)/(2*
b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^3*Log[
1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3
*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*Po
lyLog[4, E^((2*I)*(a + b*x))])/b^4
```


Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\cot^3(a+bx)\,dx = -\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(3d)\int (c+dx)^2\cot^2(a+bx)\,dx}{2b}-\int (c+dx)^3\cot(a+bx)\,dx$$

$$=\frac{i(c+dx)^4}{4d}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx$$

$$=-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}+\frac{i(c+dx)^4}{4d}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$=-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}+\frac{i(c+dx)^4}{4d}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$=-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}+\frac{i(c+dx)^4}{4d}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$=-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}+\frac{i(c+dx)^4}{4d}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$=-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}+\frac{i(c+dx)^4}{4d}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

Mathematica [B] time = 6.88083, size = 994, normalized size = 3.88

$$-\frac{\csc(a)(\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a)-bx\cos(a))c^3}{b\left(\cos^2(a)+\sin^2(a)\right)}+\frac{3d\csc(a)\sec(a)\left(b^2e^{i\tan^{-1}(\tan(a))}x^2+\frac{\left(ibx\left(2\tan^{-1}(\tan(a\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^3,x]
```

```
[Out] -(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cot[a])/4 - ((c + d*x)^3*Cs
c[a + b*x]^2)/(2*b) + (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I
)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E
^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*P
olyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*
I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyL
og[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(2*b^3) + (d^3*E^(I*a)*Csc[a]*((b
^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a +
 b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6
*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*Po
lyLog[3, -E^((-I)*(a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I
)*a) - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I
)*b*x*PolyLog[3, E^((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E
^((2*I)*a)))/(4*b^4) - (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + C
os[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*(-(b*x
*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 +
 Sin[a]^2)) + (3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] +
 d^3*x^2*Sin[b*x]))/(2*b^2) + (3*c^2*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a
]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2
*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[
Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2,
E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt
[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[T
an[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)]
 - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*
Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog
[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*
Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.363, size = 1194, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^3,x)
```

```
[Out] -3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b^
3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(
1-exp(I*(b*x+a)))*a-6*I/b^4*d^3*polylog(4,exp(I*(b*x+a)))-3*I/b^2*d^3*x^2+3
/2*I/b^4*d^3*a^4-3*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))-3*I/b^4*d^3*a^2+1/4
*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3/2*I*c^2*d*x^2-6*I*d^3*polylog(4,-exp(I*(b*
x+a)))/b^4-3/b^4*d^3*a*ln(exp(I*(b*x+a))-1)+6/b^4*d^3*a*ln(exp(I*(b*x+a)))+
3/b^3*d^2*c*ln(exp(I*(b*x+a))+1)-6/b^3*d^2*c*ln(exp(I*(b*x+a)))+3/b^3*d^2*c
*ln(exp(I*(b*x+a))-1)-1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-1/b^4*d^3*ln(1-exp(I
*(b*x+a)))*a^3-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*c*d^2*a^2*ln(exp(I*(b
*x+a)))-3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)
))+3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))-
3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x-3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/b^2*c^2
*d*ln(1-exp(I*(b*x+a)))*a+3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-6*I/b^3
*d^3*a*x-4*I/b^3*a^3*c*d^2+2*I/b^3*d^3*a^3*x+3*I/b^2*c^2*d*polylog(2,exp(I*
(b*x+a)))+3*I/b^2*a^2*c^2*d+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+3*I/b^
2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*
x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*
x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2
*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2-6
/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x
-6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)
))-2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))+1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6*I/
b*a*c^2*d*x-6*I/b^2*a^2*c*d^2*x+6*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2*x+
6*I/b^2*polylog(2,exp(I*(b*x+a)))*c*d^2*x+2/b*c^3*ln(exp(I*(b*x+a)))-1/b*c^
3*ln(exp(I*(b*x+a))-1)-1/b*c^3*ln(exp(I*(b*x+a))+1)-3*I*d^3*polylog(2,exp(I
*(b*x+a)))/b^4
```

Maxima [B] time = 6.14236, size = 5335, normalized size = 20.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 3*a*c^2*d*(1/sin(b*x +
 a)^2 + log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*
x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3 - 2*(
(b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + 4*(b*c*d^2 - a
*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - (4*
(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^2
+ 12*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*((b*x + a)^3*d
```

```
^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 8*((b*x + a)^3*
d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
 - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (4*I*(b*x + a)
^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^
2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*si
n(4*b*x + 4*a) + (-8*I*(b*x + a)^3*d^3 + 24*I*b*c*d^2 - 24*I*a*d^3 + (-24*I
*b*c*d^2 + 24*I*a*d^3)*(b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 + (-
24*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), co
s(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*c*d^2 - a*d^3)*cos(4*b*x +
 4*a) - 24*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 + 12*I*a*d^3
)*sin(4*b*x + 4*a) - (24*I*b*c*d^2 - 24*I*a*d^3)*sin(2*b*x + 2*a))*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (4*(b*x + a)^3*d^3 + 12*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*(
(b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 8*((b*x + a)^3*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12*I
*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 12*I
)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (8*I*(b*x + a)^3*d^3 + (24*I*b*c*d^2 -
 24*I*a*d^3)*(b*x + a)^2 + (24*I*b^2*c^2*d - 48*I*a*b*c*d^2 + (24*I*a^2 - 2
4*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a)
+ 1) + ((b*x + a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d -
2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(4*b*x + 4*a) - (2*(b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^
2*d^3 + (8*b*c*d^2 - (8*a - 8*I)*d^3)*(b*x + a)^3 + (12*b^2*c^2*d - (24*a -
 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a - 1)*d^3)*(b*x + a)^2 + (24*I*b^2*c^2*d -
24*(2*I*a + 1)*b*c*d^2 + (24*I*a^2 + 24*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a)
 + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*(a^2 - 1)*d^3 + 2
4*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d
^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 24*(
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I
*(b*x + a)^2*d^3 + (-12*I*a^2 + 12*I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b
*x + a))*sin(4*b*x + 4*a) - (24*I*b^2*c^2*d - 48*I*a*b*c*d^2 + 24*I*(b*x +
a)^2*d^3 + (24*I*a^2 - 24*I)*d^3 + (48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a))*s
in(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 1
2*(b*x + a)^2*d^3 + 12*(a^2 - 1)*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*
(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 24*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)
^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
(-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 12*
I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (24*I*b
^2*c^2*d - 48*I*a*b*c*d^2 + 24*I*(b*x + a)^2*d^3 + (24*I*a^2 - 24*I)*d^3 +
(48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I
```

```
*a)) - (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*
I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*
d^3)*(b*x + a) + (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*
c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a
^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (4*I*(b*x + a)^3*d^3 - 12*I*b*
c*d^2 + 12*I*a*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^
2*d - 24*I*a*b*c*d^2 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) +
 2*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a)
- 4*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^
2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-2*I*(b*x +
a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2
 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x + a) + (-2
*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(
b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x
+ a))*cos(4*b*x + 4*a) + (4*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2
 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x + a)^3*d^3
- 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 4*((b*x + a)^3*d^3
 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2
*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (24*d^3*cos(4*b*x + 4*a) - 48*d^3
*cos(2*b*x + 2*a) + 24*I*d^3*sin(4*b*x + 4*a) - 48*I*d^3*sin(2*b*x + 2*a) +
 24*d^3)*polylog(4, -e^(I*b*x + I*a)) - (24*d^3*cos(4*b*x + 4*a) - 48*d^3*c
os(2*b*x + 2*a) + 24*I*d^3*sin(4*b*x + 4*a) - 48*I*d^3*sin(2*b*x + 2*a) + 2
4*d^3)*polylog(4, e^(I*b*x + I*a)) - (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 +
24*I*a*d^3 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(4*b*x +
4*a) + (48*I*b*c*d^2 + 48*I*(b*x + a)*d^3 - 48*I*a*d^3)*cos(2*b*x + 2*a) +
24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 48*(b*c*d^2 + (b*x
+ a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (-24*I*b
*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*
d^3 + 24*I*a*d^3)*cos(4*b*x + 4*a) + (48*I*b*c*d^2 + 48*I*(b*x + a)*d^3 - 4
8*I*a*d^3)*cos(2*b*x + 2*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*
x + 4*a) - 48*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3
, e^(I*b*x + I*a)) - (-I*(b*x + a)^4*d^3 + (-4*I*b*c*d^2 + 4*I*a*d^3)*(b*x
+ a)^3 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a
)^2 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (2*I*(b*x +
 a)^4*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (8*I*b*c*d^2 -
 8*(I*a + 1)*d^3)*(b*x + a)^3 + (12*I*b^2*c^2*d - 24*(I*a + 1)*b*c*d^2 + (1
2*I*a^2 + 24*a - 12*I)*d^3)*(b*x + a)^2 - (24*b^2*c^2*d - (48*a - 24*I)*b*c
*d^2 + 24*(a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-4*I*b^3*cos(4*b*x
 + 4*a) + 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a) - 8*b^3*sin(2*b
*x + 2*a) - 4*I*b^3))/b
```

Fricas [C] time = 0.621331, size = 2637, normalized size = 10.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 + (-6*I*
b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d + 6*I*d^3 + (6*I*b^2*d^3*x^2
 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 6*I*d^3)*cos(2*b*x + 2*a))*dilog(cos(
2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x +
6*I*b^2*c^2*d - 6*I*d^3 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^
2*d + 6*I*d^3)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a
)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (
b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x
+ 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*
a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(
-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^
2*d - b*d^3)*x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-co
s(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*
x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 -
 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a) + 1) + (-3*I*d^3*cos(2*b*x + 2*a) + 3*I*d^3)*polylog
(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (3*I*d^3*cos(2*b*x + 2*a) - 3*
I*d^3)*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 6*(b*d^3*x + b*c
*d^2 - (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) +
I*sin(2*b*x + 2*a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(2*b*x
+ 2*a))*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 12*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(2*b*x + 2*a))/(b^4*cos(2*b*x + 2*a) - b^4
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \cot^3 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \cot (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^3, x)
```

3.180 $\int (c+dx)^2 \cot^3(a+bx)\,dx$

Optimal. Leaf size=168

$$\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\cot(a+bx)}{b^2}+\frac{d^2\log(\sin(a+bx))}{b^3}-\frac{(c+dx)^2\text{lo}}{}$$

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) + ((I/3)*(c + d*x)^3)/d - (d*(c + d*x)*Cot[a
 + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b) - ((c + d*x)^2*Log[1 - E^
((2*I)*(a + b*x))])/b + (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLo
g[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b
^3)
```

Rubi [A] time = 0.265509, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3475, 3717, 2190, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\cot(a+bx)}{b^2}+\frac{d^2\log(\sin(a+bx))}{b^3}-\frac{(c+dx)^2\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^3,x]
```

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) + ((I/3)*(c + d*x)^3)/d - (d*(c + d*x)*Cot[a
 + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b) - ((c + d*x)^2*Log[1 - E^
((2*I)*(a + b*x))])/b + (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLo
g[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b
^3)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\cot^3(a+bx)\,dx = -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{d\int (c+dx)\cot^2(a+bx)\,dx}{b} - \int (c+dx)^2\cot(a+bx)\,dx$$

$$= \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx -$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} - \frac{(c+dx)^2 \mathrm{lo}}{}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} - \frac{(c+dx)^2 \mathrm{lo}}{}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} - \frac{(c+dx)^2 \mathrm{lo}}{}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} - \frac{(c+dx)^2 \mathrm{lo}}{}$$

Mathematica [B] time = 6.67168, size = 540, normalized size = 3.21

$$\frac{cd\csc(a)\sec(a)\left(\frac{\tan(a)\left(i\,\mathrm{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+2\tan^{-1}(\tan(a\right.}{\sqrt{\tan^2(a)+1}}\right.}{b^2\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^3,x]
```

```
[Out] -(x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cot[a])/3 - ((c + d*x)^2*Csc[a + b*x]^2)/(2
*b) + (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*L
og[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)
*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E
^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a +
 b*x))]))/E^((2*I)*a)))/(6*b^3) - (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]
*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]
*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos
[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/
b^2 + (c*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*A
rcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[
1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]
*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a
```

```
]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2
)])
```

Maple [B] time = 0.313, size = 635, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^3,x)
```

```
[Out] -4/3*I/b^3*a^3*d^2+1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-1/b*d^2*ln(exp(I*(b*x
+a))+1)*x^2+1/3*I*d^2*x^3+I*c*d*x^2-I*c^2*x-1/b*d^2*ln(1-exp(I*(b*x+a)))*x^
2+2*I/b^2*a^2*c*d+2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+2*I/b^2*polylog(2,e
xp(I*(b*x+a)))*d^2*x+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*d*p
olylog(2,-exp(I*(b*x+a)))-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(
b*x+a))+1)+1/b^3*d^2*ln(exp(I*(b*x+a))-1)+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b
*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c
*d*exp(2*I*(b*x+a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+4*I/b*a*c*d*x
-2/b*c*d*ln(exp(I*(b*x+a))+1)*x+2/b*c^2*ln(exp(I*(b*x+a)))-1/b*c^2*ln(exp(I
*(b*x+a))+1)-1/b*c^2*ln(exp(I*(b*x+a))-1)-2*d^2*polylog(3,-exp(I*(b*x+a)))/
b^3-2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b*c*d*ln(1-exp(I*(b*x+a)))*x-2/b^
2*c*d*ln(1-exp(I*(b*x+a)))*a-2*I/b^2*a^2*d^2*x+2/b^3*d^2*a^2*ln(exp(I*(b*x+
a)))-1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)-4/
b^2*c*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 2.50918, size = 2654, normalized size = 15.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*a*c*d*(1/sin(b*x + a
)^2 + log(sin(b*x + a)^2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)
^2))/b^2 - 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)^2 + 12*b*c*d
- 12*a*d^2 - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) - 6*d^2 + 6*
((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(4*b*x + 4*a) - 12
```

```
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (
6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*sin(4*
b*x + 4*a) + (-12*I*(b*x + a)^2*d^2 + (-24*I*b*c*d + 24*I*a*d^2)*(b*x + a)
+ 12*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*
d^2*cos(4*b*x + 4*a) - 12*d^2*cos(2*b*x + 2*a) + 6*I*d^2*sin(4*b*x + 4*a) -
 12*I*d^2*sin(2*b*x + 2*a) + 6*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1)
 + (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 6*((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 12*((b*x + a)^2*d^2 + 2*(b
*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^2 + (-12*I*
b*c*d + 12*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - (12*I*(b*x + a)^2*d^2 + (
24*I*b*c*d - 24*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a),
 -cos(b*x + a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*x + a)^2 -
6*(b*x + a)*d^2)*cos(4*b*x + 4*a) - (4*(b*x + a)^3*d^2 + (12*b*c*d - (12*a
- 12*I)*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 + (24*I*b*c*d - 12*(2*I*a +
1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d
^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 24*(b*c*d + (b*x
 + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 1
2*I*a*d^2)*sin(4*b*x + 4*a) - (24*I*b*c*d + 24*I*(b*x + a)*d^2 - 24*I*a*d^2
)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (12*b*c*d + 12*(b*x + a)*d^2
- 12*a*d^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 24*(b*c*
d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)
*d^2 + 12*I*a*d^2)*sin(4*b*x + 4*a) - (24*I*b*c*d + 24*I*(b*x + a)*d^2 - 24
*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-3*I*(b*x + a)^2*d^2
+ (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*d^2 + (-
6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*cos(4*b*x + 4*a) + (6*I*(b*x +
a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*cos(2*b*x + 2*a)
+ 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x + 4*a)
- 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x + 2*a))
*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-3*I*(b*x + a
)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*
d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*cos(4*b*x + 4*a) + (6*I
*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*cos(2*b*x
 + 2*a) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x
 + 4*a) - 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x
 + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-12*I
*d^2*cos(4*b*x + 4*a) + 24*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(4*b*x + 4*a)
 - 24*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*polylog(3, -e^(I*b*x + I*a)) - (-12*
I*d^2*cos(4*b*x + 4*a) + 24*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(4*b*x + 4*a
) - 24*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*polylog(3, e^(I*b*x + I*a)) - (-2*I
*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a)^2 + 12*I*(b*x + a)*d^
2)*sin(4*b*x + 4*a) - (4*I*(b*x + a)^3*d^2 + (12*I*b*c*d - 12*(I*a + 1)*d^2
)*(b*x + a)^2 + 12*I*b*c*d - 12*I*a*d^2 - (24*b*c*d - (24*a - 12*I)*d^2)*(b
*x + a))*sin(2*b*x + 2*a))/(-6*I*b^2*cos(4*b*x + 4*a) + 12*I*b^2*cos(2*b*x
+ 2*a) + 6*b^2*sin(4*b*x + 4*a) - 12*b^2*sin(2*b*x + 2*a) - 6*I*b^2))/b
```

Fricas [C] time = 0.564182, size = 1581, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 + (-2*I*b*d^2*x - 2*I*b*c*d +
(2*I*b*d^2*x + 2*I*b*c*d)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(
2*b*x + 2*a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(2
*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(b^2*c^2 - 2*
a*b*c*d + (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(2*b*x +
 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*c
^2 - 2*a*b*c*d + (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(
2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2
*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*
d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(
2*b*x + 2*a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b
^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(
2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) - (d^2*cos(2*b*x + 2*a) - d^2)*polyl
og(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - (d^2*cos(2*b*x + 2*a) - d^2)
*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b*d^2*x + b*c*d)*si
n(2*b*x + 2*a))/(b^3*cos(2*b*x + 2*a) - b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \cot(bx+a)^3 \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^3, x)
```

3.181 $\int (c+dx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}$$

```
[Out] -(d*x)/(2*b) + ((I/2)*(c + d*x)^2)/d - (d*Cot[a + b*x])/(2*b^2) - ((c + d*x
)*Cot[a + b*x]^2)/(2*b) - ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b + ((I/
2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rubi [A] time = 0.128258, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3473, 8, 3717, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^3,x]
```

```
[Out] -(d*x)/(2*b) + ((I/2)*(c + d*x)^2)/d - (d*Cot[a + b*x])/(2*b^2) - ((c + d*x
)*Cot[a + b*x]^2)/(2*b) - ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b + ((I/
2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\cot^3(a+bx)\,dx = -\frac{(c+dx)\cot^2(a+bx)}{2b} + \frac{d\int \cot^2(a+bx)\,dx}{2b} - \int (c+dx)\cot(a+bx)\,dx$$

$$= \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx - \frac{d\int 1\,dx}{2b}$$

$$= -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} + \cdots$$

$$= -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \cdots$$

$$= -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} + \cdots$$

Mathematica [B] time = 6.14359, size = 240, normalized size = 2.2

$$\frac{d\csc(a)\sec(a)\left(\frac{\tan(a)\left(i\text{PolyLog}\left(2,e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan(a))+bx\right)}\right)+2\tan^{-1}(\tan(a))\log\right.}{\sqrt{\tan^2(a)+1}}\right.}{2b^2\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^3,x]
```

```
[Out] -(d*x^2*Cot[a])/2 - (d*x*Csc[a + b*x]^2)/(2*b) - (c*(Cot[a + b*x]^2 + 2*Log
[Cos[a + b*x]] + 2*Log[Tan[a + b*x]]))/(2*b) + (d*Csc[a]*Csc[a + b*x]*Sin[b
*x])/(2*b^2) + (d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-P
i + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]
])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[
Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTa
n[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 +
 Sin[a]^2)])
```

Maple [B] time = 0.161, size = 281, normalized size = 2.6

$$-icx+\frac{2\,idax}{b}+\frac{2\,bdx\mathrm{e}^{2\,i(bx+a)}+2\,bc\mathrm{e}^{2\,i(bx+a)}-id\mathrm{e}^{2\,i(bx+a)}+id}{b^2\left(\mathrm{e}^{2\,i(bx+a)}-1\right)^2}-\frac{c\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b}+2\,\frac{c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^3,x)
```

```
[Out] -I*c*x+2*I/b*d*a*x+(2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp
(2*I*(b*x+a))+I*d)/b^2/(exp(2*I*(b*x+a))-1)^2-1/b*c*ln(exp(I*(b*x+a))-1)+2/
b*c*ln(exp(I*(b*x+a)))-1/b*c*ln(exp(I*(b*x+a))+1)+I/b^2*d*a^2+I*d*polylog(2
,-exp(I*(b*x+a)))/b^2+1/2*I*d*x^2-1/b*d*ln(1-exp(I*(b*x+a)))*x-1/b^2*d*ln(1
-exp(I*(b*x+a)))*a+I/b^2*d*polylog(2,exp(I*(b*x+a)))-1/b*d*ln(exp(I*(b*x+a)
)+1)*x+1/b^2*d*a*ln(exp(I*(b*x+a))-1)-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 1.6957, size = 1133, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (b^2*d*x^2 + 2*b^2*c*x - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a
) - 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*
a) + (-4*I*b*d*x - 4*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x
 + a) + 1) - (2*b*c*cos(4*b*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin
(4*b*x + 4*a) - 4*I*b*c*sin(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos
(b*x + a) - 1) + (2*b*d*x*cos(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I
*b*d*x*sin(4*b*x + 4*a) - 4*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin
(b*x + a), -cos(b*x + a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) -
(2*b^2*d*x^2 + 4*I*b*c + (4*b^2*c + 4*I*b*d)*x + 2*d)*cos(2*b*x + 2*a) + (2
*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) - 4*I*d
*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*a)) + (2*d*cos(4*b*x + 4*a) -
4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*
d)*dilog(e^(I*b*x + I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*
x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x
 + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b
*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*
x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)*sin(4*b*x + 4*a)
 - (2*I*b^2*d*x^2 - 4*b*c - 4*(-I*b^2*c + b*d)*x + 2*I*d)*sin(2*b*x + 2*a)
+ 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*
b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

Fricas [B] time = 0.537357, size = 859, normalized size = 7.88

$$4\,bdx + 4\,bc + (i\,d\cos(2\,bx + 2\,a) - i\,d)\mathrm{Li}_2(\cos(2\,bx + 2\,a) + i\,\sin(2\,bx + 2\,a)) + (-i\,d\cos(2\,bx + 2\,a) + i\,d)\mathrm{Li}_2(\cos(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*d*x + 4*b*c + (I*d*cos(2*b*x + 2*a) - I*d)*dilog(cos(2*b*x + 2*a)
+ I*sin(2*b*x + 2*a)) + (-I*d*cos(2*b*x + 2*a) + I*d)*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a)) + 2*(b*c - a*d - (b*c - a*d)*cos(2*b*x + 2*a))*log(
-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*c - a*d - (b*c
 - a*d)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a
```

```
) + 1/2) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x + 2*a))*log(-cos(2*b*x
+ 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x
 + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 2*d*sin(2*b*x +
2*a))/(b^2*cos(2*b*x + 2*a) - b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*cot(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\cot(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cot(b*x + a)^3, x)
```

3.182 $\int \frac{\cot^3(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\cot^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Cot[a + b*x]^3/(c + d*x), x]
```

Rubi [A] time = 0.039856, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot^3(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/(c + d*x),x]
```

```
[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{c+dx} \, dx = \int \frac{\cot^3(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 8.05091, size = 0, normalized size = 0.

$$\int \frac{\cot^3(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/(c + d*x),x]
```

```
[Out] Integrate[Cot[a + b*x]^3/(c + d*x), x]
```

Maple [A] time = 2.391, size = 0, normalized size = 0.

$$\int \frac{(\cot(bx+a))^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(cot(b*x+a)^3/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^3/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(cot(a + b*x)**3/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^3/(d*x + c), x)
```

3.183 $\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\cot^3(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable[Cot[a + b*x]^3/(c + d*x)^2, x]
```

Rubi [A] time = 0.0373215, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/(c + d*x)^2,x]

[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 9.16524, size = 0, normalized size = 0.

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/(c + d*x)^2,x]

[Out] Integrate[Cot[a + b*x]^3/(c + d*x)^2, x]
```

Maple [A] time = 3.544, size = 0, normalized size = 0.

$$\int \frac{(\cot (bx + a))^3}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(cot(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cot (bx + a)^3}{d^2x^2 + 2\, cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3 (a + bx)}{(c + dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(cot(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot (bx+a)^3}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^3/(d*x + c)^2, x)
```

3.184 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=407

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin(4\ldots}{}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rubi [A] time = 0.796576, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin(4\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
```

```
Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{(5d)\int (c+dx)^{3/2}\,c}{64}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c+dx)^{3/2}\sin(2}{32b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos}{2048b^3}$$

Mathematica [A] time = 13.6521, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\cos(2(a+bx))-256b^3c^2\sqrt{c+dx}\cos(4(a+bx))-1024b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx))-256b^3d^2x^2\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] +
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

Maple [A] time = 0.035, size = 470, normalized size = 1.2

$$2\frac{1}{d}\left(-1/16\frac{d(dx+c)^{5/2}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+\frac{5d}{16b}\left(1/4\frac{d(dx+c)^{3/2}}{b}\sin\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)-3/4\frac{d}{b}\left(-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.31125, size = 1879, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/65536*sqrt(2)*(640*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(4*((d*x + c)*
b - b*c + a*d)/d)/abs(d) + 5120*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2*
((d*x + c)*b - b*c + a*d)/d)/abs(d) - 16*(64*sqrt(2)*(d*x + c)^(5/2)*b^2*d*
abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(4*((d*x + c
```

)*b - b*c + a*d)/d) - 256*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - ((480*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (480*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((480*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (-480*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b^3*d*abs(b))

Fricas [A] time = 0.698365, size = 944, normalized size = 2.32

$$-\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+480\pi d^3\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
+ 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x
+ 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*
x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x +
 b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x +
a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.77497, size = 2680, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16384*(64*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b
 + 16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sq
rt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x +
 c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2 + d^2*((I*sqrt(2)
*sqrt(pi)*(-64*I*b^2*c^2*d + 48*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 1
28*I*(d*x + c)^(3/2)*b^2*c*d + 64*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^
(3/2)*b*d^2 - 48*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(
d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^2 + (I*sqrt(2)*sqrt(pi)*(-64*I*b^
2*c^2*d - 48*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 128*I*(d*x + c)^(
3/2)*b^2*c*d + 64*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 48
*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I*
b*c + 4*I*a*d)/d)/b^3)/d^2 + 32*(I*sqrt(pi)*(-16*I*b^2*c^2*d + 24*b*c*d^2 +
 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I
*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*
b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt
(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 32*(
I*sqrt(pi)*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*
(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)
*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x +
c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) + 16*(I*sqrt(2)*sqrt(pi)*(8*I*b*c*d
- 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*s
qrt(2)*sqrt(pi)*(8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d + 3*d^2)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c
)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x +
```

```
 c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqr
t(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I
*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*
sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2 - 4*I*(
8*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^
((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)*c)/d
```

3.185 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\right.}{512\ldots}$$

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rubi [A] time = 0.555808, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\right.}{512\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{(3d)\int \sqrt{c+dx}\,\text{co}}{64b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+dx}\sin(2a}{32b^2}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+dx}\sin(2a}{32b^2}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+dx}\sin(2a}{32b^2}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\right.}{51}$$

Mathematica [A] time = 3.29738, size = 393, normalized size = 1.12

$$\frac{-3\sqrt{2\pi}d\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)-48\sqrt{\pi}d\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-3\sqrt{2\pi}d\cos\left(4a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
- 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

Maple [A] time = 0.035, size = 376, normalized size = 1.1

$$2\frac{1}{d}\left(-1/16\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+3/16\,\frac{d}{b}\left(1/4\,\frac{d\sqrt{dx+c}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)-1/8\,\frac{d\sqrt{\pi}}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.25609, size = 1806, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8192*sqrt(2)*(128*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(4*((d*x + c)*b
- b*c + a*d)/d)/abs(d) + 512*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(2*((d*x
 + c)*b - b*c + a*d)/d)/abs(d) - 48*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(4*
((d*x + c)*b - b*c + a*d)/d)/abs(d) - 384*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*
sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((-48*I*sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi)*sin(1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs
(d))*cos(-2*(b*c - a*d)/d) - (48*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2
```

```
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (sqrt(2)*(-3*I*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(
b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(3*I*sqrt(
pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqr
t(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sq
rt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sq
rt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*
sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(3*sqrt(pi)*cos(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/a
bs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((48*I*sq
rt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
48*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 48*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (48*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)
/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(
b^2*d*abs(b))
```

Fricas [A] time = 0.675102, size = 749, normalized size = 2.13

$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+48\,\pi d^2\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(
b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.53265, size = 1485, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2048*(8*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b +
 16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt
(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c
)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c + I*sqrt(2)*sqrt(pi)*
```

```
(8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) + I*sqrt(2)*sqrt(pi)*(8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*e
rf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*
d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8
*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(
(4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*
d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)
*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/
b^2 - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x +
 c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.186 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\right.}{64b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rubi [A] time = 0.446811, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\right.}{64b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}dx}{64b}+\frac{d\int \frac{\mathrm{co}}{}}{}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4b}{d}\right.}{\sqrt{c}}}{64b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Subst}\left(\int \mathrm{c}\right.}{32}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)C\left(\right.}{64b^{3/2}}$$

Mathematica [A] time = 0.83486, size = 264, normalized size = 0.88

$$\frac{\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\mathrm{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\right.}{128b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

Maple [A] time = 0.032, size = 286, normalized size = 1.

$$2\,\frac{1}{d}\left(-1/16\,\frac{d\sqrt{dx+c}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+1/32\,\frac{d\sqrt{\pi}}{b}\left(\cos\left(2\,\frac{ad-bc}{d}\right)\mathrm{FresnelC}\left(2\,\frac{\sqrt{dx+c}b}{d\sqrt{\pi}}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\,\frac{a}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.1839, size = 1661, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*sqrt(2)*(16*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(4*((d*x + c)*b - b*c
 + a*d)/d)/abs(d) + 64*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(2*((d*x + c)*b -
b*c + a*d)/d)/abs(d) - ((8*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 8*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + 8*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) -
 (8*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + 8*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 8*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 8*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
2*I*b/d)) - (sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(I
*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sq
rt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sq
rt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) -
```

```
(sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/
abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((8*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d
*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (-8*I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*s
in(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b*d*abs(b)
)
```

Fricas [A] time = 0.61023, size = 599, normalized size = 2.

$$\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+8\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(}{}\right.}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sqr
t(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.3384, size = 643, normalized size = 2.15

$$-\frac{\frac{\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{8\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i}{d}\right.}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*
sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x
 + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*
e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

3.187 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\right.}{64b^{3/2}}$$

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rubi [A] time = 0.454946, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\right.}{64b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```


```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx + \frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} + \frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}dx}{64b} + \frac{d\int \frac{\mathrm{co}}{}}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} + \frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4b}{d}\right.}{\sqrt{c}}}{64b}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} + \frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Subst}\left(\int \mathrm{c}\right.}{32}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)C\left(\right.}{64b^{3/2}}$$

Mathematica [A] time = 0.391485, size = 264, normalized size = 0.88

$$\frac{\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)\mathrm{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\right.}{128b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

Maple [A] time = 0.033, size = 286, normalized size = 1.

$$2\frac{1}{d}\left(-1/16\frac{d\sqrt{dx+c}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+1/32\frac{d\sqrt{\pi}}{b}\left(\cos\left(2\frac{ad-bc}{d}\right)\mathrm{FresnelC}\left(2\frac{\sqrt{dx+c}b}{d\sqrt{\pi}}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\frac{ad}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.36031, size = 1661, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*sqrt(2)*(16*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(4*((d*x + c)*b - b*c
 + a*d)/d)/abs(d) + 64*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(2*((d*x + c)*b -
b*c + a*d)/d)/abs(d) - ((8*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 8*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) + 8*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) -
 (8*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + 8*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) + 8*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 8*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
2*I*b/d)) - (sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(I
*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sq
rt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sq
rt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) -
```

```
(sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))))*d*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(-I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/
abs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((8*sqrt
(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*I*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I
*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d
*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (-8*I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*I*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 8*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 8*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*s
in(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b*d*abs(b)
)
```

Fricas [A] time = 0.613858, size = 599, normalized size = 2.

$$\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+8\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(\cdots}{\cdots}\right.}{128b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sqr
t(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.32414, size = 643, normalized size = 2.15

$$-\frac{\frac{\sqrt{2}\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{\sqrt{2}\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}+\frac{8\sqrt{\pi}d^{2}\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i}{d}\right.}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^{2}d^{2}}}+1\right)b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*
sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x
 + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*
e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

3.188 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\right.}{512b}$$

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rubi [A] time = 0.572442, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\right.}{512b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{(3d)\int\sqrt{c+dx}\cos(}{64b}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+dx}\sin(2a+}{32b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+dx}\sin(2a+}{32b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+dx}\sin(2a+}{32b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\right.}{512}$$

Mathematica [A] time = 1.61926, size = 393, normalized size = 1.12

$$\frac{-3\sqrt{2\pi}d\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)-48\sqrt{\pi}d\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-3\sqrt{2\pi}d\cos\left(4a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
- 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

Maple [A] time = 0.033, size = 376, normalized size = 1.1

$$2\,\frac{1}{d}\left(-1/16\,\frac{d\,(dx+c)^{3/2}}{b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+3/16\,\frac{d}{b}\left(1/4\,\frac{d\sqrt{dx+c}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)-1/8\,\frac{d\sqrt{\pi}}{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.2755, size = 1806, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8192*sqrt(2)*(128*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(4*((d*x + c)*b
- b*c + a*d)/d)/abs(d) + 512*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(2*((d*x
 + c)*b - b*c + a*d)/d)/abs(d) - 48*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(4*
((d*x + c)*b - b*c + a*d)/d)/abs(d) - 384*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*
sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - ((-48*I*sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*sqrt(pi)*sin(1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs
(d))*cos(-2*(b*c - a*d)/d) - (48*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2
```

```
(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (sqrt(2)*(-3*I*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*si
n(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(
b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(3*I*sqrt(
pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqr
t(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sq
rt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sq
rt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*
sqrt(abs(b)/abs(d))*cos(-4*(b*c - a*d)/d) - sqrt(2)*(3*sqrt(pi)*cos(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/a
bs(d))*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((48*I*sq
rt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I
*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
48*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 48*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (48*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 48*I*sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 48*I*sqrt(pi)*sin(
-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)
/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(
b^2*d*abs(b))
```

Fricas [A] time = 0.63998, size = 749, normalized size = 2.13

$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+48\pi d^2\sqrt{\frac{b}{\pi d}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(
b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.53742, size = 1485, normalized size = 4.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2048*(8*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b +
 16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt
(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c
)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c + I*sqrt(2)*sqrt(pi)*
```

```
(8*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) + I*sqrt(2)*sqrt(pi)*(8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*e
rf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*
d + 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8
*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(
(4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*
d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)
*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/
b^2 - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x +
 c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d
```

3.189 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=407

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4\right.}{}$$

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rubi [A] time = 0.672366, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```


```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
```

```
Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{(5d)\int (c+dx)^{3/2}\text{c}}{64}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c+dx)^{3/2}\sin(}{32b^2}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos}{2048b^3}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos}{2048b^3}$$

Mathematica [A] time = 10.9173, size = 550, normalized size = 1.35

$$-1024b^3c^2\sqrt{c+dx}\cos(2(a+bx))-256b^3c^2\sqrt{c+dx}\cos(4(a+bx))-1024b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx))-256b^3d^2x^2\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] +
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

Maple [A] time = 0.034, size = 470, normalized size = 1.2

$$2\frac{1}{d}\left(-1/16\frac{d(dx+c)^{5/2}}{b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+\frac{5d}{16b}\left(1/4\frac{d(dx+c)^{3/2}}{b}\sin\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)-3/4\frac{d}{b}\left(-1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.24238, size = 1879, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/65536*sqrt(2)*(640*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(4*((d*x + c)*
b - b*c + a*d)/d)/abs(d) + 5120*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)*sin(2*
((d*x + c)*b - b*c + a*d)/d)/abs(d) - 16*(64*sqrt(2)*(d*x + c)^(5/2)*b^2*d*
abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(4*((d*x + c
```

)*b - b*c + a*d)/d) - 256*(16*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(2)*sqrt(d*x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - ((480*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr t(d^2))) + 480*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/ sqrt(d^2))) - 480*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0 , d/sqrt(d^2))) + 480*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct an2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (480* I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^ 2))) - 480*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt (d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s qrt(2*I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2* arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/ 2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-4*(b*c - a* d)/d) - sqrt(2)*(15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2 (0, d/sqrt(d^2))) + 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc tan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*ar ctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2* arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-4*(b*c - a*d)/d))*er f(2*sqrt(d*x + c)*sqrt(I*b/d)) - (sqrt(2)*(15*sqrt(pi)*cos(1/4*pi + 1/2*arc tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*cos(-1/4*pi + 1/2*a rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*I*sqrt(pi)*sin(1/4*pi + 1/ 2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d)) *cos(-4*(b*c - a*d)/d) - sqrt(2)*(-15*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0 , b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*I*sqrt(pi)*cos(-1/4*pi + 1/2*arcta n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 15*sqrt(pi)*sin(1/4*pi + 1/2*arct an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 15*sqrt(pi)*sin(-1/4*pi + 1/2*ar ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-4* (b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((480*sqrt(pi)*cos(1/4* pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*cos(-1 /4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*I*sqrt(pi)*s in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt(p i)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt (abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) - (-480*I*sqrt(pi)*cos(1/4*pi + 1/2*a rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 480*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 480*sqrt(pi)*sin(-1/4* pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*abs(d)/(b^3*d *abs(b))

Fricas [A] time = 0.693544, size = 944, normalized size = 2.32

$$-\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+480\pi d^3\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
+ 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x
+ 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*
x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x +
 b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x +
a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.74143, size = 2680, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16384*(64*(sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*d^2*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b) + 8*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b
 + 16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sq
rt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x +
 c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2 + d^2*((I*sqrt(2)
*sqrt(pi)*(-64*I*b^2*c^2*d + 48*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 1
28*I*(d*x + c)^(3/2)*b^2*c*d + 64*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^
(3/2)*b*d^2 - 48*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(
d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^2 + (I*sqrt(2)*sqrt(pi)*(-64*I*b^
2*c^2*d - 48*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 128*I*(d*x + c)^(
3/2)*b^2*c*d + 64*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 48
*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I*
b*c + 4*I*a*d)/d)/b^3)/d^2 + 32*(I*sqrt(pi)*(-16*I*b^2*c^2*d + 24*b*c*d^2 +
 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I
*(d*x + c)^(5/2)*b^2*d - 32*I*(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*
b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt
(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 32*(
I*sqrt(pi)*(-16*I*b^2*c^2*d - 24*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 32*I*
(d*x + c)^(3/2)*b^2*c*d + 16*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)
*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x +
c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) + 16*(I*sqrt(2)*sqrt(pi)*(8*I*b*c*d
- 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*s
qrt(2)*sqrt(pi)*(8*I*b*c*d + 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 16*I*sqrt(pi)*(4*I*b*c*d + 3*d^2)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c
)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x +
```

```
 c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqr
t(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I
*a*d)/d)/b^2 - 32*I*(4*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*
sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2 - 4*I*(
8*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^
((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)*c)/d
```

3.190 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=615

$$-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a\right.}{}$$

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*
d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fr
esnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5
/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*
Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) -
(5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 1.14485, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```


```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*
d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fr
esnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5
```

```
/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*
Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) -
(5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx) - \frac{1}{16}(c+dx)^{5/2}\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{5/2}\cos(5a+5bx)\,dx$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{5/2}\sin(5a+5b}{80b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

Mathematica [C] time = 23.4923, size = 1795, normalized size = 2.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/
Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*S
qrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/
d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b
*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*
c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (
4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(
c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(64*b^5) - (c
^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c
+ d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a
- (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqr
t[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi
]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) +
 Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*C
os[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(
Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) - ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c
^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b
*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d
*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin
[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) - (c^2*(-(Sqrt[2*Pi]*Cos[5*a - (5*b*c)/
d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*FresnelC[Sqr
t[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b/d
]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*d*(Sqrt[b
/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[5*a
- (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS
[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Si
n[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)] + 10*b*
x*Sin[5*(a + b*x)])))/(800*Sqrt[5]*b^3) - ((b/d)^(3/2)*d^2*(-(Sqrt[2*Pi]*Fr
esnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*((20*b^2*c^2 - 3*d^2)*Cos[5*a -
 (5*b*c)/d] + 12*b*c*d*Sin[5*a - (5*b*c)/d])) - Sqrt[2*Pi]*FresnelC[Sqrt[b/
d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[5*a - (5*b*c)/d] + (20*b^2*c^2
 - 3*d^2)*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b
*(c - 5*d*x)*Cos[5*(a + b*x)] + d*(-3 + 20*b^2*x^2)*Sin[5*(a + b*x)])))/(32
00*Sqrt[5]*b^5)
```

Maple [A] time = 0.051, size = 716, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/16/b*d*(-1/2/b
*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1
/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1
/96/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2
)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(d*x+c)^(1/2)*b/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5/d*(d*x+c)*b+5*(a*
d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/100/b
*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2
)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.64826, size = 2940, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/1728000*sqrt(5)*sqrt(3)*(720*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*sqrt(
abs(b)/abs(d))*abs(b)*cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 2000*sqrt
(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*abs(b)*cos(3*((d*x +
c)*b - b*c + a*d)/d)/abs(d) - 36000*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*s
qrt(abs(b)/abs(d))*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) + (sqrt(3
)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
```

qrt(d^2))))*d^3*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) + sqrt(3)*(27*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I*s
qrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3
*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (s
qrt(5)*(125*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 125*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) + 125*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - 125*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(125
*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 1
25*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 125*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + 125*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))))*d^3*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) + (sqrt(5)*sqrt(3)*(-6750*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*sqrt(pi)*sin(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-(b*c - a*d)/
d)/abs(d) - sqrt(5)*sqrt(3)*(6750*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-(b*c - a*d)/d
)/abs(d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(6750*I*sqrt(pi
)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sq
rt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 675
0*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
6750*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^3*abs(b)*cos(-(b*c - a*d)/d)/abs(d) - sqrt(5)*sqrt(3)*(6750*sqrt(pi)*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi
)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 675
0*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^3*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) +
(sqrt(5)*(-125*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) - 125*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + 125*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) - 125*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(
125*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 125*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 125*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(

```
d^2))) - 125*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))))*d^3*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqr
t(-3*I*b/d)) + (sqrt(3)*(-27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d)
+ sqrt(3)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x +
 c)*sqrt(-5*I*b/d)) + 72*(20*sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqrt(abs
(b)/abs(d))*abs(b)/abs(d) - 3*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(abs(b)
/abs(d))*abs(b)/abs(d))*sin(5*((d*x + c)*b - b*c + a*d)/d) + 200*(12*sqrt(5
)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d))*abs(b)/abs(d) - 5*sqrt(
5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d))*abs(b)/abs(d))*sin(3*((d*x
 + c)*b - b*c + a*d)/d) - 3600*(4*sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqr
t(abs(b)/abs(d))*abs(b)/abs(d) - 15*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(
abs(b)/abs(d))*abs(b)/abs(d))*sin(((d*x + c)*b - b*c + a*d)/d))*abs(d)/(b^3
*d*sqrt(abs(b)/abs(d))*abs(b))
```

Fricas [A] time = 0.839235, size = 1397, normalized size = 2.27

$$-\frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-101250}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
 - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*f
resnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*
sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(90*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x
```

```
 + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*c
os(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 +
11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)

[Out] Timed out
```

Giac [C] time = 2.16739, size = 4077, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/864000*(60*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(
-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(p
i)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*s
qrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I
*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c
+ 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/
d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150
```

*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sq
rt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c^2 - d^2*(27
*(sqrt(10)*sqrt(pi)*(-20*I*b^2*c^2*d + 12*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sqr
t(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*(20*I*(d*x + c)^(5
/2)*b^2*d - 40*I*(d*x + c)^(3/2)*b^2*c*d + 20*I*sqrt(d*x + c)*b^2*c^2*d + 1
0*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 3*I*sqrt(d*x + c)*d^3)
*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^2 + 125*(sqrt(6)*sqrt(
pi)*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b
d)(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(
d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*
b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c
)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^2 + 6750*(sqrt(2)*sqrt(pi)*(4*I*b^2*c^2*
d - 12*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 8*I*(d*x + c)^(3/2)*b^2*c*d
 - 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)
*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^3)/d^2 + 6750*(sqrt(2)*sqrt(pi)*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*
x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^
2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
 c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^2 + 125*(sqrt(6)*sqrt
(pi)*(12*I*b^2*c^2*d + 12*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 24
I(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3
/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x
+ c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^2 + 27*(sqrt(10)*sqrt(pi)*(20*I*b^2*c
^2*d + 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b^3) - 10*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x + c)^(
3/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12
*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b
*c + 5*I*a*d)/d)/b^3)/d^2) - 12*(9*sqrt(10)*sqrt(pi)*(10*I*b*c*d - 3*d^2)*d
*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 125*sqrt
(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) + 2250*sqrt(2)*sqrt(pi)*(-2*I*b*c*d + 3*d^2)*d*erf(-1/
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c -
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2250*sqrt(2)*sqrt(pi)
*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr

```
t(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) + 125*sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*sqrt(10)*sqrt(pi)*(-10*I*b
*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 90*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*c*d + 3*sqrt
(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2 - 750*(-2*I*
(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I*
(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 4500*(2*I*(d*x + c)^(3/2)*b*d - 2
*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I
*a*d)/d)/b^2 - 4500*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - 3
*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 750*(2*I*(
d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*
(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 - 90*(10*I*(d*x + c)^(3/2)*b*d - 10
*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b + 5*I*b*
c - 5*I*a*d)/d)/b^2)*c)/d
```

3.191 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=534

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\cos\left(5a-\right.}{}$$

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.837616, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\cos\left(5a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
```

```
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx) - \frac{1}{16}(c+dx)^{3/2}\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{3/2}\cos(5a+5bx)\,dx$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5a+5b}{80b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5}{800b^2}$$

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$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5}{800b^2}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5}{800b^2}$$

Mathematica [C] time = 12.5847, size = 1043, normalized size = 1.95

$$-\frac{ice^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} + \frac{d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sq
rt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d]
)/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt[
```

```
2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d] +
 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sqr
t[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(32*b^3) - (c*(-(Sqrt[2*
Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sq
rt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]
+ 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqrt[3]*b*Sqrt[b
/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]
]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqrt
[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c
)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x)
] + 2*b*x*Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) - (c*(-(Sqrt[2*Pi]*Cos[5*a
- (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*Fr
esnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5
]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (d
*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*
Cos[5*a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*
FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d]
+ 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)]
 + 10*b*x*Sin[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

Maple [A] time = 0.051, size = 583, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/16/b*d*(-1/2/b
*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(-1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/160/b*d*(d*x+c)^(3/2)*s
in(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*
(d*x+c)*b+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(co
s(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*
```

```
x+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.61676, size = 2790, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288000*sqrt(5)*sqrt(3)*(240*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs
(b)/abs(d))*abs(b)*sin(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 400*sqrt(5)*
sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*abs(b)*sin(3*((d*x + c)*b -
 b*c + a*d)/d)/abs(d) - 2400*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b
)/abs(d))*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) + 72*sqrt(5)*sqrt(
3)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*cos(5*((d*x + c)*b - b*c +
a*d)/d)/abs(d) + 200*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*
abs(b)*cos(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 3600*sqrt(5)*sqrt(3)*sqr
t(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/
abs(d) - (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(
9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d^2*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/
d)) - (sqrt(5)*(25*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) + 25*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - 25*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 25*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) - sqrt(5)*
(25*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + 25*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 25*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) - 25*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))))*d^2*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) + (sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0
```

```
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-(b*c - a*d)/d)/ab
s(d) + sqrt(5)*sqrt(3)*(-450*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*sin(-(b*c - a*d)/d)/abs(d)
)*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b
)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(450*I*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*cos(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*sin
(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (sqrt(5)*(25*sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sq
rt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*
I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
25*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) - sqrt(5)*(-25*I*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*I*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*
sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*
(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(-9*I*sqrt(pi)*cos(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*co
s(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*s
in(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b^2
*d*sqrt(abs(b)/abs(d))*abs(b))
```

Fricas [A] time = 0.765724, size = 1162, normalized size = 2.18

$$\frac{27 \sqrt{10}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{5(bc-ad)}{d}\right) C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 125 \sqrt{6}\pi d^2 \sqrt{\frac{b}{\pi d}} \cos\left(-\frac{3(bc-ad)}{d}\right) C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 6750 \sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x
 + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2
*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2
)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.77981, size = 2271, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

[Out] -1/144000*(10*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c - 9*sqrt(10)*sqrt(pi)*(10*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 125*sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2250*sqrt(2)*sqrt(pi)*(-2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2250*sqrt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 125*sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*sqrt(10)*sqrt(pi)*(-10*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 90*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2 + 750*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 4500*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 + 4500*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 + 750*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 + 90*(10*I*(d*x + c)^(3/2)*b*d - 10*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d

3.192 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{F}}{8b}$$

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.692595, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{F}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```


```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
```

```
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\cos(a+bx) - \frac{1}{16}\sqrt{c+dx}\cos(3a+3bx) - \frac{1}{16}\sqrt{c+dx}\cos(5a+\right.$$

$$= -\left(\frac{1}{16}\int \sqrt{c+dx}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int \sqrt{c+dx}\cos(5a+5bx)\,dx + \frac{1}{8}\int$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b} + \frac{d}{}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b} + \frac{(d}{}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b} + \frac{\mathrm{co}}{}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}$$

Mathematica [C] time = 7.04336, size = 435, normalized size = 0.95

$$-\frac{i\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} - \frac{-\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - }{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt
[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/
Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - (-(Sqrt[2*Pi]*Cos[3*a
 - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*Fr
esnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[3]
*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)])/(96*Sqrt[3]*b*Sqrt[b/d]) - (-(Sq
rt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]
) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b
*c)/d] + 2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)])/(160*Sqrt[5]*b
*Sqrt[b/d])
```

Maple [A] time = 0.049, size = 444, normalized size = 1.

$$2\frac{1}{d}\left(1/16\,\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-1/32\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)+\sin\left(\frac{ad-b}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)
/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/160
/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2)*Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.53509, size = 2531, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/28800*sqrt(5)*sqrt(3)*(24*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/ab
s(d))*abs(b)*sin(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 40*sqrt(5)*sqrt(3)
*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*sin(3*((d*x + c)*b - b*c + a*d)
/d)/abs(d) - 240*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)
*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) + (sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-5*
(b*c - a*d)/d)/abs(d) - sqrt(3)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
```

+ 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (sqrt(5)*(-5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) - sqrt(5)*(5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(5)*sqrt(3)*(30*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(-30*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (sqrt(5)*(5*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) - sqrt(5)*(5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (sqrt(3)*(3*I*sqrt(pi)*cos

```
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*c
os(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*co
s(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/a
bs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b*d*sqrt(abs(b)/abs(d))*a
bs(b))
```

Fricas [A] time = 0.715721, size = 973, normalized size = 2.12

$$9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-450\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sq
rt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-
3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sq
rt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/
(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sq
rt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x +
 a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.44228, size = 988, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/14400*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*d^
2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6
)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c
+ 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I
*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b
- 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sq
rt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*
x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d
```

3.193 $\int \sqrt{c+dx} \cos^3(a+bx) \sin^2(a+bx)\, dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{Fre}}{8b^{3/}}$$

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.671504, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{Fre}}{8b^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```


```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
```

```
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\cos(a+bx) - \frac{1}{16}\sqrt{c+dx}\cos(3a+3bx) - \frac{1}{16}\sqrt{c+dx}\cos(5a+5$$

$$= -\left(\frac{1}{16}\int \sqrt{c+dx}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int \sqrt{c+dx}\cos(5a+5bx)\,dx + \frac{1}{8}\int$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b} + \frac{d\int}{}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b} + \frac{(d\,c}{}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b} + \frac{\cos}{}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}$$

Mathematica [C] time = 7.04373, size = 435, normalized size = 0.95

$$-\frac{i\sqrt{c+dx}e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} - \frac{-\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - \sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt
[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/
Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - (-(Sqrt[2*Pi]*Cos[3*a
 - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*Fr
esnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[3]
*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)])/(96*Sqrt[3]*b*Sqrt[b/d]) - (-(Sq
rt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]
) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b
*c)/d] + 2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)])/(160*Sqrt[5]*b
*Sqrt[b/d])
```

Maple [A] time = 0.05, size = 444, normalized size = 1.

$$2\frac{1}{d}\left(1/16\,\frac{d\sqrt{dx+c}}{b}\sin\left(\frac{(dx+c)\,b}{d}+\frac{ad-bc}{d}\right)-1/32\,\frac{d\sqrt{2}\sqrt{\pi}}{b}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{dx+c}b}{\sqrt{\pi}d}\frac{1}{\sqrt{\frac{b}{d}}}\right)+\sin\left(\frac{ad-}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)
/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/160
/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2)*Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.51465, size = 2531, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/28800*sqrt(5)*sqrt(3)*(24*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/ab
s(d))*abs(b)*sin(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 40*sqrt(5)*sqrt(3)
*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)*sin(3*((d*x + c)*b - b*c + a*d)
/d)/abs(d) - 240*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d*sqrt(abs(b)/abs(d))*abs(b)
*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) + (sqrt(3)*(-3*I*sqrt(pi)*cos(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*cos(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*sin(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*sin(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-5*
(b*c - a*d)/d)/abs(d) - sqrt(3)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
```

+ 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d)
)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (sqrt(5)*(-5*I*sqrt(pi)*cos(1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-3*(b*c -
 a*d)/d)/abs(d) - sqrt(5)*(5*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*
arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2
*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(
sqrt(d*x + c)*sqrt(3*I*b/d)) + (sqrt(5)*sqrt(3)*(30*I*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*cos(-1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*sin(1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqrt(pi)*sin(-1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*cos(-(b*
c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*sin(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b*c - a*d)/d)
/abs(d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(-30*I*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(p
i)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*sqr
t(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs
(b)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(30*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 30*I*sqrt(pi)*sin(1/4*
pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 30*I*sqrt(pi)*sin(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-(b
*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (sqrt(5)*(5*I*sqrt(p
i)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt
(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqr
t(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqr
t(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs
(b)*cos(-3*(b*c - a*d)/d)/abs(d) - sqrt(5)*(5*sqrt(pi)*cos(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*sin(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*sin(-1/4*pi + 1
/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-3*(b*c - a*d
)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (sqrt(3)*(3*I*sqrt(pi)*cos

```
(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*c
os(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*sqrt(pi)*
sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*co
s(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(3*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 3*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 3*I*sqrt(pi)*sin(-1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*abs(b)*sin(-5*(b*c - a*d)/d)/a
bs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b*d*sqrt(abs(b)/abs(d))*a
bs(b))
```

Fricas [A] time = 0.71869, size = 973, normalized size = 2.12

$$\frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-450\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sq
rt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-
3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sq
rt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/
(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sq
rt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x +
 a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

[Out] Timed out

Giac [C] time = 1.42066, size = 988, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/14400*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*d^
2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6
)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c
+ 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I
*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b
- 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sq
rt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*
x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d
```

3.194 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=534

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\right.}{}$$

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 0.857925, antiderivative size = 534, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```


```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
```

```
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx) - \frac{1}{16}(c+dx)^{3/2}\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{3/2}\cos(5a+5bx)\,dx$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5a+5b}{80b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5}{800b^2}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5}{800b^2}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5}{800b^2}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5}{800b^2}$$

Mathematica [C] time = 12.4057, size = 1043, normalized size = 1.95

$$-\frac{ice^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\text{Gamma}\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\text{Gamma}\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} + \frac{d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\text{FresnelC}\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sq
rt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d]
)/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt[
```

```
2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d] +
 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sqr
t[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(32*b^3) - (c*(-(Sqrt[2*
Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sq
rt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]
+ 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqrt[3]*b*Sqrt[b
/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]
]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqrt
[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c
)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x)
] + 2*b*x*Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) - (c*(-(Sqrt[2*Pi]*Cos[5*a
- (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*Fr
esnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5
]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (d
*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*
Cos[5*a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*
FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d]
+ 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)]
 + 10*b*x*Sin[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

Maple [A] time = 0.052, size = 583, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/16/b*d*(-1/2/b
*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(-1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/160/b*d*(d*x+c)^(3/2)*s
in(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*
(d*x+c)*b+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(co
s(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*
```

```
x+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.62542, size = 2790, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288000*sqrt(5)*sqrt(3)*(240*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs
(b)/abs(d))*abs(b)*sin(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 400*sqrt(5)*
sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b)/abs(d))*abs(b)*sin(3*((d*x + c)*b -
 b*c + a*d)/d)/abs(d) - 2400*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d*sqrt(abs(b
)/abs(d))*abs(b)*sin(((d*x + c)*b - b*c + a*d)/d)/abs(d) + 72*sqrt(5)*sqrt(
3)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*cos(5*((d*x + c)*b - b*c +
a*d)/d)/abs(d) + 200*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^2*sqrt(abs(b)/abs(d))*
abs(b)*cos(3*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 3600*sqrt(5)*sqrt(3)*sqr
t(d*x + c)*d^2*sqrt(abs(b)/abs(d))*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/
abs(d) - (sqrt(3)*(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(
9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))))*d^2*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/
d)) - (sqrt(5)*(25*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) + 25*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - 25*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) + 25*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) - sqrt(5)*
(25*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) + 25*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 25*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) - 25*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))))*d^2*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) + (sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0
```

, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*sin(-1/4*pi + 1/2*arct
an2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*cos(-(b*c - a*d)/d)/ab
s(d) + sqrt(5)*sqrt(3)*(-450*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*sin(-(b*c - a*d)/d)/abs(d)
)*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(450*sqrt(pi)*cos(1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*cos(-1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*I*sqrt(pi
)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b
)*cos(-(b*c - a*d)/d)/abs(d) + sqrt(5)*sqrt(3)*(450*I*sqrt(pi)*cos(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*I*sqrt(pi)*cos(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 450*sqrt(pi)*sin(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 450*sqrt(pi)*sin
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*sin
(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (sqrt(5)*(25*sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sq
rt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*
I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) -
25*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^2*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) - sqrt(5)*(-25*I*sqrt(pi)*cos(1/
4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*I*sqrt(pi)*cos
(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 25*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 25*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*
sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - (sqrt(3)*
(9*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 9*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 9*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
) - 9*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))))*d^2*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) - sqrt(3)*(-9*I*sqrt(pi)*cos(
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*co
s(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*s
in(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*s
in(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*abs(b)*s
in(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*abs(d)/(b^2
*d*sqrt(abs(b)/abs(d))*abs(b))

Fricas [A] time = 0.769284, size = 1162, normalized size = 2.18

$$27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-6750\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x
 + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2
*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2
)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.81203, size = 2271, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

[Out] -1/144000*(10*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c - 9*sqrt(10)*sqrt(pi)*(10*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 125*sqrt(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2250*sqrt(2)*sqrt(pi)*(-2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2250*sqrt(2)*sqrt(pi)*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 125*sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*sqrt(10)*sqrt(pi)*(-10*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 90*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2 + 750*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 + 4500*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 + 4500*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 + 750*(2*I*(d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 + 90*(10*I*(d*x + c)^(3/2)*b*d - 10*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d

3.195 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=615

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a\right.}{}$$

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*
d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fr
esnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5
/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*
Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) -
(5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rubi [A] time = 1.01548, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*
d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fr
esnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5
```

```
/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*
Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) -
(5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx) - \frac{1}{16}(c+dx)^{5/2}\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{5/2}\cos(5a+5bx)\,dx$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{5/2}\sin(5a+5}{80b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(}{160b^2}$$

Mathematica [C] time = 23.4094, size = 1795, normalized size = 2.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/
Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*S
qrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/
d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b
*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*
c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (
4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(
c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(64*b^5) - (c
^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c
+ d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a
- (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqr
t[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi
]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) +
 Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*C
os[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(
Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) - ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c
^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b
*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d
*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin
[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) - (c^2*(-(Sqrt[2*Pi]*Cos[5*a - (5*b*c)/
d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*FresnelC[Sqr
t[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b/d
]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*d*(Sqrt[b
/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[5*a
- (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS
[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Si
n[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)] + 10*b*
x*Sin[5*(a + b*x)])))/(800*Sqrt[5]*b^3) - ((b/d)^(3/2)*d^2*(-(Sqrt[2*Pi]*Fr
esnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*((20*b^2*c^2 - 3*d^2)*Cos[5*a -
 (5*b*c)/d] + 12*b*c*d*Sin[5*a - (5*b*c)/d])) - Sqrt[2*Pi]*FresnelC[Sqrt[b/
d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[5*a - (5*b*c)/d] + (20*b^2*c^2
 - 3*d^2)*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b
*(c - 5*d*x)*Cos[5*(a + b*x)] + d*(-3 + 20*b^2*x^2)*Sin[5*(a + b*x)])))/(32
00*Sqrt[5]*b^5)
```

Maple [A] time = 0.051, size = 716, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/16/b*d*(-1/2/b
*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1
/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1
/96/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2
)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(d*x+c)^(1/2)*b/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5/d*(d*x+c)*b+5*(a*
d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/100/b
*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2
)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.69643, size = 2940, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/1728000*sqrt(5)*sqrt(3)*(720*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*sqrt(
abs(b)/abs(d))*abs(b)*cos(5*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 2000*sqrt
(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*sqrt(abs(b)/abs(d))*abs(b)*cos(3*((d*x +
c)*b - b*c + a*d)/d)/abs(d) - 36000*sqrt(5)*sqrt(3)*(d*x + c)^(3/2)*b*d^2*s
qrt(abs(b)/abs(d))*abs(b)*cos(((d*x + c)*b - b*c + a*d)/d)/abs(d) + (sqrt(3
)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt
(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/s
```

qrt(d^2))))*d^3*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d) + sqrt(3)*(27*sqrt(pi)*
cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)
*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*I*s
qrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3
*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (s
qrt(5)*(125*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 125*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) + 125*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - 125*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(125
*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 1
25*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
- 125*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2
))) + 125*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqr
t(d^2))))*d^3*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) + (sqrt(5)*sqrt(3)*(-6750*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*sqrt(pi)*sin(1/4*pi + 1/2*arc
tan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*sin(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-(b*c - a*d)/
d)/abs(d) - sqrt(5)*sqrt(3)*(6750*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 6750*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-(b*c - a*d)/d
)/abs(d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + (sqrt(5)*sqrt(3)*(6750*I*sqrt(pi
)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*sq
rt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 675
0*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
6750*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^3*abs(b)*cos(-(b*c - a*d)/d)/abs(d) - sqrt(5)*sqrt(3)*(6750*sqrt(pi)*c
os(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*sqrt(pi
)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 6750*I*s
qrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 675
0*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
)*d^3*abs(b)*sin(-(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) +
(sqrt(5)*(-125*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d
/sqrt(d^2))) - 125*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2
(0, d/sqrt(d^2))) + 125*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arcta
n2(0, d/sqrt(d^2))) - 125*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*ar
ctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-3*(b*c - a*d)/d)/abs(d) + sqrt(5)*(
125*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
+ 125*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) + 125*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(

```
d^2))) - 125*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))))*d^3*abs(b)*sin(-3*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x + c)*sqr
t(-3*I*b/d)) + (sqrt(3)*(-27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/
2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*abs(b)*cos(-5*(b*c - a*d)/d)/abs(d)
+ sqrt(3)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^3*abs(b)*sin(-5*(b*c - a*d)/d)/abs(d))*erf(sqrt(d*x +
 c)*sqrt(-5*I*b/d)) + 72*(20*sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqrt(abs
(b)/abs(d))*abs(b)/abs(d) - 3*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(abs(b)
/abs(d))*abs(b)/abs(d))*sin(5*((d*x + c)*b - b*c + a*d)/d) + 200*(12*sqrt(5
)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqrt(abs(b)/abs(d))*abs(b)/abs(d) - 5*sqrt(
5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(abs(b)/abs(d))*abs(b)/abs(d))*sin(3*((d*x
 + c)*b - b*c + a*d)/d) - 3600*(4*sqrt(5)*sqrt(3)*(d*x + c)^(5/2)*b^2*d*sqr
t(abs(b)/abs(d))*abs(b)/abs(d) - 15*sqrt(5)*sqrt(3)*sqrt(d*x + c)*d^3*sqrt(
abs(b)/abs(d))*abs(b)/abs(d))*sin(((d*x + c)*b - b*c + a*d)/d))*abs(d)/(b^3
*d*sqrt(abs(b)/abs(d))*abs(b))
```

Fricas [A] time = 0.830458, size = 1397, normalized size = 2.27

$$-\frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-101250}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
 - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*f
resnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*
sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(90*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x
```

```
 + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*c
os(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 +
11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.12865, size = 4077, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/864000*(60*(9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*d^2*erf(
-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(p
i)*d^2*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*s
qrt(6)*sqrt(pi)*d^2*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*d^2*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I
*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c
+ 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/
d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150
```

*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sq
rt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c^2 - d^2*(27
*(sqrt(10)*sqrt(pi)*(-20*I*b^2*c^2*d + 12*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sqr
t(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*(20*I*(d*x + c)^(5
/2)*b^2*d - 40*I*(d*x + c)^(3/2)*b^2*c*d + 20*I*sqrt(d*x + c)*b^2*c^2*d + 1
0*(d*x + c)^(3/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 3*I*sqrt(d*x + c)*d^3)
*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^2 + 125*(sqrt(6)*sqrt(
pi)*(-12*I*b^2*c^2*d + 12*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b
d)(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*(12*I*(d*x + c)^(5/2)*b^2*d - 24*I*(
d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*
b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c
)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^2 + 6750*(sqrt(2)*sqrt(pi)*(4*I*b^2*c^2*
d - 12*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 8*I*(d*x + c)^(3/2)*b^2*c*d
 - 4*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)
*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b
^3)/d^2 + 6750*(sqrt(2)*sqrt(pi)*(-4*I*b^2*c^2*d - 12*b*c*d^2 + 15*I*d^3)*d
*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*
x + c)^(5/2)*b^2*d - 8*I*(d*x + c)^(3/2)*b^2*c*d + 4*I*sqrt(d*x + c)*b^2*c^
2*d - 10*(d*x + c)^(3/2)*b*d^2 + 12*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
 c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^2 + 125*(sqrt(6)*sqrt
(pi)*(12*I*b^2*c^2*d + 12*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 24
I(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3
/2)*b*d^2 - 12*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x
+ c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^2 + 27*(sqrt(10)*sqrt(pi)*(20*I*b^2*c
^2*d + 12*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b^3) - 10*(-20*I*(d*x + c)^(5/2)*b^2*d + 40*I*(d*x + c)^(
3/2)*b^2*c*d - 20*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 12
*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b
*c + 5*I*a*d)/d)/b^3)/d^2) - 12*(9*sqrt(10)*sqrt(pi)*(10*I*b*c*d - 3*d^2)*d
*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 125*sqrt
(6)*sqrt(pi)*(2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) + 2250*sqrt(2)*sqrt(pi)*(-2*I*b*c*d + 3*d^2)*d*erf(-1/
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c -
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2250*sqrt(2)*sqrt(pi)
*(2*I*b*c*d + 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr

```
t(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) + 125*sqrt(6)*sqrt(pi)*(-2*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*sqrt(10)*sqrt(pi)*(-10*I*b
*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 90*(-10*I*(d*x + c)^(3/2)*b*d + 10*I*sqrt(d*x + c)*b*c*d + 3*sqrt
(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2 - 750*(-2*I*
(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I*
(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2 - 4500*(2*I*(d*x + c)^(3/2)*b*d - 2
*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I
*a*d)/d)/b^2 - 4500*(-2*I*(d*x + c)^(3/2)*b*d + 2*I*sqrt(d*x + c)*b*c*d - 3
*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2 - 750*(2*I*(
d*x + c)^(3/2)*b*d - 2*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*
(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2 - 90*(10*I*(d*x + c)^(3/2)*b*d - 10
*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b + 5*I*b*
c - 5*I*a*d)/d)/b^2)*c)/d
```

3.196 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\sin\left(6a-\right.}{184}$$

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rubi [A] time = 0.896867, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\sin\left(6a-\right.}{184}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```


```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
```

```
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right) dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^{5/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} - \frac{(5d)\int (c+dx)^{3/}}{3}$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{15d(c+dx)^{3/2}\,\mathrm{s}}{256b}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos}{9216b}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos}{9216b}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos}{9216b}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos}{9216b}$$

Mathematica [A] time = 5.34667, size = 550, normalized size = 1.35

$$-2592b^3c^2\sqrt{c+dx}\cos(2(a+bx)) + 288b^3c^2\sqrt{c+dx}\cos(6(a+bx)) - 2592b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx)) + 288b^3d^2x^2\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Co
s[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d
^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a
+ b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c +
d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*
Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/P
i]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresne
lC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[
b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a -
(2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*S
qrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)]
- 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(55296*b^4)
```

Maple [A] time = 0.043, size = 477, normalized size = 1.2

$$2\frac{1}{d}\left(-\frac{3\,d\,(dx+c)^{5/2}}{128\,b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+\frac{15\,d}{128\,b}\left(1/4\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)-3/4\,\frac{d}{b}\left(-1/4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+15/128/b*d*(
1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d
*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2
*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/384/b
*d*(d*x+c)^(5/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+
c)^(3/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*
cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.24806, size = 1916, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/884736*sqrt(6)*sqrt(2)*(160*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)
*sin(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 4320*sqrt(6)*sqrt(2)*(d*x + c)
^(3/2)*b*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*(48*sqrt(
```

6)*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 5*sqrt(6)*sqrt(2)*sqrt(d*x
 + c)*d^3*abs(b)/abs(d))*cos(6*((d*x + c)*b - b*c + a*d)/d) + 216*(16*sqrt(
6)*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(6)*sqrt(2)*sqrt(d*
x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - (sqrt(2)*(5*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5
*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^3*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(5*I*sqrt(pi)*co
s(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(a
bs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (sq
rt(6)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) - 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)
*(-405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(2*I*b/d)) + (sqrt(6)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-
2*(b*c - a*d)/d) + sqrt(6)*(405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(5*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(ab
s(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(-5*I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))
sin(-6(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*abs(d)/(b^3*d*ab

```
s(b))
```

Fricas [A] time = 0.767491, size = 1089, normalized size = 2.68

$$\frac{5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6(bc-ad)}{d}\right)-1215\pi d^3\sqrt{\frac{b}{\pi d}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*f
resnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) -
1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^3*d^2*
x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c*d*x +
24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c*d*x + 4
8*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x + b^2*c*d
)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^2*d^2*x +
b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.70005, size = 2677, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/110592*(48*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)
/b - 54*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*
sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x
 + c)*d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c^2 - d^2*((I*sqrt(
3)*sqrt(pi)*(48*I*b^2*c^2*d - 24*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d +
96*I*(d*x + c)^(3/2)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^
(3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((-6*I*(d
*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^3)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*I*b^2*
c^2*d + 24*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 96*I*(d*x + c)^(3/2
)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sq
rt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((6*I*(d*x + c)*b - 6*I*b*c
 + 6*I*a*d)/d)/b^3)/d^2 + 27*(I*sqrt(pi)*(-48*I*b^2*c^2*d + 72*b*c*d^2 + 45*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*
b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*(d*
x + c)^(5/2)*b^2*d - 96*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*
c^2*d + 60*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 45*I*sqrt(d*x
 + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 27*(I*sq
rt(pi)*(-48*I*b^2*c^2*d - 72*b*c*d^2 + 45*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*(d*x + c)^(5/2)*b^2*d - 96*I*(d*x
 + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 60*(d*x + c)^(3/2)*b*d
^2 + 72*sqrt(d*x + c)*b*c*d^2 - 45*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b
 - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) - 24*(I*sqrt(3)*sqrt(pi)*(-4*I*b*c*d + d
^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(3
)*sqrt(pi)*(-4*I*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) + 9*I*sqrt(pi)*(12*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqr
```

```
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(pi)*(12*I*b*c*d + 9*d^2)*d*erf(
-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/
2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((6*I*(d*x + c)*b -
 6*I*b*c + 6*I*a*d)/d)/b^2 - 18*I*(12*I*(d*x + c)^(3/2)*b*d - 12*I*sqrt(d*x
 + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)
/d)/b^2 - 18*I*(12*I*(d*x + c)^(3/2)*b*d - 12*I*sqrt(d*x + c)*b*c*d + 9*sqr
t(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2 - 6*I*(-4*
I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6
*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2)*c)/d
```

3.197 $\int (c+dx)^{3/2} \cos^3(a+bx) \sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$\frac{\sqrt{\frac{\pi}{3}}\, d^{3/2} \sin\left(6a-\frac{6bc}{d}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536 b^{5/2}} - \frac{9\sqrt{\pi}\, d^{3/2} \sin\left(2a-\frac{2bc}{d}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{512 b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}}\, d^{3/2} \cos\left(6a-\frac{6bc}{d}\right.}{1536 b^{5}}$$

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rubi [A] time = 0.628285, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}}\, d^{3/2} \sin\left(6a-\frac{6bc}{d}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536 b^{5/2}} - \frac{9\sqrt{\pi}\, d^{3/2} \sin\left(2a-\frac{2bc}{d}\right) \operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{512 b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}}\, d^{3/2} \cos\left(6a-\frac{6bc}{d}\right.}{1536 b^{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right) dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^{3/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} - \frac{d\int \sqrt{c+dx}\cos}{128}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+dx}\sin(}{256b^2}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+dx}\sin(}{256b^2}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+dx}\sin(}{256b^2}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{d^{3/2}\sqrt{\frac{\pi}{3}}\cos\left(6a\right.}{1}$$

Mathematica [A] time = 3.08647, size = 391, normalized size = 1.11

$$\frac{\sqrt{3\pi}d\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - 81\sqrt{\pi}d\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}d\cos\left(6a-\frac{6bc}{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*
x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[
6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] +
d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*
c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/
d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(4608*b^2*Sqrt[b/d])
```

Maple [A] time = 0.041, size = 383, normalized size = 1.1

$$2\frac{1}{d}\left(-\frac{3\,d\,(dx+c)^{3/2}}{128\,b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+\frac{9\,d}{128\,b}\left(1/4\,\frac{d\sqrt{dx+c}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)-1/8\,\frac{d\sqrt{\pi}}{b}\left(\cos\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+9/128/b*d*(1
/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b
/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b
*d*(d*x+c)^(1/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.2209, size = 1829, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/73728*sqrt(6)*sqrt(2)*(32*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(
6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 288*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)
*b*d*abs(b)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*sqrt(6)*sqrt(2)*s
qrt(d*x + c)*d^2*abs(b)*sin(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 216*sqr
t(6)*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/ab
s(d) - (sqrt(2)*(-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(
pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt
```

```
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sq
rt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
 (sqrt(6)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)
*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (sqrt(6)*(-27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c -
 a*d)/d) + sqrt(6)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-6*
(b*c - a*d)/d) - sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(-6*I*b/d)))*abs(d)/(b^2*d*abs(b))
```

Fricas [A] time = 0.706436, size = 821, normalized size = 2.34

$$\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-81\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresne
l_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi
*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*
(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a
)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c
))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.07558, size = 1481, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/9216*(4*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b
- 54*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*sqr
```

```
t(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x +
c)*d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c - I*sqrt(3)*sqrt(pi)
*(-4*I*b*c*d + d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) - I*sqrt(3)*sqrt(pi)*(-4*I*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqrt(pi)*(12*I*b*c*d - 9*d^2)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqrt(pi)*(12*I*b*c*d
 + 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*I*(-4
*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((6
*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2 + 18*I*(12*I*(d*x + c)^(3/2)*b*d
 - 12*I*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b^2 + 18*I*(12*I*(d*x + c)^(3/2)*b*d - 12*I*sqrt(d*x +
c)*b*c*d + 9*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2 + 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x
+ c)*d^2)*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d
```

3.198 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)}{384b^{3/2}}$$

```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```

Rubi [A] time = 0.457678, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)}{384b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int \sqrt{c+dx}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{d\int \frac{\cos(6a+6bx)}{\sqrt{c+dx}}\,dx}{384b} + \frac{(3d}{}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\left(d\cos\left(6a-\frac{6bc}{d}\right)\right)\int \frac{\cos}{}}{384b}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\cos\left(6a-\frac{6bc}{d}\right)\text{Subst}\left(\int\right.}{1}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)C}{384b^{3/2}}$$

Mathematica [A] time = 1.38502, size = 264, normalized size = 0.88

$$\frac{-\sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) + 27\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\sin\left(6a-\frac{6bc}{d}\right)S}{1152b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*C
os[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[3/Pi]*Sqrt[c + d*x]] + 27*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b
/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*S
qrt[c + d*x]]*Sin[6*a - (6*b*c)/d] - 27*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(1152*b*Sqrt[b/d])
```

Maple [A] time = 0.041, size = 293, normalized size = 1.

$$2\frac{1}{d}\left(-\frac{3d\sqrt{dx+c}}{128b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right) + \frac{3d\sqrt{\pi}}{256b}\left(\cos\left(2\frac{ad-bc}{d}\right)\text{FresnelC}\left(2\frac{\sqrt{dx+c}b}{d\sqrt{\pi}}\frac{1}{\sqrt{\frac{b}{d}}}\right) - \sin\left(2\frac{ad-bc}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/256/b*d*Pi
^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4608/
b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.19434, size = 1681, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18432*sqrt(6)*sqrt(2)*(8*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(6*((d
*x + c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d*abs(b
)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(2)*(sqrt(pi)*cos(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*
cos(-6*(b*c - a*d)/d) - sqrt(2)*(I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (sqrt(6)*(9*sqrt(pi)*cos(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos
(-2*(b*c - a*d)/d) + sqrt(6)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d
```

```
)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (sqrt(6)*(9*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d
))*cos(-2*(b*c - a*d)/d) + sqrt(6)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(sqrt(pi)*cos(1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))
*cos(-6*(b*c - a*d)/d) - sqrt(2)*(-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 0.64598, size = 632, normalized size = 2.11

$$-\frac{\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-27\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi
*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 + b)*sq
rt(d*x + c))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.55546, size = 643, normalized size = 2.15

$$\frac{\sqrt{3}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{6i\,bc-6i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{\sqrt{3}\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-6i\,bc+6i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{27\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,a}{d}\right.}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2304*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b - 5
4*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x + c)*
d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)/d
```

3.199 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)}{384b^{3/2}}$$

```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```

Rubi [A] time = 0.448135, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)}{384b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```


```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$
$$= -\left(\frac{1}{32}\int \sqrt{c+dx}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{d\int \frac{\cos(6a+6bx)}{\sqrt{c+dx}}\,dx}{384b} + \frac{(3d}{}$$
$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\left(d\cos\left(6a-\frac{6bc}{d}\right)\right)\int \frac{\cos}{}}{384b}$$
$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\cos\left(6a-\frac{6bc}{d}\right)\text{Subst}\left(\int\right.}{1}$$
$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\text{C}}{384b^{3/2}}$$

Mathematica [A] time = 0.612968, size = 264, normalized size = 0.88

$$\frac{-\sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) + 27\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\sin\left(6a-\frac{6bc}{d}\right)S}{1152b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*C
os[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[3/Pi]*Sqrt[c + d*x]] + 27*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b
/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*S
qrt[c + d*x]]*Sin[6*a - (6*b*c)/d] - 27*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(1152*b*Sqrt[b/d])
```

Maple [A] time = 0.037, size = 293, normalized size = 1.

$$2\frac{1}{d}\left(-\frac{3d\sqrt{dx+c}}{128b}\cos\left(2\frac{(dx+c)b}{d}+2\frac{ad-bc}{d}\right)+\frac{3d\sqrt{\pi}}{256b}\left(\cos\left(2\frac{ad-bc}{d}\right)\text{FresnelC}\left(2\frac{\sqrt{dx+c}b}{d\sqrt{\pi}}\frac{1}{\sqrt{\frac{b}{d}}}\right)-\sin\left(2\frac{ad-bc}{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/256/b*d*Pi
^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4608/
b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

Maxima [C] time = 2.16878, size = 1681, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18432*sqrt(6)*sqrt(2)*(8*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d*abs(b)*cos(6*((d
*x + c)*b - b*c + a*d)/d)/abs(d) - 72*sqrt(6)*sqrt(2)*sqrt(d*x + c)*d*abs(b
)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - (sqrt(2)*(sqrt(pi)*cos(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*
cos(-6*(b*c - a*d)/d) - sqrt(2)*(I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) +
1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (sqrt(6)*(9*sqrt(pi)*cos(1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(-1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*cos
(-2*(b*c - a*d)/d) + sqrt(6)*(-9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d
```

```
)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (sqrt(6)*(9*sqrt(pi)*cos(1/4*pi +
1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*I*sqrt(pi)*sin(-1/4
*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d
))*cos(-2*(b*c - a*d)/d) + sqrt(6)*(9*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 9*sqrt(pi)*sin(1/4*pi + 1/2*arctan
2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 9*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(sqrt(pi)*cos(1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi
 + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*p
i + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))
*cos(-6*(b*c - a*d)/d) - sqrt(2)*(-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b)
 + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b)
+ 1/2*arctan2(0, d/sqrt(d^2))))*d*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*abs(d)/(b*d*abs(b))
```

Fricas [A] time = 0.651762, size = 632, normalized size = 2.11

$$-\frac{\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-27\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi
*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 + b)*sq
rt(d*x + c))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.56079, size = 643, normalized size = 2.15

$$\frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(\frac{i bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{6i bc-6i ad}{d}\right)}}{\sqrt{bd}\left(\frac{i bd}{\sqrt{b^2d^2}}+1\right)b} + \frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-6i bc+6i ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i bd}{\sqrt{b^2d^2}}+1\right)b} - \frac{27\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i bc-2i a}{d}\right.}}{\sqrt{bd}\left(\frac{i bd}{\sqrt{b^2d^2}}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2304*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b - 5
4*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x + c)*
d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)/d
```

3.200 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)S}{1536b^{5/2}}$$

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rubi [A] time = 0.55918, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)S}{1536b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```


```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```


Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right) dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^{3/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} - \frac{d\int \sqrt{c+dx}\cos(6}{128b}$$
$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+dx}\sin(2a}{256b^2}$$
$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+dx}\sin(2a}{256b^2}$$
$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+dx}\sin(2a}{256b^2}$$
$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{d^{3/2}\sqrt{\frac{\pi}{3}}\cos\left(6a-\right.}{15}$$

Mathematica [A] time = 0.216276, size = 391, normalized size = 1.11

$$\frac{\sqrt{3\pi}d\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\frac{b}{d}}\sqrt{c+dx}\right) - 81\sqrt{\pi}d\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}d\cos\left(6a-\frac{6bc}{d}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*
x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[
6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] +
d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*
c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/
d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(4608*b^2*Sqrt[b/d])
```

Maple [A] time = 0.039, size = 383, normalized size = 1.1

$$2\frac{1}{d}\left(-\frac{3\,d\,(dx+c)^{3/2}}{128\,b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+\frac{9\,d}{128\,b}\left(1/4\,\frac{d\sqrt{dx+c}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)-1/8\,\frac{d\sqrt{\pi}}{b}\left(\mathrm{co}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+9/128/b*d*(1
/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b
/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b
*d*(d*x+c)^(1/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

Maxima [C] time = 2.21766, size = 1829, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/73728*sqrt(6)*sqrt(2)*(32*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d*abs(b)*cos(
6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 288*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)
*b*d*abs(b)*cos(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*sqrt(6)*sqrt(2)*s
qrt(d*x + c)*d^2*abs(b)*sin(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) + 216*sqr
t(6)*sqrt(2)*sqrt(d*x + c)*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/ab
s(d) - (sqrt(2)*(-I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) - I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(sqr
t(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(
pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - I*sqrt
```

```
(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt
(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sq
rt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
 (sqrt(6)*(27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0
, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(
0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)
*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)))
 + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2)
)) - 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d
^2))) + 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (sqrt(6)*(-27*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
b) + 1/2*arctan2(0, d/sqrt(d^2))) + 27*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - 27*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-2*(b*c -
 a*d)/d) + sqrt(6)*(27*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))) + 27*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + 27*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*a
rctan2(0, d/sqrt(d^2))) - 27*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1
/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(I*sqrt(pi)*cos(1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + I*sqrt(pi)*cos(-1/4*pi + 1/2
*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - sqrt(pi)*sin(1/4*pi + 1/2*a
rctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + sqrt(pi)*sin(-1/4*pi + 1/2*ar
ctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*cos(-6*
(b*c - a*d)/d) - sqrt(2)*(sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))) + sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) + I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arct
an2(0, d/sqrt(d^2))) - I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arc
tan2(0, d/sqrt(d^2))))*d^2*sqrt(abs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(-6*I*b/d)))*abs(d)/(b^2*d*abs(b))
```

Fricas [A] time = 0.719563, size = 821, normalized size = 2.34

$$\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-81\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresne
l_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi
*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*
(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a
)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c
))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.07807, size = 1481, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/9216*(4*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b
- 54*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*sqr
```

```
t(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x +
c)*d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c - I*sqrt(3)*sqrt(pi)
*(-4*I*b*c*d + d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^2) - I*sqrt(3)*sqrt(pi)*(-4*I*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqrt(pi)*(12*I*b*c*d - 9*d^2)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 9*I*sqrt(pi)*(12*I*b*c*d
 + 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*I*(-4
*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((6
*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2 + 18*I*(12*I*(d*x + c)^(3/2)*b*d
 - 12*I*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b^2 + 18*I*(12*I*(d*x + c)^(3/2)*b*d - 12*I*sqrt(d*x +
c)*b*c*d + 9*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2 + 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x
+ c)*d^2)*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d
```

3.201 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\sin\left(6a-\right.}{184}$$

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rubi [A] time = 0.669301, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\sin\left(6a-\right.}{184}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
```

```
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^{5/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} - \frac{(5d)\int (c+dx)^{3/}}{3}$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{15d(c+dx)^{3/2}\,\mathrm{s}}{256b}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos}{9216b}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos}{9216b}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos}{9216b}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos}{9216b}$$

Mathematica [A] time = 3.00165, size = 550, normalized size = 1.35

$$-2592b^3c^2\sqrt{c+dx}\cos(2(a+bx)) + 288b^3c^2\sqrt{c+dx}\cos(6(a+bx)) - 2592b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx)) + 288b^3d^2x^2\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Co
s[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d
^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a
+ b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c +
d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*
Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/P
i]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresne
lC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[
b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a -
(2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*S
qrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)]
- 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(55296*b^4)
```

Maple [A] time = 0.04, size = 477, normalized size = 1.2

$$2\frac{1}{d}\left(-\frac{3\,d\,(dx+c)^{5/2}}{128\,b}\cos\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)+\frac{15\,d}{128\,b}\left(1/4\,\frac{d\,(dx+c)^{3/2}}{b}\sin\left(2\,\frac{(dx+c)\,b}{d}+2\,\frac{ad-bc}{d}\right)-3/4\,\frac{d}{b}\left(-1/4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+15/128/b*d*(
1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d
*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2
*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/384/b
*d*(d*x+c)^(5/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+
c)^(3/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*
cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

Maxima [C] time = 2.20164, size = 1916, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/884736*sqrt(6)*sqrt(2)*(160*sqrt(6)*sqrt(2)*(d*x + c)^(3/2)*b*d^2*abs(b)
*sin(6*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 4320*sqrt(6)*sqrt(2)*(d*x + c)
^(3/2)*b*d^2*abs(b)*sin(2*((d*x + c)*b - b*c + a*d)/d)/abs(d) - 8*(48*sqrt(
```

```
6)*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 5*sqrt(6)*sqrt(2)*sqrt(d*x
 + c)*d^3*abs(b)/abs(d))*cos(6*((d*x + c)*b - b*c + a*d)/d) + 216*(16*sqrt(
6)*sqrt(2)*(d*x + c)^(5/2)*b^2*d*abs(b)/abs(d) - 15*sqrt(6)*sqrt(2)*sqrt(d*
x + c)*d^3*abs(b)/abs(d))*cos(2*((d*x + c)*b - b*c + a*d)/d) - (sqrt(2)*(5*
sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*
sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5
*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) +
 5*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))
))*d^3*sqrt(abs(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(5*I*sqrt(pi)*co
s(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*
cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)
*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)
*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(a
bs(b)/abs(d))*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (sq
rt(6)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(
d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) - 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan
2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-2*(b*c - a*d)/d) + sqrt(6)
*(-405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^
2))) - 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sq
rt(d^2))) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/
sqrt(d^2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0,
 d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(2*I*b/d)) + (sqrt(6)*(405*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b
) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0
, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*sin(1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*I*sqrt(pi)*sin(-1/4*pi + 1/2*
arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*cos(-
2*(b*c - a*d)/d) + sqrt(6)*(405*I*sqrt(pi)*cos(1/4*pi + 1/2*arctan2(0, b) +
 1/2*arctan2(0, d/sqrt(d^2))) + 405*I*sqrt(pi)*cos(-1/4*pi + 1/2*arctan2(0,
 b) + 1/2*arctan2(0, d/sqrt(d^2))) - 405*sqrt(pi)*sin(1/4*pi + 1/2*arctan2(
0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 405*sqrt(pi)*sin(-1/4*pi + 1/2*arcta
n2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))*sin(-2*(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - (sqrt(2)*(5*sqrt(pi)*cos(1
/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*cos(-
1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*I*sqrt(pi)*si
n(1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*
sin(-1/4*pi + 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(ab
s(b)/abs(d))*cos(-6*(b*c - a*d)/d) - sqrt(2)*(-5*I*sqrt(pi)*cos(1/4*pi + 1/
2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*I*sqrt(pi)*cos(-1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) + 5*sqrt(pi)*sin(1/4*pi +
 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))) - 5*sqrt(pi)*sin(-1/4*pi
+ 1/2*arctan2(0, b) + 1/2*arctan2(0, d/sqrt(d^2))))*d^3*sqrt(abs(b)/abs(d))
*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*abs(d)/(b^3*d*ab
```

```
s(b))
```

Fricas [A] time = 0.764722, size = 1089, normalized size = 2.68

$$\frac{5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6(bc-ad)}{d}\right) - 1215\,\pi d^3\sqrt{\frac{b}{\pi d}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*f
resnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) -
1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^3*d^2*
x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c*d*x +
24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c*d*x + 4
8*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x + b^2*c*d
)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^2*d^2*x +
b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.74327, size = 2677, normalized size = 6.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/110592*(48*(sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*d^2*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) - 27*sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)
/b - 54*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b - 54*
sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 6*sqrt(d*x
 + c)*d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c^2 - d^2*((I*sqrt(
3)*sqrt(pi)*(48*I*b^2*c^2*d - 24*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d +
96*I*(d*x + c)^(3/2)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^
(3/2)*b*d^2 + 24*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((-6*I*(d
*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^3)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*I*b^2*
c^2*d + 24*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 96*I*(d*x + c)^(3/2
)*b^2*c*d - 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 24*sq
rt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((6*I*(d*x + c)*b - 6*I*b*c
 + 6*I*a*d)/d)/b^3)/d^2 + 27*(I*sqrt(pi)*(-48*I*b^2*c^2*d + 72*b*c*d^2 + 45*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*
b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*(d*
x + c)^(5/2)*b^2*d - 96*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*
c^2*d + 60*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 45*I*sqrt(d*x
 + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3)/d^2 + 27*(I*sq
rt(pi)*(-48*I*b^2*c^2*d - 72*b*c*d^2 + 45*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*(d*x + c)^(5/2)*b^2*d - 96*I*(d*x
 + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 60*(d*x + c)^(3/2)*b*d
^2 + 72*sqrt(d*x + c)*b*c*d^2 - 45*I*sqrt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b
 - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^2) - 24*(I*sqrt(3)*sqrt(pi)*(-4*I*b*c*d + d
^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + I*sqrt(3
)*sqrt(pi)*(-4*I*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) + 9*I*sqrt(pi)*(12*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqr
```

```
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 9*I*sqrt(pi)*(12*I*b*c*d + 9*d^2)*d*erf(
-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/
2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((6*I*(d*x + c)*b -
 6*I*b*c + 6*I*a*d)/d)/b^2 - 18*I*(12*I*(d*x + c)^(3/2)*b*d - 12*I*sqrt(d*x
 + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)
/d)/b^2 - 18*I*(12*I*(d*x + c)^(3/2)*b*d - 12*I*sqrt(d*x + c)*b*c*d + 9*sqr
t(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2 - 6*I*(-4*
I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6
*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2)*c)/d
```

3.202 $\int x^3 \cos^2(x) \cot^2(x)\, dx$

Optimal. Leaf size=112

$$-3ix\text{PolyLog}\left(2,e^{2ix}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{2ix}\right)-\frac{3x^4}{8}-ix^3+\frac{3x^2}{8}+3x^2\log\left(1-e^{2ix}\right)-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)-\frac{1}{2}x^3\sin$$

```
[Out] (3*x^2)/8 - I*x^3 - (3*x^4)/8 + (3*Cos[x]^2)/8 - (3*x^2*Cos[x]^2)/4 - x^3*C
ot[x] + 3*x^2*Log[1 - E^((2*I)*x)] - (3*I)*x*PolyLog[2, E^((2*I)*x)] + (3*P
olyLog[3, E^((2*I)*x)])/2 + (3*x*Cos[x]*Sin[x])/4 - (x^3*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.185145, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4408, 3311, 30, 3310, 3720, 3717, 2190, 2531, 2282, 6589}

$$-3ix\text{PolyLog}\left(2,e^{2ix}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{2ix}\right)-\frac{3x^4}{8}-ix^3+\frac{3x^2}{8}+3x^2\log\left(1-e^{2ix}\right)-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)-\frac{1}{2}x^3\sin$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^2,x]
```



```
[Out] (3*x^2)/8 - I*x^3 - (3*x^4)/8 + (3*Cos[x]^2)/8 - (3*x^2*Cos[x]^2)/4 - x^3*C
ot[x] + 3*x^2*Log[1 - E^((2*I)*x)] - (3*I)*x*PolyLog[2, E^((2*I)*x)] + (3*P
olyLog[3, E^((2*I)*x)])/2 + (3*x*Cos[x]*Sin[x])/4 - (x^3*Cos[x]*Sin[x])/2
```


Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\cos^2(x)\cot^2(x)\,dx = -\int x^3\cos^2(x)\,dx + \int x^3\cot^2(x)\,dx$$
$$= -\frac{3}{4}x^2\cos^2(x) - x^3\cot(x) - \frac{1}{2}x^3\cos(x)\sin(x) - \frac{\int x^3\,dx}{2} + \frac{3}{2}\int x\cos^2(x)\,dx + 3\int x^2\cot(x)$$
$$= -ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + \frac{3}{4}x\cos(x)\sin(x) - \frac{1}{2}x^3\cos(x)\sin(x) - 6$$
$$= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + 3x^2\log\left(1-e^{2ix}\right) + \frac{3}{4}x\cos(x)\sin(x$$
$$= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + 3x^2\log\left(1-e^{2ix}\right) - 3ix\text{Li}_2\left(e^{2ix}\right) +$$
$$= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + 3x^2\log\left(1-e^{2ix}\right) - 3ix\text{Li}_2\left(e^{2ix}\right) +$$
$$= \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + 3x^2\log\left(1-e^{2ix}\right) - 3ix\text{Li}_2\left(e^{2ix}\right) +$$

Mathematica [A] time = 0.175528, size = 104, normalized size = 0.93

$$\frac{1}{16}\left(48ix\text{PolyLog}\left(2,e^{-2ix}\right) + 24\text{PolyLog}\left(3,e^{-2ix}\right) - 6x^4 + 16ix^3 + 48x^2\log\left(1-e^{-2ix}\right) - 4x^3\sin(2x) - 6x^2\cos(2x) - 1$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] ((-2*I)*Pi^3 + (16*I)*x^3 - 6*x^4 + 3*Cos[2*x] - 6*x^2*Cos[2*x] - 16*x^3*Co
t[x] + 48*x^2*Log[1 - E^((-2*I)*x)] + (48*I)*x*PolyLog[2, E^((-2*I)*x)] + 2
4*PolyLog[3, E^((-2*I)*x)] + 6*x*Sin[2*x] - 4*x^3*Sin[2*x])/16
```

Maple [A] time = 0.106, size = 150, normalized size = 1.3

$$-\frac{3x^4}{8}+\frac{i}{32}\left(6ix^2+4x^3-3i-6x\right)\mathrm{e}^{2ix}-\frac{i}{32}\left(-6ix^2+4x^3+3i-6x\right)\mathrm{e}^{-2ix}-\frac{2ix^3}{\mathrm{e}^{2ix}-1}-2ix^3+3x^2\ln\left(1-\mathrm{e}^{ix}\right)-6ixp$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^2,x)
```

```
[Out] -3/8*x^4+1/32*I*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*I*(-6*I*x^2+4*x^3+3
*I-6*x)*exp(-2*I*x)-2*I*x^3/(exp(2*I*x)-1)-2*I*x^3+3*x^2*ln(1-exp(I*x))-6*I
*x*polylog(2,exp(I*x))+6*polylog(3,exp(I*x))+3*x^2*ln(1+exp(I*x))-6*I*x*pol
ylog(2,-exp(I*x))+6*polylog(3,-exp(I*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [C] time = 0.594582, size = 851, normalized size = 7.6

$$\frac{4\left(2x^3-3x\right)\cos(x)^3+24x^2\log(\cos(x)+i\sin(x)+1)\sin(x)+24x^2\log(\cos(x)-i\sin(x)+1)\sin(x)+24x^2\log(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(2*x^3 - 3*x)*cos(x)^3 + 24*x^2*log(cos(x) + I*sin(x) + 1)*sin(x) +
 24*x^2*log(cos(x) - I*sin(x) + 1)*sin(x) + 24*x^2*log(-cos(x) + I*sin(x) +
 1)*sin(x) + 24*x^2*log(-cos(x) - I*sin(x) + 1)*sin(x) - 48*I*x*dilog(cos(x
) + I*sin(x))*sin(x) + 48*I*x*dilog(cos(x) - I*sin(x))*sin(x) + 48*I*x*dilo
```

```
g(-cos(x) + I*sin(x))*sin(x) - 48*I*x*dilog(-cos(x) - I*sin(x))*sin(x) - 12
*(2*x^3 - x)*cos(x) - 3*(2*x^4 + 2*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 1)*sin(x)
 + 48*polylog(3, cos(x) + I*sin(x))*sin(x) + 48*polylog(3, cos(x) - I*sin(x
))*sin(x) + 48*polylog(3, -cos(x) + I*sin(x))*sin(x) + 48*polylog(3, -cos(x
) - I*sin(x))*sin(x))/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos^2(x) \cot^2(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x**3*cos(x)**2*cot(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \cos(x)^2 \cot(x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(x)^2*cot(x)^2, x)
```

3.203 $\int x^2 \cos^2(x) \cot^2(x) \, dx$

Optimal. Leaf size=83

$$-i\text{PolyLog}\left(2,e^{2ix}\right)-\frac{x^3}{2}-ix^2-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{x}{4}+2x\log\left(1-e^{2ix}\right)-\frac{1}{2}x\cos^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] x/4 - I*x^2 - x^3/2 - (x*Cos[x]^2)/2 - x^2*Cot[x] + 2*x*Log[1 - E^((2*I)*x)
] - I*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.169514, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4408, 3311, 30, 2635, 8, 3720, 3717, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2,e^{2ix}\right)-\frac{x^3}{2}-ix^2-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{x}{4}+2x\log\left(1-e^{2ix}\right)-\frac{1}{2}x\cos^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] x/4 - I*x^2 - x^3/2 - (x*Cos[x]^2)/2 - x^2*Cot[x] + 2*x*Log[1 - E^((2*I)*x)
] - I*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^2\cos^2(x)\cot^2(x)\,dx = -\int x^2\cos^2(x)\,dx + \int x^2\cot^2(x)\,dx$$
$$= -\frac{1}{2}x\cos^2(x) - x^2\cot(x) - \frac{1}{2}x^2\cos(x)\sin(x) - \frac{\int x^2\,dx}{2} + \frac{1}{2}\int\cos^2(x)\,dx + 2\int x\cot(x)\,dx - $$
$$= -ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) - 4i\int\frac{e^{2ix}x}{1-e^{2ix}}\,dx + $$
$$= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x$$
$$= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x$$
$$= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) - i\text{Li}_2\left(e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2$$

Mathematica [A] time = 0.10048, size = 72, normalized size = 0.87

$$\frac{1}{8}\left(-8i\text{PolyLog}\left(2,e^{2ix}\right) - 4x^3 - 8ix^2 - 2x^2\sin(2x) - 8x^2\cot(x) + 16x\log\left(1-e^{2ix}\right) + \sin(2x) - 2x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] ((-8*I)*x^2 - 4*x^3 - 2*x*Cos[2*x] - 8*x^2*Cot[x] + 16*x*Log[1 - E^((2*I)*x
)] - (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x] - 2*x^2*Sin[2*x])/8
```

Maple [A] time = 0.106, size = 112, normalized size = 1.4

$$-\frac{x^3}{2} + \frac{i}{16}\left(2\,ix + 2\,x^2 - 1\right)e^{2\,ix} - \frac{i}{16}\left(-2\,ix + 2\,x^2 - 1\right)e^{-2\,ix} - \frac{2\,ix^2}{e^{2\,ix}-1} + 2\,x\ln\left(1-e^{ix}\right) + 2\,x\ln\left(1+e^{ix}\right) - 2\,ix^2 - 2\,i\text{poly}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^2,x)
```

```
[Out] -1/2*x^3+1/16*I*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*I*(-2*I*x+2*x^2-1)*exp(-2*I
*x)-2*I*x^2/(exp(2*I*x)-1)+2*x*ln(1-exp(I*x))+2*x*ln(1+exp(I*x))-2*I*x^2-2*
I*polylog(2,exp(I*x))-2*I*polylog(2,-exp(I*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 0.558489, size = 551, normalized size = 6.64

$$\frac{(2x^2-1)\cos(x)^3 + 4x\log(\cos(x)+i\sin(x)+1)\sin(x) + 4x\log(\cos(x)-i\sin(x)+1)\sin(x) + 4x\log(-\cos(x) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*x^2 - 1)*cos(x)^3 + 4*x*log(cos(x) + I*sin(x) + 1)*sin(x) + 4*x*log
(cos(x) - I*sin(x) + 1)*sin(x) + 4*x*log(-cos(x) + I*sin(x) + 1)*sin(x) + 4
*x*log(-cos(x) - I*sin(x) + 1)*sin(x) - (6*x^2 - 1)*cos(x) - (2*x^3 + 2*x*c
os(x)^2 - x)*sin(x) - 4*I*dilog(cos(x) + I*sin(x))*sin(x) + 4*I*dilog(cos(x
) - I*sin(x))*sin(x) + 4*I*dilog(-cos(x) + I*sin(x))*sin(x) - 4*I*dilog(-co
s(x) - I*sin(x))*sin(x))/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\cos^2(x)\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x**2*cos(x)**2*cot(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\cos(x)^2\cot(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(x)^2*cot(x)^2, x)
```

3.204 $\int x\cos^2(x)\cot^2(x)\,dx$

Optimal. Leaf size=33

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0546779, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4408, 3310, 30, 3720, 3475}

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos^2(x)\cot^2(x)\,dx &= -\int x\cos^2(x)\,dx+\int x\cot^2(x)\,dx\\ &= -\frac{1}{4}\cos^2(x)-x\cot(x)-\frac{1}{2}x\cos(x)\sin(x)-\frac{\int x\,dx}{2}-\int x\,dx+\int\cot(x)\,dx\\ &= -\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0250926, size = 33, normalized size = 1.

$$-\frac{3x^2}{4}-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)-x\cot(x)+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

Maple [B] time = 0.093, size = 76, normalized size = 2.3

$$\frac{1}{2\tan(x)}\left(-x-\frac{x^2\tan(x)}{2}\right)-\frac{\ln\left((\tan(x))^2+1\right)}{2}+\ln(\tan(x))+\frac{1}{2\tan(x)\left((\tan(x))^2+1\right)}\left(-\frac{\tan(x)}{2}-x-2x(\tan(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^2,x)
```

```
[Out] 1/2*(-x-1/2*x^2*tan(x))/tan(x)-1/2*ln(tan(x)^2+1)+ln(tan(x))+1/2*(-1/2*tan(
x)-x-2*x*tan(x)^2-x^2*tan(x)-x^2*tan(x)^3)/tan(x)/(tan(x)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.5048, size = 138, normalized size = 4.18

$$\frac{4\,x\cos\left(x\right)^{3}-12\,x\cos\left(x\right)-\left(6\,x^{2}+2\,\cos\left(x\right)^{2}-1\right)\sin\left(x\right)+8\,\log\left(\frac{1}{2}\,\sin\left(x\right)\right)\sin\left(x\right)}{8\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(1
/2*sin(x))*sin(x))/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos^{2}\left(x\right)\cot^{2}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x*cos(x)**2*cot(x)**2, x)
```

Giac [B] time = 1.18014, size = 278, normalized size = 8.42

$$-\frac{6\,x^{2}\tan\left(\frac{1}{2}\,x\right)^{5}-4\,x\tan\left(\frac{1}{2}\,x\right)^{6}-4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^{2}}{\tan\left(\frac{1}{2}\,x\right)^{4}+2\,\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)\tan\left(\frac{1}{2}\,x\right)^{5}+12\,x^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-12\,x\tan\left(\frac{1}{2}\,x\right)^{4}+\tan\left(\frac{1}{2}\right.}{8\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] -1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*tan
(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*
x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(1/2*
x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x
) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

3.205 $\int x^3 \cos^2(x) \cot^3(x) \, dx$

Optimal. Leaf size=180

$$3ix^2\text{PolyLog}\left(2,e^{2ix}\right)-3x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{3}{2}i\text{PolyLog}\left(2,e^{2ix}\right)-\frac{3}{2}i\text{PolyLog}\left(4,e^{2ix}\right)+\frac{ix^4}{2}-\frac{3x^3}{4}-\frac{3ix^2}{2}-2x^3\log$$

```
[Out] (3*x)/8 - ((3*I)/2)*x^2 - (3*x^3)/4 + (I/2)*x^4 - (3*x^2*Cot[x])/2 - (x^3*C
ot[x]^2)/2 + 3*x*Log[1 - E^((2*I)*x)] - 2*x^3*Log[1 - E^((2*I)*x)] - ((3*I)
/2)*PolyLog[2, E^((2*I)*x)] + (3*I)*x^2*PolyLog[2, E^((2*I)*x)] - 3*x*PolyL
og[3, E^((2*I)*x)] - ((3*I)/2)*PolyLog[4, E^((2*I)*x)] - (3*Cos[x]*Sin[x])/
8 + (3*x^2*Cos[x]*Sin[x])/4 - (3*x*Sin[x]^2)/4 + (x^3*Sin[x]^2)/2
```

Rubi [A] time = 0.400615, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.25, Rules used = {4408, 3443, 3311, 30, 2635, 8, 3717, 2190, 2531, 6609, 2282, 6589, 3720, 2279, 2391}

$$3ix^2\text{PolyLog}\left(2,e^{2ix}\right)-3x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{3}{2}i\text{PolyLog}\left(2,e^{2ix}\right)-\frac{3}{2}i\text{PolyLog}\left(4,e^{2ix}\right)+\frac{ix^4}{2}-\frac{3x^3}{4}-\frac{3ix^2}{2}-2x^3\log$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^3,x]
```


```
[Out] (3*x)/8 - ((3*I)/2)*x^2 - (3*x^3)/4 + (I/2)*x^4 - (3*x^2*Cot[x])/2 - (x^3*C
ot[x]^2)/2 + 3*x*Log[1 - E^((2*I)*x)] - 2*x^3*Log[1 - E^((2*I)*x)] - ((3*I)
/2)*PolyLog[2, E^((2*I)*x)] + (3*I)*x^2*PolyLog[2, E^((2*I)*x)] - 3*x*PolyL
og[3, E^((2*I)*x)] - ((3*I)/2)*PolyLog[4, E^((2*I)*x)] - (3*Cos[x]*Sin[x])/
8 + (3*x^2*Cos[x]*Sin[x])/4 - (3*x*Sin[x]^2)/4 + (x^3*Sin[x]^2)/2
```


Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
```

```
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3\cos^2(x)\cot^3(x)\,dx = -\int x^3\cos^2(x)\cot(x)\,dx + \int x^3\cot^3(x)\,dx$$
$$= -\frac{1}{2}x^3\cot^2(x) + \frac{3}{2}\int x^2\cot^2(x)\,dx - 2\int x^3\cot(x)\,dx + \int x^3\cos(x)\sin(x)\,dx$$
$$= -\frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + \frac{1}{2}x^3\sin^2(x) - 2\left(-\frac{ix^4}{4} - 2i\int\frac{e^{2ix}x^3}{1-e^{2ix}}\,dx\right) - \frac{3\int x^2\,dx}{2} - \frac{3}{2}\int x^2 s$$
$$= -\frac{3ix^2}{2} - \frac{x^3}{2} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + \frac{3}{4}x^2\cos(x)\sin(x) - \frac{3}{4}x\sin^2(x) + \frac{1}{2}x^3\sin^2(x) - 6i\int$$
$$= -\frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{8}\cos(x)\sin(x) + \frac{3}{4}x^2\cos(x)\sin$$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{8}\cos(x)\sin(x) + \frac{3}{4}x^2\cos(x$$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{2}i\mathrm{Li}_2\left(e^{2ix}\right) - \frac{3}{8}\cos(x)\sin($$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{2}i\mathrm{Li}_2\left(e^{2ix}\right) - 2\left(-\frac{ix^4}{4} + x^3\right.$$

Mathematica [A] time = 0.416099, size = 159, normalized size = 0.88

$$\frac{1}{32}\left(-96ix^2\mathrm{PolyLog}\left(2,e^{-2ix}\right) - 96x\mathrm{PolyLog}\left(3,e^{-2ix}\right) - 48i\mathrm{PolyLog}\left(2,e^{2ix}\right) + 48i\mathrm{PolyLog}\left(4,e^{-2ix}\right) - 16ix^4 - 48ix^2 -\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (I*Pi^4 - (48*I)*x^2 - (16*I)*x^4 + 12*x*Cos[2*x] - 8*x^3*Cos[2*x] - 48*x^2
*Cot[x] - 16*x^3*Csc[x]^2 - 64*x^3*Log[1 - E^((-2*I)*x)] + 96*x*Log[1 - E^(
(2*I)*x)] - (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] - (48*I)*PolyLog[2, E^((2*I
)*x)] - 96*x*PolyLog[3, E^((-2*I)*x)] + (48*I)*PolyLog[4, E^((-2*I)*x)] - 6
*Sin[2*x] + 12*x^2*Sin[2*x])/32
```

Maple [A] time = 0.157, size = 240, normalized size = 1.3

$$-3\,ix^2-\frac{\left(6\,ix^2+4\,x^3-3\,i-6\,x\right)\mathrm{e}^{2\,ix}}{32}-\frac{\left(-6\,ix^2+4\,x^3+3\,i-6\,x\right)\mathrm{e}^{-2\,ix}}{32}+\frac{x^2\left(2\,x\mathrm{e}^{2\,ix}-3\,i\mathrm{e}^{2\,ix}+3\,i\right)}{\left(\mathrm{e}^{2\,ix}-1\right)^2}-2\,x^3\ln\left(1-\mathrm{e}^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^3,x)
```

```
[Out] -3*I*x^2-1/32*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*(-6*I*x^2+4*x^3+3*I-6
*x)*exp(-2*I*x)+x^2*(2*x*exp(2*I*x)-3*I*exp(2*I*x)+3*I)/(exp(2*I*x)-1)^2-2*
x^3*ln(1-exp(I*x))-2*x^3*ln(1+exp(I*x))+6*I*x^2*polylog(2,exp(I*x))+3*x*ln(
1-exp(I*x))-12*x*polylog(3,exp(I*x))-12*x*polylog(3,-exp(I*x))+3*x*ln(1+exp
(I*x))+1/2*I*x^4+6*I*x^2*polylog(2,-exp(I*x))-3*I*polylog(2,exp(I*x))-12*I*
polylog(4,exp(I*x))-12*I*polylog(4,-exp(I*x))-3*I*polylog(2,-exp(I*x))
```

Maxima [B] time = 3.47439, size = 5022, normalized size = 27.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -(4*x^3 + (4*x^3 + 6*I*x^2 - 6*x - 3*I)*cos(6*x)^2 - (-32*I*x^4 - 16*x^3 +
168*I*x^2 + 24*x + 12*I)*cos(4*x)^2 - (-32*I*x^4 + 56*x^3 + 96*I*x^2 + 12*x
)*cos(2*x)^2 - (4*x^3 + 6*I*x^2 - 6*x - 3*I)*sin(6*x)^2 - (32*I*x^4 + 16*x^
3 - 168*I*x^2 - 24*x - 12*I)*sin(4*x)^2 - (32*I*x^4 - 56*x^3 - 96*I*x^2 - 1
2*x)*sin(2*x)^2 - 6*I*x^2 - ((128*I*x^3 - 192*I*x)*cos(4*x)^2 + (128*I*x^3
- 192*I*x)*cos(2*x)^2 + (-128*I*x^3 + 192*I*x)*sin(4*x)^2 + (-128*I*x^3 + 1
92*I*x)*sin(2*x)^2 + (-64*I*x^3 + (-64*I*x^3 + 96*I*x)*cos(4*x) + (128*I*x^
3 - 192*I*x)*cos(2*x) + 32*(2*x^3 - 3*x)*sin(4*x) - 64*(2*x^3 - 3*x)*sin(2*
x) + 96*I*x)*cos(6*x) + (128*I*x^3 + (-320*I*x^3 + 480*I*x)*cos(2*x) + 160*
(2*x^3 - 3*x)*sin(2*x) - 192*I*x)*cos(4*x) + (-64*I*x^3 + 96*I*x)*cos(2*x)
+ (64*x^3 + 32*(2*x^3 - 3*x)*cos(4*x) - 64*(2*x^3 - 3*x)*cos(2*x) + (64*I*x
^3 - 96*I*x)*sin(4*x) + (-128*I*x^3 + 192*I*x)*sin(2*x) - 96*x)*sin(6*x) -
(128*x^3 + 128*(2*x^3 - 3*x)*cos(4*x) - 160*(2*x^3 - 3*x)*cos(2*x) - (320*I
*x^3 - 480*I*x)*sin(2*x) - 192*x)*sin(4*x) + 32*(2*x^3 - 4*(2*x^3 - 3*x)*co
s(2*x) - 3*x)*sin(2*x))*arctan2(sin(x), cos(x) + 1) - ((-128*I*x^3 + 192*I*
x)*cos(4*x)^2 + (-128*I*x^3 + 192*I*x)*cos(2*x)^2 + (128*I*x^3 - 192*I*x)*s
in(4*x)^2 + (128*I*x^3 - 192*I*x)*sin(2*x)^2 + (64*I*x^3 + (64*I*x^3 - 96*I
```

```
*x)*cos(4*x) + (-128*I*x^3 + 192*I*x)*cos(2*x) - 32*(2*x^3 - 3*x)*sin(4*x)
+ 64*(2*x^3 - 3*x)*sin(2*x) - 96*I*x)*cos(6*x) + (-128*I*x^3 + (320*I*x^3 -
 480*I*x)*cos(2*x) - 160*(2*x^3 - 3*x)*sin(2*x) + 192*I*x)*cos(4*x) + (64*I
*x^3 - 96*I*x)*cos(2*x) - (64*x^3 + 32*(2*x^3 - 3*x)*cos(4*x) - 64*(2*x^3 -
 3*x)*cos(2*x) - (-64*I*x^3 + 96*I*x)*sin(4*x) - (128*I*x^3 - 192*I*x)*sin(
2*x) - 96*x)*sin(6*x) + (128*x^3 + 128*(2*x^3 - 3*x)*cos(4*x) - 160*(2*x^3
- 3*x)*cos(2*x) + (-320*I*x^3 + 480*I*x)*sin(2*x) - 192*x)*sin(4*x) - 32*(2
*x^3 - 4*(2*x^3 - 3*x)*cos(2*x) - 3*x)*sin(2*x))*arctan2(sin(x), -cos(x) +
1) - (16*I*x^4 + 8*x^3 - 12*I*x^2 + (16*I*x^4 + 16*x^3 - 72*I*x^2 - 24*x -
12*I)*cos(4*x) + (-32*I*x^4 + 52*x^3 + 90*I*x^2 + 18*x + 3*I)*cos(2*x) - (1
6*x^4 - 16*I*x^3 - 72*x^2 + 24*I*x - 12)*sin(4*x) + (32*x^4 + 52*I*x^3 - 90
*x^2 + 18*I*x - 3)*sin(2*x) - 12*x + 6*I)*cos(6*x) - (-32*I*x^4 - 20*x^3 +
30*I*x^2 + (80*I*x^4 - 104*x^3 - 276*I*x^2 - 36*x - 6*I)*cos(2*x) - (80*x^4
 + 104*I*x^3 - 276*x^2 + 36*I*x - 6)*sin(2*x) + 30*x - 15*I)*cos(4*x) - (16
*I*x^4 + 16*x^3 - 24*I*x^2 - 24*x + 12*I)*cos(2*x) - ((-384*I*x^2 + 192*I)*
cos(4*x)^2 + (-384*I*x^2 + 192*I)*cos(2*x)^2 + (384*I*x^2 - 192*I)*sin(4*x)
^2 + (384*I*x^2 - 192*I)*sin(2*x)^2 + (192*I*x^2 + (192*I*x^2 - 96*I)*cos(4
*x) + (-384*I*x^2 + 192*I)*cos(2*x) - 96*(2*x^2 - 1)*sin(4*x) + 192*(2*x^2
- 1)*sin(2*x) - 96*I)*cos(6*x) + (-384*I*x^2 + (960*I*x^2 - 480*I)*cos(2*x)
 - 480*(2*x^2 - 1)*sin(2*x) + 192*I)*cos(4*x) + (192*I*x^2 - 96*I)*cos(2*x)
 - (192*x^2 + 96*(2*x^2 - 1)*cos(4*x) - 192*(2*x^2 - 1)*cos(2*x) - (-192*I*
x^2 + 96*I)*sin(4*x) - (384*I*x^2 - 192*I)*sin(2*x) - 96)*sin(6*x) + (384*x
^2 + 384*(2*x^2 - 1)*cos(4*x) - 480*(2*x^2 - 1)*cos(2*x) + (-960*I*x^2 + 48
0*I)*sin(2*x) - 192)*sin(4*x) - 96*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin
(2*x))*dilog(-e^(I*x)) - ((-384*I*x^2 + 192*I)*cos(4*x)^2 + (-384*I*x^2 + 1
92*I)*cos(2*x)^2 + (384*I*x^2 - 192*I)*sin(4*x)^2 + (384*I*x^2 - 192*I)*sin
(2*x)^2 + (192*I*x^2 + (192*I*x^2 - 96*I)*cos(4*x) + (-384*I*x^2 + 192*I)*c
os(2*x) - 96*(2*x^2 - 1)*sin(4*x) + 192*(2*x^2 - 1)*sin(2*x) - 96*I)*cos(6*
x) + (-384*I*x^2 + (960*I*x^2 - 480*I)*cos(2*x) - 480*(2*x^2 - 1)*sin(2*x)
+ 192*I)*cos(4*x) + (192*I*x^2 - 96*I)*cos(2*x) - (192*x^2 + 96*(2*x^2 - 1)
*cos(4*x) - 192*(2*x^2 - 1)*cos(2*x) - (-192*I*x^2 + 96*I)*sin(4*x) - (384*
I*x^2 - 192*I)*sin(2*x) - 96)*sin(6*x) + (384*x^2 + 384*(2*x^2 - 1)*cos(4*x
) - 480*(2*x^2 - 1)*cos(2*x) + (-960*I*x^2 + 480*I)*sin(2*x) - 192)*sin(4*x
) - 96*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin(2*x))*dilog(e^(I*x)) - (32*
(2*x^3 - 3*x)*cos(4*x)^2 + 32*(2*x^3 - 3*x)*cos(2*x)^2 - 32*(2*x^3 - 3*x)*s
in(4*x)^2 - 32*(2*x^3 - 3*x)*sin(2*x)^2 - (32*x^3 + 16*(2*x^3 - 3*x)*cos(4*
x) - 32*(2*x^3 - 3*x)*cos(2*x) - (-32*I*x^3 + 48*I*x)*sin(4*x) - (64*I*x^3
- 96*I*x)*sin(2*x) - 48*x)*cos(6*x) + (64*x^3 - 80*(2*x^3 - 3*x)*cos(2*x) +
 (-160*I*x^3 + 240*I*x)*sin(2*x) - 96*x)*cos(4*x) - 16*(2*x^3 - 3*x)*cos(2*
x) + (-32*I*x^3 + (-32*I*x^3 + 48*I*x)*cos(4*x) + (64*I*x^3 - 96*I*x)*cos(2
*x) + 16*(2*x^3 - 3*x)*sin(4*x) - 32*(2*x^3 - 3*x)*sin(2*x) + 48*I*x)*sin(6
*x) + (64*I*x^3 + (128*I*x^3 - 192*I*x)*cos(4*x) + (-160*I*x^3 + 240*I*x)*c
os(2*x) + 80*(2*x^3 - 3*x)*sin(2*x) - 96*I*x)*sin(4*x) + (-32*I*x^3 + (128*
I*x^3 - 192*I*x)*cos(2*x) + 48*I*x)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*c
os(x) + 1) - (32*(2*x^3 - 3*x)*cos(4*x)^2 + 32*(2*x^3 - 3*x)*cos(2*x)^2 - 3
```

```
2*(2*x^3 - 3*x)*sin(4*x)^2 - 32*(2*x^3 - 3*x)*sin(2*x)^2 - (32*x^3 + 16*(2*
x^3 - 3*x)*cos(4*x) - 32*(2*x^3 - 3*x)*cos(2*x) - (-32*I*x^3 + 48*I*x)*sin(
4*x) - (64*I*x^3 - 96*I*x)*sin(2*x) - 48*x)*cos(6*x) + (64*x^3 - 80*(2*x^3
- 3*x)*cos(2*x) + (-160*I*x^3 + 240*I*x)*sin(2*x) - 96*x)*cos(4*x) - 16*(2*
x^3 - 3*x)*cos(2*x) + (-32*I*x^3 + (-32*I*x^3 + 48*I*x)*cos(4*x) + (64*I*x^
3 - 96*I*x)*cos(2*x) + 16*(2*x^3 - 3*x)*sin(4*x) - 32*(2*x^3 - 3*x)*sin(2*x
) + 48*I*x)*sin(6*x) + (64*I*x^3 + (128*I*x^3 - 192*I*x)*cos(4*x) + (-160*I
*x^3 + 240*I*x)*cos(2*x) + 80*(2*x^3 - 3*x)*sin(2*x) - 96*I*x)*sin(4*x) + (
-32*I*x^3 + (128*I*x^3 - 192*I*x)*cos(2*x) + 48*I*x)*sin(2*x))*log(cos(x)^2
 + sin(x)^2 - 2*cos(x) + 1) - ((-384*I*cos(4*x) + 768*I*cos(2*x) + 384*sin(
4*x) - 768*sin(2*x) - 384*I)*cos(6*x) + (-1920*I*cos(2*x) + 1920*sin(2*x) +
 768*I)*cos(4*x) + 768*I*cos(4*x)^2 + 768*I*cos(2*x)^2 + (384*cos(4*x) - 76
8*cos(2*x) + 384*I*sin(4*x) - 768*I*sin(2*x) + 384)*sin(6*x) - 384*(4*cos(4
*x) - 5*cos(2*x) - 5*I*sin(2*x) + 2)*sin(4*x) - 768*I*sin(4*x)^2 - 384*(4*c
os(2*x) - 1)*sin(2*x) - 768*I*sin(2*x)^2 - 384*I*cos(2*x))*polylog(4, -e^(I
*x)) - ((-384*I*cos(4*x) + 768*I*cos(2*x) + 384*sin(4*x) - 768*sin(2*x) - 3
84*I)*cos(6*x) + (-1920*I*cos(2*x) + 1920*sin(2*x) + 768*I)*cos(4*x) + 768*
I*cos(4*x)^2 + 768*I*cos(2*x)^2 + (384*cos(4*x) - 768*cos(2*x) + 384*I*sin(
4*x) - 768*I*sin(2*x) + 384)*sin(6*x) - 384*(4*cos(4*x) - 5*cos(2*x) - 5*I*
sin(2*x) + 2)*sin(4*x) - 768*I*sin(4*x)^2 - 384*(4*cos(2*x) - 1)*sin(2*x) -
 768*I*sin(2*x)^2 - 384*I*cos(2*x))*polylog(4, e^(I*x)) - (768*x*cos(4*x)^2
 + 768*x*cos(2*x)^2 - 768*x*sin(4*x)^2 - 768*x*sin(2*x)^2 - (384*x*cos(4*x)
 - 768*x*cos(2*x) + 384*I*x*sin(4*x) - 768*I*x*sin(2*x) + 384*x)*cos(6*x) -
 (1920*x*cos(2*x) + 1920*I*x*sin(2*x) - 768*x)*cos(4*x) - 384*x*cos(2*x) +
(-384*I*x*cos(4*x) + 768*I*x*cos(2*x) + 384*x*sin(4*x) - 768*x*sin(2*x) - 3
84*I*x)*sin(6*x) + (1536*I*x*cos(4*x) - 1920*I*x*cos(2*x) + 1920*x*sin(2*x)
 + 768*I*x)*sin(4*x) + (1536*I*x*cos(2*x) - 384*I*x)*sin(2*x))*polylog(3, -
e^(I*x)) - (768*x*cos(4*x)^2 + 768*x*cos(2*x)^2 - 768*x*sin(4*x)^2 - 768*x*
sin(2*x)^2 - (384*x*cos(4*x) - 768*x*cos(2*x) + 384*I*x*sin(4*x) - 768*I*x*
sin(2*x) + 384*x)*cos(6*x) - (1920*x*cos(2*x) + 1920*I*x*sin(2*x) - 768*x)*
cos(4*x) - 384*x*cos(2*x) + (-384*I*x*cos(4*x) + 768*I*x*cos(2*x) + 384*x*s
in(4*x) - 768*x*sin(2*x) - 384*I*x)*sin(6*x) + (1536*I*x*cos(4*x) - 1920*I*
x*cos(2*x) + 1920*x*sin(2*x) + 768*I*x)*sin(4*x) + (1536*I*x*cos(2*x) - 384
*I*x)*sin(2*x))*polylog(3, e^(I*x)) + (16*x^4 - 8*I*x^3 - 12*x^2 - (-8*I*x^
3 + 12*x^2 + 12*I*x - 6)*cos(6*x) + (16*x^4 - 16*I*x^3 - 72*x^2 + 24*I*x -
12)*cos(4*x) - (32*x^4 + 52*I*x^3 - 90*x^2 + 18*I*x - 3)*cos(2*x) - (-16*I*
x^4 - 16*x^3 + 72*I*x^2 + 24*x + 12*I)*sin(4*x) - (32*I*x^4 - 52*x^3 - 90*I
*x^2 - 18*x - 3*I)*sin(2*x) + 12*I*x + 6)*sin(6*x) - (32*x^4 - 20*I*x^3 - 3
0*x^2 + (64*x^4 - 32*I*x^3 - 336*x^2 + 48*I*x - 24)*cos(4*x) - (80*x^4 + 10
4*I*x^3 - 276*x^2 + 36*I*x - 6)*cos(2*x) + (-80*I*x^4 + 104*x^3 + 276*I*x^2
 + 36*x + 6*I)*sin(2*x) + 30*I*x + 15)*sin(4*x) + (16*x^4 - 16*I*x^3 - 24*x
^2 - (64*x^4 + 112*I*x^3 - 192*x^2 + 24*I*x)*cos(2*x) + 24*I*x + 12)*sin(2*
x) - 6*x + 3*I)/((32*cos(4*x) - 64*cos(2*x) + 32*I*sin(4*x) - 64*I*sin(2*x)
 + 32)*cos(6*x) + 32*(5*cos(2*x) + 5*I*sin(2*x) - 2)*cos(4*x) - 64*cos(4*x)
^2 - 64*cos(2*x)^2 - (-32*I*cos(4*x) + 64*I*cos(2*x) + 32*sin(4*x) - 64*sin
```

```
(2*x) - 32*I)*sin(6*x) - (128*I*cos(4*x) - 160*I*cos(2*x) + 160*sin(2*x) +
64*I)*sin(4*x) + 64*sin(4*x)^2 - (128*I*cos(2*x) - 32*I)*sin(2*x) + 64*sin(
2*x)^2 + 32*cos(2*x))
```

Fricas [C] time = 0.643887, size = 1566, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*x^3 - 3*x)*cos(x)^4 - 2*x^3 - 3*(2*x^3 - 3*x)*cos(x)^2 - ((24*I*
x^2 - 12*I)*cos(x)^2 - 24*I*x^2 + 12*I)*dilog(cos(x) + I*sin(x)) - ((-24*I*
x^2 + 12*I)*cos(x)^2 + 24*I*x^2 - 12*I)*dilog(cos(x) - I*sin(x)) - ((-24*I*
x^2 + 12*I)*cos(x)^2 + 24*I*x^2 - 12*I)*dilog(-cos(x) + I*sin(x)) - ((24*I*
x^2 - 12*I)*cos(x)^2 - 24*I*x^2 + 12*I)*dilog(-cos(x) - I*sin(x)) - 4*(2*x^
3 - (2*x^3 - 3*x)*cos(x)^2 - 3*x)*log(cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (
2*x^3 - 3*x)*cos(x)^2 - 3*x)*log(cos(x) - I*sin(x) + 1) - 4*(2*x^3 - (2*x^3
 - 3*x)*cos(x)^2 - 3*x)*log(-cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3
*x)*cos(x)^2 - 3*x)*log(-cos(x) - I*sin(x) + 1) - (-48*I*cos(x)^2 + 48*I)*p
olylog(4, cos(x) + I*sin(x)) - (48*I*cos(x)^2 - 48*I)*polylog(4, cos(x) - I
*sin(x)) - (48*I*cos(x)^2 - 48*I)*polylog(4, -cos(x) + I*sin(x)) - (-48*I*c
os(x)^2 + 48*I)*polylog(4, -cos(x) - I*sin(x)) + 48*(x*cos(x)^2 - x)*polylo
g(3, cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, cos(x) - I*sin(x))
 + 48*(x*cos(x)^2 - x)*polylog(3, -cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)
*polylog(3, -cos(x) - I*sin(x)) - 3*((2*x^2 - 1)*cos(x)^3 + (2*x^2 + 1)*cos
(x))*sin(x) - 3*x)/(cos(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x)**2*cot(x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\cos(x)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(x)^2*cot(x)^3, x)
```

3.206 $\int x^2 \cos^2(x) \cot^3(x) \, dx$

Optimal. Leaf size=106

$$2ix\text{PolyLog}\left(2,e^{2ix}\right)-\text{PolyLog}\left(3,e^{2ix}\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}-2x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot$$

```
[Out] (-3*x^2)/4 + ((2*I)/3)*x^3 - x*Cot[x] - (x^2*Cot[x]^2)/2 - 2*x^2*Log[1 - E^
((2*I)*x)] + Log[Sin[x]] + (2*I)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^(
(2*I)*x)] + (x*Cos[x]*Sin[x])/2 - Sin[x]^2/4 + (x^2*Sin[x]^2)/2
```

Rubi [A] time = 0.278309, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {4408, 3443, 3310, 30, 3717, 2190, 2531, 2282, 6589, 3720, 3475}

$$2ix\text{PolyLog}\left(2,e^{2ix}\right)-\text{PolyLog}\left(3,e^{2ix}\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}-2x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2*Cot[x]^3,x]
```


```
[Out] (-3*x^2)/4 + ((2*I)/3)*x^3 - x*Cot[x] - (x^2*Cot[x]^2)/2 - 2*x^2*Log[1 - E^
((2*I)*x)] + Log[Sin[x]] + (2*I)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^(
(2*I)*x)] + (x*Cos[x]*Sin[x])/2 - Sin[x]^2/4 + (x^2*Sin[x]^2)/2
```


Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\cos^2(x)\cot^3(x)\,dx = -\int x^2\cos^2(x)\cot(x)\,dx + \int x^2\cot^3(x)\,dx$$
$$= -\frac{1}{2}x^2\cot^2(x) - 2\int x^2\cot(x)\,dx + \int x\cot^2(x)\,dx + \int x^2\cos(x)\sin(x)\,dx$$
$$= -x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \frac{1}{2}x^2\sin^2(x) - 2\left(-\frac{ix^3}{3} - 2i\int\frac{e^{2ix}x^2}{1-e^{2ix}}\,dx\right) - \int x\,dx + \int\cot(x)\,dx$$
$$= -\frac{x^2}{2} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin^2(x) - \frac{\int x}{2}$$
$$= -\frac{3x^2}{4} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin^2(x) - 2\Big($$
$$= -\frac{3x^2}{4} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin^2(x) - 2\Big($$
$$= -\frac{3x^2}{4} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) - 2\left(-\frac{ix^3}{3} + x^2\log\left(1-e^{2ix}\right) - ix\mathrm{Li}_2\left(e^{2ix}\right) + \frac{1}{2}\mathrm{Li}\right.$$

Mathematica [A] time = 0.323761, size = 108, normalized size = 1.02

$$-2ix\mathrm{PolyLog}\left(2,e^{-2ix}\right) - \mathrm{PolyLog}\left(3,e^{-2ix}\right) - \frac{2ix^3}{3} - 2x^2\log\left(1-e^{-2ix}\right) - \frac{1}{4}x^2\cos(2x) - \frac{1}{2}x^2\csc^2(x) + \frac{1}{4}x\sin(2x) + \frac{1}{8}c$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (I/12)*Pi^3 - ((2*I)/3)*x^3 + Cos[2*x]/8 - (x^2*Cos[2*x])/4 - x*Cot[x] - (x
^2*Csc[x]^2)/2 - 2*x^2*Log[1 - E^((-2*I)*x)] + Log[Sin[x]] - (2*I)*x*PolyLo
g[2, E^((-2*I)*x)] - PolyLog[3, E^((-2*I)*x)] + (x*Sin[2*x])/4
```

Maple [A] time = 0.224, size = 170, normalized size = 1.6

$$\frac{2i}{3}x^3 - \frac{\left(2ix+2x^2-1\right)e^{2ix}}{16} - \frac{\left(-2ix+2x^2-1\right)e^{-2ix}}{16} + 2\frac{x\left(xe^{2ix}-ie^{2ix}+i\right)}{\left(e^{2ix}-1\right)^2} + \ln\left(e^{ix}-1\right) - 2\ln\left(e^{ix}\right) + \ln\left(1+e^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^3,x)
```

```
[Out] 2/3*I*x^3-1/16*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*(-2*I*x+2*x^2-1)*exp(-2*I*x)
+2*x*(x*exp(2*I*x)-I*exp(2*I*x)+I)/(exp(2*I*x)-1)^2+ln(exp(I*x)-1)-2*ln(exp
(I*x))+ln(1+exp(I*x))-2*x^2*ln(1-exp(I*x))+4*I*x*polylog(2,exp(I*x))-4*poly
log(3,exp(I*x))-2*x^2*ln(1+exp(I*x))+4*I*x*polylog(2,-exp(I*x))-4*polylog(3
,-exp(I*x))
```

Maxima [B] time = 2.63055, size = 3856, normalized size = 36.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -(3*(2*x^2 + 2*I*x - 1)*cos(6*x)^2 - (-64*I*x^3 - 24*x^2 + 168*I*x + 12)*co
s(4*x)^2 - (-64*I*x^3 + 84*x^2 + 96*I*x + 6)*cos(2*x)^2 - 3*(2*x^2 + 2*I*x
- 1)*sin(6*x)^2 - (64*I*x^3 + 24*x^2 - 168*I*x - 12)*sin(4*x)^2 - (64*I*x^3
 - 84*x^2 - 96*I*x - 6)*sin(2*x)^2 + 6*x^2 - ((192*I*x^2 - 96*I)*cos(4*x)^2
 + (192*I*x^2 - 96*I)*cos(2*x)^2 + (-192*I*x^2 + 96*I)*sin(4*x)^2 + (-192*I
*x^2 + 96*I)*sin(2*x)^2 + (-96*I*x^2 + (-96*I*x^2 + 48*I)*cos(4*x) + (192*I
*x^2 - 96*I)*cos(2*x) + 48*(2*x^2 - 1)*sin(4*x) - 96*(2*x^2 - 1)*sin(2*x) +
 48*I)*cos(6*x) + (192*I*x^2 + (-480*I*x^2 + 240*I)*cos(2*x) + 240*(2*x^2 -
 1)*sin(2*x) - 96*I)*cos(4*x) + (-96*I*x^2 + 48*I)*cos(2*x) + (96*x^2 + 48*
```

```
(2*x^2 - 1)*cos(4*x) - 96*(2*x^2 - 1)*cos(2*x) + (96*I*x^2 - 48*I)*sin(4*x)
 + (-192*I*x^2 + 96*I)*sin(2*x) - 48)*sin(6*x) - (192*x^2 + 192*(2*x^2 - 1)
*cos(4*x) - 240*(2*x^2 - 1)*cos(2*x) - (480*I*x^2 - 240*I)*sin(2*x) - 96)*s
in(4*x) + 48*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin(2*x))*arctan2(sin(x),
 cos(x) + 1) - ((48*I*cos(4*x) - 96*I*cos(2*x) - 48*sin(4*x) + 96*sin(2*x)
+ 48*I)*cos(6*x) + (240*I*cos(2*x) - 240*sin(2*x) - 96*I)*cos(4*x) - 96*I*c
os(4*x)^2 - 96*I*cos(2*x)^2 - (48*cos(4*x) - 96*cos(2*x) + 48*I*sin(4*x) -
96*I*sin(2*x) + 48)*sin(6*x) + 48*(4*cos(4*x) - 5*cos(2*x) - 5*I*sin(2*x) +
 2)*sin(4*x) + 96*I*sin(4*x)^2 + 48*(4*cos(2*x) - 1)*sin(2*x) + 96*I*sin(2*
x)^2 + 48*I*cos(2*x))*arctan2(sin(x), cos(x) - 1) - (-192*I*x^2*cos(4*x)^2
- 192*I*x^2*cos(2*x)^2 + 192*I*x^2*sin(4*x)^2 + 192*I*x^2*sin(2*x)^2 + 96*I
*x^2*cos(2*x) + (96*I*x^2*cos(4*x) - 192*I*x^2*cos(2*x) - 96*x^2*sin(4*x) +
 192*x^2*sin(2*x) + 96*I*x^2)*cos(6*x) + (480*I*x^2*cos(2*x) - 480*x^2*sin(
2*x) - 192*I*x^2)*cos(4*x) - (96*x^2*cos(4*x) - 192*x^2*cos(2*x) + 96*I*x^2
*sin(4*x) - 192*I*x^2*sin(2*x) + 96*x^2)*sin(6*x) + (384*x^2*cos(4*x) - 480
*x^2*cos(2*x) - 480*I*x^2*sin(2*x) + 192*x^2)*sin(4*x) + 96*(4*x^2*cos(2*x)
 - x^2)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) - (32*I*x^3 + 12*x^2 + (32*I
*x^3 + 24*x^2 - 72*I*x - 12)*cos(4*x) + (-64*I*x^3 + 78*x^2 + 90*I*x + 9)*c
os(2*x) - (32*x^3 - 24*I*x^2 - 72*x + 12*I)*sin(4*x) + (64*x^3 + 78*I*x^2 -
 90*x + 9*I)*sin(2*x) - 12*I*x - 6)*cos(6*x) - (-64*I*x^3 - 30*x^2 + (160*I
*x^3 - 156*x^2 - 276*I*x - 18)*cos(2*x) - (160*x^3 + 156*I*x^2 - 276*x + 18
*I)*sin(2*x) + 30*I*x + 15)*cos(4*x) - (32*I*x^3 + 24*x^2 - 24*I*x - 12)*co
s(2*x) - (-384*I*x*cos(4*x)^2 - 384*I*x*cos(2*x)^2 + 384*I*x*sin(4*x)^2 + 3
84*I*x*sin(2*x)^2 + (192*I*x*cos(4*x) - 384*I*x*cos(2*x) - 192*x*sin(4*x) +
 384*x*sin(2*x) + 192*I*x)*cos(6*x) + (960*I*x*cos(2*x) - 960*x*sin(2*x) -
384*I*x)*cos(4*x) + 192*I*x*cos(2*x) - (192*x*cos(4*x) - 384*x*cos(2*x) + 1
92*I*x*sin(4*x) - 384*I*x*sin(2*x) + 192*x)*sin(6*x) + (768*x*cos(4*x) - 96
0*x*cos(2*x) - 960*I*x*sin(2*x) + 384*x)*sin(4*x) + 192*(4*x*cos(2*x) - x)*
sin(2*x))*dilog(-e^(I*x)) - (-384*I*x*cos(4*x)^2 - 384*I*x*cos(2*x)^2 + 384
*I*x*sin(4*x)^2 + 384*I*x*sin(2*x)^2 + (192*I*x*cos(4*x) - 384*I*x*cos(2*x)
 - 192*x*sin(4*x) + 384*x*sin(2*x) + 192*I*x)*cos(6*x) + (960*I*x*cos(2*x)
- 960*x*sin(2*x) - 384*I*x)*cos(4*x) + 192*I*x*cos(2*x) - (192*x*cos(4*x) -
 384*x*cos(2*x) + 192*I*x*sin(4*x) - 384*I*x*sin(2*x) + 192*x)*sin(6*x) + (
768*x*cos(4*x) - 960*x*cos(2*x) - 960*I*x*sin(2*x) + 384*x)*sin(4*x) + 192*
(4*x*cos(2*x) - x)*sin(2*x))*dilog(e^(I*x)) - (48*(2*x^2 - 1)*cos(4*x)^2 +
48*(2*x^2 - 1)*cos(2*x)^2 - 48*(2*x^2 - 1)*sin(4*x)^2 - 48*(2*x^2 - 1)*sin(
2*x)^2 - (48*x^2 + 24*(2*x^2 - 1)*cos(4*x) - 48*(2*x^2 - 1)*cos(2*x) - (-48
*I*x^2 + 24*I)*sin(4*x) - (96*I*x^2 - 48*I)*sin(2*x) - 24)*cos(6*x) + (96*x
^2 - 120*(2*x^2 - 1)*cos(2*x) + (-240*I*x^2 + 120*I)*sin(2*x) - 48)*cos(4*x
) - 24*(2*x^2 - 1)*cos(2*x) + (-48*I*x^2 + (-48*I*x^2 + 24*I)*cos(4*x) + (9
6*I*x^2 - 48*I)*cos(2*x) + 24*(2*x^2 - 1)*sin(4*x) - 48*(2*x^2 - 1)*sin(2*x
) + 24*I)*sin(6*x) + (96*I*x^2 + (192*I*x^2 - 96*I)*cos(4*x) + (-240*I*x^2
+ 120*I)*cos(2*x) + 120*(2*x^2 - 1)*sin(2*x) - 48*I)*sin(4*x) + (-48*I*x^2
+ (192*I*x^2 - 96*I)*cos(2*x) + 24*I)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2
*cos(x) + 1) - (48*(2*x^2 - 1)*cos(4*x)^2 + 48*(2*x^2 - 1)*cos(2*x)^2 - 48*
```

```
(2*x^2 - 1)*sin(4*x)^2 - 48*(2*x^2 - 1)*sin(2*x)^2 - (48*x^2 + 24*(2*x^2 -
1)*cos(4*x) - 48*(2*x^2 - 1)*cos(2*x) - (-48*I*x^2 + 24*I)*sin(4*x) - (96*I
*x^2 - 48*I)*sin(2*x) - 24)*cos(6*x) + (96*x^2 - 120*(2*x^2 - 1)*cos(2*x) +
 (-240*I*x^2 + 120*I)*sin(2*x) - 48)*cos(4*x) - 24*(2*x^2 - 1)*cos(2*x) + (
-48*I*x^2 + (-48*I*x^2 + 24*I)*cos(4*x) + (96*I*x^2 - 48*I)*cos(2*x) + 24*(
2*x^2 - 1)*sin(4*x) - 48*(2*x^2 - 1)*sin(2*x) + 24*I)*sin(6*x) + (96*I*x^2
+ (192*I*x^2 - 96*I)*cos(4*x) + (-240*I*x^2 + 120*I)*cos(2*x) + 120*(2*x^2
- 1)*sin(2*x) - 48*I)*sin(4*x) + (-48*I*x^2 + (192*I*x^2 - 96*I)*cos(2*x) +
 24*I)*sin(2*x))*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + ((192*cos(4*x) -
 384*cos(2*x) + 192*I*sin(4*x) - 384*I*sin(2*x) + 192)*cos(6*x) + 192*(5*co
s(2*x) + 5*I*sin(2*x) - 2)*cos(4*x) - 384*cos(4*x)^2 - 384*cos(2*x)^2 - (-1
92*I*cos(4*x) + 384*I*cos(2*x) + 192*sin(4*x) - 384*sin(2*x) - 192*I)*sin(6
*x) - (768*I*cos(4*x) - 960*I*cos(2*x) + 960*sin(2*x) + 384*I)*sin(4*x) + 3
84*sin(4*x)^2 - (768*I*cos(2*x) - 192*I)*sin(2*x) + 384*sin(2*x)^2 + 192*co
s(2*x))*polylog(3, -e^(I*x)) + ((192*cos(4*x) - 384*cos(2*x) + 192*I*sin(4*
x) - 384*I*sin(2*x) + 192)*cos(6*x) + 192*(5*cos(2*x) + 5*I*sin(2*x) - 2)*c
os(4*x) - 384*cos(4*x)^2 - 384*cos(2*x)^2 - (-192*I*cos(4*x) + 384*I*cos(2*
x) + 192*sin(4*x) - 384*sin(2*x) - 192*I)*sin(6*x) - (768*I*cos(4*x) - 960*
I*cos(2*x) + 960*sin(2*x) + 384*I)*sin(4*x) + 384*sin(4*x)^2 - (768*I*cos(2
*x) - 192*I)*sin(2*x) + 384*sin(2*x)^2 + 192*cos(2*x))*polylog(3, e^(I*x))
+ (32*x^3 - 12*I*x^2 - (-12*I*x^2 + 12*x + 6*I)*cos(6*x) + (32*x^3 - 24*I*x
^2 - 72*x + 12*I)*cos(4*x) - (64*x^3 + 78*I*x^2 - 90*x + 9*I)*cos(2*x) - (-
32*I*x^3 - 24*x^2 + 72*I*x + 12)*sin(4*x) - (64*I*x^3 - 78*x^2 - 90*I*x - 9
)*sin(2*x) - 12*x + 6*I)*sin(6*x) - (64*x^3 - 30*I*x^2 + (128*x^3 - 48*I*x^
2 - 336*x + 24*I)*cos(4*x) - (160*x^3 + 156*I*x^2 - 276*x + 18*I)*cos(2*x)
+ (-160*I*x^3 + 156*x^2 + 276*I*x + 18)*sin(2*x) - 30*x + 15*I)*sin(4*x) +
(32*x^3 - 24*I*x^2 - (128*x^3 + 168*I*x^2 - 192*x + 12*I)*cos(2*x) - 24*x +
 12*I)*sin(2*x) - 6*I*x - 3)/((48*cos(4*x) - 96*cos(2*x) + 48*I*sin(4*x) -
96*I*sin(2*x) + 48)*cos(6*x) + 48*(5*cos(2*x) + 5*I*sin(2*x) - 2)*cos(4*x)
- 96*cos(4*x)^2 - 96*cos(2*x)^2 - (-48*I*cos(4*x) + 96*I*cos(2*x) + 48*sin(
4*x) - 96*sin(2*x) - 48*I)*sin(6*x) - (192*I*cos(4*x) - 240*I*cos(2*x) + 24
0*sin(2*x) + 96*I)*sin(4*x) + 96*sin(4*x)^2 - (192*I*cos(2*x) - 48*I)*sin(2
*x) + 96*sin(2*x)^2 + 48*cos(2*x))
```

Fricas [C] time = 0.613128, size = 1204, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*x^2 - 1)*cos(x)^4 - 3*(2*x^2 - 1)*cos(x)^2 - 2*x^2 - (16*I*x*cos
(x)^2 - 16*I*x)*dilog(cos(x) + I*sin(x)) - (-16*I*x*cos(x)^2 + 16*I*x)*dilo
g(cos(x) - I*sin(x)) - (-16*I*x*cos(x)^2 + 16*I*x)*dilog(-cos(x) + I*sin(x)
) - (16*I*x*cos(x)^2 - 16*I*x)*dilog(-cos(x) - I*sin(x)) + 4*((2*x^2 - 1)*c
os(x)^2 - 2*x^2 + 1)*log(cos(x) + I*sin(x) + 1) + 4*((2*x^2 - 1)*cos(x)^2 -
 2*x^2 + 1)*log(cos(x) - I*sin(x) + 1) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) +
 1/2*I*sin(x) + 1/2) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) - 1/2*I*sin(x) + 1/
2) + 8*(x^2*cos(x)^2 - x^2)*log(-cos(x) + I*sin(x) + 1) + 8*(x^2*cos(x)^2 -
 x^2)*log(-cos(x) - I*sin(x) + 1) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) + I
*sin(x)) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) - I*sin(x)) + 16*(cos(x)^2 -
 1)*polylog(3, -cos(x) + I*sin(x)) + 16*(cos(x)^2 - 1)*polylog(3, -cos(x) -
 I*sin(x)) - 4*(x*cos(x)^3 + x*cos(x))*sin(x) - 1)/(cos(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos^2(x) \cot^3(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x**2*cos(x)**2*cot(x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \cos(x)^2 \cot(x)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(x)^2*cot(x)^3, x)
```

3.207 $\int x\cos^2(x)\cot^3(x)\,dx$

Optimal. Leaf size=73

$$i\text{PolyLog}\left(2,e^{2ix}\right)+ix^2-\frac{3x}{4}-2x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)-\frac{\cot(x)}{2}+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] (-3*x)/4 + I*x^2 - Cot[x]/2 - (x*Cot[x]^2)/2 - 2*x*Log[1 - E^((2*I)*x)] + I
*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rubi [A] time = 0.163592, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4408, 3443, 2635, 8, 3717, 2190, 2279, 2391, 3720, 3473}

$$i\text{PolyLog}\left(2,e^{2ix}\right)+ix^2-\frac{3x}{4}-2x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)-\frac{\cot(x)}{2}+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-3*x)/4 + I*x^2 - Cot[x]/2 - (x*Cot[x]^2)/2 - 2*x*Log[1 - E^((2*I)*x)] + I
*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int x\cos^2(x)\cot^3(x)\,dx = -\int x\cos^2(x)\cot(x)\,dx + \int x\cot^3(x)\,dx$$
$$= -\frac{1}{2}x\cot^2(x) + \frac{1}{2}\int \cot^2(x)\,dx - 2\int x\cot(x)\,dx + \int x\cos(x)\sin(x)\,dx$$
$$= -\frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{2}x\sin^2(x) - 2\left(-\frac{ix^2}{2} - 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\right) - \frac{\int 1\,dx}{2} - \frac{1}{2}\int \sin^2(x)\,dx$$
$$= -\frac{x}{2} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - \frac{\int 1\,dx}{4} - 2\left(-\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right.\right.$$
$$= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - 2\left(-\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) + \frac{1}{2}i\,\mathrm{S}\right.$$
$$= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) - 2\left(-\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) - \frac{1}{2}i\mathrm{Li}_2\left(e^{2ix}\right)\right) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}$$

Mathematica [A] time = 0.107574, size = 62, normalized size = 0.85

$$\frac{1}{8}\left(8i\mathrm{PolyLog}\left(2,e^{2ix}\right) + 8ix^2 - 16x\log\left(1-e^{2ix}\right) + \sin(2x) - 2x\cos(2x) - 4\cot(x) - 4x\csc^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] ((8*I)*x^2 - 2*x*Cos[2*x] - 4*Cot[x] - 4*x*Csc[x]^2 - 16*x*Log[1 - E^((2*I)
*x)] + (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x])/8
```

Maple [A] time = 0.126, size = 109, normalized size = 1.5

$$ix^2 - \frac{(i+2x)\,e^{2ix}}{16} - \frac{(2x-i)\,e^{-2ix}}{16} + \frac{2xe^{2ix} - ie^{2ix} + i}{\left(e^{2ix}-1\right)^2} - 2x\ln\left(1-e^{ix}\right) - 2x\ln\left(1+e^{ix}\right) + 2i\mathrm{polylog}\left(2,e^{ix}\right) + 2i\mathrm{pol}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^3,x)
```

```
[Out] I*x^2-1/16*(I+2*x)*exp(2*I*x)-1/16*(2*x-I)*exp(-2*I*x)+(2*x*exp(2*I*x)-I*ex
p(2*I*x)+I)/(exp(2*I*x)-1)^2-2*x*ln(1-exp(I*x))-2*x*ln(1+exp(I*x))+2*I*poly
log(2,exp(I*x))+2*I*polylog(2,-exp(I*x))
```

Maxima [B] time = 1.88111, size = 2349, normalized size = 32.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -((2*x + I)*cos(6*x)^2 - (-32*I*x^2 - 8*x - 4*I)*cos(4*x)^2 - (-32*I*x^2 +
28*x - 16*I)*cos(2*x)^2 - (2*x + I)*sin(6*x)^2 - (32*I*x^2 + 8*x + 4*I)*sin
(4*x)^2 - (32*I*x^2 - 28*x + 16*I)*sin(2*x)^2 - (64*I*x*cos(4*x)^2 + 64*I*x
*cos(2*x)^2 - 64*I*x*sin(4*x)^2 - 64*I*x*sin(2*x)^2 + (-32*I*x*cos(4*x) + 6
4*I*x*cos(2*x) + 32*x*sin(4*x) - 64*x*sin(2*x) - 32*I*x)*cos(6*x) + (-160*I
*x*cos(2*x) + 160*x*sin(2*x) + 64*I*x)*cos(4*x) - 32*I*x*cos(2*x) + (32*x*c
os(4*x) - 64*x*cos(2*x) + 32*I*x*sin(4*x) - 64*I*x*sin(2*x) + 32*x)*sin(6*x
) - (128*x*cos(4*x) - 160*x*cos(2*x) - 160*I*x*sin(2*x) + 64*x)*sin(4*x) -
32*(4*x*cos(2*x) - x)*sin(2*x))*arctan2(sin(x), cos(x) + 1) - (-64*I*x*cos(
4*x)^2 - 64*I*x*cos(2*x)^2 + 64*I*x*sin(4*x)^2 + 64*I*x*sin(2*x)^2 + (32*I*
x*cos(4*x) - 64*I*x*cos(2*x) - 32*x*sin(4*x) + 64*x*sin(2*x) + 32*I*x)*cos(
6*x) + (160*I*x*cos(2*x) - 160*x*sin(2*x) - 64*I*x)*cos(4*x) + 32*I*x*cos(2
*x) - (32*x*cos(4*x) - 64*x*cos(2*x) + 32*I*x*sin(4*x) - 64*I*x*sin(2*x) +
32*x)*sin(6*x) + (128*x*cos(4*x) - 160*x*cos(2*x) - 160*I*x*sin(2*x) + 64*x
)*sin(4*x) + 32*(4*x*cos(2*x) - x)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) -
 (16*I*x^2 + (16*I*x^2 + 8*x + 4*I)*cos(4*x) + (-32*I*x^2 + 26*x - 17*I)*co
s(2*x) - 4*(4*x^2 - 2*I*x + 1)*sin(4*x) + (32*x^2 + 26*I*x + 17)*sin(2*x) +
 4*x + 14*I)*cos(6*x) - (-32*I*x^2 + (80*I*x^2 - 52*x + 34*I)*cos(2*x) - 2*
(40*x^2 + 26*I*x + 17)*sin(2*x) - 10*x - 27*I)*cos(4*x) - (16*I*x^2 + 8*x +
 12*I)*cos(2*x) - ((32*I*cos(4*x) - 64*I*cos(2*x) - 32*sin(4*x) + 64*sin(2*
x) + 32*I)*cos(6*x) + (160*I*cos(2*x) - 160*sin(2*x) - 64*I)*cos(4*x) - 64*
I*cos(4*x)^2 - 64*I*cos(2*x)^2 - (32*cos(4*x) - 64*cos(2*x) + 32*I*sin(4*x)
 - 64*I*sin(2*x) + 32)*sin(6*x) + 32*(4*cos(4*x) - 5*cos(2*x) - 5*I*sin(2*x
) + 2)*sin(4*x) + 64*I*sin(4*x)^2 + 32*(4*cos(2*x) - 1)*sin(2*x) + 64*I*sin
(2*x)^2 + 32*I*cos(2*x))*dilog(-e^(I*x)) - ((32*I*cos(4*x) - 64*I*cos(2*x)
- 32*sin(4*x) + 64*sin(2*x) + 32*I)*cos(6*x) + (160*I*cos(2*x) - 160*sin(2*
x) - 64*I)*cos(4*x) - 64*I*cos(4*x)^2 - 64*I*cos(2*x)^2 - (32*cos(4*x) - 64
*cos(2*x) + 32*I*sin(4*x) - 64*I*sin(2*x) + 32)*sin(6*x) + 32*(4*cos(4*x) -
 5*cos(2*x) - 5*I*sin(2*x) + 2)*sin(4*x) + 64*I*sin(4*x)^2 + 32*(4*cos(2*x)
 - 1)*sin(2*x) + 64*I*sin(2*x)^2 + 32*I*cos(2*x))*dilog(e^(I*x)) - (32*x*co
```

```
s(4*x)^2 + 32*x*cos(2*x)^2 - 32*x*sin(4*x)^2 - 32*x*sin(2*x)^2 - (16*x*cos(
4*x) - 32*x*cos(2*x) + 16*I*x*sin(4*x) - 32*I*x*sin(2*x) + 16*x)*cos(6*x) -
 (80*x*cos(2*x) + 80*I*x*sin(2*x) - 32*x)*cos(4*x) - 16*x*cos(2*x) + (-16*I
*x*cos(4*x) + 32*I*x*cos(2*x) + 16*x*sin(4*x) - 32*x*sin(2*x) - 16*I*x)*sin
(6*x) + (64*I*x*cos(4*x) - 80*I*x*cos(2*x) + 80*x*sin(2*x) + 32*I*x)*sin(4*
x) + (64*I*x*cos(2*x) - 16*I*x)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*cos(x
) + 1) - (32*x*cos(4*x)^2 + 32*x*cos(2*x)^2 - 32*x*sin(4*x)^2 - 32*x*sin(2*
x)^2 - (16*x*cos(4*x) - 32*x*cos(2*x) + 16*I*x*sin(4*x) - 32*I*x*sin(2*x) +
 16*x)*cos(6*x) - (80*x*cos(2*x) + 80*I*x*sin(2*x) - 32*x)*cos(4*x) - 16*x*
cos(2*x) + (-16*I*x*cos(4*x) + 32*I*x*cos(2*x) + 16*x*sin(4*x) - 32*x*sin(2
*x) - 16*I*x)*sin(6*x) + (64*I*x*cos(4*x) - 80*I*x*cos(2*x) + 80*x*sin(2*x)
 + 32*I*x)*sin(4*x) + (64*I*x*cos(2*x) - 16*I*x)*sin(2*x))*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1) + (16*x^2 + 2*(2*I*x - 1)*cos(6*x) + 4*(4*x^2 - 2*I
*x + 1)*cos(4*x) - (32*x^2 + 26*I*x + 17)*cos(2*x) - (-16*I*x^2 - 8*x - 4*I
)*sin(4*x) - (32*I*x^2 - 26*x + 17*I)*sin(2*x) - 4*I*x + 14)*sin(6*x) - (32
*x^2 + 8*(8*x^2 - 2*I*x + 1)*cos(4*x) - 2*(40*x^2 + 26*I*x + 17)*cos(2*x) +
 (-80*I*x^2 + 52*x - 34*I)*sin(2*x) - 10*I*x + 27)*sin(4*x) + (16*x^2 - 8*(
8*x^2 + 7*I*x + 4)*cos(2*x) - 8*I*x + 12)*sin(2*x) + 2*x - I)/((16*cos(4*x)
 - 32*cos(2*x) + 16*I*sin(4*x) - 32*I*sin(2*x) + 16)*cos(6*x) + 16*(5*cos(2
*x) + 5*I*sin(2*x) - 2)*cos(4*x) - 32*cos(4*x)^2 - 32*cos(2*x)^2 - (-16*I*c
os(4*x) + 32*I*cos(2*x) + 16*sin(4*x) - 32*sin(2*x) - 16*I)*sin(6*x) - (64*
I*cos(4*x) - 80*I*cos(2*x) + 80*sin(2*x) + 32*I)*sin(4*x) + 32*sin(4*x)^2 -
 (64*I*cos(2*x) - 16*I)*sin(2*x) + 32*sin(2*x)^2 + 16*cos(2*x))
```

Fricas [B] time = 0.581824, size = 640, normalized size = 8.77

$$-\frac{2\,x\cos(x)^4 - 3\,x\cos(x)^2 - \left(4i\,\cos(x)^2 - 4i\right)\mathrm{Li}_2\left(\cos(x) + i\,\sin(x)\right) - \left(-4i\,\cos(x)^2 + 4i\right)\mathrm{Li}_2\left(\cos(x) - i\,\sin(x)\right) - \left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*x*cos(x)^4 - 3*x*cos(x)^2 - (4*I*cos(x)^2 - 4*I)*dilog(cos(x) + I*s
in(x)) - (-4*I*cos(x)^2 + 4*I)*dilog(cos(x) - I*sin(x)) - (-4*I*cos(x)^2 +
4*I)*dilog(-cos(x) + I*sin(x)) - (4*I*cos(x)^2 - 4*I)*dilog(-cos(x) - I*sin
(x)) + 4*(x*cos(x)^2 - x)*log(cos(x) + I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*l
og(cos(x) - I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(-cos(x) + I*sin(x) + 1)
+ 4*(x*cos(x)^2 - x)*log(-cos(x) - I*sin(x) + 1) - (cos(x)^3 + cos(x))*sin(
x) - x)/(cos(x)^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos^2(x)\cot^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x*cos(x)**2*cot(x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(x)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)^2*cot(x)^3, x)
```

3.208 $\int (c+dx)^m \tan(a+bx)\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\tan(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Tan[a + b*x], x]
```

Rubi [A] time = 0.0177912, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x],x]

[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (c+dx)^m \tan(a+bx)\,dx$$

Mathematica [A] time = 2.51709, size = 0, normalized size = 0.

$$\int (c+dx)^m \tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x],x]

[Out] Integrate[(c + d*x)^m*Tan[a + b*x], x]
```

Maple [A] time = 0.2, size = 0, normalized size = 0.

$$\int (dx+c)^m \sec (bx+a) \sin (bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec (bx+a) \sin (bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \sec (bx+a) \sin (bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

3.209 $\int (c+dx)^4 \tan(a+bx)\,dx$

Optimal. Leaf size=158

$$-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{3d^4}{}$$

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*
I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5)
```

Rubi [A] time = 0.209578, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{3d^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Tan[a + b*x],x]
```

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*
I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4 \tan(a+bx)\,dx = \frac{i(c+dx)^5}{5d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1+e^{2i(a+bx)}}\,dx$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{(4d)\int (c+dx)^3\log\left(1+e^{2i(a+bx)}\right)dx}{b}$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{\left(6id^2\right)\int (c+dx)^2\text{I}}{b^2}$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)^2\text{Li}_3\left(-\right.}{b^3}$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)^2\text{Li}_3\left(-\right.}{b^3}$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)^2\text{Li}_3\left(-\right.}{b^3}$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)^2\text{Li}_3\left(-\right.}{b^3}$$

Mathematica [A] time = 0.0687408, size = 157, normalized size = 0.99

$$-\frac{3d^2\left(2b^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)+d\left(2ib(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)-d\text{PolyLog}\left(5,-e^{2i(a+bx)}\right)\right)\right)}{2b^5} + \frac{2id(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Tan[a + b*x],x]
```

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*
I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(2*b^2*(c +
 d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c + d*x)*PolyLog[4,
-E^((2*I)*(a + b*x))] - d*PolyLog[5, -E^((2*I)*(a + b*x))])))/(2*b^5)
```

Maple [B] time = 0.315, size = 616, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+1/5*I*d^4*x^5+I*c*d^3*x^4-3*I/b^4*
c*d^3*polylog(4,-exp(2*I*(b*x+a)))-3/b^3*c^2*d^2*polylog(3,-exp(2*I*(b*x+a)
))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2-4/b*c*d^3*ln(exp(2*I*(b*x+a))
+1)*x^3-1/b*c^4*ln(exp(2*I*(b*x+a))+1)-I*c^4*x-6/b*c^2*d^2*ln(exp(2*I*(b*x+
a))+1)*x^2-12*I/b^2*a^2*c^2*d^2*x+8*I/b^3*c*d^3*a^3*x+6*I/b^2*c*d^3*polylog
(2,-exp(2*I*(b*x+a)))*x^2+6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-6/
b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-1/b*d^4*ln(exp(2*I*(b*x+a))+1)*x^4
-8/5*I/b^5*d^4*a^5+8*I/b*a*c^3*d*x-4/b*c^3*d*ln(exp(2*I*(b*x+a))+1)*x-2*I/b
^4*d^4*a^4*x-8*I/b^3*a^3*c^2*d^2+4*I/b^2*a^2*c^3*d+6*I/b^4*c*d^3*a^4+2*I*c^
2*d^2*x^3+2*I*c^3*d*x^2+2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-8/b^2*c^3*d*a*ln(e
xp(I*(b*x+a)))+12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-8/b^4*c*d^3*a^3*ln(exp
(I*(b*x+a)))+2/b*c^4*ln(exp(I*(b*x+a)))+2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b
*x+a)))-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+2*I/b^2*d^4*polylog(2,-e
xp(2*I*(b*x+a)))*x^3
```

Maxima [B] time = 2.05808, size = 1069, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/30*(15*c^4*log(-sin(b*x + a)^2 + 1) - 60*a*c^3*d*log(-sin(b*x + a)^2 + 1
)/b + 90*a^2*c^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 60*a^3*c*d^3*log(-sin(b
*x + a)^2 + 1)/b^3 + 15*a^4*d^4*log(-sin(b*x + a)^2 + 1)/b^4 + 2*(-3*I*(b*x
 + a)^5*d^4 + (-15*I*b*c*d^3 + 15*I*a*d^4)*(b*x + a)^4 - 45*d^4*polylog(5,
-e^(2*I*b*x + 2*I*a)) + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2*d^4)
*(b*x + a)^3 + (-30*I*b^3*c^3*d + 90*I*a*b^2*c^2*d^2 - 90*I*a^2*b*c*d^3 + 3
0*I*a^3*d^4)*(b*x + a)^2 + (30*I*(b*x + a)^4*d^4 + (80*I*b*c*d^3 - 80*I*a*d
^4)*(b*x + a)^3 + (90*I*b^2*c^2*d^2 - 180*I*a*b*c*d^3 + 90*I*a^2*d^4)*(b*x
+ a)^2 + (60*I*b^3*c^3*d - 180*I*a*b^2*c^2*d^2 + 180*I*a^2*b*c*d^3 - 60*I*a
^3*d^4)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (-30*I
*b^3*c^3*d + 90*I*a*b^2*c^2*d^2 - 90*I*a^2*b*c*d^3 - 60*I*(b*x + a)^3*d^4 +
 30*I*a^3*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a)^2 + (-90*I*b^2*c^2
*d^2 + 180*I*a*b*c*d^3 - 90*I*a^2*d^4)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a
)) + 5*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a) + 1) + (60*I*b*c*d^3 + 90*I*(b*x + a)*d^4 - 60*I*a
*d^4)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 15*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 +
6*(b*x + a)^2*d^4 + 3*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -
```

```
e^(2*I*b*x + 2*I*a)))/b^4)/b
```

Fricas [C] time = 0.741923, size = 3355, normalized size = 21.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*polylog(5, I
*cos(b*x + a) - sin(b*x + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) + sin(b*x
 + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) - sin(b*x + a)) + (-4*I*b^3*d^4*
x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(
b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3
*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (4*I*b^3
*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 1
2*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) -
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log
(cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) + I) -
 (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b
^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a)
+ sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 +
4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*
a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^4*d^4*x
^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d -
6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(b
*x + a) + I) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) + si
n(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) - si
n(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + s
in(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - s
in(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*
cos(b*x + a) + sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^
2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^
3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(b^2*d^4
```

```
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x +
a)))/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \sec(bx+a)\sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(b*x + a), x)
```

3.210 $\int (c+dx)^3 \tan(a+bx)\, dx$

Optimal. Leaf size=132

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}-\frac{(c+dx)^3\log}{}$$

```
[Out] ((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3
*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*
x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4
```

Rubi [A] time = 0.182795, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}-\frac{(c+dx)^3\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x],x]
```

```
[Out] ((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3
*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*
x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\tan(a+bx)\,dx = \frac{i(c+dx)^4}{4d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}\,dx$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{(3d)\int (c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{b}$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{\left(3id^2\right)\int (c+dx)\mathrm{L}}{b^2}$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx)\mathrm{Li}_3\left(-e\right.}{2b^3}$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx)\mathrm{Li}_3\left(-e\right.}{2b^3}$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx)\mathrm{Li}_3\left(-e\right.}{2b^3}$$

Mathematica [A] time = 0.0874103, size = 126, normalized size = 0.95

$$\frac{1}{4}i\left(\frac{3d\left(2b^2(c+dx)^2\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)+d\left(2ib(c+dx)\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)-d\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)\right)\right)}{b^4} + \frac{4i(c+}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x],x]
```

```
[Out] (I/4)*((c + d*x)^4/d + ((4*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b +
 (3*d*(2*b^2*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c +
 d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] - d*PolyLog[4, -E^((2*I)*(a + b*x))]
)))/b^4)
```

Maple [B] time = 0.289, size = 423, normalized size = 3.2

$$-\frac{c^3\ln\left(e^{2i(bx+a)}+1\right)}{b} - \frac{3d^2c\,\mathrm{polylog}\left(3,-e^{2i(bx+a)}\right)}{2b^3} - \frac{3d^3\mathrm{polylog}\left(3,-e^{2i(bx+a)}\right)x}{2b^3} + \frac{\frac{3i}{2}d^3a^4}{b^4} + \frac{i}{4}d^3x^4 + icd^2x^3 - ic^3x + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] -1/b*c^3*ln(exp(2*I*(b*x+a))+1)-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-
3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+3/2*I/b^4*d^3*a^4+1/4*I*d^3*x^4+
I*c*d^2*x^3-I*c^3*x+3/2*I*c^2*d*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x
+a)))-3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x
^2-1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x
+a)))*x^2+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)
))-4*I/b^3*a^3*c*d^2+2*I/b^3*d^3*a^3*x+3*I/b^2*a^2*c^2*d-3/4*I*d^3*polylog(
4,-exp(2*I*(b*x+a)))/b^4+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-2/b^4
*d^3*a^3*ln(exp(I*(b*x+a)))+6*I/b*a*c^2*d*x-6*I/b^2*a^2*c*d^2*x+2/b*c^3*ln(
exp(I*(b*x+a)))
```

Maxima [B] time = 1.857, size = 662, normalized size = 5.02

$$-\frac{6\,c^3\log\left(-\sin(bx+a)^2+1\right) - \frac{18\,ac^2d\log\left(-\sin(bx+a)^2+1\right)}{b} + \frac{18\,a^2cd^2\log\left(-\sin(bx+a)^2+1\right)}{b^2} - \frac{6\,a^3d^3\log\left(-\sin(bx+a)^2+1\right)}{b^3} + \frac{-3i\,(bx+a)^4d^3+\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*(6*c^3*log(-sin(b*x + a)^2 + 1) - 18*a*c^2*d*log(-sin(b*x + a)^2 + 1)
/b + 18*a^2*c*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 6*a^3*d^3*log(-sin(b*x + a
)^2 + 1)/b^3 + (-3*I*(b*x + a)^4*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x +
a)^3 + 12*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) + (-18*I*b^2*c^2*d + 36*I*
a*b*c*d^2 - 18*I*a^2*d^3)*(b*x + a)^2 + (16*I*(b*x + a)^3*d^3 + (36*I*b*c*d
^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*a^2*
d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (-18*I*b^
2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 - 18*I*a^2*d^3 + (-36*I*b*c
*d^2 + 36*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(4*(b*x + a)^
3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
 2*a) + 1) + 6*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b
*x + 2*I*a)))/b^3)/b
```

Fricas [C] time = 0.664148, size = 2367, normalized size = 17.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*
d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x +
 a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(I*cos(b*x
+ a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*
dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x
 - 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*s
in(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) -
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) -
sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*
d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*poly
log(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) - sin(b*x + a)))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \sec(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a), x)
```

3.211 $\int (c+dx)^2 \tan(a+bx)\,dx$

Optimal. Leaf size=96

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^3}{3d}$$

```
[Out] ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*d
*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, -E^((2*I
)*(a + b*x))])/(2*b^3)
```

Rubi [A] time = 0.152983, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3719, 2190, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x],x]
```

```
[Out] ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*d
*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, -E^((2*I
)*(a + b*x))])/(2*b^3)
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\tan(a+bx)\,dx &= \frac{i(c+dx)^3}{3d}-2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx\\ &= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(2d)\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}\\ &= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{\left(id^2\right)\int \mathrm{Li}_2\left(-e^{2i(a+\right.}}{b^2}\\ &= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\,\mathrm{Subst}\left(\int \frac{\mathrm{Li}_2(-x)}{x}\right.}{2b^3}\\ &= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\mathrm{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.0409372, size = 100, normalized size = 1.04

$$\frac{6ibd^2(c+dx)\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)-3d^3\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)+2ib^2(c+dx)^2\left(b(c+dx)+3id\log\left(1+e^{2i(a+bx)}\right)\right)}{6b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x],x]
```

```
[Out] ((2*I)*b^2*(c + d*x)^2*(b*(c + d*x) + (3*I)*d*Log[1 + E^((2*I)*(a + b*x))])
 + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*d^3*PolyLog[3
, -E^((2*I)*(a + b*x))])/(6*b^3*d)
```

Maple [B] time = 0.277, size = 257, normalized size = 2.7

$$icdx^2 + \frac{2\,ia^2cd}{b^2} - ic^2x - \frac{c^2\ln\left(e^{2\,i(bx+a)}+1\right)}{b} + 2\,\frac{c^2\ln\left(e^{i(bx+a)}\right)}{b} + 2\,\frac{a^2d^2\ln\left(e^{i(bx+a)}\right)}{b^3} - \frac{2\,ia^2d^2x}{b^2} + \frac{i}{3}d^2x^3 - 2\,\frac{cd\ln\left(e^{2\,i(bx-\right.}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] I*c*d*x^2+2*I/b^2*a^2*c*d-I*c^2*x-1/b*c^2*ln(exp(2*I*(b*x+a))+1)+2/b*c^2*ln
(exp(I*(b*x+a)))+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-2*I/b^2*a^2*d^2*x+1/3*I*d
^2*x^3-2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a
)))*x-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+
a)))-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c*d*a*ln(exp(I*(b*x+a))
)-4/3*I/b^3*a^3*d^2+4*I/b*a*c*d*x
```

Maple [B] time = 1.85091, size = 378, normalized size = 3.94

$$-\frac{3\,c^2\log\left(-\sin\left(bx+a\right)^2+1\right) - \frac{6\,acd\log\left(-\sin(bx+a)^2+1\right)}{b} + \frac{3\,a^2d^2\log\left(-\sin(bx+a)^2+1\right)}{b^2} + \frac{-2i\,(bx+a)^3d^2+\left(-6i\,bcd+6i\,ad^2\right)(bx+a)^2+3\,d^2\mathrm{Li}_3(-e^{(2\cdots}}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^2*log(-sin(b*x + a)^2 + 1) - 6*a*c*d*log(-sin(b*x + a)^2 + 1)/b +
 3*a^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 + (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c
*d + 6*I*a*d^2)*(b*x + a)^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) + (6*I
*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) + (-6*I*b*c*d - 6*I*(b*x + a)*d^2 + 6*I*a*d^2)
*dilog(-e^(2*I*b*x + 2*I*a)) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
```

```
)/b^2)/b
```

Fricas [C] time = 0.597114, size = 1534, normalized size = 15.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - (-2*I*b*d^2*x - 2
*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*
dilog(I*cos(b*x + a) - sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-I*cos(b*x +
 a) - sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*
sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*si
n(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*c
os(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2
*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a
) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a)
 + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \sin(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec (bx + a) \sin (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a), x)
```

3.212 $\int (c+dx)\tan(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^2}{2d}$$

```
[Out] ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b + ((I/2)
*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

Rubi [A] time = 0.0925894, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3719, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x],x]
```


```
[Out] ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b + ((I/2)
*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```


Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\tan(a+bx)\,dx &= \frac{i(c+dx)^2}{2d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx \\
&= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\int \log\left(1+e^{2i(a+bx)}\right)\,dx}{b} \\
&= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{(id)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i(a+bx)}\right)}{2b^2} \\
&= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id\,\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.0147743, size = 70, normalized size = 1.06

$$\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{c\log(\cos(a+bx))}{b} - \frac{dx\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{1}{2}idx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Tan[a + b*x],x]
```

```
[Out] (I/2)*d*x^2 - (d*x*Log[1 + E^((2*I)*(a + b*x))])/b - (c*Log[Cos[a + b*x]])/
b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

Maple [B] time = 0.252, size = 123, normalized size = 1.9

$$\frac{i}{2}dx^2 - icx + 2\frac{c\ln\left(e^{i(bx+a)}\right)}{b} - \frac{c\ln\left(e^{2i(bx+a)}+1\right)}{b} + \frac{2idax}{b} + \frac{ida^2}{b^2} - \frac{d\ln\left(e^{2i(bx+a)}+1\right)x}{b} + \frac{\frac{i}{2}d\,\text{polylog}\left(2,-e^{2i(bx+a)}\right)}{b^2} - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/2*I*d*x^2-I*c*x+2/b*c*ln(exp(I*(b*x+a)))-1/b*c*ln(exp(2*I*(b*x+a))+1)+2*I
/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(2,-exp(
2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 1.72612, size = 154, normalized size = 2.33

$$-\frac{-i\,b^2dx^2 - 2i\,b^2cx + (2i\,bdx + 2i\,bc)\arctan\left(\sin\left(2\,bx + 2\,a\right), \cos\left(2\,bx + 2\,a\right) + 1\right) - i\,d\mathrm{Li}_2\left(-e^{(2i\,bx+2i\,a)}\right) + (bdx + bc)\,\mathrm{lo}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(2*b*x
+ 2*a), cos(2*b*x + 2*a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + (b*d*x +
b*c)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1))
/b^2
```

Fricas [B] time = 0.571796, size = 837, normalized size = 12.68

$$\frac{-i\,d\mathrm{Li}_2\left(i\,\cos\left(bx + a\right) + \sin\left(bx + a\right)\right) + i\,d\mathrm{Li}_2\left(i\,\cos\left(bx + a\right) - \sin\left(bx + a\right)\right) + i\,d\mathrm{Li}_2\left(-i\,\cos\left(bx + a\right) + \sin\left(bx + a\right)\right) - i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(-I*c
os(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) -
 sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)
- (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*log(-
cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(
b*x + a) + I))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \sin (a + bx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec (bx + a) \sin (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a), x)
```

3.213 $\int \frac{\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Tan[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.0199929, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Tan[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 3.56978, size = 0, normalized size = 0.

$$\int \frac{\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Tan[a + b*x]/(c + d*x), x]
```

Maple [A] time = 0.222, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)\sin(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

3.214 $\int \frac{\tan(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.0196721, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx = \int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 5.36351, size = 0, normalized size = 0.

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Tan[a + b*x]/(c + d*x)^2, x]
```

Maple [A] time = 0.235, size = 0, normalized size = 0.

$$\int \frac{\sec (bx + a) \sin (bx + a)}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (bx + a) \sin (bx + a)}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec (bx + a) \sin (bx + a)}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin (a + bx) \sec (a + bx)}{(c + dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

3.215 $\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=147

$$\text{Unintegrable}\left(\sec(a+bx)(c+dx)^m,x\right)+\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)$$

```
[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*
x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Unintegrable[(c + d
*x)^m*Sec[a + b*x], x]
```

Rubi [A] time = 0.135594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*
x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x
)^m*Sec[a + b*x], x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)^m \cos(a+bx)\,dx+\int (c+dx)^m \sec(a+bx)\,dx\\
&= -\left(\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx\right)-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx)^m \sec(a\\
&= \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+}{d}\right.}{2}
\end{aligned}$$

Mathematica [A] time = 6.58576, size = 0, normalized size = 0.

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x], x]
```

Maple [A] time = 0.323, size = 0, normalized size = 0.

$$\int (dx+c)^{m} \sec (bx+a) (\sin (bx+a))^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{m} \sec (bx+a) \sin (bx+a)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos (bx+a)^{2}-1\right)(dx+c)^{m} \sec (bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

3.216 $\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=275

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+$$

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (6*d^3*Cos[a + b*x])/b^4 -
 (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*
E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (6*d^2*(c +
 d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*
(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 + (6*d^2*(
c + d*x)*Sin[a + b*x])/b^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rubi [A] time = 0.214907, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4407, 3296, 2638, 4181, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```


```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (6*d^3*Cos[a + b*x])/b^4 -
 (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*
E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (6*d^2*(c +
 d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*
(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 + (6*d^2*(
c + d*x)*Sin[a + b*x])/b^3 - ((c + d*x)^3*Sin[a + b*x])/b
```


Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^3\cos(a+bx)\,dx + \int (c+dx)^3\sec(a+bx)\,dx$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-i\right.}{b}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{3id(c+dx)^2\text{Li}_2\left(-ie^{i(a}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{3id}{}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{3id}{}$$

Mathematica [B] time = 1.5463, size = 557, normalized size = 2.03

$$-\frac{-3ib^2d(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right) + 3ib^2d(c+dx)^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right) + 6bcd^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right) - 6bcd^2\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -(((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^2*c^2*d*Cos[a + b*x] - 6*d^3
*Cos[a + b*x] + 6*b^2*c*d^2*x*Cos[a + b*x] + 3*b^2*d^3*x^2*Cos[a + b*x] - 3
*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a
 + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 +
I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x
^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(
I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] + 6*b
*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a
 + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3,
I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)*d^3
*PolyLog[4, I*E^(I*(a + b*x))] + b^3*c^3*Sin[a + b*x] - 6*b*c*d^2*Sin[a + b
*x] + 3*b^3*c^2*d*x*Sin[a + b*x] - 6*b*d^3*x*Sin[a + b*x] + 3*b^3*c*d^2*x^2
```

```
*Sin[a + b*x] + b^3*d^3*x^3*Sin[a + b*x])/b^4)
```

Maple [B] time = 0.418, size = 901, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 6*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))+6*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*
x+a)))*x-6*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x-6*I/b^3*c*d^2*a^2*arct
an(exp(I*(b*x+a)))-1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3
-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)
/b^4*exp(-I*(b*x+a))-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-3*I/b^2*c^2*d
*polylog(2,I*exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2-3
*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*
x+a)))+3*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))+1/2*I*(d^3*x^3*b^3+3*b^3*
c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6
*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-1/b*d^3*ln(1+I*exp(I*(b*
x+a)))*x^3+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+6/b^3*d^3*polylog(3,I*exp(I*(
b*x+a)))*x-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+1/b^4*a^3*d^3*ln(1-I*ex
p(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-I*exp(I*(b*x+a)))-1/b^4*a^3*d^3*ln(1+I
*exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,I*exp(I*(b*x+a)))-2*I/b*c^3*arctan(e
xp(I*(b*x+a)))+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/b^3*a^2*c*d^2*ln(1
-I*exp(I*(b*x+a)))+3/b*c*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*c*d^2*ln(1+I*ex
p(I*(b*x+a)))*x^2+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))+3/b*c^2*d*ln(1-I*e
xp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3/b*c^2*d*ln(1+I*exp(
I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a
```

Maxima [B] time = 2.24289, size = 1247, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a)) -
 3*a*c^2*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
```

```
/b + 3*a^2*c*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x
 + a))/b^2 - a^3*d^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin
(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x + I*a)) - 12*I*d^3*polylog
(4, -I*e^(I*b*x + I*a)) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3
)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*
arctan2(cos(b*x + a), sin(b*x + a) + 1) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c
*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*
d^3)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 6*(b^2*c^2*d - 2
*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*cos(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 -
6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*
a)) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*dilog(-I*e^(I*b*x + I*a)) + ((b*x +
 a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) +
1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin
(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, I*e^(I*b*x
 + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -I*e^(I*b*x + I*
a)) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x +
 a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(b*x + a)
)/b^3)/b
```

Fricas [C] time = 0.77632, size = 2591, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3
*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + (-3*I*b^2*d^3*x^2
- 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (
-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) -
sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b
^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x +
```

```
 a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*
x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*pol
ylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*
(b^3*c^2*d - 2*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \sec(bx+a)\sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^2, x)
```

3.217 $\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=186

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{PolyLog}\left(3,ie\right.}{b^3}$$

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (2*d*(c + d*x)*Cos[a + b*x
])/b^2 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*
d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (2*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (2*d^2*Sin
[a + b*x])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```

Rubi [A] time = 0.146066, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {4407, 3296, 2637, 4181, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{PolyLog}\left(3,ie\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```


```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (2*d*(c + d*x)*Cos[a + b*x
])/b^2 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*
d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (2*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (2*d^2*Sin
[a + b*x])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```


Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\,dx + \int (c+dx)^2\sec(a+bx)\,dx$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b} - \frac{(2d)\int (c+dx)\log\left(1-ie\right.}{b}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+}\right.}{b^2}$$

Mathematica [A] time = 0.883213, size = 315, normalized size = 1.69

$$-\frac{-2ibd(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)+2ibd(c+dx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)+2d^2\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)-2d^2\mathrm{PolyLog}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -(((2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b*c*d*Cos[a + b*x] + 2*b*d^2*x
*Cos[a + b*x] - 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1
- I*E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + b^2*d^2*x^2
*Log[1 + I*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a +
 b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 2*d^2*PolyLog
[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] + b^2*c^2*S
in[a + b*x] - 2*d^2*Sin[a + b*x] + 2*b^2*c*d*x*Sin[a + b*x] + b^2*d^2*x^2*S
in[a + b*x])/b^3)
```

Maple [B] time = 0.361, size = 512, normalized size = 2.8

$$\frac{2\,id^2\mathrm{polylog}\left(2,-ie^{i(bx+a)}\right)x}{b^2} - \frac{2\,id^2a^2\arctan\left(e^{i(bx+a)}\right)}{b^3} - 2\,\frac{cd\ln\left(1+ie^{i(bx+a)}\right)a}{b^2} - \frac{d^2\ln\left(1+ie^{i(bx+a)}\right)x^2}{b} + 2\,\frac{d^2\mathrm{polylog}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-2*I/b^3*d^2*a^2*arctan(exp(I*(b*
x+a)))-2/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^
2+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+
1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a))
)-1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3*e
xp(-I*(b*x+a))-2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x+2*I/b^2*c*d*polylo
g(2,-I*exp(I*(b*x+a)))+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))+2/b*c*d*ln(1-I*
exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a-2*d^2*polylog(3,-I*exp
(I*(b*x+a)))/b^3-2*I/b*c^2*arctan(exp(I*(b*x+a)))-2/b*c*d*ln(1+I*exp(I*(b*x
+a)))*x-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))+1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x
+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*(b*x+a))
```

Maxima [B] time = 1.98873, size = 689, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a)) -
 2*a*c*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))/b
 + a^2*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
/b^2 + (4*d^2*polylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*b*x
+ I*a)) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*arcta
n2(cos(b*x + a), sin(b*x + a) + 1) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +
4*I*a*d^2)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 4*(b*c*d +
 (b*x + a)*d^2 - a*d^2)*cos(b*x + a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*
I*a*d^2)*dilog(I*e^(I*b*x + I*a)) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*
d^2)*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*sin(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) -
2*d^2)*sin(b*x + a))/b^2)/b
```

Fricas [C] time = 0.676838, size = 1666, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 4*(b*d^2*x + b*c*
d)*cos(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x
 + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) -
(2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (2*I*b*d^
2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c
*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d
^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x +
a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2
)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2 - 2*d^2)*sin(b*x + a))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \sec(bx+a)\sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^2, x)
```

3.218 $\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] ((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (d*Cos[a + b*x])/b^2 + (I*d*
PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - ((c + d*x)*Sin[a + b*x])/b
```

Rubi [A] time = 0.0675388, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4407, 3296, 2638, 4181, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (d*Cos[a + b*x])/b^2 + (I*d*
PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - ((c + d*x)*Sin[a + b*x])/b
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)\cos(a+bx)\,dx + \int (c+dx)\sec(a+bx)\,dx\\ &= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b} + \cdots\\ &= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b} + \frac{(id)\,\mathrm{Subst}(\cdots}{\cdots}\\ &= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} + \frac{id\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{id\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}\cdots\end{aligned}$$

Mathematica [B] time = 0.423804, size = 213, normalized size = 2.07

$$\frac{d\left(i\left(\mathrm{PolyLog}\left(2,-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\mathrm{PolyLog}\left(2,e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)+\left(-a-bx+\frac{\pi}{2}\right)\left(\log\left(1-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\log\left(1+e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\cdots\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (c*ArcTanh[Sin[a + b*x]])/b + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a + Pi/
2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-a +
Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2, E^(I
*(-a + Pi/2 - b*x))])))/b^2 - (d*Cos[b*x]*(Cos[a] + b*x*Sin[a]))/b^2 - (d*(
b*x*Cos[a] - Sin[a])*Sin[b*x])/b^2 - (c*Sin[a + b*x])/b
```

Maple [B] time = 0.227, size = 221, normalized size = 2.2

$$\frac{\frac{i}{2}\left(dxb+bc+id\right)\mathrm{e}^{i(bx+a)}}{b^2}-\frac{\frac{i}{2}\left(dxb+bc-id\right)\mathrm{e}^{-i(bx+a)}}{b^2}-\frac{2\,ic\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)x}{b}-\frac{d\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d*x*b+b*c+I*d)/b^2*exp(I*(b*x+a))-1/2*I*(d*x*b+b*c-I*d)/b^2*exp(-I*(
b*x+a))-2*I/b*c*arctan(exp(I*(b*x+a)))-1/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/b^2
*d*ln(1+I*exp(I*(b*x+a)))*a+1/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/b^2*d*ln(1-I*e
xp(I*(b*x+a)))*a+I/b^2*d*dilog(1+I*exp(I*(b*x+a)))-I/b^2*d*dilog(1-I*exp(I*
(b*x+a)))+2*I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.594791, size = 905, normalized size = 8.79

$$-\frac{2\,d\cos\left(bx+a\right)+i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)-i\,d\mathrm{Li}_2\left(-i\,\cos\left(bx+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dil
og(I*cos(b*x + a) - sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a
)) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x +
a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) +
I) - (b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*l
og(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
- (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d*x + b*c)*sin(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \sin^2 (a + bx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)**2*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec (bx + a) \sin (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^2, x)
```

3.219 $\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=68

$$\text{Unintegrable}\left(\frac{\sec(a+bx)}{c+dx},x\right)-\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Unintegrable[Sec[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.110089, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][Sec[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)}{c+dx}\,dx+\int \frac{\sec(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)+\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int \frac{\sec(a+bx)}{c+dx}\,dx\\ &= -\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\sec(a+bx)}{c+dx}\,dx\end{aligned}$$

Mathematica [A] time = 6.07811, size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Maple [A] time = 0.34, size = 0, normalized size = 0.

$$\int \frac{\sec (bx + a) (\sin (bx + a))^2}{dx + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos (bx + a)^2 - 1\right) \sec (bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c), x)
```

3.220 $\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=86

$$\text{Unintegrable}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\cos(a+bx)}{d(c+dx)}$$

```
[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Unintegrable[
Sec[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.138904, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][Se
c[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d}+\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \frac{\sec(a+bx)}{(c+dx)^2}
\end{aligned}$$

Mathematica [A] time = 7.23992, size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.607, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)(\sin(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos(bx+a)^2-1\right)\sec(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c)^2, x)
```

3.221 $\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=151

$$\text{Unintegrable}\left(\tan(a+bx)(c+dx)^m,x\right)+\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-3}e^{-}}{}$$

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Unintegrable[(c + d*x)^m*Tan[a + b*x], x]
```

Rubi [A] time = 0.170835, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Defer[Int][(c + d*x)^m*Tan[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)^m\cos(a+bx)\sin(a+bx)\,dx+\int (c+dx)^m\tan(a+bx)\,dx\\ &= -\int \frac{1}{2}(c+dx)^m\sin(2a+2bx)\,dx+\int (c+dx)^m\tan(a+bx)\,dx\\ &= -\left(\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)\,dx\right)+\int (c+dx)^m\tan(a+bx)\,dx\\ &= -\left(\frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx\right)+\frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx+\int (c+dx)^m\\ &= \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(}{}\end{aligned}$$

Mathematica [A] time = 7.77205, size = 0, normalized size = 0.

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x], x]
```

Maple [A] time = 0.274, size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)(\sin(bx+a))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos(bx+a)^2-1\right)(dx+c)^m\sec(bx+a)\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)\sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

3.222 $\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx) \, dx$

Optimal. Leaf size=251

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\,\mathrm{s}}{4b}$$

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) + ((I/4)*(c + d*x)^4)/d - ((c + d*x)
^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E
^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))
])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (3*d^3*
Cos[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a +
b*x])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin
[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.297539, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4407, 4404, 3311, 32, 2635, 8, 3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\,\mathrm{s}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```


```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) + ((I/4)*(c + d*x)^4)/d - ((c + d*x)
^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E
^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))
])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (3*d^3*
Cos[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a +
b*x])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin
[a + b*x]^2)/(2*b)
```


Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)^3\tan(a+bx)\,dx$$

$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}\,dx + \frac{(3d)\int (c+$$

$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx}{4b^2}$$

$$= \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}_2\left(-e^{2i(}\right.}{2b^2}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2}{2}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2}{2}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2}{2}$$

Mathematica [B] time = 7.14703, size = 1720, normalized size = 6.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((-I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)
*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
 (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^
(I*a)) - (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))
*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*Pol
yLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))
] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] - (c^3*Se
c[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[
a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[
a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x -
ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]
] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)
*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^
2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)*Sin
[2*a + 2*b*x])/b^4)*(8*b^3*c^3*Cos[a] - (12*I)*b^2*c^2*d*Cos[a] - 12*b*c*d^
2*Cos[a] + (6*I)*d^3*Cos[a] + 24*b^3*c^2*d*x*Cos[a] - (24*I)*b^2*c*d^2*x*Co
s[a] - 12*b*d^3*x*Cos[a] + 24*b^3*c*d^2*x^2*Cos[a] - (12*I)*b^2*d^3*x^2*Cos
```

```
[a] + 8*b^3*d^3*x^3*Cos[a] + (32*I)*b^4*c^3*x*Cos[a + 2*b*x] + (48*I)*b^4*c
^2*d*x^2*Cos[a + 2*b*x] + (32*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] + (8*I)*b^4*d
^3*x^4*Cos[a + 2*b*x] - (32*I)*b^4*c^3*x*Cos[3*a + 2*b*x] - (48*I)*b^4*c^2*
d*x^2*Cos[3*a + 2*b*x] - (32*I)*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] - (8*I)*b^4*
d^3*x^4*Cos[3*a + 2*b*x] + 4*b^3*c^3*Cos[3*a + 4*b*x] + (6*I)*b^2*c^2*d*Cos
[3*a + 4*b*x] - 6*b*c*d^2*Cos[3*a + 4*b*x] - (3*I)*d^3*Cos[3*a + 4*b*x] + 1
2*b^3*c^2*d*x*Cos[3*a + 4*b*x] + (12*I)*b^2*c*d^2*x*Cos[3*a + 4*b*x] - 6*b*
d^3*x*Cos[3*a + 4*b*x] + 12*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] + (6*I)*b^2*d^3*
x^2*Cos[3*a + 4*b*x] + 4*b^3*d^3*x^3*Cos[3*a + 4*b*x] + 4*b^3*c^3*Cos[5*a +
 4*b*x] + (6*I)*b^2*c^2*d*Cos[5*a + 4*b*x] - 6*b*c*d^2*Cos[5*a + 4*b*x] - (
3*I)*d^3*Cos[5*a + 4*b*x] + 12*b^3*c^2*d*x*Cos[5*a + 4*b*x] + (12*I)*b^2*c*
d^2*x*Cos[5*a + 4*b*x] - 6*b*d^3*x*Cos[5*a + 4*b*x] + 12*b^3*c*d^2*x^2*Cos[
5*a + 4*b*x] + (6*I)*b^2*d^3*x^2*Cos[5*a + 4*b*x] + 4*b^3*d^3*x^3*Cos[5*a +
 4*b*x] - 32*b^4*c^3*x*Sin[a + 2*b*x] - 48*b^4*c^2*d*x^2*Sin[a + 2*b*x] - 3
2*b^4*c*d^2*x^3*Sin[a + 2*b*x] - 8*b^4*d^3*x^4*Sin[a + 2*b*x] + 32*b^4*c^3*
x*Sin[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] + 32*b^4*c*d^2*x^3*S
in[3*a + 2*b*x] + 8*b^4*d^3*x^4*Sin[3*a + 2*b*x] + (4*I)*b^3*c^3*Sin[3*a +
4*b*x] - 6*b^2*c^2*d*Sin[3*a + 4*b*x] - (6*I)*b*c*d^2*Sin[3*a + 4*b*x] + 3*
d^3*Sin[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*Sin[3*a + 4*b*x] - 12*b^2*c*d^2*x
*Sin[3*a + 4*b*x] - (6*I)*b*d^3*x*Sin[3*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2*S
in[3*a + 4*b*x] - 6*b^2*d^3*x^2*Sin[3*a + 4*b*x] + (4*I)*b^3*d^3*x^3*Sin[3*
a + 4*b*x] + (4*I)*b^3*c^3*Sin[5*a + 4*b*x] - 6*b^2*c^2*d*Sin[5*a + 4*b*x]
- (6*I)*b*c*d^2*Sin[5*a + 4*b*x] + 3*d^3*Sin[5*a + 4*b*x] + (12*I)*b^3*c^2*
d*x*Sin[5*a + 4*b*x] - 12*b^2*c*d^2*x*Sin[5*a + 4*b*x] - (6*I)*b*d^3*x*Sin[
5*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] - 6*b^2*d^3*x^2*Sin[5*
a + 4*b*x] + (4*I)*b^3*d^3*x^3*Sin[5*a + 4*b*x])
```

Maple [B] time = 0.285, size = 641, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] -1/b*c^3*ln(exp(2*I*(b*x+a))+1)-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-
3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+3/2*I/b^4*d^3*a^4+1/4*I*d^3*x^4+
I*c*d^2*x^3-I*c^3*x+3/2*I*c^2*d*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x
+a)))-3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x
^2-1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x
+a)))*x^2+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)
))-4*I/b^3*a^3*c*d^2+2*I/b^3*d^3*a^3*x+3*I/b^2*a^2*c^2*d-3/4*I*d^3*polylog(
```

```
4,-exp(2*I*(b*x+a)))/b^4+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-2/b^4
*d^3*a^3*ln(exp(I*(b*x+a)))+6*I/b*a*c^2*d*x-6*I/b^2*a^2*c*d^2*x+1/32*(4*d^3
*x^3*b^3+6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6
*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*I*(b*x+a))+2/
b*c^3*ln(exp(I*(b*x+a)))+1/32*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*x
^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*d^
3-6*c*d^2*b)/b^4*exp(-2*I*(b*x+a))
```

Maxima [B] time = 2.09739, size = 925, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^3 - 72*(sin(b*x + a)
^2 + log(sin(b*x + a)^2 - 1))*a*c^2*d/b + 72*(sin(b*x + a)^2 + log(sin(b*x
+ a)^2 - 1))*a^2*c*d^2/b^2 - 24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*
a^3*d^3/b^3 + (-12*I*(b*x + a)^4*d^3 + (-48*I*b*c*d^2 + 48*I*a*d^3)*(b*x +
a)^3 + 48*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) + (-72*I*b^2*c^2*d + 144*I
*a*b*c*d^2 - 72*I*a^2*d^3)*(b*x + a)^2 + (64*I*(b*x + a)^3*d^3 + (144*I*b*c
*d^2 - 144*I*a*d^3)*(b*x + a)^2 + (144*I*b^2*c^2*d - 288*I*a*b*c*d^2 + 144*
I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(
2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a
) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 96*I*(b*x + a)^2*d^3 - 72*I*a^2*d^
3 + (-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) +
 8*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1) + 24*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*pol
ylog(3, -e^(2*I*b*x + 2*I*a)) + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^
2*d^3 + (2*a^2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))/
b^3)/b
```

Fricas [C] time = 0.843416, size = 2753, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, I*cos(b*x + a)
+ sin(b*x + a)) + 24*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 24*I
*d^3*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^3*polylog(4, -I*co
s(b*x + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*
c^2*d - b*d^3)*x - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*
dilog(I*cos(b*x + a) + sin(b*x + a)) - (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x
 + 12*I*b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (12*I*b^2*d^3*x^2
 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))
 - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*dilog(-I*cos(b*x
 + a) - sin(b*x + a)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3
*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b
*x + a) - sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*
d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*
x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + 4*
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*s
in(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*lo
g(-cos(b*x + a) - I*sin(b*x + a) + I) + 24*(b*d^3*x + b*c*d^2)*polylog(3, I
*cos(b*x + a) + sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x
 + a) - sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*
x + a)))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \sec(bx+a)\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^3, x)
```

3.223 $\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx)\,dx$

Optimal. Leaf size=184

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{d^2\sin^2(a+bx)}{4b^3}-\frac{(c$$

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) + ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[
1 + E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x)
)])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)
^2*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.228075, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4407, 4404, 3310, 3719, 2190, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{d^2\sin^2(a+bx)}{4b^3}-\frac{(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) + ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[
1 + E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x)
)])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)
^2*Sin[a + b*x]^2)/(2*b)
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
```

```
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)^2\tan(a+bx)\,dx$$
$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx + \frac{d\int (c+dx)\,\mathrm{s}}{}$$
$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} +$$
$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+}\right.}{b^2}$$
$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+}\right.}{b^2}$$
$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+}\right.}{b^2}$$

Mathematica [B] time = 6.4593, size = 518, normalized size = 2.82

$$-\frac{cd\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\mathrm{PolyLog}\left(2,e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((-I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
(3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(
I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Si
n[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a
]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)]
- 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*L
og[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[
2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqr
t[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a] - d^2*
Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*Sin[2*a]
```

```
 - 2*b*d^2*x*Sin[2*a]))/(8*b^3) - ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Cos[2*a] +
 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d^2*x^2*S
in[2*a])*Sin[2*b*x])/(8*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

Maple [B] time = 0.353, size = 379, normalized size = 2.1

$$\frac{2\,ia^2cd}{b^2}+icdx^2-\frac{\frac{4\,i}{3}a^3d^2}{b^3}+\frac{\left(2\,d^2x^2b^2+2\,ibd^2x+4\,b^2cdx+2\,ibcd+2\,b^2c^2-d^2\right)\mathrm{e}^{2\,i(bx+a)}}{16\,b^3}+\frac{\left(2\,d^2x^2b^2-2\,ibd^2x+4\,b^2c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2*I/b^2*a^2*c*d+I*c*d*x^2-4/3*I/b^3*a^3*d^2+1/16*(2*d^2*x^2*b^2+2*I*b*d^2*x
+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*I*(b*x+a))+1/16*(2*d^2*x^2*
b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(-2*I*(b*x+a))-
1/b*c^2*ln(exp(2*I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a)))+2/b^3*d^2*a^2*ln(
exp(I*(b*x+a)))+4*I/b*a*c*d*x+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-I*c^2*
x+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x
^2-2*I/b^2*a^2*d^2*x-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c*d*a*l
n(exp(I*(b*x+a)))+1/3*I*d^2*x^3-2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x
```

Maxima [B] time = 1.83483, size = 512, normalized size = 2.78

$$-\frac{12\left(\sin\left(bx+a\right)^2+\log\left(\sin\left(bx+a\right)^2-1\right)\right)c^2-\frac{24\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)acd}{b}+\frac{12\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)a^2d^2}{b^2}+\frac{-8i}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^2 - 24*(sin(b*x + a)
^2 + log(sin(b*x + a)^2 - 1))*a*c*d/b + 12*(sin(b*x + a)^2 + log(sin(b*x +
a)^2 - 1))*a^2*d^2/b^2 + (-8*I*(b*x + a)^3*d^2 + (-24*I*b*c*d + 24*I*a*d^2)
*(b*x + a)^2 + 12*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) + (24*I*(b*x + a)^2*
d^2 + (48*I*b*c*d - 48*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*
b*x + 2*a) + 1) - 3*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)
*cos(2*b*x + 2*a) + (-24*I*b*c*d - 24*I*(b*x + a)*d^2 + 24*I*a*d^2)*dilog(-
e^(2*I*b*x + 2*I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*l
```

```
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(b
*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

Fricas [C] time = 0.706088, size = 1746, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 + 4*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 4
*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*
x + a) + sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) +
 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (-4*I*b*d^2*x - 4*I*b*c*d)
*dilog(I*cos(b*x + a) + sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(I*c
os(b*x + a) - sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(-I*cos(b*x +
a) + sin(b*x + a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*dilog(-I*cos(b*x + a) - sin
(b*x + a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x
 + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x
 + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a
^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d
*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x
 + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*
x + a) + I))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \sec (bx+a) \sin (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^3, x)
```

3.224 $\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=115

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}+\frac{i(c+d$$

```
[Out] (d*x)/(4*b) + ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Si
n[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.127852, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4407, 4404, 2635, 8, 3719, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}+\frac{i(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```


```
[Out] (d*x)/(4*b) + ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Si
n[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```


Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)\tan(a+bx)\,dx$$
$$= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx + \frac{d\int \sin^2(a+bx)}{2b}$$
$$= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{(c+dx)\mathrm{si}}{2}$$
$$= \frac{dx}{4b} + \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{(c+}{}$$
$$= \frac{dx}{4b} + \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{d\cos(a+bx}{4}$$

Mathematica [A] time = 0.303667, size = 134, normalized size = 1.17

$$\frac{d\left(\frac{1}{2}i\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)+\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)\right)}{b^2} - \frac{d\sin(2(a+bx))}{8b^2} + \frac{ad\log(\cos(a+bx))}{b^2} - \frac{c\left(\mathrm{lo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (a*d*Log[Cos[a + b*x]])/b^2 - (c*(-Cos[a + b
*x]^2/2 + Log[Cos[a + b*x]]))/b + (d*((I/2)*(a + b*x)^2 - (a + b*x)*Log[1 +
 E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2 - (d*S
in[2*(a + b*x)])/(8*b^2)
```

Maple [A] time = 0.355, size = 179, normalized size = 1.6

$$\frac{i}{2}dx^2 - icx + \frac{(2\,dxb+id+2\,bc)\,e^{2\,i(bx+a)}}{16\,b^2} + \frac{(2\,dxb-id+2\,bc)\,e^{-2\,i(bx+a)}}{16\,b^2} + 2\,\frac{c\ln\left(e^{i(bx+a)}\right)}{b} - \frac{c\ln\left(e^{2\,i(bx+a)}+1\right)}{b} + \frac{2\,idax}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/2*I*d*x^2-I*c*x+1/16*(2*d*x*b+I*d+2*b*c)/b^2*exp(2*I*(b*x+a))+1/16*(2*d*x
*b-I*d+2*b*c)/b^2*exp(-2*I*(b*x+a))+2/b*c*ln(exp(I*(b*x+a)))-1/b*c*ln(exp(2
*I*(b*x+a))+1)+2*I/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I
```

```
*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [A] time = 1.77019, size = 196, normalized size = 1.7

$$-\frac{-4i\,b^2dx^2-8i\,b^2cx+(8i\,bdx+8i\,bc)\arctan\left(\sin\left(2\,bx+2\,a\right),\cos\left(2\,bx+2\,a\right)+1\right)-2\,(bdx+bc)\cos\left(2\,bx+2\,a\right)-4i\,\ldots}{8\,\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x + (8*I*b*d*x + 8*I*b*c)*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 4*I*d*
dilog(-e^(2*I*b*x + 2*I*a)) + 4*(b*d*x + b*c)*log(cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + d*sin(2*b*x + 2*a))/b^2
```

Fricas [B] time = 0.61366, size = 965, normalized size = 8.39

$$-\frac{bdx-2\,(bdx+bc)\cos\left(bx+a\right)^2+d\cos\left(bx+a\right)\sin\left(bx+a\right)+2i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)-2i\,d\mathrm{Li}_2\left(i\,\cos\left(\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
+ 2*I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) - 2*I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*I*d*dilog(
-I*cos(b*x + a) - sin(b*x + a)) + 2*(b*c - a*d)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + 2*(b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d
*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(I*co
s(b*x + a) - sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) + sin(
b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + 2
*(b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b*c - a*d)*log(-c
os(b*x + a) - I*sin(b*x + a) + I))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec(bx + a) \sin(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^3, x)
```

3.225 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=81

$$\text{Unintegrable}\left(\frac{\tan(a+bx)}{c+dx},x\right)-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Unintegrable[Tan[a + b*x]
/(c + d*x), x]
```

Rubi [A] time = 0.151151, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Tan[a + b*x]/(
c + d*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx + \int \frac{\tan(a+bx)}{c+dx}\,dx \\
&= -\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx + \int \frac{\tan(a+bx)}{c+dx}\,dx \\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\right) + \int \frac{\tan(a+bx)}{c+dx}\,dx \\
&= -\left(\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\right) - \frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx + \\
&= -\frac{\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} - \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d} + \int \frac{\tan(a+bx)}{c+dx}\,dx
\end{aligned}$$

Mathematica [A] time = 0.779359, size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Maple [A] time = 0.488, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)(\sin(bx+a))^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\sec\left(bx+a\right)\sin\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^{3}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c), x)
```

3.226 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=101

$$\text{Unintegrable}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a}{2d(c}$$

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Unintegrable[Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.176326, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = -\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx + \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$
$$= -\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx + \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$
$$= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\right) + \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d} + \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}}{d}$$
$$= -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} + \frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2} + $$

Mathematica [A] time = 2.50299, size = 0, normalized size = 0.

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.812, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)(\sin(bx+a))^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\sec\left(bx+a\right)\sin\left(bx+a\right)}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x
 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^{3}}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c)^2, x)
```

3.227 $\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\csc(a+bx)\sec(a+bx)(c+dx)^m, x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

Rubi [A] time = 0.183179, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]

[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Mathematica [A] time = 5.94163, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

Maple [A] time = 0.141, size = 0, normalized size = 0.

$$\int (dx+c)^m \csc (bx+a) \sec (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc (bx+a) \sec (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \csc (bx+a) \sec (bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

3.228 $\int (c + dx)^4 \csc(a + bx) \sec(a + bx)\, dx$

Optimal. Leaf size=247

$$-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c}{}$$

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^((2*I)*(a + b*x))])/b + ((2*I)*d*(c + d*x)^3*Poly
Log[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I
)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b
^3 + (3*d^2*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(
c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/
(2*b^5) - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rubi [A] time = 0.229067, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4419, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```



```
[Out] (-2*(c + d*x)^4*ArcTanh[E^((2*I)*(a + b*x))])/b + ((2*I)*d*(c + d*x)^3*Poly
Log[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I
)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b
^3 + (3*d^2*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(
c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/
(2*b^5) - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```


Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4\csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^4\csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(4d)\int(c+dx)^3\log\left(1-e^{i(2a+2bx)}\right)dx}{b} + \frac{(4d)\int(}{}$$
$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)^3\text{Li}_2\left(\right.}{b^2}$$
$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)^3\text{Li}_2\left(\right.}{b^2}$$
$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)^3\text{Li}_2\left(\right.}{b^2}$$
$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)^3\text{Li}_2\left(\right.}{b^2}$$
$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)^3\text{Li}_2\left(\right.}{b^2}$$

Mathematica [B] time = 1.37853, size = 578, normalized size = 2.34

$$\frac{-6b^2c^2d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)+6b^2c^2d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)-12b^2cd^3x\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)+12b^2cd^3x\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-4*b^4*c^4*ArcTanh[E^((2*I)*(a + b*x))] + 8*b^4*c^3*d*x*Log[1 - E^((2*I)*(
a + b*x))] + 12*b^4*c^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 8*b^4*c*d^3*
x^3*Log[1 - E^((2*I)*(a + b*x))] + 2*b^4*d^4*x^4*Log[1 - E^((2*I)*(a + b*x)
)] - 8*b^4*c^3*d*x*Log[1 + E^((2*I)*(a + b*x))] - 12*b^4*c^2*d^2*x^2*Log[1
+ E^((2*I)*(a + b*x))] - 8*b^4*c*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] - 2*b
^4*d^4*x^4*Log[1 + E^((2*I)*(a + b*x))] + (4*I)*b^3*d*(c + d*x)^3*PolyLog[2
, -E^((2*I)*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a +
b*x))] - 6*b^2*c^2*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - 12*b^2*c*d^3*x*Po
lyLog[3, -E^((2*I)*(a + b*x))] - 6*b^2*d^4*x^2*PolyLog[3, -E^((2*I)*(a + b*
x))] + 6*b^2*c^2*d^2*PolyLog[3, E^((2*I)*(a + b*x))] + 12*b^2*c*d^3*x*PolyL
og[3, E^((2*I)*(a + b*x))] + 6*b^2*d^4*x^2*PolyLog[3, E^((2*I)*(a + b*x))]
- (6*I)*b*c*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (6*I)*b*d^4*x*PolyLog[4,
 -E^((2*I)*(a + b*x))] + (6*I)*b*c*d^3*PolyLog[4, E^((2*I)*(a + b*x))] + (6
*I)*b*d^4*x*PolyLog[4, E^((2*I)*(a + b*x))] + 3*d^4*PolyLog[5, -E^((2*I)*(a
 + b*x))] - 3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Maple [B] time = 0.329, size = 1242, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] 3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1
/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4-3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a))
)-3/b^3*c^2*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(
b*x+a)))*x^2-4/b*c*d^3*ln(exp(2*I*(b*x+a))+1)*x^3+4/b*c*d^3*ln(1-exp(I*(b*x
+a)))*x^3-1/b*c^4*ln(exp(2*I*(b*x+a))+1)+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a
^3-6/b*c^2*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^
3+6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+6*I/b^2*c^2*d^2*polylog(2,
-exp(2*I*(b*x+a)))*x+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+12/b^3*c^2*d^
2*polylog(3,exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^
3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)
))*x-1/b*d^4*ln(exp(2*I*(b*x+a))+1)*x^4+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^
2+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+4
/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a+4/b*c^3*
d*ln(exp(I*(b*x+a))+1)*x+24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b*c^2*
d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x-4/b*c
^3*d*ln(exp(2*I*(b*x+a))+1)*x+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)+6/b^3*c^2*
d^2*a^2*ln(exp(I*(b*x+a))-1)-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-4/b^2*c^3
*d*a*ln(exp(I*(b*x+a))-1)-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-4*I/b^2*c
^3*d*polylog(2,-exp(I*(b*x+a)))+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*
I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a
)))*x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,
exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-24*d^4*polylog(5,
-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-12*I/b^2*c*d^3*po
lylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-1
2*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^4*ln(exp(I*(b*x+a))+1)+1
/b*c^4*ln(exp(I*(b*x+a))-1)-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+2*
I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*
x+a)))*x+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3
```

Maxima [B] time = 2.50605, size = 2402, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^4*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 12*a*c^3*d*(l
og(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 18*a^2*c^2*d^2*(log(sin(b
*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 12*a^3*c*d^3*(log(sin(b*x + a)^
2 - 1) - log(sin(b*x + a)^2))/b^3 + 3*a^4*d^4*(log(sin(b*x + a)^2 - 1) - lo
g(sin(b*x + a)^2))/b^4 - (18*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 144*d^4
*polylog(5, -e^(I*b*x + I*a)) - 144*d^4*polylog(5, e^(I*b*x + I*a)) - (12*I
*(b*x + a)^4*d^4 + (32*I*b*c*d^3 - 32*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*
d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a
*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) - (-6*I*(b*x + a)^4*d^4 + (-24*I*b*c*d^3 +
24*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^
4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a*b^2*c^2*d^2 - 72*I*a^2*b*c*d^3 +
 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*I*(b
*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2
 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^
2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a
), -cos(b*x + a) + 1) - (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 - 36*I*a^2*b*
c*d^3 - 24*I*(b*x + a)^3*d^4 + 12*I*a^3*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*
(b*x + a)^2 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4)*(b*x + a)
)*dilog(-e^(2*I*b*x + 2*I*a)) - (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I
*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I*a
*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(b*
x + a))*dilog(-e^(I*b*x + I*a)) - (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72
*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I
*a*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(
b*x + a))*dilog(e^(I*b*x + I*a)) - 2*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^
4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b
^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^
4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) +
3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*
c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x
 + a) + 1) - (24*I*b*c*d^3 + 36*I*(b*x + a)*d^4 - 24*I*a*d^4)*polylog(4, -e
^(2*I*b*x + 2*I*a)) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*
polylog(4, -e^(I*b*x + I*a)) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*
I*a*d^4)*polylog(4, e^(I*b*x + I*a)) - 6*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(
b*x + a)^2*d^4 + 3*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(
```

```
2*I*b*x + 2*I*a)) + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d
^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 72*(b^2*
c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*
x + a))*polylog(3, e^(I*b*x + I*a)))/b^4)/b
```

Fricas [C] time = 1.03994, size = 6280, normalized size = 25.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x
 + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*polylog(
5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a) - si
n(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*po
lylog(5, -cos(b*x + a) - I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d
^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(cos(b*x + a) + I*sin(b*x
 + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b
^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b
^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) + s
in(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x +
 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 1
2*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-I*cos(b*x +
a) + sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*
d^2*x - 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (4*I*b^3*d^4
*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-cos(
b*x + a) + I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*
b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4
*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*l
og(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4
*c^4)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x +
a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*
x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d
^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3
- a^4*d^4)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*
```

```
d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2
*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (
b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3
*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) -
 sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*
c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2
*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
+ 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4
*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x
 + a) + I*sin(b*x + a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^
2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 -
 a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(
b*x + a) + I) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*s
in(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*s
in(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) + si
n(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) - si
n(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + s
in(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - s
in(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*
sin(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*
sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, c
os(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d
^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d
^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a
)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x +
 a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylo
g(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^
2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2
*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \csc(bx+a)\sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a), x)
```

3.229 $\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=197

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c+\cdots}{}$$

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*
PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2,
 E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x
))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) -
(((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*d^3*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4
```

Rubi [A] time = 0.166281, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4419, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```


```
[Out] (-2*(c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*
PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2,
 E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x
))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) -
(((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*d^3*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4
```


Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^3 \csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2 \log\left(1-e^{i(2a+2bx)}\right)dx}{b} + \frac{(3d)\int(}{}$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2 \text{Li}_2\left(\right.}{2b^2}$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2 \text{Li}_2\left(\right.}{2b^2}$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2 \text{Li}_2\left(\right.}{2b^2}$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2 \text{Li}_2\left(\right.}{2b^2}$$

Mathematica [A] time = 1.0162, size = 350, normalized size = 1.78

$$\frac{6ib^2d(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right) - 6ib^2d(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right) - 6bd^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right) + 6bc}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*I)*
(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3
*Log[1 - E^((2*I)*(a + b*x))] - 12*b^3*c^2*d*x*Log[1 + E^((2*I)*(a + b*x))]
 - 12*b^3*c*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - 4*b^3*d^3*x^3*Log[1 + E^
((2*I)*(a + b*x))] + (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x)
)] - (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))] - 6*b*d^2*(c +
 d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^((2*I)*(a +
 b*x))] + 6*b*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] - (3*I)*d^3*PolyLog[4,
-E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/(4*b^4)
```

Maple [B] time = 0.313, size = 816, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] 3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3*I/b
^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))
*x^2-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I
*(b*x+a)))-1/b*c^3*ln(exp(2*I*(b*x+a))+1)-3/2/b^3*c*d^2*polylog(3,-exp(2*I*
(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(
I*(b*x+a)))/b^4+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-3/b*c^2*d*ln(e
xp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-1/b*d^3*ln(exp(2*
I*(b*x+a))+1)*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+1/b*d^3*ln
(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*
(b*x+a))+1)*x^3+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*c^2*d*a*ln(exp(I
*(b*x+a))-1)-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+3/b*c^2*d*ln(exp(I*(b*x+a
))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a
-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3*I/b^2*c*d^2*polylog(2,-exp(2*
I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-ex
p(I*(b*x+a)))*x+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3
,-exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6*I/b^4*d^3*polylog(4,
-exp(I*(b*x+a)))-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*po
lylog(2,-exp(I*(b*x+a)))*x+1/b*c^3*ln(exp(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b
*x+a))+1)
```

<u>Maxima</u> [B] time = 2.20054, size = 1435, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^3*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 9*a*c^2*d*(lo
g(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 9*a^2*c*d^2*(log(sin(b*x +
 a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 3*a^3*d^3*(log(sin(b*x + a)^2 - 1)
- log(sin(b*x + a)^2))/b^3 + (6*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 36
*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I*a))
+ (8*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^
2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a
), cos(2*b*x + 2*a) + 1) + (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I*a*
d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3)*(b*x +
 a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*I*(b*x + a)^3*d^3 + (18*I
*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*
I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-9*I*b^2*
```

```
c^2*d + 18*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 9*I*a^2*d^3 + (-18*I*b*c*d^
2 + 18*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + (18*I*b^2*c^2*d -
36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I
*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (18*I*b^2*c^2*d - 36*I*a*b*c*d
^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x
 + a))*dilog(e^(I*b*x + I*a)) + (4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 3*((b*x + a)^3*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(
(b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x +
 a) + 1) + 3*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b*x
 + 2*I*a)) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*
a)) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3
)/b
```

Fricas [C] time = 0.864937, size = 4410, normalized size = 22.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b
*x + a)) - 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*poly
log(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x + a)
 - sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + 6*I
*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b
^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2
*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x
+ a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*
x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d
)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x
 + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
 (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilo
g(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*
```

```
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x +
a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a)
+ 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d
^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) +
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x +
 a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x
 + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*
x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a
) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin
(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)
) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*
d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x +
b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)
*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog
(3, -cos(b*x + a) - I*sin(b*x + a)))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \csc (bx + a) \sec (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a), x)
```

3.230 $\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=127

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2,
 -E^((2*I)*(a + b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E
^((2*I)*(a + b*x))])/(2*b^3)
```

Rubi [A] time = 0.116645, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4419, 4183, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```


```
[Out] (-2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2,
 -E^((2*I)*(a + b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E
^((2*I)*(a + b*x))])/(2*b^3)
```


Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^2\csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(2d)\int(c+dx)\log\left(1-e^{i(2a+2bx)}\right)dx}{b} + \frac{(2d)\int(c$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a}\right.}{b^2}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a}\right.}{b^2}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a}\right.}{b^2}$$

Mathematica [A] time = 0.628039, size = 213, normalized size = 1.68

$$\frac{2ibd(c+dx)\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right) - 2ibd(c+dx)\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right) - d^2\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right) + d^2\mathrm{PolyLog}\left(3,e^{2i(}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 - E^((2*I)*(a
+ b*x))] + 2*b^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 +
 E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] + (2*I)*
b*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLo
g[2, E^((2*I)*(a + b*x))] - d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d^2*Poly
Log[3, E^((2*I)*(a + b*x))])/(2*b^3)
```

Maple [B] time = 0.297, size = 469, normalized size = 3.7

$$-\frac{d^2\mathrm{polylog}\left(3,-\mathrm{e}^{2\,i(bx+a)}\right)}{2\,b^3}-\frac{d^2\ln\left(\mathrm{e}^{2\,i(bx+a)}+1\right)x^2}{b}-\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^2}{b^3}+\frac{d^2\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x^2}{b}+\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] -1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^
2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d
^2*ln(1-exp(I*(b*x+a)))*x^2-1/b*c^2*ln(exp(2*I*(b*x+a))+1)-2*I/b^2*c*d*poly
log(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+I/b^2*d^2*poly
log(2,-exp(2*I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+I/b^2*c
*d*polylog(2,-exp(2*I*(b*x+a)))+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+1/b*c^2*ln(e
xp(I*(b*x+a))+1)+1/b*c^2*ln(exp(I*(b*x+a))-1)+2*d^2*polylog(3,-exp(I*(b*x+a
)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2*I/b^2*d^2*polylog(2,exp(I*(b*
x+a)))*x-2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/
b^2*c*d*ln(1-exp(I*(b*x+a)))*a+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-2/b^2*c*d
*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 1.97525, size = 797, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 2*a*c*d*(log(si
n(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + a^2*d^2*(log(sin(b*x + a)^2 -
```

```
1) - log(sin(b*x + a)^2))/b^2 + (d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 4*d
^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*a)) + (2*I*
(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*
a), cos(2*b*x + 2*a) + 1) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2
)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^2*d^2
 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) +
 1) + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*dilog(-e^(2*I*b*x + 2*I*
a)) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*dilog(-e^(I*b*x + I*a)) +
 (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2
)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((
b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

Fricas [C] time = 0.737504, size = 2850, normalized size = 22.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a
)) - 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*polylog(3, -I*
cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x +
 a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3,
-cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(cos(b*x
+ a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(cos(b*x + a) - I*s
in(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x +
a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (2*I
*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x
 - 2*I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*
c*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dil
og(-cos(b*x + a) - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b
*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b
^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b
```

```
^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b
*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos
(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*co
s(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*
x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
 + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \csc(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc(bx+a) \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a), x)
```

3.231 $\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=71

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

```
[Out] (-2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b + ((I/2)*d*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rubi [A] time = 0.0555391, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4419, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b + ((I/2)*d*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)\csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-e^{i(2a+2bx)}\right)dx}{b} + \frac{d\int \log\left(1+e^{i(2a+2b}\right.}{b}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{(id)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{i(2a+2bx)}\right)}{2b^2} - \frac{(id)\,\text{Su}}{}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id\,\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\,\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}$$

Mathematica [A] time = 0.11855, size = 141, normalized size = 1.99

$$\frac{d\left(i\left(\text{PolyLog}\left(2,-e^{i(2a+2bx)}\right)-\text{PolyLog}\left(2,e^{i(2a+2bx)}\right)\right)+(2a+2bx)\left(\log\left(1-e^{i(2a+2bx)}\right)-\log\left(1+e^{i(2a+2bx)}\right)\right)-2a\log\left(\right.\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] -((c*Log[Cos[a + b*x]])/b) + (c*Log[Sin[a + b*x]])/b + (d*((2*a + 2*b*x)*(L
og[1 - E^(I*(2*a + 2*b*x))] - Log[1 + E^(I*(2*a + 2*b*x))]) - 2*a*Log[Tan[(
2*a + 2*b*x)/2]] + I*(PolyLog[2, -E^(I*(2*a + 2*b*x))] - PolyLog[2, E^(I*(2
*a + 2*b*x))])))/(2*b^2)
```

Maple [B] time = 0.234, size = 208, normalized size = 2.9

$$-\frac{c\ln\left(e^{2i(bx+a)}+1\right)}{b} + \frac{c\ln\left(e^{i(bx+a)}-1\right)}{b} + \frac{c\ln\left(e^{i(bx+a)}+1\right)}{b} + \frac{d\ln\left(1-e^{i(bx+a)}\right)x}{b} + \frac{d\ln\left(1-e^{i(bx+a)}\right)a}{b^2} - \frac{id\,\text{polylog}\left(2\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] -1/b*c*ln(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*
x+a))+1)+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*po
lylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(
2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*
(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 1.94968, size = 360, normalized size = 5.07

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)+\left(2i\,bdx+2i\,bc\right)\arctan\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) + (-2*I*b*d*x - 2*I*b*c)*arctan2(sin(b*x + a),
 cos(b*x + a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + 2*I*d*dilog(-e^(I*b*
x + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b*d*x + b*c)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

Fricas [B] time = 0.630837, size = 1544, normalized size = 21.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos
(b*x + a) - sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d
*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*sin(b*
x + a)) - I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*log(cos
```

```
(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a
*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*log(I*cos(b*x +
a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) - sin(b*x + a) +
1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*
log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a)
 + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*si
n(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) -
 (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x + a*d)*log(-c
os(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(b
*x + a) + I))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\csc(bx+a)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a), x)
```

3.232 $\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=21

$$2\text{Unintegrable}\left(\frac{\csc(2a+2bx)}{c+dx},x\right)$$

```
[Out] 2*Unintegrable[Csc[2*a + 2*b*x]/(c + d*x), x]
```

Rubi [A] time = 0.04482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = 2\int \frac{\csc(2a+2bx)}{c+dx}\,dx$$

Mathematica [A] time = 4.52966, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x), x]
```

Maple [A] time = 0.2, size = 0, normalized size = 0.

$$\int \frac{\csc (bx + a) \sec (bx + a)}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx + a) \sec (bx + a)}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc (bx + a) \sec (bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (a + bx) \sec (a + bx)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

3.233 $\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=21

$$2\text{Unintegrable}\left(\frac{\csc(2a+2bx)}{(c+dx)^2},x\right)$$

```
[Out] 2*Unintegrable[Csc[2*a + 2*b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.0410168, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = 2\int \frac{\csc(2a+2bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 5.88274, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.197, size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)\sec (bx+a)}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)\sec (bx+a)}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx+a)\sec (bx+a)}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (a+bx)\sec (a+bx)}{(c+dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

3.234 $\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\csc^2(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

Rubi [A] time = 0.196717, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Mathematica [A] time = 9.15385, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

Maple [A] time = 0.163, size = 0, normalized size = 0.

$$\int (dx+c)^m (\csc(bx+a))^2 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \csc(bx+a)^2 \sec(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

3.235 $\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=350

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+$$

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E
^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I)*d^2*(c + d*x)*
PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*E^
(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b
^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog
[3, -E^(I*(a + b*x))])/b^4 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x
))])/b^3 + (6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d^3*Pol
yLog[3, E^(I*(a + b*x))])/b^4 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))]
)/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```

Rubi [A] time = 0.640135, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2621, 321, 207, 4420, 6741, 12, 6742, 6273, 4181, 2531, 6609, 2282, 6589, 4183}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```


```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E
^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I)*d^2*(c + d*x)*
PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*E^
(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b
^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog
[3, -E^(I*(a + b*x))])/b^4 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x
))])/b^3 + (6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d^3*Pol
yLog[3, E^(I*(a + b*x))])/b^4 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))]
)/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```


Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
```

```
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\int (c+dx)^3\csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - (3d)\int (c+dx)^2\Bigg(\frac{\tanh}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - (3d)\int \frac{(c+dx)^2\left(\tanh\right.}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\left(\tanh\right.}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int \left((c+dx)^2\tanh\right.}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\tanh^{-}}{b}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+bx)}{b} + \frac{\int b(c+dx)^3\sec(a+b}{b}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+bx)}{b} + \frac{6id^2(c+dx)\mathrm{Li}_2\left(-e^{i(a}\right.}{b^3}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+}{b}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+}{b}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+}{b}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+}{b}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+}{b}$$

Mathematica [B] time = 6.41717, size = 739, normalized size = 2.11

$$\frac{3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right) - 3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right) - 6bcd^2\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right) + 6bcd^2\mathrm{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -(((c + d*x)^3*Csc[a])/b) + (3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c
 + d*x)^2*Log[1 + E^(I*(a + b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I
*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d
*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2))/b^2
 + ((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(I*(
a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1
 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3*c*
d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))]
 + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*d*(
c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(
a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog
[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)*d^
3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))
])/b^4 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^3*Sin[(b*x)/2]) - 3*c^2*d*x*Sin[
(b*x)/2] - 3*c*d^2*x^2*Sin[(b*x)/2] - d^3*x^3*Sin[(b*x)/2]))/(2*b) + (Csc[a
/2]*Csc[a/2 + (b*x)/2]*(c^3*Sin[(b*x)/2] + 3*c^2*d*x*Sin[(b*x)/2] + 3*c*d^2
*x^2*Sin[(b*x)/2] + d^3*x^3*Sin[(b*x)/2]))/(2*b)
```

Maple [B] time = 0.679, size = 1158, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] -6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+3
*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/
b^2*ln(1-exp(I*(b*x+a)))*x^2+6/b^3*d^3*ln(1-exp(I*(b*x+a)))*a*x-6/b^3*c*d^2
*a*ln(exp(I*(b*x+a))-1)-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+
a))/b/(exp(2*I*(b*x+a))-1)+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))+6*I/b^2*c
*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)
))*x-6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))-6*I/b^4*d^3*a*dilog(exp(I*(b*
x+a))+1)-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-6*I*d^3/b^3*polylog(2,exp(I*(b*
x+a)))*x-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-3*I/b^2*c^2*d*polylog(2,I
*exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*p
olylog(2,I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+3*I/b
^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)+3/b^
2*c^2*d*ln(exp(I*(b*x+a))-1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)-1/b*d^3*ln(
1+I*exp(I*(b*x+a)))*x^3+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+6/b^3*d^3*polylo
```

```
g(3,I*exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+1/b^4*a^3*
d^3*ln(1-I*exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-I*exp(I*(b*x+a)))-1/b^4*a
^3*d^3*ln(1+I*exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,I*exp(I*(b*x+a)))-2*I/b
*c^3*arctan(exp(I*(b*x+a)))+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/b^3*a
^2*c*d^2*ln(1-I*exp(I*(b*x+a)))+3/b*c*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*c*
d^2*ln(1+I*exp(I*(b*x+a)))*x^2+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))+3/b*c
^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3/b*c^2*
d*ln(1+I*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+6*I*d^3/b^3
*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^4*d^3*a*dilog(exp(I*(b*x+a)))+6*I/b^3*d
^2*c*dilog(exp(I*(b*x+a)))+6*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))*a+6*I/b^3
*d^2*c*dilog(exp(I*(b*x+a))+1)-6*I/b^4*d^3*polylog(2,exp(I*(b*x+a)))*a
```

<u>Maxima</u> [B] time = 4.38934, size = 4374, normalized size = 12.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
- 3*a*c^2*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)
)/b + 3*a^2*c*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a
) - 1))/b^2 - a^3*d^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x
 + a) - 1))/b^3 - 2*((2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) - 2*((b*x + a)^3*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*
(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*si
n(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^3*d^
3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*(b*x + a) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a
)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a
*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a),
-sin(b*x + a) + 1) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^
2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x
+ a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-
6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I
*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), c
os(b*x + a) + 1) - (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*a^2*d^3 - 6*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*cos(2*b*x + 2*a) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2
```

+ 6*I*a^2*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) +
(6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 +
2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^2*d^3 + (
-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x +
a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)
^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(b*x + a) + (6*b^2
*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3
)*(b*x + a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b
*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*
d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x
 + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (6*b^2*c^2*d - 12*a*b*c
d^2 + 6(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a
)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x +
2*a))*dilog(-I*e^(I*b*x + I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3
 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (12*I*b*c*d^2 +
12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a))
+ (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 -
 a*d^3)*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3
)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^
2 - 3*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a
) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6
*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*(b^2*c^2*d - 2*a*b*
c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3*I*
b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^
2 - 6*I*a*d^3)*(b*x + a) + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^
2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a
) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a
d^3)(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*
cos(b*x + a) + 1) + (I*(b*x + a)^3*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a
)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)*(b*x + a) + (-I*(b*x +
a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a
*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 3*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a
) + 1) + (-I*(b*x + a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3
*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3
+ (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 +
3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 12
*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3)*polylog(4, I*e^(I*b*
x + I*a)) - 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3)*polylo

```
g(4, -I*e^(I*b*x + I*a)) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3
+ (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(
b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x +
I*a)) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (12*I*b*c*d^2 +
12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)
*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x + I*a)) + (12*I*d^3*
cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, -e^(I*b*x
 + I*a)) + (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3
)*polylog(3, e^(I*b*x + I*a)) + (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12
*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3)*(
b*x + a))*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a)
 + 2*I*b^3))/b
```

Fricas [C] time = 0.951221, size = 4446, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*I*d^3
*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*I*d^3*polylog(4
, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b
*x + a) + sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b*x + a) -
 sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) + I*sin(b*x + a
))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3
*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x -
6*I*b*c*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3
*x + 6*I*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-3*I
*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(
b*x + a))*sin(b*x + a) - (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*
d)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - (3*I*b^2*d^3*x^2 + 6
*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b
*x + a) - (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*cos(
b*x + a) - sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(
-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*
dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*s
in(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)
```

```
*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(
b*x + a) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x +
a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2
)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a
) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x +
 a) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a)
 + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*
b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*si
n(b*x + a) + I)*sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x +
 a) + sin(b*x + a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b
*x + a) - sin(b*x + a))*sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*
cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3
, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a))/(b^4*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \csc(bx+a)^2 \sec(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a), x)
```

3.236 $\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=226

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,\right.}{b^3}$$

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (4*d*(c + d*x)*ArcTanh[E^(
I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -
E^(I*(a + b*x))])/b^3 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))]
)/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*
PolyLog[2, E^(I*(a + b*x))])/b^3 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])
/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3
```

Rubi [A] time = 0.383878, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2621, 321, 207, 4420, 6741, 12, 6742, 6273, 4181, 2531, 2282, 6589, 4183, 2279, 2391}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```


```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (4*d*(c + d*x)*ArcTanh[E^(
I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -
E^(I*(a + b*x))])/b^3 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))]
)/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*
PolyLog[2, E^(I*(a + b*x))])/b^3 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])
/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3
```


Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2\csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - (2d)\int (c+dx)\Big(\frac{\tanh}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - (2d)\int \frac{(c+dx)\left(\tanh\right.}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int (c+dx)\left(\tanh^{-}\right.}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int \left((c+dx)\tanh\right.}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int (c+dx)\tanh^{-1}}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{\int b(c+dx)^2\sec(a+bx}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{\left(2id^2\right)\mathrm{Subst}\left(\int \frac{\log(1-}{x}\right.}{b^3}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+}{b}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+}{b}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+}{b}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+}{b}$$

Mathematica [B] time = 6.29071, size = 593, normalized size = 2.62

$$\frac{2ibd(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right) - 2ibd(c+dx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right) - 2d^2\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right) + 2d^2\mathrm{PolyLog}\left(3,i\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -(((c + d*x)^2*Csc[a])/b) + ((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2
*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))]
- 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a +
b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(
c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*
x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((4*I)*c*d*ArcTan[(I*Cos[
a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sqrt[Cos[a]^2
+ Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]) - 2*c*d*x*
Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*
(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]))/(2*b) + (
2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[
a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2] + (((b*x + ArcTan[Tan[a]])*(Lo
g[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x + ArcTan[Tan[a]]))]
) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - PolyLog[2, E^(I*(b*x + A
rcTan[Tan[a]]))]))*Sec[a])/Sqrt[1 + Tan[a]^2]))/b^3
```

Maple [B] time = 0.449, size = 556, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] 2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)
+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+2*d/b^2*c*ln(exp(I*(b*x+a))-1)-1/b^3*a^
2*d^2*ln(1-I*exp(I*(b*x+a)))-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+1/b^3*a^2*d
^2*ln(1+I*exp(I*(b*x+a)))-2/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+2*I/b^2*c*d*poly
log(2,-I*exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x+2*I/b^3*
d^2*dilog(exp(I*(b*x+a))+1)+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-2*d/b^2*c*
ln(exp(I*(b*x+a))+1)+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+2*d^2*polylog(3,I*e
xp(I*(b*x+a)))/b^3+2*I/b^3*d^2*dilog(exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,
I*exp(I*(b*x+a)))-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-2*I*(d^2*x^2+2*c*d
*x+c^2)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-2*d^2/b^2*ln(exp(I*(b*x+a))+1
)*x-2/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x-2*I
/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-2*I/b*c^2*arctan(exp(I*(b*x+a)))
```

Maxima [B] time = 2.35497, size = 2202, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
- 2*a*c*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/
b + a^2*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)
)/b^2 - 2*((2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 2*((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a)^2*d
^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x
+ a), sin(b*x + a) + 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a)
- 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (-2*
I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*a
rctan2(cos(b*x + a), -sin(b*x + a) + 1) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*
d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-4*I*b*c*d - 4*
I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*
x + a) + 1) - (4*b*c*d - 4*a*d^2 - 4*(b*c*d - a*d^2)*cos(2*b*x + 2*a) - (4*
I*b*c*d - 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) -
 1) - 4*((b*x + a)*d^2*cos(2*b*x + 2*a) + I*(b*x + a)*d^2*sin(2*b*x + 2*a)
- (b*x + a)*d^2)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(b*x + a) + (4*b*c*d + 4*(b*x + a)*d^
2 - 4*a*d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-4*I*b*
c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I
*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*
d^2)*cos(2*b*x + 2*a) - (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(2*b
*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + 4*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(
2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 4*(d^2*cos(2*b*x + 2*a) + I*d
^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) + (-2*I*b*c*d - 2*I*(b*x
+ a)*d^2 + 2*I*a*d^2 + (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2)*cos(2*b*
x + 2*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*I*b*c*d + 2*I*(b*x + a)*
d^2 - 2*I*a*d^2 + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*cos(2*b*x +
2*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d
 - 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b
*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))
*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1
) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (I*(b*x + a)
^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 +
```

```
sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (-4*I*d^2*cos(2*b*x + 2*a) + 4*d^2*s
in(2*b*x + 2*a) + 4*I*d^2)*polylog(3, I*e^(I*b*x + I*a)) + (4*I*d^2*cos(2*b
*x + 2*a) - 4*d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, -I*e^(I*b*x + I*a)
) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(b*x + a
))/(-2*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(2*b*x + 2*a) + 2*I*b^2))/b
```

Fricas [C] time = 0.764123, size = 2844, normalized size = 12.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*dilog(cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) - 2*I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a
))*sin(b*x + a) + 2*I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - 2*I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 2*d^2*poly
log(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, I*cos
(b*x + a) - sin(b*x + a))*sin(b*x + a) + 2*d^2*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x +
a))*sin(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*
x + a))*sin(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) - si
n(b*x + a))*sin(b*x + a) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a)
+ sin(b*x + a))*sin(b*x + a) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x +
 a) - sin(b*x + a))*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*x + a) + I
*sin(b*x + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b
*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*
x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2
*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*
x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x +
a) - sin(b*x + a) + 1)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*
d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x +
a) + 1)*sin(b*x + a) - 2*(b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(
b*x + a) + 1/2)*sin(b*x + a) - 2*(b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/
2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b*d^2*x + a*d^2)*log(-cos(b*x + a
) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(
-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) - 2*(b*d^2*x + a*d^2)*log(
-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a))/(b^3*sin(b*x
```

```
+ a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")

[Out] Timed out
```

3.237 $\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=131

$$\frac{idPolyLog\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{idPolyLog\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}+\frac{(c+dx)\tanh^{-1}(}{b}$$

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (d
*x*ArcTanh[Sin[a + b*x]])/b + ((c + d*x)*ArcTanh[Sin[a + b*x]])/b - ((c + d
*x)*Csc[a + b*x])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*Pol
yLog[2, I*E^(I*(a + b*x))])/b^2
```

Rubi [A] time = 0.134538, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2621, 321, 207, 4420, 6271, 12, 4181, 2279, 2391, 3770}

$$\frac{idPolyLog\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{idPolyLog\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}+\frac{(c+dx)\tanh^{-1}(}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (d
*x*ArcTanh[Sin[a + b*x]])/b + ((c + d*x)*ArcTanh[Sin[a + b*x]])/b - ((c + d
*x)*Csc[a + b*x])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*Pol
yLog[2, I*E^(I*(a + b*x))])/b^2
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)\csc(a+bx)}{b} - d\int\left(\frac{\tanh^{-1}(\sin(a+b\right.$$

$$= \frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)\csc(a+bx)}{b} - \frac{d\int\tanh^{-1}(\sin(a+bx)}{b}$$

$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c+dx)\tanh^{-1}(\sin(a+}{b}$$

$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c+dx)\tanh^{-1}(\sin(a+}{b}$$

$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c}{}$$

$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c}{}$$

$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c}{}$$

Mathematica [C] time = 2.94677, size = 517, normalized size = 3.95

$$-\frac{c\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{b} - \frac{dx\left(-i\left(\text{PolyLog}\left(2,\frac{1}{2}\left((1+i)-(1-i)\tan\left(\frac{1}{2}(a+bx)\right)\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] (d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b
^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b -
(d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (d*x*(a*Log
[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*Tan[(a
+ b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1 + I) -
(1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 + I
/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2]
```

```
)]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2
])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 + I*Ta
n[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1
- I) + (1 + I)*Tan[(a + b*x)/2])/2]))/(b*(a - I*Log[1 - I*Tan[(a + b*x)/2]
] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*Sec[(a + b*x)/2]*(a*Sin[(a + b*x)/
2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)
```

Maple [A] time = 0.299, size = 235, normalized size = 1.8

$$\frac{-2\,i\mathrm{e}^{i(bx+a)}\,(dx+c)}{b\left(\mathrm{e}^{2\,i(bx+a)}-1\right)}+\frac{2\,iad\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}-\frac{d\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b^2}+\frac{d\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^2}-\frac{2\,ic\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1+i\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] -2*I*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))-1)+2*I/b^2*d*a*arctan(exp(I
*(b*x+a)))-d/b^2*ln(exp(I*(b*x+a))+1)+d/b^2*ln(exp(I*(b*x+a))-1)-2*I/b*c*ar
ctan(exp(I*(b*x+a)))-1/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/b^2*d*ln(1+I*exp(I*(b
*x+a)))*a-I/b^2*d*dilog(1-I*exp(I*(b*x+a)))+I/b^2*d*dilog(1+I*exp(I*(b*x+a)
))+1/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/b^2*d*ln(1-I*exp(I*(b*x+a)))*a
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.607381, size = 1215, normalized size = 9.27

$$-\frac{2\,bdx+i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)\sin\left(bx+a\right)+i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-i\,d\mathrm{Li}_2\left(-i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + I*d*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + I*d
*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x +
 a) + sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)
)*sin(b*x + a) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
 + a) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + d
*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*d*x + a*d)*log(I*cos(b*x + a
) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(I*cos(b*x + a) - sin
(b*x + a) + 1)*sin(b*x + a) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x +
 a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) +
1)*sin(b*x + a) - d*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*c - a*d)
*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*c - a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\csc(bx+a)^2\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a), x)
```

3.238 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

Rubi [A] time = 0.159846, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]

[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx=\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 11.6999, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]

[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

Maple [A] time = 2.356, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx + a))^2 \sec (bx + a)}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx + a)^2 \sec (bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2 (a + bx) \sec (a + bx)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)^2 \sec (bx+a)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

3.239 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi [A] time = 0.189883, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 11.9084, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 2.649, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2 \sec(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2 \sec(bx+a)}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.240 $\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\csc^3(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]
```

Rubi [A] time = 0.236741, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Mathematica [A] time = 11.0166, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]
```

Maple [A] time = 0.178, size = 0, normalized size = 0.

$$\int (dx+c)^m (\csc (bx+a))^3 \sec (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc (bx+a)^3 \sec (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \csc (bx+a)^3 \sec (bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

3.241 $\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=325

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c$$

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh
[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d
*x)^3*Cot[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((
(3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^
2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2
*I)*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/
4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rubi [A] time = 0.82345, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2620, 14, 4420, 6741, 12, 6742, 3720, 3717, 2190, 2279, 2391, 32, 2551, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```


```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh
[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d
*x)^3*Cot[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((
(3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^
2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2
*I)*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/
4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```


Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - (3d)\int (c+dx)^2\Big(-\frac{\text{co}}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - (3d)\int \frac{(c+dx)^2\big(-\text{c}}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - \frac{(3d)\int (c+dx)^2\big(-\text{co}}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - \frac{(3d)\int \big(-(c+dx)^2\,\text{co}}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(3d)\int (c+dx)^2\cot^2(}{2b}$$

$$= -\frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^3\csc(2a+2}{b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cot(a}{2b^2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cot(a}{2b^2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cot(a}{2b^2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cot(a}{2b^2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cot(a}{2b^2}$$

Mathematica [B] time = 6.99256, size = 1477, normalized size = 4.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -((c + d*x)^3*Csc[a + b*x]^2)/(2*b) - (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^
((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^(
(2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a +
```

```
 b*x))]]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a +
b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(2*b^3) - (d^3*E^
(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[
1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*
(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))
] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x))] - 2*PolyLog[4, -E^((-I)*(a +
b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a
 + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)
*(a + b*x))]))/E^((2*I)*a)))/(4*b^4) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2
+ d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E
^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLo
g[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)
*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a
) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1
+ E^((2*I)*a))*(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*Pol
yLog[3, -E^((-2*I)*(a + b*x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E
^((2*I)*a)))*Sec[a] - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[
b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c^3*Csc[a]*(-(b*x*Cos[a])
 + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)
) + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]
*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*Arc
Tan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*
I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])
)] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I
*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])
/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (3*Csc[a]*Csc[a + b*x]*(c^2
*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (3*c^2*d*Cs
c[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]
) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*
(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x
+ ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a]
)/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^
3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan
[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2
*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[
b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Ta
n[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.421, size = 1223, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] 3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3*I/b
^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))
*x^2-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I
*(b*x+a)))-1/b*c^3*ln(exp(2*I*(b*x+a))+1)-3/2/b^3*c*d^2*polylog(3,-exp(2*I*
(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+3/b^3*d^3*ln(exp(I*(b*
x+a))+1)*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*
a-3*I/b^2*d^3*x^2-3*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))-3*I/b^4*d^3*a^2+6*
I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x
+a)))-3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x
^2-1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x
+a)))*x^2-3/b^4*d^3*a*ln(exp(I*(b*x+a))-1)+6/b^4*d^3*a*ln(exp(I*(b*x+a)))+3
/b^3*d^2*c*ln(exp(I*(b*x+a))+1)-6/b^3*d^2*c*ln(exp(I*(b*x+a)))+3/b^3*d^2*c*
ln(exp(I*(b*x+a))-1)+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*
(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/b^3*c*d^2*a^2*ln(exp(I*(b*
x+a))-1)-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x
+a)))+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b
^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-6*I/b^3*d^3*a*x+(2*b*d^3*x^3*exp(2*I*(b*x+a
))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*
exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))+3*
I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(exp(2*I*(b*x
+a))-1)^2-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3*I/b^2*c*d^2*polylog(
2,-exp(2*I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3*d^3*poly
log(3,-exp(I*(b*x+a)))*x+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c*d^2*
polylog(3,-exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6*I/b^4*d^3*p
olylog(4,-exp(I*(b*x+a)))-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2
*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^3*ln(exp(I*(b*x+a))-1)+1/b*c^3*ln
(exp(I*(b*x+a))+1)-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4
```

Maxima [B] time = 7.78119, size = 6939, normalized size = 21.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)
) - 3*a*c^2*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a
)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin
(b*x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - l
```

og(sin(b*x + a)^2))/b^3 - 2*(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 - (8*
(b*x + a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*(b*x + a) + 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a
) - 4*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (8*I*(b*x + a)^3*d^3
+ (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^
2 + 18*I*a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-16*I*(b*x + a)^3*d^3 + (-
36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2
- 36*I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(
2*b*x + 2*a) + 1) + (6*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x
+ a) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x +
 4*a) - 12*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) - (-6*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^
2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^
2 - 18*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (12*I*(b*x + a)^3*d^3 + 36*I*b
*c*d^2 - 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c
^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 36*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (18*b*c*d^2 - 18*a*d^3 + 18*(b*c
*d^2 - a*d^3)*cos(4*b*x + 4*a) - 36*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-
18*I*b*c*d^2 + 18*I*a*d^3)*sin(4*b*x + 4*a) - (36*I*b*c*d^2 - 36*I*a*d^3)*s
in(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^3*d
^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
+ 1)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
+ 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
 12*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a
*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^
3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*
d^2 + (18*I*a^2 + 18*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*(b*x + a)
^3*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I
*a*b*c*d^2 + (-36*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(s
in(b*x + a), -cos(b*x + a) + 1) - 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)
*(b*x + a))*cos(4*b*x + 4*a) - (12*I*(b*x + a)^3*d^3 + 18*b^2*c^2*d - 36*a*
b*c*d^2 + 18*a^2*d^3 + (36*I*b*c*d^2 - 18*(2*I*a + 1)*d^3)*(b*x + a)^2 + (3
6*I*b^2*c^2*d - 36*(2*I*a + 1)*b*c*d^2 + (36*I*a^2 + 36*a)*d^3)*(b*x + a))*
cos(2*b*x + 2*a) + (9*b^2*c^2*d - 18*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 9*a^2
d^3 + 18(b*c*d^2 - a*d^3)*(b*x + a) + 3*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b
*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a)
 - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 -
 12*I*(b*x + a)^2*d^3 - 9*I*a^2*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a
))*sin(4*b*x + 4*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 24*I*(b*x + a)^2*d

```
^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*dilog(-e^(2*I*b*x + 2*I*a)) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)
^2*d^3 + 18*(a^2 + 1)*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
+ a))*cos(4*b*x + 4*a) - 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a
^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c
^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 18*I)*d^3 + (36*
I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-36*I*b^2*c^2*d + 72
*I*a*b*c*d^2 - 36*I*(b*x + a)^2*d^3 + (-36*I*a^2 - 36*I)*d^3 + (-72*I*b*c*d
^2 + 72*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (18
*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 18*(a^2 + 1)*d^3 + 36*(b*c
*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (
a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 36*(b^2*c^
2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x +
 a)^2*d^3 + (18*I*a^2 + 18*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*
sin(4*b*x + 4*a) + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*(b*x + a)^2*d^3
 + (-36*I*a^2 - 36*I)*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a))*sin(2*b
*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-4*I*(b*x + a)^3*d^3 + (-9*I*b*c*d^2 +
 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 9*I*a^2*d^3)*(
b*x + a) + (-4*I*(b*x + a)^3*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 +
 (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 9*I*a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a
) + (8*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*
b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (4
*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(4*(b*x + a)^3*d^3 + 9*(b*c
*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a) + 1) - (3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*
d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 +
 9*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*
I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I
*a^2 + 9*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 18*I
*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b
^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 18*I)*d^3)*(b*x + a))*cos(2*b*x +
2*a) - 3*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(4*b*x +
 4*a) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
 + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(2*b*x
+ 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b
*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x +
a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 9*I)*d^3)*(b*x + a) + (
3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(
b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 9*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 +
```

```
(-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^
2 + (-18*I*a^2 - 18*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^
3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6*((b*x + a)^3*d
^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (6*d^3*cos(4*b*x + 4*a) - 12*d^
3*cos(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) - 12*I*d^3*sin(2*b*x + 2*a) +
 6*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + (36*d^3*cos(4*b*x + 4*a) - 72*d^
3*cos(2*b*x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) - 72*I*d^3*sin(2*b*x + 2*a)
+ 36*d^3)*polylog(4, -e^(I*b*x + I*a)) + (36*d^3*cos(4*b*x + 4*a) - 72*d^3*
cos(2*b*x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) - 72*I*d^3*sin(2*b*x + 2*a) +
36*d^3)*polylog(4, e^(I*b*x + I*a)) - (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 +
9*I*a*d^3 + (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3)*cos(4*b*x + 4*a
) + (18*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 18*I*a*d^3)*cos(2*b*x + 2*a) + 3*(
3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(4*b*x + 4*a) - 6*(3*b*c*d^2 + 4*
(b*x + a)*d^3 - 3*a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a))
 - (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(
b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (-72*I*b*c*d^2 - 72*I*(b*x +
a)*d^3 + 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3
)*sin(4*b*x + 4*a) + 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))
*polylog(3, -e^(I*b*x + I*a)) - (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a
*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) +
(-72*I*b*c*d^2 - 72*I*(b*x + a)*d^3 + 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*
c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 72*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - (18*I*(b*x + a)
^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*(b*x
 + a)^3*d^3 - 18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3 + (36*b*c*d^2
- (36*a - 18*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a - 36*I)*b*c*d^2 +
36*(a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^3*cos(4*b*x + 4*a)
 + 12*I*b^3*cos(2*b*x + 2*a) + 6*b^3*sin(4*b*x + 4*a) - 12*b^3*sin(2*b*x +
2*a) - 6*I*b^3))/b
```

Fricas [C] time = 1.26092, size = 8092, normalized size = 24.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + (3*I*b^2*d^3*x^
```

2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2
*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*
sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d
^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*x
+ a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c
*d^2*x + 3*I*b^2*c^2*d + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*
d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3
*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*
b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*
c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x +
a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + (-3*I*b^2*d^3*x^
2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a)
- sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I
*d^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*
x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^
2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x -
 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x +
 a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 - (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x
+ a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a) + 1) +
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*d^3)*x)*cos
(b*x + a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) - I*sin(b*x + a) +
1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin
(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2)*log(I*cos(b
*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3
*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*c
os(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2 + a^3*d^3)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3
*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3 - (b^3*c^3 - 3
*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-
1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^3*c^3 - 3*a*b^2*c^2*d + 3
*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 +

```
1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I
*sin(b*x + a) + 1/2) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + (a^3 + 3*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a
)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x +
 a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 (a^3 + 3*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3
*c^2*d + b*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (
6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a))
 + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x
+ a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylog(4, I*cos(b*x + a) + sin(
b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4, I*cos(b*x + a) -
 sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4, -I*cos(b*x
+ a) + sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylog(4, -I*cos
(b*x + a) - sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4,
-cos(b*x + a) + I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylo
g(4, -cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*
c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3
*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a)
- I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)
^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d
^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) +
 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -I*c
os(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*co
s(b*x + a)^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c
*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin
(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*pol
ylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a)^2 - b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \csc (bx+a)^3 \sec (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a), x)
```

3.242 $\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=201

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+b}\right.}{2b^3}$$

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))
])/b - (d*(c + d*x)*Cot[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b)
+ (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*
x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Poly
Log[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x))
])/(2*b^3)
```

Rubi [A] time = 0.442508, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2620, 14, 4420, 6741, 12, 6742, 3720, 3475, 2551, 4183, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+b}\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```


```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))
])/b - (d*(c + d*x)*Cot[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b)
+ (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*
x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Poly
Log[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x))
])/(2*b^3)
```


Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
```

```
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}-(2d)\int (c+dx)\Big(-\frac{\cot}{}$$
$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}-(2d)\int\frac{(c+dx)\big(-\cot}{}$$
$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}-\frac{d\int (c+dx)\big(-\cot^2(a}{}$$
$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}-\frac{d\int\big(-(c+dx)\cot^2(a}{}$$
$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}+\frac{d\int (c+dx)\cot^2(a+}{b}$$
$$= -\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{\int 2b(c+dx)^2\csc(2a+2b}{b}$$
$$= -\frac{cdx}{b}-\frac{d^2x^2}{2b}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{d^2\log(\sin(a}{b^3}$$
$$= -\frac{cdx}{b}-\frac{d^2x^2}{2b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+d}{}$$
$$= -\frac{cdx}{b}-\frac{d^2x^2}{2b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+d}{}$$
$$= -\frac{cdx}{b}-\frac{d^2x^2}{2b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+d}{}$$
$$= -\frac{cdx}{b}-\frac{d^2x^2}{2b}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+d}{}$$

Mathematica [B] time = 6.75669, size = 872, normalized size = 4.34

$$-\frac{\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))c^2}{b\left(\cos^2(a)+\sin^2(a)\right)}+\frac{\csc(a)(\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a)}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -((c + d*x)^2*Csc[a + b*x]^2)/(2*b) - (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((
2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3
*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2
```

```
*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b
*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*
x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(6*b^3) + (x*(3*c^2
 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/3 - ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3
*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a)
)*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3,
-E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos
[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) +
(c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a])
)/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin
[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*(
(b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) -
Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x
 - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - Ar
cTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 +
Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Csc[a]*Csc
[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/b^2 - (c*d*Csc[a]*Sec[a]*(b^2*E^
(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a
]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2
]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.369, size = 632, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] -1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^
2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d
^2*ln(1-exp(I*(b*x+a)))*x^2-1/b*c^2*ln(exp(2*I*(b*x+a))+1)+I/b^2*c*d*polylo
g(2,-exp(2*I*(b*x+a)))-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*d^2*
polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-2*I/b^2*
c*d*polylog(2,-exp(I*(b*x+a)))-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(ex
p(I*(b*x+a))+1)+1/b^3*d^2*ln(exp(I*(b*x+a))-1)+2*(b*d^2*x^2*exp(2*I*(b*x+a)
)+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a)
)-I*c*d*exp(2*I*(b*x+a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+2/b*c*d*
ln(exp(I*(b*x+a))+1)*x+1/b*c^2*ln(exp(I*(b*x+a))+1)+1/b*c^2*ln(exp(I*(b*x+a
))-1)+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/
```

```
b^3+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-2/b*c*d*ln(exp(2*I*(b*x+a))+1)
*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+1/b^3*d^
2*a^2*ln(exp(I*(b*x+a))-1)-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 2.94517, size = 3405, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)
) - 2*a*c*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^
2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2))/b^2 + 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(b*x + a)*d^2*sin(4
*b*x + 4*a) - 4*b*c*d + 4*a*d^2 + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b
*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a
) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (2
*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) +
(-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a
))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2*(b*x + a)^2*d^2 + 4
*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) + d^2)*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) + d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +
4*I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) - (4*I*(b*x + a)^2*d^2 + (
8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b
*x + a), cos(b*x + a) + 1) - (2*d^2*cos(4*b*x + 4*a) - 4*d^2*cos(2*b*x + 2*
a) + 2*I*d^2*sin(4*b*x + 4*a) - 4*I*d^2*sin(2*b*x + 2*a) + 2*d^2)*arctan2(s
in(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*
x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a)
 - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (2*
I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) + (
-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a)
)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (4*I*(b*x + a)^2*d^2 + 4*b*c*d
 - 4*a*d^2 + (8*I*b*c*d - 4*(2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) -
(2*b*c*d + 2*(b*x + a)*d^2 - 2*a*d^2 + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*co
s(4*b*x + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-2*I
*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*sin(4*b*x + 4*a) - (4*I*b*c*d + 4*I
*(b*x + a)*d^2 - 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) +
 (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*c
os(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I
```

```
*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*
I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (4
*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(
4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*
c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*I*(
b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b
*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 +
(-2*I*b*c*d + 2*I*a*d^2)*(b*x + a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2
 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*
c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*
I*a*d^2)*(b*x + a) + I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(
b*x + a) + I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +
4*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*
c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*
c*d - a*d^2)*(b*x + a) + d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^
2)*(b*x + a) + I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x +
a) + I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*
d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a) + d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 - 2*cos(b*x + a) + 1) + (-I*d^2*cos(4*b*x + 4*a) + 2*I*d^2*cos(2*b*x + 2
*a) + d^2*sin(4*b*x + 4*a) - 2*d^2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -e^
(2*I*b*x + 2*I*a)) + (4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) -
 4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -e^(
I*b*x + I*a)) + (4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) - 4*d^
2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, e^(I*b*x
+ I*a)) - (4*(b*x + a)^2*d^2 - 4*I*b*c*d + 4*I*a*d^2 + (8*b*c*d - (8*a - 4*
I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-2*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*c
os(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2
))/b
```

Fricas [C] time = 0.915492, size = 4868, normalized size = 24.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)
*sin(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b
*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d
 + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x
 + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)
^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d + (2*I
*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a))
+ (-2*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dil
og(-I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2
*x - 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-2
*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-c
os(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x -
2*I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*
cos(b*x + a)^2 + d^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(co
s(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*cos(b*x + a)^2 + d^2)*log(cos(b*
x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I)
 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x +
 a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2
+ 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) - s
in(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-I*cos(b*x
 + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^
2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-
I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2 -
 (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2 - (b^2
*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-c
os(b*x + a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a
) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*si
n(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(d^2*co
s(b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(
b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 2*(d^2*cos(b*
x + a)^2 - d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*(d^2*cos(b*x
+ a)^2 - d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*(d^2*cos(b*x +
a)^2 - d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*(d^2*cos(b*x + a
```

```
)^2 - d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(d^2*cos(b*x + a)
^2 - d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^
2 - d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^2 -
b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a), x)
```

3.243 $\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=141

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx$$

```
[Out] -(d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Cot[a + b*x])/(2
*b^2) - ((c + d*x)*Cot[a + b*x]^2)/(2*b) - (d*x*Log[Tan[a + b*x]])/b + ((c
+ d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^
2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rubi [A] time = 0.138571, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2620, 14, 4420, 3473, 8, 2548, 12, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```


```
[Out] -(d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Cot[a + b*x])/(2
*b^2) - ((c + d*x)*Cot[a + b*x]^2)/(2*b) - (d*x*Log[Tan[a + b*x]])/b + ((c
+ d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^
2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```


Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)\cot^2(a+bx)}{2b} + \frac{(c+dx)\log(\tan(a+bx))}{b} - d\int\left(-\frac{\cot^2(a+bx)}{2b} + \ldots\right.$$

$$= -\frac{(c+dx)\cot^2(a+bx)}{2b} + \frac{(c+dx)\log(\tan(a+bx))}{b} + \frac{d\int\cot^2(a+bx)\,dx}{2b} - \ldots$$

$$= -\frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{dx\log(\tan(a+bx))}{b} + \frac{(c+dx)\log(\ldots}{b}$$

$$= -\frac{dx}{2b} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{dx\log(\tan(a+bx))}{b} + \frac{(c+dx)\ldots}{}$$

$$= -\frac{dx}{2b} - \frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{dx\log\ldots}{}$$

$$= -\frac{dx}{2b} - \frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{dx\log\ldots}{}$$

$$= -\frac{dx}{2b} - \frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{dx\log\ldots}{}$$

Mathematica [A] time = 0.957523, size = 210, normalized size = 1.49

$$\frac{d\left(\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)+\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)\right)}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2 \ldots\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -(d*Cot[a + b*x])/(2*b^2) - (d*x*Csc[a + b*x]^2)/(2*b) + (a*d*Log[Cos[a + b
*x]])/b^2 - (c*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])
)/(2*b) - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a + b*x)^2 - (a + b*x)*L
og[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2
+ (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2 + PolyLog
[2, E^((2*I)*(a + b*x))])))/b^2
```

Maple [B] time = 0.173, size = 270, normalized size = 1.9

$$\frac{2bdxe^{2i(bx+a)}+2bce^{2i(bx+a)}-ide^{2i(bx+a)}+id}{b^2\left(e^{2i(bx+a)}-1\right)^2}-\frac{c\ln\left(e^{2i(bx+a)}+1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}+\frac{d\ln\left(1-e\ldots\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+I*d)/
b^2/(exp(2*I*(b*x+a))-1)^2-1/b*c*ln(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*x
+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(
1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(2*I*(b*x
+a))+1)*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+
1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 2.21863, size = 1397, normalized size = 9.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos
(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*
b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2
*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos(2*b
*x + 2*a) - (-2*I*b*d*x - 2*I*b*c)*sin(4*b*x + 4*a) - (4*I*b*d*x + 4*I*b*c)
*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(4*b
*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) - 4*I*b*c*sin
(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a) - 4
*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) - 2
*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*di
log(-e^(2*I*b*x + 2*I*a)) + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) +
2*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*
a)) + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a)
 - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c
 + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x +
2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*l
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*
```

```
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 2*(2
*b*d*x + 2*b*c - I*d)*sin(2*b*x + 2*a) - 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) +
4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a)
- 2*I*b^2)
```

Fricas [B] time = 0.722204, size = 2457, normalized size = 17.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c + (-I*d*cos(b*x + a)^2 + I*d
)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(c
os(b*x + a) - I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(I*cos(b*x
 + a) + sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(I*cos(b*x + a) - s
in(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-I*cos(b*x + a) + sin(b*x +
 a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
(I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-I*d*co
s(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x - (b*d*x
 + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - ((b*
c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) - I*sin(b*
x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) - I
*sin(b*x + a) + I) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(I*cos
(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x +
 a)^2 + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*
d)*cos(b*x + a)^2 + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + ((b*c -
a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
 + 1/2) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) - (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c +
 a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*d*x - (b*d*x + a*d)*cos(
b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - ((b*c - a*d)*co
s(b*x + a)^2 - b*c + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^2*cos
(b*x + a)^2 - b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\csc(bx+a)^3\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a), x)
```

3.244 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

Rubi [A] time = 0.146745, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 14.3861, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

Maple [A] time = 3.076, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx + a))^3 \sec (bx + a)}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx + a)^3 \sec (bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3 (a + bx) \sec (a + bx)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

3.245 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi [A] time = 0.191978, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 16.889, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 4.781, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx+a))^{3} \sec (bx+a)}{(dx+c)^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx+a)^{3} \sec (bx+a)}{d^{2}x^{2}+2\,cdx+c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{3}(a+bx)\sec (a+bx)}{(c+dx)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)/(d*x + c)^2, x)
```

3.246 $\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\tan(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

Rubi [A] time = 0.142017, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]

[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

Mathematica [A] time = 0.930321, size = 0, normalized size = 0.

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]

[Out] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

Maple [A] time = 0.162, size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a)\tan(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sec(bx + a)\tan(bx + a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m \sec(bx + a)\tan(bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \tan(a + bx)\sec(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

3.247 $\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=227

$$\frac{24d^3(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}-\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+12id$$

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - ((12*I)*d^2*(c + d*x)^2
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)*d^2*(c + d*x)^2*PolyLog[2,
 I*E^(I*(a + b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x)
)])/b^4 - (24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((24*I)*d^
4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I)*d^4*PolyLog[4, I*E^(I*(a
+ b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rubi [A] time = 0.185714, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4409, 4181, 2531, 6609, 2282, 6589}

$$\frac{24d^3(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}-\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+12id$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - ((12*I)*d^2*(c + d*x)^2
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)*d^2*(c + d*x)^2*PolyLog[2,
 I*E^(I*(a + b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x)
)])/b^4 - (24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((24*I)*d^
4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I)*d^4*PolyLog[4, I*E^(I*(a
+ b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^4\sec(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\sec(a+bx)\,dx}{b}$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^4\sec(a+bx)}{b} + \frac{\left(12d^2\right)\int (c+dx)^2\log\left(1\right.}{b^2}$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+dx)^2\mathrm{Li}}{b^3}$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+dx)^2\mathrm{Li}}{b^3}$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+dx)^2\mathrm{Li}}{b^3}$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+dx)^2\mathrm{Li}}{b^3}$$

Mathematica [A] time = 1.23976, size = 428, normalized size = 1.89

$$\frac{(c+dx)^4\sec(a+bx)}{b} - \frac{4d\left(3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right) - 3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right) - 6bcd^2\mathrm{PolyLog}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(
I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Lo
g[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3
*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x
))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*
d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*Poly
Log[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)
*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*
x))]))/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Maple [B] time = 0.315, size = 767, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 24*I*d^3/b^3*c*polylog(2,I*exp(I*(b*x+a)))*x+24*I*d^3/b^4*c*a^2*arctan(exp(
I*(b*x+a)))-24*I*d^2/b^3*c^2*a*arctan(exp(I*(b*x+a)))+24*I*d^4*polylog(4,-I
*exp(I*(b*x+a)))/b^5+2*exp(I*(b*x+a))*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*
c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))+1)-24*d^4/b^4*polylog(3,I*exp(I*(b*x+a)))*
x+24*d^4/b^4*polylog(3,-I*exp(I*(b*x+a)))*x-4*d^4/b^5*a^3*ln(1-I*exp(I*(b*x
+a)))+4*d^4/b^2*ln(1+I*exp(I*(b*x+a)))*x^3-4*d^4/b^2*ln(1-I*exp(I*(b*x+a)))
*x^3-24*d^3/b^4*c*polylog(3,I*exp(I*(b*x+a)))+4*d^4/b^5*a^3*ln(1+I*exp(I*(b
*x+a)))+24*d^3/b^4*c*polylog(3,-I*exp(I*(b*x+a)))-24*I*d^4*polylog(4,I*exp(
I*(b*x+a)))/b^5-24*I*d^3/b^3*c*polylog(2,-I*exp(I*(b*x+a)))*x-12*d^2/b^3*c^
2*ln(1-I*exp(I*(b*x+a)))*a+12*d^3/b^2*c*ln(1+I*exp(I*(b*x+a)))*x^2-12*d^3/b
^2*c*ln(1-I*exp(I*(b*x+a)))*x^2+12*I*d^2/b^3*c^2*polylog(2,I*exp(I*(b*x+a))
)-12*I*d^4/b^3*polylog(2,-I*exp(I*(b*x+a)))*x^2+12*I*d^4/b^3*polylog(2,I*ex
p(I*(b*x+a)))*x^2-12*I*d^2/b^3*c^2*polylog(2,-I*exp(I*(b*x+a)))-8*I*d^4/b^5
*a^3*arctan(exp(I*(b*x+a)))+8*I*d/b^2*c^3*arctan(exp(I*(b*x+a)))+12*d^3/b^4
*c*a^2*ln(1-I*exp(I*(b*x+a)))+12*d^2/b^2*c^2*ln(1+I*exp(I*(b*x+a)))*x+12*d^
2/b^3*c^2*ln(1+I*exp(I*(b*x+a)))*a-12*d^3/b^4*c*a^2*ln(1+I*exp(I*(b*x+a)))-
12*d^2/b^2*c^2*ln(1-I*exp(I*(b*x+a)))*x
```

Maxima [B] time = 3.0425, size = 3974, normalized size = 17.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] (2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*a
)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*c^3
*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) -
 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*a
)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*a*c
^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b^2) + 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x
 + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin
```

(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos
(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1
))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b^3) - 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin
(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x +
a) + 1))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1)*b^4) + c^4/cos(b*x + a) - 4*a*c^3*d/(b*cos(b*x + a)) + 6*a^2*c^2*
d^2/(b^2*cos(b*x + a)) - 4*a^3*c*d^3/(b^3*cos(b*x + a)) + a^4*d^4/(b^4*cos(
b*x + a)) + ((4*(b*x + a)^3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 4*((b*x + a)^3*d^4 + 3*(b*c*
d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a
))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*
(b*x + a)^2 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a)
)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (4*(b*x + a)^
3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*(b*x + a) + 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-4*
I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*b^2*c
^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan
2(cos(b*x + a), -sin(b*x + a) + 1) - (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 -
8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)
*(b*x + a)^2)*cos(b*x + a) + (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^
2*d^4 + 12*a^2*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a) + 12*(b^2*c^2*d^2 - 2*a
*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2
*b*x + 2*a) - (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*(b*x + a)^2*d^4 -
12*I*a^2*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*di
log(I*e^(I*b*x + I*a)) - (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^
4 + 12*a^2*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a) + 12*(b^2*c^2*d^2 - 2*a*b*c
*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x
 + 2*a) + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*(b*x + a)^2*d^4 + 12*I*
a^2*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I
*e^(I*b*x + I*a)) - (-2*I*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x
 + a)^2 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a) + (-2
I(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2
*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x
+ a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) - (2*I*(b*x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*
(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a) +
(2*I*(b*x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2
*d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 2*((b*x

```
+ a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 - 2*sin(b*x + a) + 1) - 24*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a)
 + d^4)*polylog(4, I*e^(I*b*x + I*a)) + 24*(d^4*cos(2*b*x + 2*a) + I*d^4*si
n(2*b*x + 2*a) + d^4)*polylog(4, -I*e^(I*b*x + I*a)) - (-24*I*b*c*d^3 - 24*
I*(b*x + a)*d^4 + 24*I*a*d^4 + (-24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a
*d^4)*cos(2*b*x + 2*a) + 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2
*a))*polylog(3, I*e^(I*b*x + I*a)) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 2
4*I*a*d^4 + (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4)*cos(2*b*x + 2*
a) - 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -I*e
^(I*b*x + I*a)) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*sin(b*x + a))/(-I*b^4*co
s(2*b*x + 2*a) + b^4*sin(2*b*x + 2*a) - I*b^4))/b
```

Fricas [C] time = 0.784666, size = 2930, normalized size = 12.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^
4 - 12*I*d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 12*I*
d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 12*I*d^4*cos(b
*x + a)*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 12*I*d^4*cos(b*x + a)*
polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (6*I*b^2*d^4*x^2 + 12*I*b^2*c*
d^3*x + 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))
+ (6*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog
(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*
I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b
^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(
b*x + a) - sin(b*x + a)) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^3*
d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + I) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*
x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*
x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x +
 a) - sin(b*x + a) + 1) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*
x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2
```

```
*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x
 + a) - sin(b*x + a) + 1) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
- a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^
3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x
+ a) - I*sin(b*x + a) + I) + 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3,
 I*cos(b*x + a) + sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polyl
og(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*
polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*cos(b*x
 + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/(b^5*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^4 \tan (a+bx) \sec (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \sec (bx+a) \tan (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*tan(b*x + a), x)
```

3.248 $\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=159

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{PolyLog}}{b^4}$$

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(
I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*
PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rubi [A] time = 0.125766, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4409, 4181, 2531, 2282, 6589}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{PolyLog}}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(
I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*
PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\sec(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)^3\sec(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\sec(a+bx)\,dx}{b}\\ &= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\sec(a+bx)}{b}+\frac{\left(6d^2\right)\int (c+dx)\log\left(1-\right.}{b^2}\\ &= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\mathrm{Li}_2\left(ie\right.}{b^3}\\ &= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\mathrm{Li}_2\left(ie\right.}{b^3}\\ &= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\mathrm{Li}_2\left(ie\right.}{b^3}\end{aligned}$$

Mathematica [A] time = 0.854534, size = 256, normalized size = 1.61

$$\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{3d\left(2ibd(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)-2ibd(c+dx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)-2d^2\mathrm{PolyLog}\left(3,-i\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*
(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 +
I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c
+ d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3
, I*E^(I*(a + b*x))]))/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Maple [B] time = 0.274, size = 463, normalized size = 2.9

$$2\,\frac{\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2\,i(bx+a)}+1\right)}-3\,\frac{d^3\ln\left(1-i\mathrm{e}^{i(bx+a)}\right)x^2}{b^2}+\frac{6\,id^3a^2\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^4}+\frac{6\,id^3x\mathrm{polylog}\left(2,i\mathrm{e}^{i(bx+}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))+1)
-3*d^3/b^2*ln(1-I*exp(I*(b*x+a)))*x^2+6*I*d^3/b^4*a^2*arctan(exp(I*(b*x+a))
)+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,I*exp(I*(b*x+a)
))/b^4+6*I*d/b^2*c^2*arctan(exp(I*(b*x+a)))+6*d^2/b^3*c*ln(1+I*exp(I*(b*x+a
)))*a-3*d^3/b^4*a^2*ln(1+I*exp(I*(b*x+a)))-6*I*c*d^2*polylog(2,-I*exp(I*(b*
x+a)))/b^3+6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^2/b^2*c*ln(1-I*exp
(I*(b*x+a)))*x+3*d^3/b^2*ln(1+I*exp(I*(b*x+a)))*x^2-6*I*d^3*x*polylog(2,-I*
exp(I*(b*x+a)))/b^3-6*d^2/b^3*c*ln(1-I*exp(I*(b*x+a)))*a+3*d^3/b^4*a^2*ln(1
-I*exp(I*(b*x+a)))+6*d^2/b^2*c*ln(1+I*exp(I*(b*x+a)))*x-12*I*d^2/b^3*c*a*ar
ctan(exp(I*(b*x+a)))+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4
```

Maxima [B] time = 2.26116, size = 2395, normalized size = 15.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))
*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b) - 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))
*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
)*b^2) + 3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b
*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c
os(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) +
 1))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b^3) + 2*c^3/cos(b*x + a) - 6*a*c^2*d/(b*cos(b*x + a)) + 6*a^2*c*d^2/
(b^2*cos(b*x + a)) - 2*a^3*d^3/(b^3*cos(b*x + a)) + 2*((6*(b*x + a)^2*d^3 +
 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I
*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1
) + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^
3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3
 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*
x + a), -sin(b*x + a) + 1) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*
d^3)*(b*x + a)^2)*cos(b*x + a) + (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3
+ 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 -
12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a))
 - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3
- a*d^3)*cos(2*b*x + 2*a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3
)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) - (-3*I*(b*x + a)^2*d^3 + (-6
*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 +
 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 -
 a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
2*sin(b*x + a) + 1) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x
 + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b
*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-12*I*d
^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, I*e^(I
*b*x + I*a)) - (12*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) + 12*I*
d^3)*polylog(3, -I*e^(I*b*x + I*a)) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2)*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x
 + 2*a) - 2*I*b^3))/b
```

Fricas [C] time = 0.669218, size = 1993, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
a) - sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b
*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*
cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*di
log(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x
+ a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I
) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*lo
g(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a
*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 3
*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*
cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c
*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*
x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b
*x + a) - I*sin(b*x + a) + I))/(b^4*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \tan (a + bx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \sec(bx+a)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a), x)
```

3.249 $\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I
)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + (
(c + d*x)^2*Sec[a + b*x])/b
```

Rubi [A] time = 0.0684212, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4409, 4181, 2279, 2391}

$$-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```


```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I
)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + (
(c + d*x)^2*Sec[a + b*x])/b
```


Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\sec(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)^2\sec(a+bx)}{b}-\frac{(2d)\int (c+dx)\sec(a+bx)\,dx}{b}\\ &= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}+\frac{\left(2d^2\right)\int \log\left(1-ie^{i(a+bx)}\right)}{b^2}\\ &= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{\left(2id^2\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,c\right.}{b^3}\\ &= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{(c+dx}{}\end{aligned}$$

Mathematica [A] time = 1.71315, size = 174, normalized size = 1.79

$$\frac{2d^2\csc(a)\left(i\text{PolyLog}\left(2,-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-i\text{PolyLog}\left(2,e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+\left(bx-\tan^{-1}(\cot(a))\right)\left(\log\left(1-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-\log\left(1+e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)\right)\right)}{\sqrt{\csc^2(a)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(
Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])
)]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a + b*x])/b^3
```

Maple [B] time = 0.135, size = 227, normalized size = 2.3

$$2\frac{e^{i(bx+a)}\left(d^2x^2+2\,cdx+c^2\right)}{b\left(e^{2\,i(bx+a)}+1\right)}+\frac{4\,idc\arctan\left(e^{i(bx+a)}\right)}{b^2}+2\,\frac{d^2\ln\left(1+ie^{i(bx+a)}\right)x}{b^2}+2\,\frac{d^2\ln\left(1+ie^{i(bx+a)}\right)a}{b^3}-2\,\frac{d^2\ln\left(1-i\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)+4*I*d/b^2*c*a
rctan(exp(I*(b*x+a)))+2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1+I*e
xp(I*(b*x+a)))*a-2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*d^2/b^3*ln(1-I*exp(I*
(b*x+a)))*a-2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))+2*I*d^2/b^3*dilog(1-I*exp
(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.595668, size = 1166, normalized size = 12.02

$$\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2+i\,d^2\cos\left(bx+a\right)\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+i\,d^2\cos\left(bx+a\right)\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)-\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x +
 a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a
)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b
*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c*d - a*d^2)*cos(b*x + a
```

```
)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log
(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*
cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos
(b*x + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x
 + a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a)
+ I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*
sin(b*x + a) + I))/(b^3*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \tan(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \sec(bx+a) \tan(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a), x)
```

3.250 $\int (c + dx) \sec(a + bx) \tan(a + bx) \, dx$

Optimal. Leaf size=29

$$\frac{(c + dx)\sec(a + bx)}{b} - \frac{d\tanh^{-1}(\sin(a + bx))}{b^2}$$

```
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Sec[a + b*x])/b
```

Rubi [A] time = 0.0193704, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4409, 3770}

$$\frac{(c + dx)\sec(a + bx)}{b} - \frac{d\tanh^{-1}(\sin(a + bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Sec[a + b*x])/b
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (c + dx)\sec(a + bx)\tan(a + bx)\,dx &= \frac{(c + dx)\sec(a + bx)}{b} - \frac{d\int \sec(a + bx)\,dx}{b} \\ &= -\frac{d\tanh^{-1}(\sin(a + bx))}{b^2} + \frac{(c + dx)\sec(a + bx)}{b} \end{aligned}$$

Mathematica [B] time = 0.0471758, size = 93, normalized size = 3.21

$$\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{d\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)+\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{c\sec(a+bx)}{b}+\frac{dx\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (d*Log[Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2]])/b^2 - (d*Log[Cos[a/2 + (b*
x)/2] + Sin[a/2 + (b*x)/2]])/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b*x])/
b
```

Maple [A] time = 0.022, size = 49, normalized size = 1.7

$$\frac{dx}{b\cos(bx+a)}-\frac{d\ln(\sec(bx+a)+\tan(bx+a))}{b^2}+\frac{c}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 1/b*d/cos(b*x+a)*x-1/b^2*d*ln(sec(b*x+a)+tan(b*x+a))+1/b*c/cos(b*x+a)
```

Maxima [B] time = 1.49983, size = 350, normalized size = 12.07

$$\frac{\frac{\left(4\,(bx+a)\cos(2\,bx+2\,a)\cos(bx+a)+4\,(bx+a)\sin(2\,bx+2\,a)\sin(bx+a)+4\,(bx+a)\cos(bx+a)-\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos(bx+a)^2+\right.}{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\right.}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2
*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*d
```

```
/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) + 2
*c/cos(b*x + a) - 2*a*d/(b*cos(b*x + a)))/b
```

Fricas [B] time = 0.492755, size = 163, normalized size = 5.62

$$\frac{2\,bdx - d\cos\left(bx + a\right)\log\left(\sin\left(bx + a\right) + 1\right) + d\cos\left(bx + a\right)\log\left(-\sin\left(bx + a\right) + 1\right) + 2\,bc}{2\,b^{2}\cos\left(bx + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-s
in(b*x + a) + 1) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)\tan\left(a + bx\right)\sec\left(a + bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)*sec(a + b*x), x)
```

Giac [B] time = 1.68498, size = 2075, normalized size = 71.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2*tan(1/2*a)^
2 + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x
)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2
*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*t
```

an(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*b*x)^2 + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)^2 - 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*a)^2 + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*a)^2 + 2*b*d*x + 2*b*c + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)) - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)))/(b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*tan(1/2*b*x)^2 - 4*b^2*tan(1/2*b*x)*tan(1/2*a) - b^2*tan(1/2*a)^2 + b^2)

3.251 $\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Rubi [A] time = 0.0940087, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 11.2622, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Maple [A] time = 0.268, size = 0, normalized size = 0.

$$\int \frac{\sec (bx+a) \tan (bx+a)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec (bx+a) \tan (bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan (a+bx) \sec (a+bx)}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

3.252 $\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi [A] time = 0.114617, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]

[Out] Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 19.3385, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]

[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.32, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec(bx+a)\tan(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c)^2, x)
```

3.253 $\int (c+dx)^m \tan^2(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\tan^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Tan[a + b*x]^2, x]
```

Rubi [A] time = 0.0357392, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x]^2,x]

[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (c+dx)^m \tan^2(a+bx)\,dx$$

Mathematica [A] time = 3.14041, size = 0, normalized size = 0.

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x]^2,x]

[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^2, x]
```

Maple [A] time = 0.166, size = 0, normalized size = 0.

$$\int (dx+c)^m (\tan(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*tan(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*tan(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \tan(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*tan(b*x + a)^2, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \tan^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

3.254 $\int (c+dx)^3 \tan^2(a+bx)\,dx$

Optimal. Leaf size=128

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tan(a+b}{b}$$

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) + (3*d*(c + d*x)^2*Log[1 + E^((2*I
)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])
/b^3 + (3*d^3*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*Tan[
a + b*x])/b
```

Rubi [A] time = 0.210269, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3719, 2190, 2531, 2282, 6589, 32}

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tan(a+b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) + (3*d*(c + d*x)^2*Log[1 + E^((2*I
)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])
/b^3 + (3*d^3*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*Tan[
a + b*x])/b
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
```

```
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\tan(a+bx)\,dx}{b} - \int (c+dx)^3\,dx$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{(c+dx)^3\tan(a+bx)}{b} + \frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{(6d^2)\,\int}{}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \ldots$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \ldots$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \ldots$$

Mathematica [B] time = 6.56442, size = 424, normalized size = 3.31

$$\frac{3cd^2\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\text{PolyLog}\left(2,e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan\ldots\right)}\right)\ldots\right)}{\sqrt{\cot^2(a)+1}}\ldots\right)}{b^3\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x]^2,x]
```

```
[Out] -(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + ((I/4)*d^3*(2*b^2*x^2*
(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 +
E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*
PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^4*E^(I*a)) + (3*c^2*d*Sec[a]*
(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a]^
2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*
(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - Arc
Tan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] -
 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b
*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[Csc[a]^2*(Co
s[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(c^3*Sin[b*x] + 3*c^2*d*x*Sin[b
*x] + 3*c*d^2*x^2*Sin[b*x] + d^3*x^3*Sin[b*x]))/b
```

Maple [B] time = 0.197, size = 348, normalized size = 2.7

$$-\frac{d^3x^4}{4} - cd^2x^3 - \frac{3\,c^2dx^2}{2} - c^3x - \frac{3\,id^3\mathrm{polylog}\left(2, -\mathrm{e}^{2\,i(bx+a)}\right)x}{b^3} + 3\,\frac{c^2d\ln\left(\mathrm{e}^{2\,i(bx+a)}+1\right)}{b^2} - 6\,\frac{c^2d\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^2} - 6\,\frac{d^3a^2\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tan(b*x+a)^2,x)
```

```
[Out] -1/4*d^3*x^4-c*d^2*x^3-3/2*c^2*d*x^2-c^3*x-3*I*d^3/b^3*polylog(2,-exp(2*I*(
b*x+a)))*x+3*d/b^2*c^2*ln(exp(2*I*(b*x+a))+1)-6*d/b^2*c^2*ln(exp(I*(b*x+a))
)-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))-3*I*d^2/b^3*c*polylog(2,-exp(2*I*(b*x+a)
))+6*I*d^3/b^3*a^2*x-6*I*d^2/b^3*c*a^2+2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c
^3)/b/(exp(2*I*(b*x+a))+1)+3*d^3/b^2*ln(exp(2*I*(b*x+a))+1)*x^2-2*I*d^3/b*x
^3+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+12*d^2/b^3*c*a*ln(exp(I*(b*x+a)
))+6*d^2/b^2*c*ln(exp(2*I*(b*x+a))+1)*x+4*I*d^3/b^4*a^3-12*I*d^2/b^2*c*a*x-
6*I*d^2/b*c*x^2
```

Maxima [B] time = 1.93025, size = 1840, normalized size = 14.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a - tan(b*x + a))*c^3 - 6*(b*x + a - tan(b*x + a))*a*c^2*d/b
 + 6*(b*x + a - tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a - tan(b*x + a))*a^
3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a
)^2 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 +
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^2
*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) -
 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*
x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^2/((cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^2) + 3*((b*
x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)^
2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*
```

```
cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x +
 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(I*(b*x + a
)^4*d^3 + (4*I*b*c*d^2 - 4*I*a*d^3)*(b*x + a)^3 + (12*(b*x + a)^2*d^3 + 24*
(b*c*d^2 - a*d^3)*(b*x + a) + 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*
x + a))*cos(2*b*x + 2*a) + (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a)
 + 1) + (I*(b*x + a)^4*d^3 + (4*I*b*c*d^2 - 4*(I*a + 2)*d^3)*(b*x + a)^3 -
24*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) - (12*b*c*d^2 + 12*(b*x
+ a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a)
 - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilo
g(-e^(2*I*b*x + 2*I*a)) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d
^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*
x + 2*a) + 1) + (-6*I*d^3*cos(2*b*x + 2*a) + 6*d^3*sin(2*b*x + 2*a) - 6*I*d
^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) - ((b*x + a)^4*d^3 + (4*b*c*d^2 - (4*a
 - 8*I)*d^3)*(b*x + a)^3 - (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a)^2)*sin(2*
b*x + 2*a))/(-4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(2*b*x + 2*a) - 4*I*b^3))
/b
```

Fricas [C] time = 0.502994, size = 913, normalized size = 7.13

$$-\frac{b^4d^3x^4+4\,b^4cd^2x^3+6\,b^4c^2dx^2+4\,b^4c^3x-3\,d^3\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)-3\,d^3\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2-2i\,\tan(bx\cdots}{\tan(bx+a)^2+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x - 3*d^3
*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) -
 3*d^3*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 +
 1)) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x +
a)^2 + 1) + 1) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(2*(-I*tan(b*x + a) - 1)
/(tan(b*x + a)^2 + 1) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*lo
g(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^3*x^2 + 2*b^2*c*
d^2*x + b^2*c^2*d)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 4*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tan(b*x + a)^2, x)
```

3.255 $\int (c+dx)^2 \tan^2(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{id^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) + (2*d*(c + d*x)*Log[1 + E^((2*I)*
(a + b*x))])/b^2 - (I*d^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + ((c + d*x
)^2*Tan[a + b*x])/b
```

Rubi [A] time = 0.141087, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3720, 3719, 2190, 2279, 2391, 32}

$$-\frac{id^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) + (2*d*(c + d*x)*Log[1 + E^((2*I)*
(a + b*x))])/b^2 - (I*d^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + ((c + d*x
)^2*Tan[a + b*x])/b
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^2\tan^2(a+bx)\,dx = \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{(2d)\int (c+dx)\tan(a+bx)\,dx}{b} - \int (c+dx)^2\,dx$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\tan(a+bx)}{b} + \frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{\left(2d^2\right)\ldots}{\ldots}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+bx)}{b} + \frac{\left(id^2\right)\ldots}{\ldots}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\ldots}{\ldots}$$

Mathematica [B] time = 6.38096, size = 276, normalized size = 2.88

$$\frac{d^2\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))}-\frac{\cot(a)\left(i\text{PolyLog}\left(2,e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^3\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x]^2,x]
```

```
[Out] -(x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 + (2*c*d*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b
*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (d^2*Cs
c[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a
]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I
)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*
x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqr
t[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[
a]*Sec[a + b*x]*(c^2*Sin[b*x] + 2*c*d*x*Sin[b*x] + d^2*x^2*Sin[b*x]))/b
```

Maple [B] time = 0.162, size = 191, normalized size = 2.

$$-\frac{d^2x^3}{3}-cdx^2-c^2x+\frac{2i\left(d^2x^2+2cdx+c^2\right)}{b\left(e^{2i(bx+a)}+1\right)}-4\frac{cd\ln\left(e^{i(bx+a)}\right)}{b^2}+2\frac{cd\ln\left(e^{2i(bx+a)}+1\right)}{b^2}-\frac{2id^2x^2}{b}-\frac{4id^2ax}{b^2}-\frac{2id^2a^2}{b^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tan(b*x+a)^2,x)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2-c^2*x+2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)
-4*d/b^2*c*ln(exp(I*(b*x+a)))+2*d/b^2*c*ln(exp(2*I*(b*x+a))+1)-2*I*d^2/b*x^
2-4*I*d^2/b^2*a*x-2*I*d^2/b^3*a^2+2*d^2/b^2*ln(exp(2*I*(b*x+a))+1)*x-I*d^2*
polylog(2,-exp(2*I*(b*x+a)))/b^3+4*d^2/b^3*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 1.84466, size = 563, normalized size = 5.86

$$\frac{ib^3d^2x^3+3ib^3cdx^2+3ib^3c^2x+6b^2c^2+\left(6bd^2x+6bcd+6\left(bd^2x+bcd\right)\cos(2bx+2a)+\left(6ibd^2x+6ibcd\right)\sin(2bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (I*b^3*d^2*x^3 + 3*I*b^3*c*d*x^2 + 3*I*b^3*c^2*x + 6*b^2*c^2 + (6*b*d^2*x +
 6*b*c*d + 6*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (6*I*b*d^2*x + 6*I*b*c*d)
*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (I*b^3
*d^2*x^3 + (3*I*b^3*c*d - 6*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*co
s(2*b*x + 2*a) - 3*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*di
log(-e^(2*I*b*x + 2*I*a)) + (-3*I*b*d^2*x - 3*I*b*c*d + (-3*I*b*d^2*x - 3*I
*b*c*d)*cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b^3*d^2*x^3
+ 3*(b^3*c*d + 2*I*b^2*d^2)*x^2 + (3*b^3*c^2 + 12*I*b^2*c*d)*x)*sin(2*b*x +
 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) - 3*I*b^3)
```

Fricas [B] time = 0.490464, size = 522, normalized size = 5.44

$$-\frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,b^3c^2x-3i\,d^2\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+3i\,d^2\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-6\left(bd^2x+bcd\right)\log\left(-\right.}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*I*d^2*dilog(2*(I*tan(
b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 3*I*d^2*dilog(2*(-I*tan(b*x + a)
- 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(b*d^2*x + b*c*d)*log(-2*(I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1)) - 6*(b*d^2*x + b*c*d)*log(-2*(-I*tan(b*x + a) -
 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*tan(b*x
 + a))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2\tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \tan (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tan(b*x + a)^2, x)
```

3.256 $\int (c+dx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-cx-\frac{dx^2}{2}$$

```
[Out] -(c*x) - (d*x^2)/2 + (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

Rubi [A] time = 0.0283952, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3720, 3475}

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-cx-\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x]^2,x]
```

```
[Out] -(c*x) - (d*x^2)/2 + (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\tan^2(a+bx)\,dx &= \frac{(c+dx)\tan(a+bx)}{b}-\frac{d\int \tan(a+bx)\,dx}{b}-\int (c+dx)\,dx\\ &= -cx-\frac{dx^2}{2}+\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.265031, size = 76, normalized size = 1.9

$$\frac{d\log(\cos(a+bx))}{b^2}-\frac{c\tan^{-1}(\tan(a+bx))}{b}+\frac{c\tan(a+bx)}{b}+\frac{dx\sec(a)\sin(bx)\sec(a+bx)}{b}-\frac{dx\sec(a)(bx\cos(a)-2\sin}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Tan[a + b*x]^2,x]
```

```
[Out] -((c*ArcTan[Tan[a + b*x]])/b) + (d*Log[Cos[a + b*x]])/b^2 - (d*x*Sec[a]*(b*
x*Cos[a] - 2*Sin[a]))/(2*b) + (d*x*Sec[a]*Sec[a + b*x]*Sin[b*x])/b + (c*Tan
[a + b*x])/b
```

Maple [A] time = 0.044, size = 47, normalized size = 1.2

$$-\frac{dx^2}{2}-cx+\frac{d\tan(bx+a)x}{b}+\frac{d\ln(\cos(bx+a))}{b^2}+\frac{c\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tan(b*x+a)^2,x)
```

```
[Out] -1/2*d*x^2-c*x+1/b*d*tan(b*x+a)*x+d*ln(cos(b*x+a))/b^2+1/b*c*tan(b*x+a)
```

Maxima [B] time = 1.4674, size = 320, normalized size = 8.

$$-\frac{2(bx+a-\tan(bx+a))c-\frac{2(bx+a-\tan(bx+a))ad}{b}+\frac{\left((bx+a)^2\cos(2bx+2a)^2+(bx+a)^2\sin(2bx+2a)^2+2(bx+a)^2\cos(2bx+2a)+(bx+a)^2-(\cos(2bx\cdots\right.}{(\cos(2bx+\cdots}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a - tan(b*x + a))*c - 2*(b*x + a - tan(b*x + a))*a*d/b + ((b
*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)
^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)
```

```
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

Fricas [A] time = 0.471681, size = 131, normalized size = 3.28

$$-\frac{b^2dx^2 + 2\,b^2cx - d\log\left(\frac{1}{\tan(bx+a)^2+1}\right) - 2\,(bdx+bc)\tan(bx+a)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*x^2 + 2*b^2*c*x - d*log(1/(tan(b*x + a)^2 + 1)) - 2*(b*d*x + b*
c)*tan(b*x + a))/b^2
```

Sympy [A] time = 0.372713, size = 65, normalized size = 1.62

$$\begin{cases} -cx - \frac{dx^2}{2} + \frac{c\tan(a+bx)}{b} + \frac{dx\tan(a+bx)}{b} - \frac{d\log\left(\tan^2(a+bx)+1\right)}{2b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\tan^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)**2,x)
```

```
[Out] Piecewise((-c*x - d*x**2/2 + c*tan(a + b*x)/b + d*x*tan(a + b*x)/b - d*log(
tan(a + b*x)**2 + 1)/(2*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*tan(a)**2, True
))
```

Giac [B] time = 1.40319, size = 301, normalized size = 7.52

$$-\frac{b^2dx^2\tan(bx)\tan(a) + 2\,b^2cx\tan(bx)\tan(a) - b^2dx^2 - 2\,b^2cx + 2\,bdx\tan(bx) + 2\,bdx\tan(a) - d\log\left(\frac{}{\tan(bx)^4\tan(a)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*x^2*tan(b*x)*tan(a) + 2*b^2*c*x*tan(b*x)*tan(a) - b^2*d*x^2 - 2
*b^2*c*x + 2*b*d*x*tan(b*x) + 2*b*d*x*tan(a) - d*log(4*(tan(a)^2 + 1)/(tan(
b*x)^4*tan(a)^2 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + tan(b*x)^2 -
2*tan(b*x)*tan(a) + 1))*tan(b*x)*tan(a) + 2*b*c*tan(b*x) + 2*b*c*tan(a) + d
*log(4*(tan(a)^2 + 1)/(tan(b*x)^4*tan(a)^2 - 2*tan(b*x)^3*tan(a) + tan(b*x)
^2*tan(a)^2 + tan(b*x)^2 - 2*tan(b*x)*tan(a) + 1)))/(b^2*tan(b*x)*tan(a) -
b^2)
```

3.257 $\int \frac{\tan^2(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\tan^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Tan[a + b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0398115, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan^2(a+bx)}{c+dx} \, dx = \int \frac{\tan^2(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 3.95696, size = 0, normalized size = 0.

$$\int \frac{\tan^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Tan[a + b*x]^2/(c + d*x), x]
```

Maple [A] time = 0.264, size = 0, normalized size = 0.

$$\int \frac{(\tan (bx + a))^2}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(tan(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\tan (bx + a)^2}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2 (a + bx)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^2/(d*x + c), x)
```

3.258 $\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\tan^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Tan[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0385174, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx = \int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 5.26563, size = 0, normalized size = 0.

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Tan[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A] time = 0.324, size = 0, normalized size = 0.

$$\int \frac{(\tan (bx + a))^2}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(tan(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan (bx + a)^2}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2 (a + bx)}{(c + dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan{(bx+a)}^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^2/(d*x + c)^2, x)
```

3.259 $\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=149

$$\text{CannotIntegrate}\left(\tan(a+bx)\sec(a+bx)(c+dx)^m,x\right)+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+$$

```
[Out] CannotIntegrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x] + (E^(I*(a - (b*c
)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(((-I)*b*(c + d*
x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2*b*E^(I*(a - (b
*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rubi [A] time = 0.157146, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(
((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2
*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Sec[
a + b*x]*Tan[a + b*x], x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx &= -\int (c+dx)^m \sin(a+bx)\,dx+\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx\\
&= -\left(\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx\right)+\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx)^m \sec(\\
&= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+}{d}\right.}{2}
\end{aligned}$$

Mathematica [A] time = 5.09666, size = 0, normalized size = 0.

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2, x]
```

Maple [A] time = 0.17, size = 0, normalized size = 0.

$$\int (dx+c)^m \sin(bx+a) (\tan(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sin(bx+a) \tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \sin(bx+a) \tan(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sin (bx+a) \tan (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

3.260 $\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx$

Optimal. Leaf size=228

$$-\frac{6id^2(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d^3\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\mathrm{PolyLog}\left(3\right.}{b^4}$$

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*Cos[a
+ b*x])/b^3 + ((c + d*x)^3*Cos[a + b*x])/b - ((6*I)*d^2*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog
[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b + (6*d^3*Sin[a +
 b*x])/b^4 - (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.21102, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4407, 3296, 2637, 4409, 4181, 2531, 2282, 6589}

$$-\frac{6id^2(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d^3\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\mathrm{PolyLog}\left(3\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```


```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*Cos[a
+ b*x])/b^3 + ((c + d*x)^3*Cos[a + b*x])/b - ((6*I)*d^2*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog
[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b + (6*d^3*Sin[a +
 b*x])/b^4 - (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```


Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx = -\int (c+dx)^3 \sin(a+bx)\,dx + \int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\cos(a+bx)\,dx}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} -$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(a+bx)}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(a+bx)}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(a+bx)}{b}$$

Mathematica [B] time = 1.60966, size = 532, normalized size = 2.33

$$\frac{\sec(a+bx)\left(-12ibd^2(c+dx)\cos(a+bx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)+12ibd^2(c+dx)\cos(a+bx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)+12d^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x]*(3*b^3*c^3 - 6*b*c*d^2 + 9*b^3*c^2*d*x - 6*b*d^3*x + 9*b^3*c*
d^2*x^2 + 3*b^3*d^3*x^3 + (12*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))]*Cos[a +
b*x] + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a + b*x)] + 3*b^3*c^2*d*
x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3*c*d^2*x^2*Cos[2*(a
+ b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] - 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1
- I*E^(I*(a + b*x))] - 6*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*E^(I*(a + b*x))
] + 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] + 6*b^2*d^3*x^2*
Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] - (12*I)*b*d^2*(c + d*x)*Cos[a + b*
x]*PolyLog[2, (-I)*E^(I*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Cos[a + b*x]*P
olyLog[2, I*E^(I*(a + b*x))] + 12*d^3*Cos[a + b*x]*PolyLog[3, (-I)*E^(I*(a
+ b*x))] - 12*d^3*Cos[a + b*x]*PolyLog[3, I*E^(I*(a + b*x))] - 3*b^2*c^2*d*
Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a + b*x)] - 6*b^2*c*d^2*x*Sin[2*(a + b*x)]
- 3*b^2*d^3*x^2*Sin[2*(a + b*x)]))/(2*b^4)
```

Maple [B] time = 0.296, size = 677, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*
d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))+1
/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d
^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))+2
*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))+1)-
3*d^3/b^4*a^2*ln(1+I*exp(I*(b*x+a)))-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))
/b^3+6*d^2/b^3*c*ln(1+I*exp(I*(b*x+a)))*a+6*d^2/b^2*c*ln(1+I*exp(I*(b*x+a))
)*x-6*d^2/b^2*c*ln(1-I*exp(I*(b*x+a)))*x+3*d^3/b^2*ln(1+I*exp(I*(b*x+a)))*x
^2-6*d^2/b^3*c*ln(1-I*exp(I*(b*x+a)))*a-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a
)))/b^3-3*d^3/b^2*ln(1-I*exp(I*(b*x+a)))*x^2+3*d^3/b^4*a^2*ln(1-I*exp(I*(b*
x+a)))+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,I*exp(I*(b
*x+a)))/b^4+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4+6*I*d^3/b^4*a^2*arctan(e
xp(I*(b*x+a)))+6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I*d/b^2*c^2*arct
an(exp(I*(b*x+a)))-12*I*d^2/b^3*c*a*arctan(exp(I*(b*x+a)))
```

Maxima [B] time = 6.93555, size = 14923, normalized size = 65.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*(1/cos(b*x + a) + cos(b*x + a)) - 6*a*c^2*d*(1/cos(b*x + a) + co
s(b*x + a))/b + 6*a^2*c*d^2*(1/cos(b*x + a) + cos(b*x + a))/b^2 - 2*a^3*d^3
*(1/cos(b*x + a) + cos(b*x + a))/b^3 + 3*((b*x + (b*x + a)*cos(2*b*x + 2*a)
 + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 +
((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*
(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos
(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x
+ a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(
b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) +
```

```
(b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)
 + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x +
a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x +
 a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13
*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*
x + a)^2 + b*x + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 +
2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + c
os(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*
(3*(b*x + a)*cos(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*
x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x +
a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*co
s(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x
 + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2
*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 +
sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*s
in(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x
+ a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
 a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos
(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(
b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*
x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) +
 sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
+ (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 +
 12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(
b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*
b*x + 2*a) + (b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*
(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*
cos(2*b*x + 2*a) - cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x
 + a)^2 + 13*(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^
2 - sin(b*x + a)^2 - 1)*sin(3*b*x + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(
b*x + a) + (b*x + a)*sin(b*x + a)^3)*sin(2*b*x + 2*a) - sin(b*x + a))*c^2*d
/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x +
3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*
x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin
(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(
```

```
b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a
) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b) - 6*((b*x + (b*x +
a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a
)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(
3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*c
os(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(
b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*
cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x +
2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a
))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x
 + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + (
(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(
2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2
 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a
)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x +
a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*co
s(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*s
in(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (
b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos
(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x
+ 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2
*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(c
os(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*
b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + ((cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (
cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x
 + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*
x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a)
+ cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(
2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*
b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))
*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2
*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*c
os(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + 2*(((b*x + a)*
sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)*cos(b*x + a) + s
in(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(
3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - s
in(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 +
 ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a)
- sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b*x + 3*a) + 6*((b*x + a)*
```

```
cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x + a)^3)*sin(2*b*x + 2*a) -
sin(b*x + a))*a*c*d^2/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*
b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(
2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x +
 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*
(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(co
s(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b
*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b
^2) + 3*((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*
x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - cos
(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x +
a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x +
 a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x
 + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8
*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a
) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a) -
sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a))
*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2
*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*x
+ a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*co
s(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (
12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)*s
in(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3*(
b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x + a
))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^
2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin
(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(
b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a
))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)
+ cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*
sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x +
3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a)
 + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2
+ (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*
b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(co
s(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2
 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^
2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b
*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)
```

```
^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a) -
cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(b*
x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((b*
x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x +
a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x +
a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x + 2*
a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x +
a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b*x
 + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x + a)
^3)*sin(2*b*x + 2*a) - sin(b*x + a))*a^2*d^3/(((cos(2*b*x + 2*a)^2 + sin(2*
b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2
 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(
b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b
*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a)
)*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) +
 cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*s
in(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3
*a) + sin(b*x + a)^2)*b^3) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a + 6*I)*d
^3 + (3*b*c*d^2 - (3*a + 3*I)*d^3)*(b*x + a)^2 + ((b*x + a)^3*d^3 - 6*b*c*d
^2 + (6*a - 6*I)*d^3 + (3*b*c*d^2 - (3*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c
*d^2 - 6*(I*a + 1)*d^3)*(b*x + a))*cos(3*b*x + 3*a)^2 + 6*((b*x + a)^3*d^3
- 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*
cos(b*x + a)^2 - ((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a - 6*I)*d^3 + (3*b*c*d^
2 - (3*a - 3*I)*d^3)*(b*x + a)^2 - (-6*I*b*c*d^2 - 6*(-I*a - 1)*d^3)*(b*x +
 a))*sin(3*b*x + 3*a)^2 + (12*I*(b*x + a)^3*d^3 - 24*I*b*c*d^2 - 24*I*(b*x
+ a)*d^3 + 24*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2)*cos(b*x +
a)*sin(b*x + a) - 6*((b*x + a)^3*d^3 - 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*sin(b*x + a)^2 + (-6*I*b*c*d^2 - 6*(-I
*a + 1)*d^3)*(b*x + a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3
)*(b*x + a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))
*cos(2*b*x + 2*a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin
(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 1
2*I*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^
3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^3 + (12*I
*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - (6*(b*x + a)^2*d^3 + 12*(b
*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (6*((b*x + a)^2*d^3 + 2*(
b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) - 6*((b*x +
a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*arctan2(cos(b*x + a
), sin(b*x + a) + 1) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*
(b*x + a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*c
os(2*b*x + 2*a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2
```

*b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) + ((7*(b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c*d^2 - 6*(I*a + 3)*d^3)*(b*x + a))*cos(b*x + a) + (7*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 - 6*(-3*I*a - 1)*d^3 + (21*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x + a)^2 - (6*b*c*d^2 - (6*a - 18*I)*d^3)*(b*x + a))*sin(b*x + a))*cos(3*b*x + 3*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a + 6*I)*d^3 + (3*b*c*d^2 - (3*a + 3*I)*d^3)*(b*x + a)^2 + (-6*I*b*c*d^2 - 6*(-I*a + 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(b*x + a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*cos(2*b*x + 2*a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(b*x + a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(b*x + a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(b*x + a))*sin(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*dilog(I*e^(I*b*x + I*a)) + ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(b*x + a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(b*x + a) + (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) + (12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(b*x + a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(b*x + a))*sin(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*dilog(-I*e^(I*b*x + I*a)) - ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) +

```
(-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(2*b*x +
2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*sin(3*b*x + 3*a) - ((-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b
*x + a))*cos(b*x + a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*sin(b*x + a))*sin(2*b*x + 2*a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6
*I*a*d^3)*(b*x + a))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
sin(b*x + a) + 1) + ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a) + 3
*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (3*I*
(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*co
s(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b
*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*sin(b
*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
 a))*cos(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x +
 a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2
*a))*sin(3*b*x + 3*a) + ((3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(
b*x + a))*cos(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*sin(b*x + a))*sin(2*b*x + 2*a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*
I*a*d^3)*(b*x + a))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*s
in(b*x + a) + 1) - (12*d^3*cos(b*x + a) + 12*I*d^3*sin(b*x + a) + 12*(d^3*c
os(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*cos(3*b*x + 3*a) + 12*(d^3*
cos(b*x + a) + I*d^3*sin(b*x + a))*cos(2*b*x + 2*a) - (-12*I*d^3*cos(2*b*x
+ 2*a) + 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*sin(3*b*x + 3*a) - (-12*I*d^3*
cos(b*x + a) + 12*d^3*sin(b*x + a))*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x
 + I*a)) + (12*d^3*cos(b*x + a) + 12*I*d^3*sin(b*x + a) + 12*(d^3*cos(2*b*x
 + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*cos(3*b*x + 3*a) + 12*(d^3*cos(b*x
+ a) + I*d^3*sin(b*x + a))*cos(2*b*x + 2*a) + (12*I*d^3*cos(2*b*x + 2*a) -
12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*sin(3*b*x + 3*a) + (12*I*d^3*cos(b*x +
a) - 12*d^3*sin(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x + I*a))
+ ((2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 - 12*(-I*a - 1)*d^3 + (6*I*b*c*d^2 -
 6*(I*a + 1)*d^3)*(b*x + a)^2 - (12*b*c*d^2 - (12*a - 12*I)*d^3)*(b*x + a))
*cos(3*b*x + 3*a) + (7*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 - 6*(-3*I*a - 1)*d^
3 + (21*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x + a)^2 - (6*b*c*d^2 - (6*a - 18
*I)*d^3)*(b*x + a))*cos(b*x + a) - (7*(b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a
- 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 - (-6*I*b*c*d^2 -
6*(-I*a - 3)*d^3)*(b*x + a))*sin(b*x + a))*sin(3*b*x + 3*a) + (I*(b*x + a)^
3*d^3 - 6*I*b*c*d^2 - 6*(-I*a + 1)*d^3 + (3*I*b*c*d^2 - 3*(I*a - 1)*d^3)*(b
*x + a)^2 + (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(2*b
^3*cos(b*x + a) + 2*I*b^3*sin(b*x + a) + (2*b^3*cos(2*b*x + 2*a) + 2*I*b^3*
sin(2*b*x + 2*a) + 2*b^3)*cos(3*b*x + 3*a) + (2*b^3*cos(b*x + a) + 2*I*b^3*
sin(b*x + a))*cos(2*b*x + 2*a) - (-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b
*x + 2*a) - 2*I*b^3)*sin(3*b*x + 3*a) + 2*(I*b^3*cos(b*x + a) - b^3*sin(b*x
 + a))*sin(2*b*x + 2*a)))/b
```

Fricas [C] time = 0.745178, size = 2237, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
a) - sin(b*x + a)) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2
 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + (6*I*b*d^3*x + 6*I*b*c*d^2)*
cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*
d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*
I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3
*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x
 + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x
 + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2
- a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3
*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x +
a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2
*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a)
- sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^3*x^2 +
2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*x
+ a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sin(a + b*x)*tan(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \sin(bx+a)\tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sin(b*x + a)*tan(b*x + a)^2, x)
```

3.261 $\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx) \, dx$

Optimal. Leaf size=145

$$-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d^2 \mathrm{c}}{}$$

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*d^2*Cos[a + b*x])/b^3
+ ((c + d*x)^2*Cos[a + b*x])/b - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))
])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + ((c + d*x)^2*Sec[a
 + b*x])/b - (2*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.131128, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4407, 3296, 2638, 4409, 4181, 2279, 2391}

$$-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d^2 \mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```


```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*d^2*Cos[a + b*x])/b^3
+ ((c + d*x)^2*Cos[a + b*x])/b - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))
])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + ((c + d*x)^2*Sec[a
 + b*x])/b - (2*d*(c + d*x)*Sin[a + b*x])/b^2
```


Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2 \sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^2 \sin(a+bx)\,dx + \int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int (c+dx)\cos(a+bx)\,dx}{b}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} -$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(c+d}{}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b} - \frac{2id^2\mathrm{L}}{}$$

Mathematica [B] time = 3.11789, size = 362, normalized size = 2.5

$$\frac{2d^2\csc(a)\left(i\text{PolyLog}\left(2,-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-i\text{PolyLog}\left(2,e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+\left(bx-\tan^{-1}(\cot(a))\right)\left(\log\left(1-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-\log\left(1+e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)\right)\right)}{\sqrt{\csc^2(a)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(
Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])
)]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a] + Cos[b*x]*((
-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*
x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + (b^2*(c + d*x)^2*
Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2]))
 - (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2]
+ Sin[(a + b*x)/2])))/b^3
```

Maple [B] time = 0.158, size = 345, normalized size = 2.4

$$\frac{\left(d^2x^2b^2+2\,b^2cdx+b^2c^2+2\,ibd^2x-2\,d^2+2\,ibcd\right)e^{i(bx+a)}}{2\,b^3}+\frac{\left(d^2x^2b^2+2\,b^2cdx+b^2c^2-2\,ibd^2x-2\,d^2-2\,ibcd\right)e^{-i(bx+a)}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I
*(b*x+a))+1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)
/b^3*exp(-I*(b*x+a))+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x
+a))+1)+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))+2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))
*x+2*d^2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*
d^2/b^3*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))+2*I*
d^2/b^3*dilog(1-I*exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.637048, size = 1316, normalized size = 9.08

$$\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,\cos(bx+a) + \sin(bx+a)) + i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,\cos(bx+a) - \sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x +
 a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a
)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b
*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*
x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) -
```

```
sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + si
n(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(
b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a)
 + I) - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \sin(a + bx) \tan^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)*tan(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sin(bx + a) \tan(bx + a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

3.262 $\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=56

$$-\frac{d\sin(a+bx)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

```
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Cos[a + b*x])/b + ((c + d*x)*
Sec[a + b*x])/b - (d*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.0537627, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4407, 3296, 2637, 4409, 3770}

$$-\frac{d\sin(a+bx)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```


```
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Cos[a + b*x])/b + ((c + d*x)*
Sec[a + b*x])/b - (d*Sin[a + b*x])/b^2
```


Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)\sin(a+bx)\,dx + \int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a+bx)}{b} - \frac{d\int\cos(a+bx)\,dx}{b} - \frac{d\int\sec(a+ \ldots}{b}$$
$$= -\frac{d\tanh^{-1}(\sin(a+bx))}{b^2} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a+bx)}{b} - \frac{d\sin(a \ldots}{b \ldots}$$

Mathematica [A] time = 0.316159, size = 107, normalized size = 1.91

$$\frac{\sec(a+bx)\left(b(c+dx)\cos(2(a+bx)) - d\sin(2(a+bx)) + 2d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right) - \sin\left(\frac{1}{2}(a+bx)\right)\right) - \log\left(\ldots\right.\right.\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x]*(3*b*c + 3*b*d*x + b*(c + d*x)*Cos[2*(a + b*x)] + 2*d*Cos[a +
 b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] + Si
n[(a + b*x)/2]]) - d*Sin[2*(a + b*x)]))/(2*b^2)
```

Maple [C] time = 0.129, size = 123, normalized size = 2.2

$$\frac{(dxb+bc+id)\,e^{i(bx+a)}}{2\,b^2} + \frac{(dxb+bc-id)\,e^{-i(bx+a)}}{2\,b^2} + 2\,\frac{e^{i(bx+a)}\,(dx+c)}{b\left(e^{2\,i(bx+a)}+1\right)} - \frac{d\ln\left(e^{i(bx+a)}+i\right)}{b^2} + \frac{d\ln\left(e^{i(bx+a)}-i\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 1/2*(d*x*b+b*c+I*d)/b^2*exp(I*(b*x+a))+1/2*(d*x*b+b*c-I*d)/b^2*exp(-I*(b*x+
a))+2*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))+1)-d/b^2*ln(exp(I*(b*x+a))
+I)+d/b^2*ln(exp(I*(b*x+a))-I)
```

Maxima [B] time = 1.76264, size = 2866, normalized size = 51.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*(1/cos(b*x + a) + cos(b*x + a)) - 2*a*d*(1/cos(b*x + a) + cos(b*x
+ a))/b + ((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*
b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - c
os(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x
+ a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x
 + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b
*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 +
(8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2
*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)
- sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a
))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x +
 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*
x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*
cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) +
 (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)
*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3
*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x +
 a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a
)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + co
s(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x +
 a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a
) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^
2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
```

```
+ 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x +
a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) +
1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^
2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(
3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(
cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)
^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a
)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin
(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a)
- cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(
b*x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((
b*x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x
+ a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x
+ a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x +
2*a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x
+ a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b
*x + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x +
a)^3)*sin(2*b*x + 2*a) - sin(b*x + a))*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
+ a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
 a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*co
s(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos
(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b
*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a)
+ sin(b*x + a)^2)*b))/b
```

Fricas [A] time = 0.511821, size = 251, normalized size = 4.48

$$\frac{2\,bdx + 2\,(bdx + bc)\cos\left(bx + a\right)^2 - d\cos\left(bx + a\right)\log\left(\sin\left(bx + a\right) + 1\right) + d\cos\left(bx + a\right)\log\left(-\sin\left(bx + a\right) + 1\right) - 2\,d\cos}{2\,b^2\cos\left(bx + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x + 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
+ a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 2*d*cos(b*x + a)*sin(b*
x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)*tan(a + b*x)**2, x)
```

Giac [B] time = 2.52883, size = 3729, normalized size = 66.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b*c*tan(1/2*b*x)^4*tan(1/2*a)^
4 + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x
)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2
*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*t
an(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*
b*x)^4*tan(1/2*a)^4 - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)
^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*
b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*
tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2
*a) + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 - 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^
3 - 16*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(
1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*ta
n(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x
)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan
(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*log(2*(tan
(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) -
 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 +
tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)^3*tan(1/2*a
)^3 + 4*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 4
*b*d*x*tan(1/2*b*x)^4 + 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) + 48*b*d*x*tan(1
/2*b*x)^2*tan(1/2*a)^2 + 16*b*d*x*tan(1/2*b*x)*tan(1/2*a)^3 + 4*b*d*x*tan(1
```

/2*a)^4 + 4*b*c*tan(1/2*b*x)^4 - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4
*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2
+ tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*ta
n(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x)
- 2*tan(1/2*a) + 1))*tan(1/2*b*x)^4 + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b
*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2
*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 -
2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*
b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)^4 + 16*b*c*tan(1/2*b*x)^3*tan(1/2*a)
- 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*
x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/
2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*
tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2
*b*x)^3*tan(1/2*a) + 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a
)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2
*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)
*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/
2*a) + 1))*tan(1/2*b*x)^3*tan(1/2*a) - 4*d*tan(1/2*b*x)^4*tan(1/2*a) + 48*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 16*b*c*
tan(1/2*b*x)*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*ta
n(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 +
tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1
/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2
*tan(1/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a)^3 + 4*d*log(2*(tan(1/2*a)^2 + 1)/
(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)
^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/
2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 +
2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a)^3 - 24*d*tan(1/
2*b*x)^2*tan(1/2*a)^3 + 4*b*c*tan(1/2*a)^4 - d*log(2*(tan(1/2*a)^2 + 1)/(ta
n(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*
tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b
*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*t
an(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*a)^4 + d*log(2*(tan(1/2*a)^2 + 1)/
(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)
^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/
2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 +
2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*a)^4 - 4*d*tan(1/2*b*x)*tan(1/2
*a)^4 - 16*b*d*x*tan(1/2*b*x)*tan(1/2*a) + 4*d*tan(1/2*b*x)^3 - 16*b*c*tan(
1/2*b*x)*tan(1/2*a) - 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/
2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x
)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1
/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1
/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3

```
 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/
2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a) + 24*d*tan(1/2*b*x)^2*t
an(1/2*a) + 24*d*tan(1/2*b*x)*tan(1/2*a)^2 + 4*d*tan(1/2*a)^3 + 4*b*d*x + 4
*b*c + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*
b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(
1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 +
2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)) - d*l
og(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan
(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2
*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*
b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)) - 4*d*tan(1/2*b
*x) - 4*d*tan(1/2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)
^3*tan(1/2*a)^3 - b^2*tan(1/2*b*x)^4 - 4*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*
b^2*tan(1/2*b*x)*tan(1/2*a)^3 - b^2*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)*tan(1
/2*a) + b^2)
```

3.263 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=75

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec(a+bx)}{c+dx},x\right)-\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x] - (CosIntegral[(b
*c)/d + b*x]*Sin[a - (b*c)/d])/d - (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d +
b*x])/d
```

Rubi [A] time = 0.150145, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x
), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx &= -\int \frac{\sin(a+bx)}{c+dx}\,dx+\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx\\
&= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int \frac{\sec(a+b\text{...}}{c\text{...}}\\
&= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx
\end{aligned}$$

Mathematica [A] time = 3.81839, size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

Maple [A] time = 0.299, size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)(\tan(bx+a))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(bx+a)\tan(bx+a)^2}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)\tan(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

3.264 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=93

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+$$

```
[Out] CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x] - (b*Cos[a - (b
*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 + Sin[a + b*x]/(d*(c + d*x)) + (b*Si
n[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```

Rubi [A] time = 0.181405, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c
 + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int]
[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx\\ &= \frac{\sin(a+bx)}{d(c+dx)}-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx\\ &= \frac{\sin(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\int\\ &= -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \frac{\sec(a+}{}\end{aligned}$$

Mathematica [A] time = 4.18268, size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A] time = 0.378, size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)\left(\tan(bx+a)\right)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin(bx+a)\tan(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

3.265 $\int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\csc(a+bx)\sec^2(a+bx)(c+dx)^m, x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

Rubi [A] time = 0.24846, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx) \, dx = \int (c+dx)^m \csc(a+bx)\sec^2(a+bx) \, dx$$

Mathematica [A] time = 8.94658, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

Maple [A] time = 0.16, size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)(\sec(bx+a))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \csc(bx+a)\sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

3.266 $\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\,dx$

Optimal. Leaf size=469

$$\frac{24d^3(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}-\frac{24id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{24id}{}$$

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^4*ArcTanh[
E^(I*(a + b*x))])/b + ((4*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((12*I)*d^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)
*d^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((4*I)*d*(c + d*x)^3*
PolyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a
+ b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (
24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (12*d^2*(c + d*x)^2*P
olyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a
 + b*x))])/b^4 + ((24*I)*d^4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I
)*d^4*PolyLog[4, I*E^(I*(a + b*x))])/b^5 + ((24*I)*d^3*(c + d*x)*PolyLog[4,
 E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5 - (24*d^
4*PolyLog[5, E^(I*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rubi [A] time = 0.794557, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$\frac{24d^3(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}-\frac{24id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{24id}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```



```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^4*ArcTanh[
E^(I*(a + b*x))])/b + ((4*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((12*I)*d^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)
*d^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((4*I)*d*(c + d*x)^3*
PolyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a
+ b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (
24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (12*d^2*(c + d*x)^2*P
olyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a
 + b*x))])/b^4 + ((24*I)*d^4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I
)*d^4*PolyLog[4, I*E^(I*(a + b*x))])/b^5 + ((24*I)*d^3*(c + d*x)*PolyLog[4,
```

```
 E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5 - (24*d^
4*PolyLog[5, E^(I*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} - (4d)\int (c+dx)^3 \Big(-$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} - (4d)\int \frac{(c+dx)^3\big(-}{}$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\big(-}{}$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} - \frac{(4d)\int \big(-(c+dx)^3}{}$$

$$= -\frac{(c+dx)^4\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4\sec(a+bx)}{b} + \frac{(4d)\int (c+dx)^3\,\mathrm{ta}}{}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^4\sec(a+bx)}{b} + \frac{\int b(c+dx)^4\csc(a+b}{b}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+dx)^2}{b^3}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{12id^2(c+dx)^2\mathrm{L}}{b^3}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx)^3\mathrm{Li}_2}{b^2}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx)^3\mathrm{Li}_2}{b^2}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx)^3\mathrm{Li}_2}{b^2}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx)^3\mathrm{Li}_2}{b^2}$$

$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx)^3\mathrm{Li}_2}{b^2}$$

Mathematica [A] time = 3.62998, size = 694, normalized size = 1.48

$$\frac{-4d\left(3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right) - 3ib^2d(c+dx)^2\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right) - 6bcd^2\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right) + 6bcd}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (b^4*(c + d*x)^4*Log[1 - E^(I*(a + b*x))] - b^4*(c + d*x)^4*Log[1 + E^(I*(a
 + b*x))] - 4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log
```

```
[1 - I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*
d^3*x^3*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x)
)] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(
I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] -
(3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3
, (-I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*
c*d^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x)
)] - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^
(I*(a + b*x))]) + (4*I)*d*(b^3*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] + (
3*I)*b^2*d*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*Po
lyLog[4, -E^(I*(a + b*x))] + I*d*PolyLog[5, -E^(I*(a + b*x))])) - (4*I)*d*(
b^3*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyL
og[3, E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*PolyLog[4, E^(I*(a + b*x))] + I
*d*PolyLog[5, E^(I*(a + b*x))])) + b^4*(c + d*x)^4*Sec[a + b*x])/b^5
```

Maple [B] time = 0.758, size = 1866, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+24*I*d^
3/b^3*c*polylog(2,I*exp(I*(b*x+a)))*x+24*I*d^3/b^4*c*a^2*arctan(exp(I*(b*x+
a)))-24*I*d^2/b^3*c^2*a*arctan(exp(I*(b*x+a)))+4/b*c*d^3*ln(1-exp(I*(b*x+a)
))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3+24*I/b^4*c*d^3*polylog(2,I*exp(
I*(b*x+a)))*a-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+12*I/b^2*polylog(2,-exp(I*
(b*x+a)))*c^2*d^2*x+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+12/b^3*d^
4*polylog(3,exp(I*(b*x+a)))*x^2+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-12
/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)
))*x^2+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^
4-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4
/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x-24/b^3*c
*d^3*polylog(3,-exp(I*(b*x+a)))*x-6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b
^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/
b^5+2*exp(I*(b*x+a))*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)/b/(e
xp(2*I*(b*x+a))+1)-24*d^4/b^4*polylog(3,I*exp(I*(b*x+a)))*x+24*d^4/b^4*poly
log(3,-I*exp(I*(b*x+a)))*x-4*d^4/b^5*a^3*ln(1-I*exp(I*(b*x+a)))+4*d^4/b^2*l
n(1+I*exp(I*(b*x+a)))*x^3-4*d^4/b^2*ln(1-I*exp(I*(b*x+a)))*x^3-24*d^3/b^4*c
*polylog(3,I*exp(I*(b*x+a)))+4*d^4/b^5*a^3*ln(1+I*exp(I*(b*x+a)))+24*d^3/b^
4*c*polylog(3,-I*exp(I*(b*x+a)))-24*I*d^4*polylog(4,I*exp(I*(b*x+a)))/b^5+4
```

```
*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a
)))*x^3-24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(4,
-exp(I*(b*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-12*I/b^3*c^2*d^2*dilog(
1+I*exp(I*(b*x+a)))+12*I/b^3*c^2*d^2*dilog(1-I*exp(I*(b*x+a)))-12*I/b^5*d^4
*a^2*dilog(1+I*exp(I*(b*x+a)))+12*I/b^5*d^4*a^2*dilog(1-I*exp(I*(b*x+a)))+1
2*I/b^5*d^4*a^2*polylog(2,-I*exp(I*(b*x+a)))-12*I/b^5*d^4*a^2*polylog(2,I*e
xp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)-24*I*d^3/b^3*c*polylo
g(2,-I*exp(I*(b*x+a)))*x-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-4/b^2*c^3*d*a
*ln(exp(I*(b*x+a))-1)-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+24*I/b^4*d^4*
polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/
b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-12*d^2/b^3*c^2*ln(1-I*exp(I*(b*x+a)))*a
+12*d^3/b^2*c*ln(1+I*exp(I*(b*x+a)))*x^2-12*d^3/b^2*c*ln(1-I*exp(I*(b*x+a))
)*x^2-12*I*d^4/b^3*polylog(2,-I*exp(I*(b*x+a)))*x^2+12*I*d^4/b^3*polylog(2,
I*exp(I*(b*x+a)))*x^2-8*I*d^4/b^5*a^3*arctan(exp(I*(b*x+a)))+8*I*d/b^2*c^3*
arctan(exp(I*(b*x+a)))+12*d^3/b^4*c*a^2*ln(1-I*exp(I*(b*x+a)))+12*d^2/b^2*c
^2*ln(1+I*exp(I*(b*x+a)))*x+12*d^2/b^3*c^2*ln(1+I*exp(I*(b*x+a)))*a-12*d^3/
b^4*c*a^2*ln(1+I*exp(I*(b*x+a)))-12*d^2/b^2*c^2*ln(1-I*exp(I*(b*x+a)))*x+24
*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-12
*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-1/b*c^4*ln(exp(I*(b*x+a))+1)+1/b
*c^4*ln(exp(I*(b*x+a))-1)-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-24*I
/b^4*c*d^3*polylog(2,-I*exp(I*(b*x+a)))*a+24*I/b^4*c*d^3*a*dilog(1+I*exp(I*
(b*x+a)))-24*I/b^4*c*d^3*a*dilog(1-I*exp(I*(b*x+a)))
```

Maxima [B] time = 9.45849, size = 7688, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 4*a*c^3*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 6*a^2*c^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
a) - 1))/b^2 - 4*a^3*c*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(co
s(b*x + a) - 1))/b^3 + a^4*d^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^4 + 2*((8*b^3*c^3*d - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*
d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24
*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b
*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x +
2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x +
```

a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I
*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*
arctan2(cos(b*x + a), sin(b*x + a) + 1) + (8*b^3*c^3*d - 24*a*b^2*c^2*d^2 +
24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x
+ a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 8*(b^3*c^3*d
- 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3
- a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*
cos(2*b*x + 2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 - 24*I*a^2*b*c*d^3
- 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a
)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4)*(b*x + a))*sin(2*
b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (2*(b*x + a)^4*d^4 +
8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
x + a) + 2((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2
*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2
+ 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d
^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c
*d^3 + 12*I*a^2*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24
*I*a^2*b*c*d^3 - 8*I*a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), cos(b*x + a) + 1) - (2*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a
)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 8*(b^3*c^3*d -
3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + 2*((b*x + a)^4*d^4
+ 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4
)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a
)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*a^2*b*c*d^3 - 8*I*a^3*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (4
I(b*x + a)^4*d^4 + (16*I*b*c*d^3 - 16*I*a*d^4)*(b*x + a)^3 + (24*I*b^2*c^
2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)*(b*x + a)^2 + (16*I*b^3*c^3*d - 48*I
*a*b^2*c^2*d^2 + 48*I*a^2*b*c*d^3 - 16*I*a^3*d^4)*(b*x + a))*cos(b*x + a) +
(24*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*(b*x + a)^2*d^4 + 24*a^2*d^4 + 48*(b*c
*d^3 - a*d^4)*(b*x + a) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 +
a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-24*I*b^2*c^2
*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 - 24*I*a^2*d^4 + (-48*I*b*c*d^
3 + 48*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (24
*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*(b*x + a)^2*d^4 + 24*a^2*d^4 + 48*(b*c*d^3
- a*d^4)*(b*x + a) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2
d^4 + 2(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (24*I*b^2*c^2*d^2
- 48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + 24*I*a^2*d^4 + (48*I*b*c*d^3 - 48
*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (8*b^3*c
^3*d - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 +
24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d
^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d

^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^
2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d
^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a
^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (8*b^3*c^3*d
 - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(
b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*
x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 -
 a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
 a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2
 + 24*I*a^2*b*c*d^3 + 8*I*(b*x + a)^3*d^4 - 8*I*a^3*d^4 + (24*I*b*c*d^3 - 2
4*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^4*d^4
+ (-4*I*b*c*d^3 + 4*I*a*d^4)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d
^3 - 6*I*a^2*d^4)*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I
*a^2*b*c*d^3 + 4*I*a^3*d^4)*(b*x + a) + (-I*(b*x + a)^4*d^4 + (-4*I*b*c*d^3
 + 4*I*a*d^4)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^
4)*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 +
4*I*a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^4*d^4 + 4*(b*c*d^3 -
a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 +
4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(2*
b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*
(b*x + a)^4*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^2*c^2*d^2
- 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a)^2 + (4*I*b^3*c^3*d - 12*I*a*b^2*c
^2*d^2 + 12*I*a^2*b*c*d^3 - 4*I*a^3*d^4)*(b*x + a) + (I*(b*x + a)^4*d^4 + (
4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 +
6*I*a^2*d^4)*(b*x + a)^2 + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b
*c*d^3 - 4*I*a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^4*d^4 + 4*(b
*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x
+ a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a
))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) +
 1) - (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 - 4*I*(b*x +
a)^3*d^4 + 4*I*a^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*
b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a) + (-4*I*b^3*c^3*d +
12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 - 4*I*(b*x + a)^3*d^4 + 4*I*a^3*d^4 +
 (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c
*d^3 - 12*I*a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 4*(b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b
x + a)^2 + 3(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x +
2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (4*I*b^3*
c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b*c*d^3 + 4*I*(b*x + a)^3*d^4 - 4*I*a
^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I
*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a) + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2
+ 12*I*a^2*b*c*d^3 + 4*I*(b*x + a)^3*d^4 - 4*I*a^3*d^4 + (12*I*b*c*d^3 - 12
*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*
(b*x + a))*cos(2*b*x + 2*a) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^

```
3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a
)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (48*I*d^4*cos(2*b*x + 2*a) - 4
8*d^4*sin(2*b*x + 2*a) + 48*I*d^4)*polylog(5, -e^(I*b*x + I*a)) - (-48*I*d^
4*cos(2*b*x + 2*a) + 48*d^4*sin(2*b*x + 2*a) - 48*I*d^4)*polylog(5, e^(I*b*
x + I*a)) - 48*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) + d^4)*polylo
g(4, I*e^(I*b*x + I*a)) + 48*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a)
 + d^4)*polylog(4, -I*e^(I*b*x + I*a)) - (48*b*c*d^3 + 48*(b*x + a)*d^4 - 4
8*a*d^4 + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) + (48*I*b*c
*d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(I*
b*x + I*a)) + (48*b*c*d^3 + 48*(b*x + a)*d^4 - 48*a*d^4 + 48*(b*c*d^3 + (b*
x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-48*I*b*c*d^3 - 48*I*(b*x + a)*d^4
+ 48*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) - (-48*I*b*c*d^
3 - 48*I*(b*x + a)*d^4 + 48*I*a*d^4 + (-48*I*b*c*d^3 - 48*I*(b*x + a)*d^4 +
 48*I*a*d^4)*cos(2*b*x + 2*a) + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*
b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)) - (48*I*b*c*d^3 + 48*I*(b*x + a)*
d^4 - 48*I*a*d^4 + (48*I*b*c*d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*cos(2*b
*x + 2*a) - 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(
3, -I*e^(I*b*x + I*a)) - (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x +
a)^2*d^4 - 24*I*a^2*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-24*I*b
^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 - 24*I*a^2*d^4 + (-48*I*
b*c*d^3 + 48*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 24*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b
*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^
3 + 24*I*(b*x + a)^2*d^4 + 24*I*a^2*d^4 + (48*I*b*c*d^3 - 48*I*a*d^4)*(b*x
+ a) + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + 24*I*a^2
*d^4 + (48*I*b*c*d^3 - 48*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 24*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x
+ a))*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + 4*((b*x + a)^4*d^4 +
4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(
b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x
 + a))*sin(b*x + a))/(-2*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin(2*b*x + 2*a) -
2*I*b^4))/b
```

Fricas [C] time = 1.35805, size = 6263, normalized size = 13.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x +
 2*b^4*c^4 - 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) + I*sin(b*x + a))
- 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*co
s(b*x + a)*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*cos(b*x + a)
*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*polylog
(4, I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*polylog(4, I*cos
(b*x + a) - sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a
) + sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a) - sin(
b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4
*I*b^3*c^3*d)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (4*I*b^3*
d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*cos(b*x
+ a)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (12*I*b^2*d^4*x^2 + 24*I*b^2*c*
d^3*x + 12*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))
 + (12*I*b^2*d^4*x^2 + 24*I*b^2*c*d^3*x + 12*I*b^2*c^2*d^2)*cos(b*x + a)*di
log(I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b^2*d^4*x^2 - 24*I*b^2*c*d^3*x
- 12*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-
12*I*b^2*d^4*x^2 - 24*I*b^2*c*d^3*x - 12*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(
-I*cos(b*x + a) - sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 -
12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*cos(b*x + a)*dilog(-cos(b*x + a) + I*si
n(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x +
4*I*b^3*c^3*d)*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4*d^
4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(
b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*
d*x + b^4*c^4)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 4*(b^3
*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*
x + a) - I*sin(b*x + a) + I) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos
(b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^
2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*co
s(b*x + a) - sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) +
1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*
log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) +
I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4
*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*lo
```

```
g(-cos(b*x + a) - I*sin(b*x + a) + 1) + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*
a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I)
 + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)*polylog(4, cos(b*x + a) + I*s
in(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)*polylog(4, cos(b
*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)*poly
log(4, -cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos
(b*x + a)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x
 + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylo
g(3, cos(b*x + a) - I*sin(b*x + a)) + 24*(b*d^4*x + b*c*d^3)*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) + sin(b*x + a)) - 24*(b*d^4*x + b*c*d^3)*cos(b*x +
 a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 24*(b*d^4*x + b*c*d^3)*cos(
b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 24*(b*d^4*x + b*c*d^3
)*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2
 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*s
in(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*
polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^5*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.267 $\int (c+dx)^3 \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=343

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c$$

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^3*ArcTanh[
E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2
*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(c + d*x)^2*PolyLo
g[2, E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])
/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E
^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 -
((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(
a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rubi [A] time = 0.572627, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```


```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^3*ArcTanh[
E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2
*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(c + d*x)^2*PolyLo
g[2, E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])
/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E
^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 -
((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(
a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
```

```
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}-(3d)\int (c+dx)^2\Big(-\text{t}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}-(3d)\int\frac{(c+dx)^2\left(-\right.}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\left(-\text{ta}\right.}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{(3d)\int\left(-(c+dx)^2\text{ta}\right.}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\tanh}{b}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{\int b(-c-dx)^3\csc(a+bx}{b}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{Li}_2\left(i\right.}{b^3}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{6id^2(c+dx)\text{Li}_2\left(-\right.}{b^3}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\text{Li}_2\left(-\right.}{b^2}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\text{Li}_2\left(-\right.}{b^2}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\text{Li}_2\left(-\right.}{b^2}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3id(c+dx)^2\text{Li}_2\left(-\right.}{b^2}$$

Mathematica [A] time = 1.43863, size = 473, normalized size = 1.38

$$\frac{-3d\left(2ibd(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)-2ibd(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)-2d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)+2d^2\text{PolyLog}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] - 3*d*((-2*I)*b^
2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^
2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x)
)] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))]
 - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*
x))]) + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]]
 + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*P
olyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyL
og[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a
 + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]
) + b^3*(c + d*x)^3*Sec[a + b*x])/b^4
```

Maple [B] time = 0.651, size = 1152, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] -3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3*d^
3/b^2*ln(1-I*exp(I*(b*x+a)))*x^2-3*d^3/b^4*a^2*ln(1+I*exp(I*(b*x+a)))+3*d^3
/b^2*ln(1+I*exp(I*(b*x+a)))*x^2+3*d^3/b^4*a^2*ln(1-I*exp(I*(b*x+a)))+6*I*d^
3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-
6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)
*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))+1)+6*I*d/b^2*c^2*arctan(exp(I*(b*x+a)))
-6*I/b^2*d^2*c*polylog(2,exp(I*(b*x+a)))*x-6*I*d^3*polylog(4,-exp(I*(b*x+a)
))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+1/b*d^3*ln(1-exp(I*(b*x+a)))*x
^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/b^
3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-3/b^3*c
*d^2*a^2*ln(1-exp(I*(b*x+a)))-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln
(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+3*I/b^2*c^2*d*polyl
og(2,-exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-6*I*d^3*x*
polylog(2,-I*exp(I*(b*x+a)))/b^3+6*d^2/b^3*c*ln(1+I*exp(I*(b*x+a)))*a-6*d^2
/b^2*c*ln(1-I*exp(I*(b*x+a)))*x-6*d^2/b^3*c*ln(1-I*exp(I*(b*x+a)))*a+6*d^2/
b^2*c*ln(1+I*exp(I*(b*x+a)))*x+6*I*d^3/b^4*a^2*arctan(exp(I*(b*x+a)))-12*I*
d^2/b^3*c*a*arctan(exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/
b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-
6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6
```

```
*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2*x+6*I/b^4*d^3*polylog(2,I*exp(I*(b*
x+a)))*a+6*I/b^3*d^2*c*dilog(1-I*exp(I*(b*x+a)))-6*I/b^3*d^2*c*dilog(1+I*ex
p(I*(b*x+a)))+6*I/b^4*d^3*a*dilog(1+I*exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2
,exp(I*(b*x+a)))*x^2-6*I/b^4*d^3*polylog(2,-I*exp(I*(b*x+a)))*a-3*I/b^2*c^2
*d*polylog(2,exp(I*(b*x+a)))-6*I/b^4*d^3*a*dilog(1-I*exp(I*(b*x+a)))+1/b*c^
3*ln(exp(I*(b*x+a))-1)-1/b*c^3*ln(exp(I*(b*x+a))+1)
```

Maxima [B] time = 4.01994, size = 4327, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 3*a*c^2*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 3*a^2*c*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a)
 - 1))/b^2 - a^3*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
+ a) - 1))/b^3 + 2*((6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2
*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x +
 a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6
*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*
b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), si
n(b*x + a) + 1) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d
^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a
)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I
*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*
c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin
(b*x + a) + 1) - (2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 3*(b*c*
d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))
*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x +
 a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x + a))*sin(2*b*x
 + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^3*d^3 + 6*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
 a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3
+ (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 +
 6*I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x +
 a) + 1) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 +
 (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*(b*x + a))*cos(b*x + a) +
```

(12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3) *sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*(b*x + a)^3*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3) *(b*x + a) + (I*(b*x + a)^3*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*polylog(4, -e^(I*b*x + I*a)) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*polylog(4, e^(I*b*x + I*a)) - (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, I*e^(I*b*x + I*a)) - (12*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*polylog(3, -I*e^(I*b*x + I*a)) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (12*I*b*c*d^2 + 12

```
*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a
*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2
*a))*polylog(3, e^(I*b*x + I*a)) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(b*x + a
))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) - 2*I*b^3))/b
```

Fricas [C] time = 1.00506, size = 4352, normalized size = 12.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*I*d^3*
cos(b*x + a)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x +
a)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)*polylog
(4, -cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)*polylog(4, -cos(
b*x + a) - I*sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) +
 sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a
)) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*
cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin
(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x +
a)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b
*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*
cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c
*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3*x -
6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2
*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)*dilog(-cos(b*x + a
) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*c
os(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*
x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*
x + a) + I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x +
a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x +
 a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) +
1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*l
og(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2
```

```
*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
+ (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a
) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) +
 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a)
+ I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*s
in(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-
cos(b*x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*pol
ylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)
*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x
+ a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos
(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \csc(bx+a) \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^2, x)
```

3.268 $\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=219

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie\right.}{b^3}$$

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^2*ArcTanh[E^
(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 -
((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*
E^(I*(a + b*x))])/b^3 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2
 - (2*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b
*x))])/b^3 + ((c + d*x)^2*Sec[a + b*x])/b
```

Rubi [A] time = 0.37738, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 2282, 6589, 4181, 2279, 2391}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```


```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^2*ArcTanh[E^
(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 -
((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*
E^(I*(a + b*x))])/b^3 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2
 - (2*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b
*x))])/b^3 + ((c + d*x)^2*Sec[a + b*x])/b
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}-(2d)\int (c+dx)\Big(-$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}-(2d)\int \frac{(c+dx)\left(-\right.}{}$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{(2d)\int (c+dx)\left(-\right.}{}$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{(2d)\int \left(-(c+dx)\right.}{}$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}+\frac{(2d)\int (c+dx)\tan}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}+\frac{\int b(c+dx)^2\csc(a+bx}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{\left(2id^2\right)\mathrm{Subst}\left(\int \frac{\log(1-ix}{x}\right.}{b^3}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{2id^2\mathrm{Li}_2\left(-ie^{i(a+bx}\right.}{b^3}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{2id(c+dx)\mathrm{Li}_2\left(-\right.}{b^2}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{2id(c+dx)\mathrm{Li}_2\left(-\right.}{b^2}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{2id(c+dx)\mathrm{Li}_2\left(-\right.}{b^2}$$

Mathematica [A] time = 2.58814, size = 317, normalized size = 1.45

$$\frac{2id\left(b(c+dx)\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)+id\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)\right)+2d\left(d\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)-ib(c+dx)\mathrm{PolyLog}\left(2,e^{i\right.\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))
] - b^2*(c + d*x)^2*Log[1 + E^(I*(a + b*x))] + (2*d^2*Csc[a]*((b*x - ArcTan
[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcT
an[Cot[a]]))]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2,
 E^(I*(b*x - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + (2*I)*d*(b*(c + d*x)*Poly
Log[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]) + 2*d*((-I)*b*
(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x))]) + b^
2*(c + d*x)^2*Sec[a + b*x])/b^3
```

Maple [B] time = 0.448, size = 568, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2*d^2
/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*d^2/b^3*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^
3*dilog(1+I*exp(I*(b*x+a)))+2*I*d^2/b^3*dilog(1-I*exp(I*(b*x+a)))-1/b^3*d^2
*ln(1-exp(I*(b*x+a)))*a^2-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp
(I*(b*x+a)))*x^2+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))
+1)+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-exp(I*(
b*x+a)))-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,exp(
I*(b*x+a)))+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(
b*x+a)))-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-1/b*c^2*ln(exp(I*(b*x+a))+1)+1/b*c^
2*ln(exp(I*(b*x+a))-1)-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3
,exp(I*(b*x+a)))/b^3+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b
*x+a)))*a+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-2/b^2*c*d*a*ln(exp(I*(b*x+a))-
1)
```

Maxima [B] time = 2.26547, size = 2157, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 2*a*c*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
 + a^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b^2 + 2*((4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 -
 a*d^2)*cos(2*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin
(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (4*b*c*d + 4*(b*x
+ a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (
-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b
*x + a), -sin(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x +
 a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) +
(2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d -
a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*
b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (4*I*(b*x + a)^2*d
^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*cos(b*x + a) + 4*(d^2*cos(2*b*x + 2
*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(I*e^(I*b*x + I*a)) - 4*(d^2*cos(2
*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(-I*e^(I*b*x + I*a)) + (4*
b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2
*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a)
)*dilog(-e^(I*b*x + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d
 + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2
 - 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^2*d^
2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d
+ 2*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*
d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*c
os(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) +
 (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) -
 ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-2*I*b*c*d - 2*I*(b*x
+ a)*d^2 + 2*I*a*d^2 + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*cos(2*b
*x + 2*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (2*I*b*c*d + 2*I*(b*x + a)
*d^2 - 2*I*a*d^2 + (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2)*cos(2*b*x +
2*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-4*I*d^2*cos(2*b*x + 2*a) + 4*
d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, -e^(I*b*x + I*a)) - (4*I*d^2*cos
(2*b*x + 2*a) - 4*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, e^(I*b*x + I*a
)) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(b*x + a))/(-2*I*
b^2*cos(2*b*x + 2*a) + 2*b^2*sin(2*b*x + 2*a) - 2*I*b^2))/b
```

Fricas [C] time = 0.776298, size = 2773, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*cos(b*x + a)*dilog(I
*cos(b*x + a) + sin(b*x + a)) + 2*I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) -
 sin(b*x + a)) - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))
 - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b
*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)*poly
log(3, cos(b*x + a) - I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(
b*x + a) + I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(b*x + a) -
I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)*dilog(cos(b*x + a
) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)*dilog(cos(b*x
+ a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)*dilog(-cos
(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)*dilog(
-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2)*cos(b*
x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 2*(b*c*d -
 a*d^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*d^2*x +
a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d^2*x + a
*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 2*(b*d^2*x + a*
d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d^2*x + a*
d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) +
1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2
)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b
^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) + 2*(b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x +
a) + I))/(b^3*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.269 $\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=113

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{c\sec(a+bx)}{b}-\frac{c\tanh^{-1}(\cos(a+bx))}{b}+\frac{d}{}$$

```
[Out] (-2*d*x*ArcTanh[E^(I*(a + b*x))])/b - (c*ArcTanh[Cos[a + b*x]])/b - (d*ArcT
anh[Sin[a + b*x]])/b^2 + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*Poly
Log[2, E^(I*(a + b*x))])/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b*x])/b
```

Rubi [A] time = 0.13045, antiderivative size = 122, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2622, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\sec(a+bx)}{b}-\frac{(c+dx)\tanh^{-1}(\cos}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*d*x*ArcTanh[E^(I*(a + b*x))])/b + (d*x*ArcTanh[Cos[a + b*x]])/b - ((c +
 d*x)*ArcTanh[Cos[a + b*x]])/b - (d*ArcTanh[Sin[a + b*x]])/b^2 + (I*d*PolyL
og[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + ((c
+ d*x)*Sec[a + b*x])/b
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx &= -\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)\sec(a+bx)}{b}-d\int\Bigg(-\frac{\tanh^{-1}(\cos(a+\\
&= -\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b}+\frac{(c+dx)\sec(a+bx)}{b}+\frac{d\int\tanh^{-1}(\cos(a+bx}{b}\\
&= \frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b}-\frac{d\tanh^{-1}(\sin(a+bx)}{b^2}\\
&= \frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b}-\frac{d\tanh^{-1}(\sin(a+bx)}{b^2}\\
&= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(\cos(a+bx}{b}\\
&= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(\cos(a+bx}{b}\\
&= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{dx\tanh^{-1}(\cos(a+bx))}{b}-\frac{(c+dx)\tanh^{-1}(\cos(a+bx}{b}
\end{aligned}$$

Mathematica [A] time = 0.504331, size = 212, normalized size = 1.88

$$\frac{d\left(i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{b^2}-\frac{ad\log\left(\tan\left(\frac{1}{2}(a\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] -((c*Log[Cos[(a + b*x)/2]])/b) + (d*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]
])/b^2 + (c*Log[Sin[(a + b*x)/2]])/b - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b
*x)/2]])/b^2 - (a*d*Log[Tan[(a + b*x)/2]])/b^2 + (d*((a + b*x)*(Log[1 - E^(
I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))]
 - PolyLog[2, E^(I*(a + b*x))])))/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b
*x])/b
```

Maple [A] time = 0.29, size = 160, normalized size = 1.4

$$2\frac{e^{i(bx+a)}(dx+c)}{b\left(e^{2i(bx+a)}+1\right)}-\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{2id\arctan\left(e^{i(bx+a)}\right)}{b^2}+\frac{id\mathrm{dilog}\left(e^{i(bx+a)}+1\right)}{b^2}-\frac{d\ln\left(e^{i(bx+a}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 2*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))+1)-1/b*c*ln(exp(I*(b*x+a))+1)+
1/b*c*ln(exp(I*(b*x+a))-1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+I/b^2*d*dilog(e
xp(I*(b*x+a))+1)-1/b*d*ln(exp(I*(b*x+a))+1)*x+I/b^2*d*dilog(exp(I*(b*x+a)))
-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 2.04559, size = 1085, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*arctan2(2*(cos(b*x + 2*
a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*
sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (c
os(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2
 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)
*sin(a) + sin(a)^2)) + (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a)
- (-2*I*b*d*x - 2*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x +
a) + 1) - 2*(b*c*cos(2*b*x + 2*a) + I*b*c*sin(2*b*x + 2*a) + b*c)*arctan2(s
in(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(
2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (-4*I*b*
d*x - 4*I*b*c)*cos(b*x + a) - 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a)
+ d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a)
 + d)*dilog(e^(I*b*x + I*a)) - (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(2*b
*x + 2*a) - (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(2*b
*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*cos(b*x + a) + 1) - (-I*d*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a) -
I*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2
```

```
+ 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + si
n(a)^2)) - 4*(b*d*x + b*c)*sin(b*x + a))/(-2*I*b^2*cos(2*b*x + 2*a) + 2*b^2
*sin(2*b*x + 2*a) - 2*I*b^2)
```

Fricas [B] time = 0.581003, size = 1046, normalized size = 9.26

$$\frac{2\,bdx - i\,d\cos\left(bx + a\right)\mathrm{Li}_2\left(\cos\left(bx + a\right) + i\,\sin\left(bx + a\right)\right) + i\,d\cos\left(bx + a\right)\mathrm{Li}_2\left(\cos\left(bx + a\right) - i\,\sin\left(bx + a\right)\right) - i\,d\cos\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - I*d*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*
cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x + a)*dilog(-cos(b*x + a) - I*
sin(b*x + a)) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a
) + 1) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1)
+ (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2
) + (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1
/2) + (b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) +
(b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - d*cos(
b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) + 2*
b*c)/(b^2*cos(b*x + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)\csc(a + bx)\sec^2(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\csc(bx+a)\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^2, x)
```

3.270 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi [A] time = 0.153201, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx=\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 9.84684, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]
```

Maple [A] time = 2.332, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)(\sec(bx+a))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

3.271 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [A] time = 0.180374, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 10.0946, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A] time = 2.663, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)(\sec(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.272 $\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\csc^2(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

Rubi [A] time = 0.189649, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

Mathematica [A] time = 2.78455, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

Maple [A] time = 0.173, size = 0, normalized size = 0.

$$\int (dx+c)^m (\csc (bx+a))^2 (\sec (bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc (bx+a)^2 \sec (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \csc (bx+a)^2 \sec (bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

3.273 $\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=118

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{4i(a+bx)}\right)}{8b^4}+\frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\cot(2a+}{b}$$

```
[Out] ((-2*I)*(c + d*x)^3)/b - (2*(c + d*x)^3*Cot[2*a + 2*b*x])/b + (3*d*(c + d*x
)^2*Log[1 - E^((4*I)*(a + b*x))])/b^2 - (((3*I)/2)*d^2*(c + d*x)*PolyLog[2,
 E^((4*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((4*I)*(a + b*x))])/(8*b^4
)
```

Rubi [A] time = 0.280106, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4419, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{4i(a+bx)}\right)}{8b^4}+\frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\cot(2a+}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-2*I)*(c + d*x)^3)/b - (2*(c + d*x)^3*Cot[2*a + 2*b*x])/b + (3*d*(c + d*x
)^2*Log[1 - E^((4*I)*(a + b*x))])/b^2 - (((3*I)/2)*d^2*(c + d*x)*PolyLog[2,
 E^((4*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((4*I)*(a + b*x))])/(8*b^4
)
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx &= 4\int (c+dx)^3\csc^2(2a+2bx)\,dx \\
&= -\frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{(6d)\int (c+dx)^2\cot(2a+2bx)\,dx}{b} \\
&= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} - \frac{(12id)\int \frac{e^{2i(2a+2bx)}(c+dx)^2}{1-e^{2i(2a+2bx)}}\,dx}{b} \\
&= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2} - \\
&= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2} - \\
&= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2} - \\
&= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2} -
\end{aligned}
$$

Mathematica [B] time = 2.23132, size = 285, normalized size = 2.42

$$
\frac{12ibd^2(c+dx)\text{PolyLog}\left(2,-e^{-i(a+bx)}\right)+12ibd^2(c+dx)\text{PolyLog}\left(2,e^{-i(a+bx)}\right)+6ibd^2(c+dx)\text{PolyLog}\left(2,-e^{-2i(a+bx)}\right)+}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (((-8*I)*b^3*(c + d*x)^3)/(-1 + E^((4*I)*a)) + 6*b^2*d*(c + d*x)^2*Log[1 -
E^((-I)*(a + b*x))] + 6*b^2*d*(c + d*x)^2*Log[1 + E^((-I)*(a + b*x))] + 6*b
^2*d*(c + d*x)^2*Log[1 + E^((-2*I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Pol
yLog[2, -E^((-I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^((-I)*(a
 + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((-2*I)*(a + b*x))] + 12*d^
3*PolyLog[3, -E^((-I)*(a + b*x))] + 12*d^3*PolyLog[3, E^((-I)*(a + b*x))] +
 3*d^3*PolyLog[3, -E^((-2*I)*(a + b*x))] + 4*b^3*(c + d*x)^3*Csc[2*a]*Csc[2
*(a + b*x)]*Sin[2*b*x])/(2*b^4)
```

Maple [B] time = 0.336, size = 687, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] 6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-3*
d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/b
^2*ln(1-exp(I*(b*x+a)))*x^2-12*d^3/b^4*a^2*ln(exp(I*(b*x+a)))-12*d/b^2*c^2*
ln(exp(I*(b*x+a)))+3*d/b^2*c^2*ln(exp(2*I*(b*x+a))+1)+3*d^3/b^2*ln(exp(2*I*
(b*x+a))+1)*x^2+6*d^2/b^2*c*ln(exp(2*I*(b*x+a))+1)*x-6*I*d^2/b^3*c*polylog(
2,-exp(I*(b*x+a)))-12*I*d^2/b*c*x^2+12*I*d^3/b^3*a^2*x-12*I*d^2/b^3*c*a^2-3
*I*d^2/b^3*c*polylog(2,-exp(2*I*(b*x+a)))-3*I*d^3/b^3*polylog(2,-exp(2*I*(b
*x+a)))*x-6/b^3*c*d^2*a*ln(exp(I*(b*x+a))-1)+24*d^2/b^3*c*a*ln(exp(I*(b*x+a
)))+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+8*I*d^3/b^4*a^3-4*I*d^3/b*x^3-
24*I*d^2/b^2*c*a*x-6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x+6*d^2/b^2*c*ln(
exp(I*(b*x+a))+1)*x+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x+6*d^2/b^3*c*ln(1-exp
(I*(b*x+a)))*a-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(exp(I
*(b*x+a))+1)+3/b^2*c^2*d*ln(exp(I*(b*x+a))-1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a
))-1)-4*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))+1)/(exp(2
*I*(b*x+a))-1)-6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))
```

<u>Maxima</u> [B] time = 2.35505, size = 3179, normalized size = 26.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^3*(1/tan(b*x + a) - tan(b*x + a)) - 6*a*c^2*d*(1/tan(b*x + a) - t
an(b*x + a))/b + 6*a^2*c*d^2*(1/tan(b*x + a) - tan(b*x + a))/b^2 - 2*a^3*d^
3*(1/tan(b*x + a) - tan(b*x + a))/b^3 - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x
+ 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 -
2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a
) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*si
n(4*b*x + 4*a))*c^2*d/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b
*x + 4*a) + 1)*b) + 6*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b
*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) +
 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2
```

```
+ sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*a*c*
d^2/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b^2
) - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*l
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos
(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4
*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*a^2*d^3/((cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b^3) + 2*((6*(b*x +
 a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d
^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos
(2*b*x + 2*a) + 1) + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) -
6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (6*I
*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2
 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*c
os(4*b*x + 4*a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x
 + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 8*((b*x
 + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(4*b*x + 4*a) - (6*b*c*d^
2 + 6*(b*x + a)*d^3 - 6*a*d^3 - 6*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b
*x + 4*a) + (-6*I*b*c*d^2 - 6*I*(b*x + a)*d^3 + 6*I*a*d^3)*sin(4*b*x + 4*a)
)*dilog(-e^(2*I*b*x + 2*I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 -
 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x + 4*a) + (-12*I*b*c*d^2 - 1
2*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(4*b*x + 4*a))*dilog(-e^(I*b*x + I*a)) -
 (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 -
a*d^3)*cos(4*b*x + 4*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)
*sin(4*b*x + 4*a))*dilog(e^(I*b*x + I*a)) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c
*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a
*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3
)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
+ 2*cos(2*b*x + 2*a) + 1) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3
)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))
*cos(4*b*x + 4*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin
(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
(3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a
)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 3*((b*x
+ a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-3*I*d^3*cos(4*b*x + 4*a) +
 3*d^3*sin(4*b*x + 4*a) + 3*I*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) - (-12*
I*d^3*cos(4*b*x + 4*a) + 12*d^3*sin(4*b*x + 4*a) + 12*I*d^3)*polylog(3, -e^
(I*b*x + I*a)) - (-12*I*d^3*cos(4*b*x + 4*a) + 12*d^3*sin(4*b*x + 4*a) + 12
*I*d^3)*polylog(3, e^(I*b*x + I*a)) - (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^
2 + 24*I*a*d^3)*(b*x + a)^2)*sin(4*b*x + 4*a))/(-2*I*b^3*cos(4*b*x + 4*a) +
 2*b^3*sin(4*b*x + 4*a) + 2*I*b^3))/b
```

Fricas [C] time = 0.898168, size = 4263, normalized size = 36.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*c
os(b*x + a)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*
cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*d^3
*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*d^
3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*
d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a)
+ 6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a
) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*
x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(cos(b
*x + a) - I*sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*
x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x -
6*I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a)
 + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*
x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*co
s(b*x + a) - sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b
*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x
- 6*I*b*c*d^2)*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x
+ a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x + a) - I*
sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(
b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a)
 + sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*
c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*
x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x +
a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 +
2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - s
in(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b
*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*
```

```
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/
2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a
*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*si
n(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*
x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
+ 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) +
1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-c
os(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - 4*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2)/(b^4*cos(b*x + a)*sin(b*x
 + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \csc (bx+a)^2 \sec (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

3.274 $\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{id^2\text{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}-\frac{2i(c+dx)^2}{b}$$

```
[Out] ((-2*I)*(c + d*x)^2)/b - (2*(c + d*x)^2*Cot[2*a + 2*b*x])/b + (2*d*(c + d*x
)*Log[1 - E^((4*I)*(a + b*x))])/b^2 - ((I/2)*d^2*PolyLog[2, E^((4*I)*(a + b
*x))])/b^3
```

Rubi [A] time = 0.191444, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4419, 4184, 3717, 2190, 2279, 2391}

$$-\frac{id^2\text{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}-\frac{2i(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-2*I)*(c + d*x)^2)/b - (2*(c + d*x)^2*Cot[2*a + 2*b*x])/b + (2*d*(c + d*x
)*Log[1 - E^((4*I)*(a + b*x))])/b^2 - ((I/2)*d^2*PolyLog[2, E^((4*I)*(a + b
*x))])/b^3
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
```

```
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx = 4\int (c+dx)^2\csc^2(2a+2bx)\,dx$$

$$= -\frac{2(c+dx)^2\cot(2a+2bx)}{b} + \frac{(4d)\int (c+dx)\cot(2a+2bx)\,dx}{b}$$

$$= -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2\cot(2a+2bx)}{b} - \frac{(8id)\int \frac{e^{2i(2a+2bx)}(c+dx)}{1-e^{2i(2a+2bx)}}\,dx}{b}$$

$$= -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2\cot(2a+2bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2} - \frac{(2$$

$$= -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2\cot(2a+2bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2} + \frac{(i$$

$$= -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2\cot(2a+2bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2} - \frac{id$$

Mathematica [B] time = 1.75939, size = 277, normalized size = 3.15

$$\frac{2b^2\csc(2a)\sin(2bx)(c+dx)^2\csc(2(a+bx)) - \frac{ie^{4ia}\left(-2\left(1-e^{-4ia}\right)d^2\text{PolyLog}\left(2,-e^{-i(a+bx)}\right)-2\left(1-e^{-4ia}\right)d^2\text{PolyLog}\left(2,e^{-i(a+bx)}\right)-\left(1-e^{-4ia}\right)d^2\text{PolyL}}{}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (((-I)*E^((4*I)*a)*((4*b^2*(c + d*x)^2)/E^((4*I)*a) + (2*I)*b*d*(1 - E^((-4
*I)*a))*(c + d*x)*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a)
)*(c + d*x)*Log[1 + E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a))*(c +
 d*x)*Log[1 + E^((-2*I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, -
E^((-I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, E^((-I)*(a + b*x)
)] - d^2*(1 - E^((-4*I)*a))*PolyLog[2, -E^((-2*I)*(a + b*x))]))/(-1 + E^((4
*I)*a)) + 2*b^2*(c + d*x)^2*Csc[2*a]*Csc[2*(a + b*x)]*Sin[2*b*x])/b^3
```

Maple [B] time = 0.332, size = 351, normalized size = 4.

$$\frac{-4\,id^2x^2}{b}+2\,\frac{cd\ln\left(e^{i(bx+a)}-1\right)}{b^2}+2\,\frac{cd\ln\left(e^{i(bx+a)}+1\right)}{b^2}+2\,\frac{cd\ln\left(e^{2\,i(bx+a)}+1\right)}{b^2}-8\,\frac{cd\ln\left(e^{i(bx+a)}\right)}{b^2}-\frac{2\,id^2\mathrm{polylog}\left(2,-e^{i(}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] -4*I*d^2/b*x^2+2*d/b^2*c*ln(exp(I*(b*x+a))-1)+2*d/b^2*c*ln(exp(I*(b*x+a))+1
)+2*d/b^2*c*ln(exp(2*I*(b*x+a))+1)-8*d/b^2*c*ln(exp(I*(b*x+a)))-2*I*d^2/b^3
*polylog(2,-exp(I*(b*x+a)))+2*d^2/b^2*ln(exp(2*I*(b*x+a))+1)*x-I*d^2*polylo
g(2,-exp(2*I*(b*x+a)))/b^3+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-
exp(I*(b*x+a)))*a-8*I*d^2/b^2*a*x-4*I*d^2/b^3*a^2-4*I*(d^2*x^2+2*c*d*x+c^2)
/b/(exp(2*I*(b*x+a))+1)/(exp(2*I*(b*x+a))-1)-2*I*d^2*polylog(2,exp(I*(b*x+a
)))/b^3+2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)+8
*d^2/b^3*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 2.12905, size = 1049, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(4*b^2*c^2 + (2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a) -
 (2*I*b*d^2*x + 2*I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(
2*b*x + 2*a) + 1) + (2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(4*b*x +
4*a) - (2*I*b*d^2*x + 2*I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), co
s(b*x + a) + 1) - (2*b*c*d*cos(4*b*x + 4*a) + 2*I*b*c*d*sin(4*b*x + 4*a) -
2*b*c*d)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d^2*x*cos(4*b*x + 4
*a) + 2*I*b*d^2*x*sin(4*b*x + 4*a) - 2*b*d^2*x)*arctan2(sin(b*x + a), -cos(
b*x + a) + 1) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(4*b*x + 4*a) + (d^2*cos(4
*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(2*I*b*x + 2*I*a)) + 2
*(d^2*cos(4*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(I*b*x + I*
a)) + 2*(d^2*cos(4*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(e^(I*b*
x + I*a)) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(4*b*x + 4*a)
+ (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b
*c*d)*cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*
d^2*x - I*b*c*d)*cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-4*I*b^2*d^2*x^2
- 8*I*b^2*c*d*x)*sin(4*b*x + 4*a))/(-I*b^3*cos(4*b*x + 4*a) + b^3*sin(4*b*x
 + 4*a) + I*b^3)
```

Fricas [B] time = 0.746377, size = 2573, normalized size = 29.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x - I*d^2*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(
b*x + a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)
)*sin(b*x + a) - I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*si
n(b*x + a) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b
*x + a) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x
+ a) + I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) +
 b^2*c^2 + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a)
 + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) + I*sin(
b*x + a) + I)*sin(b*x + a) + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos
(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b*x +
```

```
a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + a*d^2)*
cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x
 + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a)
 + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*s
in(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b
*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d
^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*
d^2*x + a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
 + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I
)*sin(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2)/(b
^3*cos(b*x + a)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc (bx+a)^2 \sec (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

3.275 $\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{d\log(\sin(2a+2bx))}{b^2}-\frac{2(c+dx)\cot(2a+2bx)}{b}$$

```
[Out] (-2*(c + d*x)*Cot[2*a + 2*b*x])/b + (d*Log[Sin[2*a + 2*b*x]])/b^2
```

Rubi [A] time = 0.0594156, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4419, 4184, 3475}

$$\frac{d\log(\sin(2a+2bx))}{b^2}-\frac{2(c+dx)\cot(2a+2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (-2*(c + d*x)*Cot[2*a + 2*b*x])/b + (d*Log[Sin[2*a + 2*b*x]])/b^2
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = 4\int (c+dx)\csc^2(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)\cot(2a+2bx)}{b} + \frac{(2d)\int \cot(2a+2bx)\,dx}{b}$$
$$= -\frac{2(c+dx)\cot(2a+2bx)}{b} + \frac{d\log(\sin(2a+2bx))}{b^2}$$

Mathematica [A] time = 0.203898, size = 32, normalized size = 0.91

$$\frac{d\log(\sin(2(a+bx))) - 2b(c+dx)\cot(2(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (-2*b*(c + d*x)*Cot[2*(a + b*x)] + d*Log[Sin[2*(a + b*x)]])/b^2
```

Maple [B] time = 0.074, size = 182, normalized size = 5.2

$$\left(\frac{c}{2b} - 3\frac{c(\tan(1/2\,bx+a/2))^2}{b} + \frac{c}{2b}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4 + \frac{dx}{2b} - 3\frac{dx(\tan(1/2\,bx+a/2))^2}{b} + \frac{dx}{2b}\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4\right)\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] (1/2/b*c-3/b*c*tan(1/2*b*x+1/2*a)^2+1/2/b*c*tan(1/2*b*x+1/2*a)^4+1/2*d*x/b-
3*d/b*x*tan(1/2*b*x+1/2*a)^2+1/2*d/b*x*tan(1/2*b*x+1/2*a)^4)/tan(1/2*b*x+1/
2*a)/(tan(1/2*b*x+1/2*a)^2-1)+d/b^2*ln(tan(1/2*b*x+1/2*a))+d/b^2*ln(tan(1/2
*b*x+1/2*a)-1)+d/b^2*ln(tan(1/2*b*x+1/2*a)+1)-2*d/b^2*ln(1+tan(1/2*b*x+1/2*
a)^2)
```

Maxima [B] time = 1.50139, size = 416, normalized size = 11.89

$$-\frac{2c\left(\frac{1}{\tan(bx+a)} - \tan(bx+a)\right) - \frac{2ad\left(\frac{1}{\tan(bx+a)} - \tan(bx+a)\right)}{b} - \frac{\left(\left(\cos(4bx+4a)^2 + \sin(4bx+4a)^2 - 2\cos(4bx+4a) + 1\right)\log\left(\cos(2bx+2a)^2 + \sin(2bx+2\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*(1/tan(b*x + a) - tan(b*x + a)) - 2*a*d*(1/tan(b*x + a) - tan(b*x
 + a))/b - ((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) +
 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) +
 (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*d/((cos(4*b*x +
 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b))/b
```

Fricas [B] time = 0.49963, size = 197, normalized size = 5.63

$$\frac{d\cos\left(bx+a\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)\sin\left(bx+a\right)\right)\sin\left(bx+a\right)+bdx-2\left(bdx+bc\right)\cos\left(bx+a\right)^{2}+bc}{b^{2}\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a)*log(-1/2*cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + b*d*x -
2*(b*d*x + b*c)*cos(b*x + a)^2 + b*c)/(b^2*cos(b*x + a)*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.276 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=23

$$4\text{Unintegrable}\left(\frac{\csc^2(2a+2bx)}{c+dx},x\right)$$

```
[Out] 4*Unintegrable[Csc[2*a + 2*b*x]^2/(c + d*x), x]
```

Rubi [A] time = 0.0936632, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = 4\int \frac{\csc^2(2a+2bx)}{c+dx}\,dx$$

Mathematica [A] time = 7.3347, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x), x]
```

Maple [A] time = 0.518, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx+a))^{2} (\sec (bx+a))^{2}}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc (bx+a)^{2} \sec (bx+a)^{2}}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{2}(a+bx)\sec^{2}(a+bx)}{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

3.277 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$4\text{Unintegrable}\left(\frac{\csc^2(2a+2bx)}{(c+dx)^2},x\right)$$

```
[Out] 4*Unintegrable[Csc[2*a + 2*b*x]^2/(c + d*x)^2, x]
```

Rubi [A] time = 0.0857983, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = 4\int \frac{\csc^2(2a+2bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 7.58574, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A] time = 0.811, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx + a))^2 (\sec (bx + a))^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx + a)^2 \sec (bx + a)^2}{d^2x^2 + 2\, cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2 (a + bx) \sec^2 (a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c)^2, x)
```

3.278 $\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\csc^3(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi [A] time = 0.215213, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx) \, dx = \int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx) \, dx$$

Mathematica [A] time = 11.7618, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Maple [A] time = 0.21, size = 0, normalized size = 0.

$$\int (dx+c)^{m} (\csc (bx+a))^{3} (\sec (bx+a))^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^{m} \csc (bx+a)^{3} \sec (bx+a)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^{m} \csc (bx+a)^{3} \sec (bx+a)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

3.279 $\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=601

$$-\frac{6icd^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6icd^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{9d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{9d^2(c+dx)\text{PolyLog}}{b^3}$$

```
[Out] ((12*I)*c*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 + ((6*I)*d^3*x^2*ArcTan[E^(I*(
a + b*x))])/b^2 - (6*d^3*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (3*(c + d*x)^3*A
rcTanh[E^(I*(a + b*x))])/b - (3*c*d^2*ArcTanh[Cos[a + b*x]])/b^3 - (3*c^2*d
*ArcTanh[Sin[a + b*x]])/b^2 - (3*c^2*d*Csc[a + b*x])/(2*b^2) - (3*c*d^2*x*C
sc[a + b*x])/b^2 - (3*d^3*x^2*Csc[a + b*x])/(2*b^2) + ((3*I)*d^3*PolyLog[2,
 -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*
x))])/b^2 - ((6*I)*c*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3
*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*c*d^2*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 + ((6*I)*d^3*x*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*
d^3*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2,
E^(I*(a + b*x))])/b^2 - (9*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3
+ (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E^(I*(
a + b*x))])/b^4 + (9*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I
)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, E^(I*(a + b
*x))])/b^4 + (3*(c + d*x)^3*Sec[a + b*x])/(2*b) - ((c + d*x)^3*Csc[a + b*x]
^2*Sec[a + b*x])/(2*b)
```

Rubi [A] time = 2.31269, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 64, number of rules used = 24, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {2622, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4133, 453, 206, 4181, 2279, 2391, 2621, 6271, 3770, 14}

$$-\frac{6icd^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6icd^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{9d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{9d^2(c+dx)\text{PolyLog}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```


```
[Out] ((12*I)*c*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 + ((6*I)*d^3*x^2*ArcTan[E^(I*(
a + b*x))])/b^2 - (6*d^3*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (3*(c + d*x)^3*A
rcTanh[E^(I*(a + b*x))])/b - (3*c*d^2*ArcTanh[Cos[a + b*x]])/b^3 - (3*c^2*d
*ArcTanh[Sin[a + b*x]])/b^2 - (3*c^2*d*Csc[a + b*x])/(2*b^2) - (3*c*d^2*x*C
sc[a + b*x])/b^2 - (3*d^3*x^2*Csc[a + b*x])/(2*b^2) + ((3*I)*d^3*PolyLog[2,
 -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*
```

```
x))])/b^2 - ((6*I)*c*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3
*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*c*d^2*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 + ((6*I)*d^3*x*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*
d^3*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2,
E^(I*(a + b*x))])/b^2 - (9*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3
+ (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E^(I*(
a + b*x))])/b^4 + (9*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I
)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, E^(I*(a + b
*x))])/b^4 + (3*(c + d*x)^3*Sec[a + b*x])/(2*b) - ((c + d*x)^3*Csc[a + b*x]
^2*Sec[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
```

```
x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(\cdots}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(\cdots}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(\cdots}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(\cdots}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(\cdots}{}$$

$$= \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b} - \frac{3\int b(-c-dx)\cdots}{\cdots}$$

$$= \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b} - \frac{3}{2}\int (-c-dx)^{3}\cdots$$

$$= -\frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(a+\cdots}{2b}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3c^2d\csc(a+bx)}{2b^2} + \frac{9id(c+dx)^2\text{Li}_2\left(-e^{i(a+bx)}\cdots\right.}{2b^2}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\cdots\right.}{b}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\cdots\right.}{b}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\cdots\right.}{b}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \cdots$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \cdots$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \cdots$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \cdots$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \cdots$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \cdots$$

Mathematica [A] time = 8.31608, size = 907, normalized size = 1.51

$$-\frac{\sec(a+bx)\left(-bc^3+3b\cos(2a+2bx)c^3-3bdxc^2+9bdx\cos(2a+2bx)c^2+3d\sin(2a+2bx)c^2-3bd^2x^2c+9bd^2x^2\cos(\right.}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*
(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 +
I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c
+ d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3
, I*E^(I*(a + b*x))]))/b^4 + (3*(b^3*c^3*Log[1 - E^(I*(a + b*x))] + 2*b*c*d
^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - E^(I*(a + b*x))] + 2*b*
d^3*x*Log[1 - E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] +
 b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^3*c^3*Log[1 + E^(I*(a + b*x))] -
2*b*c*d^2*Log[1 + E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))]
 - 2*b*d^3*x*Log[1 + E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b
*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)
^2)*PolyLog[2, -E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[
2, E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*Po
lyLog[3, -E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^
3*x*PolyLog[3, E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] +
(6*I)*d^3*PolyLog[4, E^(I*(a + b*x))]))/(2*b^4) - (Csc[a + b*x]^2*Sec[a + b
*x]*(-(b*c^3) - 3*b*c^2*d*x - 3*b*c*d^2*x^2 - b*d^3*x^3 + 3*b*c^3*Cos[2*a +
 2*b*x] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*Cos[2*a + 2*b*x] + 3
*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin[2*a
+ 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

Maple [B] time = 0.727, size = 1613, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] -9/2/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+9/2/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-
9*I/b^2*d^2*c*polylog(2,exp(I*(b*x+a)))*x+9*I/b^2*d^2*c*polylog(2,-exp(I*(b
*x+a)))*x+3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4-3*d^3/b^2*ln(1-I*exp(I*(b*
```

```
x+a)))*x^2-3*d^3/b^4*a^2*ln(1+I*exp(I*(b*x+a)))+3*d^3/b^2*ln(1+I*exp(I*(b*x
+a)))*x^2+3*d^3/b^4*a^2*ln(1-I*exp(I*(b*x+a)))+6*I*d^3*x*polylog(2,I*exp(I*
(b*x+a)))/b^3+9*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-3/b^3*d^3*ln(exp(I*(b*x
+a))+1)*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a
+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b
^4+6*I*d/b^2*c^2*arctan(exp(I*(b*x+a)))-9*I*d^3*polylog(4,-exp(I*(b*x+a)))/
b^4-3/b^4*d^3*a*ln(exp(I*(b*x+a))-1)-3/b^3*d^2*c*ln(exp(I*(b*x+a))+1)+3/b^3
*d^2*c*ln(exp(I*(b*x+a))-1)+3/2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+3/2/b^4*d^3*
ln(1-exp(I*(b*x+a)))*a^3-3/2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+9/2/b^3*c*d^2*a
^2*ln(exp(I*(b*x+a))-1)-9/2/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-9/2/b^3*c*d^2*
a^2*ln(1-exp(I*(b*x+a)))-9/2/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+9/2/b*c^2*d*ln(
1-exp(I*(b*x+a)))*x+9/2/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-6*I*d^3*x*polylog(
2,-I*exp(I*(b*x+a)))/b^3+6*d^2/b^3*c*ln(1+I*exp(I*(b*x+a)))*a-6*d^2/b^2*c*l
n(1-I*exp(I*(b*x+a)))*x-6*d^2/b^3*c*ln(1-I*exp(I*(b*x+a)))*a+6*d^2/b^2*c*ln
(1+I*exp(I*(b*x+a)))*x+6*I*d^3/b^4*a^2*arctan(exp(I*(b*x+a)))-12*I*d^2/b^3*
c*a*arctan(exp(I*(b*x+a)))+9/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-9/b^3*d^3*
polylog(3,-exp(I*(b*x+a)))*x+9/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-9/b^3*c*
d^2*polylog(3,-exp(I*(b*x+a)))-3/2/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)-9/2*I/b
^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+9/2*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)
))*x^2-9/2*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+9/2*I/b^2*c^2*d*polylog(2,
-exp(I*(b*x+a)))+6*I/b^4*d^3*polylog(2,I*exp(I*(b*x+a)))*a+6*I/b^3*d^2*c*di
log(1-I*exp(I*(b*x+a)))-6*I/b^3*d^2*c*dilog(1+I*exp(I*(b*x+a)))+6*I/b^4*d^3
*a*dilog(1+I*exp(I*(b*x+a)))-6*I/b^4*d^3*polylog(2,-I*exp(I*(b*x+a)))*a-6*I
/b^4*d^3*a*dilog(1-I*exp(I*(b*x+a)))+3/2/b*c^3*ln(exp(I*(b*x+a))-1)-3/2/b*c
^3*ln(exp(I*(b*x+a))+1)+1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(
3*d^3*x^3*b*exp(5*I*(b*x+a))+9*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp
(5*I*(b*x+a))-2*d^3*x^3*b*exp(3*I*(b*x+a))+3*c^3*b*exp(5*I*(b*x+a))-6*c*d^2
*x^2*b*exp(3*I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))-6*c^2*d*x*b*exp(3*I*(b*x
+a))+3*d^3*x^3*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(5*I*(b*x+a))-2*c^3*b*exp(3*
I*(b*x+a))+9*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*x+a))+9*c^2*
d*x*b*exp(I*(b*x+a))+3*c^3*b*exp(I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a))-3*I*
c^2*d*exp(5*I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))-3*I*d^3*polylog(2,exp(I*(b
*x+a)))/b^4
```

Maxima [B] time = 18.6152, size = 10858, normalized size = 18.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*cos(b*x + a)
^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(c
os(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3
 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 -
 a^3*d^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^3 + 4*((12*b^2*c^2*d - 24*a*
b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*a^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a)
+ 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^
2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 12*(b^2
*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
 + a))*cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a
)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(6*b*x
+ 6*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2
*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (12*I*b^2*
c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^
2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x
+ a) + 1) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*a^2*d^3
+ 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^
2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*(b^2
*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
 + a))*cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a
^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d
 + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 +
24*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2
+ 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x +
a))*sin(4*b*x + 4*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*
d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a
))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (6*(b*x + a)^3*d^3 + 12*b*c*d
^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 +
 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a
*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^
2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 -
2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3
*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + 12*I*b*
c*d^2 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^
2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a) +
 (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*
a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)
*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 +
12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d +
36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arcta
```

n2(sin(b*x + a), cos(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*c*d^2 - a*d^3)*cos(6*b*x + 6*a) - 12*(b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 12*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 + 12*I*a*d^3)*sin(6*b*x + 6*a) - (12*I*b*c*d^2 - 12*I*a*d^3)*sin(4*b*x + 4*a) - (12*I*b*c*d^2 - 12*I*a*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + (36*I*b*c*d^2 - 12*(3*I*a - 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 24*(3*I*a - 1)*b*c*d^2 + (36*I*a^2 - 24*a)*d^3)*(b*x + a))*cos(5*b*x + 5*a) - (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3)*(b*x + a))*cos(3*b*x + 3*a) - (12*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*d^2 - 12*a^2*d^3 + (36*I*b*c*d^2 - 12*(3*I*a + 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 24*(3*I*a + 1)*b*c*d^2 + (36*I*a^2 + 24*a)*d^3)*(b*x + a))*cos(b*x + a) + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x + 4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(6*b*x + 6*a) - (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(4*b*x + 4*a) - (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x + 4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(6*b*x + 6*a) + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(4*b*x + 4*a) + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 1

2*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (18*I
*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3
+ (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (18*I*b^2*c^2*d
 - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 + (36*I*b*
c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) -
(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*
(b*c*d^2 - a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*
d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6
*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*
d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3
*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b
*x + 2*a) + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I
*a^2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a)
+ (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 1
2*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-18*
I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I)*d^
3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*
x + I*a)) - (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2
 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 -
6*I)*d^3)*(b*x + a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9
*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-
9*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (3*I*(b*x + a)^3*d^3 + 6*
I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^
2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(
b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x
 + a))*sin(6*b*x + 6*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c
*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(
b*x + a))*sin(4*b*x + 4*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(
b*x + a) + 1) - (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d
^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 +
6*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I
*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*
a^2 + 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b
*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2
*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)
*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*
x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*
(b*x + a))*sin(6*b*x + 6*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*

(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x + 6*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (36*d^3*cos(6*b*x + 6*a) - 36*d^3*cos(4*b*x + 4*a) - 36*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) - 36*I*d^3*sin(4*b*x + 4*a) - 36*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*polylog(4, -e^(I*b*x + I*a)) + (36*d^3*cos(6*b*x + 6*a) - 36*d^3*cos(4*b*x + 4*a) - 36*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) - 36*I*d^3*sin(4*b*x + 4*a) - 36*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*polylog(4, e^(I*b*x + I*a)) - (-24*I*d^3*cos(6*b*x + 6*a) + 24*I*d^3*cos(4*b*x + 4*a) + 24*I*d^3*cos(2*b*x + 2*a) + 24*d^3*sin(6*b*x + 6*a) - 24*d^3*sin(4*b*x + 4*a) - 24*d^3*sin(2*b*x + 2*a) - 24*I*d^3)*polylog(3, I*e^(I*b*x + I*a)) - (24*I*d^3*cos(6*b*x + 6*a) - 24*I*d^3*cos(4*b*x + 4*a) - 24*I*d^3*cos(2*b*x + 2*a) - 24*d^3*sin(6*b*x + 6*a) + 24*d^3*sin(4*b*x + 4*a) + 24*d^3*sin(2*b*x + 2*a) + 24*I*d^3)*polylog(3, -I*e^(I*b*x + I*a)) - (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3 + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*cos(6*b*x + 6*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(2*b*x + 2*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(6*b*x + 6*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (36*I*b*c*d^2 + 36*I*(b*x + a)

```
*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(6*
b*x + 6*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*cos(4*b*x +
4*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*cos(2*b*x + 2*a) -
 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(6*b*x + 6*a) + 36*(b*c*d^2 + (b*x
 + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*
sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (12*(b*x + a)^3*d^3 - 12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (36*b*c*d^2 - (36*a + 12*I)*d^3
)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a + 24*I)*b*c*d^2 + 12*(3*a^2 + 2*I*a)*
d^3)*(b*x + a))*sin(5*b*x + 5*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(3*b*x +
 3*a) + (12*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^
3 + (36*b*c*d^2 - (36*a - 12*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a -
24*I)*b*c*d^2 + 12*(3*a^2 - 2*I*a)*d^3)*(b*x + a))*sin(b*x + a))/(-4*I*b^3*
cos(6*b*x + 6*a) + 4*I*b^3*cos(4*b*x + 4*a) + 4*I*b^3*cos(2*b*x + 2*a) + 4*
b^3*sin(6*b*x + 6*a) - 4*b^3*sin(4*b*x + 4*a) - 4*b^3*sin(2*b*x + 2*a) - 4*
I*b^3))/b
```

Fricas [C] time = 1.3474, size = 7588, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 6*(b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 6*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - ((-9*I
*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^3 +
 (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a
))*dilog(cos(b*x + a) + I*sin(b*x + a)) - ((9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^
2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^3 + (-9*I*b^2*d^3*x^2 - 18*I*b^
2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a))*dilog(cos(b*x + a) - I*s
in(b*x + a)) - ((12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a)^3 + (-12*I*b*d^3
*x - 12*I*b*c*d^2)*cos(b*x + a))*dilog(I*cos(b*x + a) + sin(b*x + a)) - ((1
2*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a)^3 + (-12*I*b*d^3*x - 12*I*b*c*d^2)
*cos(b*x + a))*dilog(I*cos(b*x + a) - sin(b*x + a)) - ((-12*I*b*d^3*x - 12*
I*b*c*d^2)*cos(b*x + a)^3 + (12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a))*dil
og(-I*cos(b*x + a) + sin(b*x + a)) - ((-12*I*b*d^3*x - 12*I*b*c*d^2)*cos(b*
x + a)^3 + (12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a))*dilog(-I*cos(b*x + a
) - sin(b*x + a)) - ((-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d -
 6*I*d^3)*cos(b*x + a)^3 + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^
```

2*d + 6*I*d^3)*cos(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((9*I*
b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^3 +
(-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a
))*dilog(-cos(b*x + a) - I*sin(b*x + a)) + 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^3 - (b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x
)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 6*((b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*c
os(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*((b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a
)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d +
 2*b*d^3)*x)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 6*((b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 +
 a^2*d^3)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) + 6*((b^2*d^
3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^3 - (b^2*d^3*x^
2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a))*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) - 6*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*
d^3)*cos(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)
*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 6*((b^2*d^3*x^2 + 2
*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a))*log(-I*cos(b*x + a) + sin(b
x + a) + 1) - 6((b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos
(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x
 + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*((b^3*c^3 - 3*a*b^2*c^2*
d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^3 - (b^3*c^3 - 3*a*
b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a))*log(-1/2*c
os(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*((b^3*c^3 - 3*a*b^2*c^2*d + (3*
a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^3 - (b^3*c^3 - 3*a*b^2*c^2
*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a))*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) - 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*co
s(b*x + a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a))*log(-cos(b*
x + a) + I*sin(b*x + a) + 1) + 6*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b
*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a))*log(-cos(b*x
+ a) + I*sin(b*x + a) + I) - 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x
+ a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (
a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a))*log(-cos(b*x + a)
 - I*sin(b*x + a) + 1) - 6*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a
)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a))*log(-cos(b*x + a) -
 I*sin(b*x + a) + I) - (18*I*d^3*cos(b*x + a)^3 - 18*I*d^3*cos(b*x + a))*po
lylog(4, cos(b*x + a) + I*sin(b*x + a)) - (-18*I*d^3*cos(b*x + a)^3 + 18*I*
d^3*cos(b*x + a))*polylog(4, cos(b*x + a) - I*sin(b*x + a)) - (18*I*d^3*cos
(b*x + a)^3 - 18*I*d^3*cos(b*x + a))*polylog(4, -cos(b*x + a) + I*sin(b*x +

```
 a)) - (-18*I*d^3*cos(b*x + a)^3 + 18*I*d^3*cos(b*x + a))*polylog(4, -cos(b
*x + a) - I*sin(b*x + a)) - 18*((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3
*x + b*c*d^2)*cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 18*
((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a))*pol
ylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(d^3*cos(b*x + a)^3 - d^3*cos(b
*x + a))*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 12*(d^3*cos(b*x + a)^3
 - d^3*cos(b*x + a))*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(d^3*co
s(b*x + a)^3 - d^3*cos(b*x + a))*polylog(3, -I*cos(b*x + a) + sin(b*x + a))
 + 12*(d^3*cos(b*x + a)^3 - d^3*cos(b*x + a))*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)) + 18*((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2
)*cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 18*((b*d^3*x +
 b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a))*polylog(3, -co
s(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a)^3 - b^4*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.280 $\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=305

$$\frac{3id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,\right.}{b^3}$$

```
[Out] ((4*I)*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*(c + d*x)^2*ArcTanh[E^(I*(a
+ b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3 - (2*c*d*ArcTanh[Sin[a + b*x]
])/b^2 - (c*d*Csc[a + b*x])/b^2 - (d^2*x*Csc[a + b*x])/b^2 + ((3*I)*d*(c +
d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (3*d^2*PolyLog[3, -E^(I*(a + b*
x))])/b^3 + (3*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 + (3*(c + d*x)^2*Sec[a
+ b*x])/(2*b) - ((c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rubi [A] time = 0.865244, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 22, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {2622, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4183, 2531, 2282, 6589, 4133, 453, 206, 4181, 2279, 2391, 2621, 6271, 3770}

$$\frac{3id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```


```
[Out] ((4*I)*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*(c + d*x)^2*ArcTanh[E^(I*(a
+ b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3 - (2*c*d*ArcTanh[Sin[a + b*x]
])/b^2 - (c*d*Csc[a + b*x])/b^2 - (d^2*x*Csc[a + b*x])/b^2 + ((3*I)*d*(c +
d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (3*d^2*PolyLog[3, -E^(I*(a + b*
x))])/b^3 + (3*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 + (3*(c + d*x)^2*Sec[a
+ b*x])/(2*b) - ((c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
```

```
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
```

```
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a$$

$$= \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\int b(c+dx)^2 c}{2b}$$

$$= \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3}{2}\int (c+dx)^2 \mathrm{cs}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a+bx}{2b}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{cd\csc(a+bx)}{b^2} + \frac{3id(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} -$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2cd\tanh^{-1}(\sin(a+bx)}{b^2}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2cd\tanh^{-1}(\sin(a+bx)}{b^2}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}$$

Mathematica [B] time = 7.9231, size = 889, normalized size = 2.91

$$-\frac{2\left(\frac{2\tan^{-1}(\cot(a))\tanh^{-1}\left(\frac{\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}-\frac{\csc(a)\left(\left(bx-\tan^{-1}(\cot(a))\right)\left(\log\left(1-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-\log\left(1+e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)\right)+i\left(\mathrm{PolyLog}\left(2,-\right.\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2]^2)/(8*b) + (3*b^2*c^2*Log[1
- E^(I*(a + b*x))] + 2*d^2*Log[1 - E^(I*(a + b*x))] + 6*b^2*c*d*x*Log[1 - E
^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*c^2*Log[1
+ E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x))] - 6*b^2*c*d*x*Log[1 + E
^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (6*I)*b*d*(c + d
*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a +
 b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*d^2*PolyLog[3, E^(I*(a + b
*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/2 + (b*x)/2]^2)/(8*b) + (
(c + d*x)*Csc[a]*Sec[a]*(-(d*Cos[a]) + b*c*Sin[a] + b*d*x*Sin[a]))/b^2 - ((
4*I)*c*d*ArcTan[((-I)*Sin[a] - I*Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a
]^2]])/(b^2*Sqrt[Cos[a]^2 + Sin[a]^2]) - (2*d^2*(-((Csc[a]*((b*x - ArcTan[C
ot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan
[Cot[a]]))]) + I*(PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - PolyLog[2, E^
(I*(b*x - ArcTan[Cot[a]]))])))/Sqrt[1 + Cot[a]^2]) + (2*ArcTan[Cot[a]]*ArcT
anh[(Sin[a] + Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^
2 + Sin[a]^2]))/b^3 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)/2]) - d
^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)/2
] + d^2*x*Sin[(b*x)/2]))/(2*b^2) + (c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2]
 + d^2*x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin
[a/2 + (b*x)/2])) + (-(c^2*Sin[(b*x)/2]) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*S
in[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)
/2]))
```

Maple [B] time = 0.508, size = 802, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2*d^2
/b^2*ln(1-I*exp(I*(b*x+a)))*x-2*d^2/b^3*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^
3*dilog(1+I*exp(I*(b*x+a)))+2*I*d^2/b^3*dilog(1-I*exp(I*(b*x+a)))-3/2/b^3*d
^2*ln(1-exp(I*(b*x+a)))*a^2-3/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/2/b*d^2*ln
(1-exp(I*(b*x+a)))*x^2+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2/b^3*a*arc
tan(exp(I*(b*x+a)))-1/b^3*d^2*ln(exp(I*(b*x+a))+1)+1/b^3*d^2*ln(exp(I*(b*x+
a))-1)-3/b*c*d*ln(exp(I*(b*x+a))+1)*x-3/2/b*c^2*ln(exp(I*(b*x+a))+1)+3/2/b*
c^2*ln(exp(I*(b*x+a))-1)-3*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+3*d^2*polylog
(3,exp(I*(b*x+a)))/b^3+3/b*c*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c*d*ln(1-exp(I*
(b*x+a)))*a+1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*d^2*x^2*b*
exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*x+a))+3*c^2*b*exp(5*I*(b*x+a))-2*d^2*
x^2*b*exp(3*I*(b*x+a))-4*c*d*x*b*exp(3*I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x+a)
)-2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b*exp(I*(b*x+a))-2*I*d*c*exp(5*I*(b*x+
a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*exp(I*(b*x+a))+2*I*d^2*x*exp(I*(b*x+a)
)+2*I*d*c*exp(I*(b*x+a)))+3/2/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*c*d*a*
ln(exp(I*(b*x+a))-1)-3*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+3*I/b^2*d^2*poly
log(2,-exp(I*(b*x+a)))*x-3*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+3*I/b^2*c*
d*polylog(2,-exp(I*(b*x+a)))
```

Maxima [B] time = 5.04162, size = 5157, normalized size = 16.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 2*a*c*d*(2*(3*cos(b*x + a)^2
 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos
(b*x + a) - 1))/b + a^2*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos
(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 + 4*((8
*b*c*d + 8*(b*x + a)*d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(
6*b*x + 6*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*
d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-8*I*b*c*d - 8*I*(b*x + a)*d
^2 + 8*I*a*d^2)*sin(6*b*x + 6*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d
^2)*sin(4*b*x + 4*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*
x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (8*b*c*d + 8*(b*x + a)*
d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 8*(b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a
```

```
*d^2)*cos(2*b*x + 2*a) - (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(6
*b*x + 6*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(4*b*x + 4*a)
- (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos
(b*x + a), -sin(b*x + a) + 1) - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*
x + a) + 4*d^2 + 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2
)*cos(6*b*x + 6*a) - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2
*d^2)*cos(4*b*x + 4*a) - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)
 + 2*d^2)*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^
2)*(b*x + a) + 4*I*d^2)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b
*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^
2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(2*b*x + 2*a))*a
rctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*d^2*cos(6*b*x + 6*a) - 4*d^2*co
s(4*b*x + 4*a) - 4*d^2*cos(2*b*x + 2*a) + 4*I*d^2*sin(6*b*x + 6*a) - 4*I*d^
2*sin(4*b*x + 4*a) - 4*I*d^2*sin(2*b*x + 2*a) + 4*d^2)*arctan2(sin(b*x + a)
, cos(b*x + a) - 1) - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 6
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 6*((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^
2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*sin(6*b*x + 6*a) + (-6*I*(b*x + a)
^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*
x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arct
an2(sin(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^2*d^2 + 8*b*c*d - 8*
a*d^2 + (24*I*b*c*d - 8*(3*I*a - 1)*d^2)*(b*x + a))*cos(5*b*x + 5*a) - (-8*
I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a))*cos(3*b*x + 3*a)
- (12*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (24*I*b*c*d - 8*(3*I*a + 1)*d
^2)*(b*x + a))*cos(b*x + a) + (8*d^2*cos(6*b*x + 6*a) - 8*d^2*cos(4*b*x + 4
*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I*d^2*sin(6*b*x + 6*a) - 8*I*d^2*sin(4*b*x
 + 4*a) - 8*I*d^2*sin(2*b*x + 2*a) + 8*d^2)*dilog(I*e^(I*b*x + I*a)) - (8*d
^2*cos(6*b*x + 6*a) - 8*d^2*cos(4*b*x + 4*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I
*d^2*sin(6*b*x + 6*a) - 8*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x + 2*a)
 + 8*d^2)*dilog(-I*e^(I*b*x + I*a)) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d
^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x
 + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*co
s(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(6*b*x
+ 6*a) - (12*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(4*b*x + 4*a) -
(12*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(
I*b*x + I*a)) - (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 + 12*(b*c*d + (b*x
+ a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos
(4*b*x + 4*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (12*I
*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(6*b*x + 6*a) + (-12*I*b*c*d -
 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(4*b*x + 4*a) + (-12*I*b*c*d - 12*I*(b
*x + a)*d^2 + 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-3*I*
(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2 + (-3*I*(b*x
 + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(6*b*x + 6*a
) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2)*cos
```

```
(4*b*x + 4*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) +
2*I*d^2)*cos(2*b*x + 2*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a
) + 2*d^2)*sin(6*b*x + 6*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x +
 a) + 2*d^2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x
 + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) - (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a)
+ 2*I*d^2 + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*
d^2)*cos(6*b*x + 6*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b
*x + a) - 2*I*d^2)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d +
 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - (3*(b*x + a)^2*d^2 + 6*
(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2*d^2 +
6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) + (3*(b*x + a)^2*d^2
+ 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 +
 4*I*a*d^2 + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(6*b*x + 6*a)
+ (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(4*b*x + 4*a) + (4*I*b*c*d
 + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(2*b*x + 2*a) + 4*(b*c*d + (b*x + a)*d
^2 - a*d^2)*sin(6*b*x + 6*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(4*b*x
+ 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (4*I*b*c*d + 4*I*(b*x + a)*d^
2 - 4*I*a*d^2 + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(6*b*x + 6*a
) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(4*b*x + 4*a) + (-4*I*b
*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(2*b*x + 2*a) - 4*(b*c*d + (b*x +
a)*d^2 - a*d^2)*sin(6*b*x + 6*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(4*
b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-12*I*d^2*cos(6*b*x + 6*
a) + 12*I*d^2*cos(4*b*x + 4*a) + 12*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(6*b
*x + 6*a) - 12*d^2*sin(4*b*x + 4*a) - 12*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*p
olylog(3, -e^(I*b*x + I*a)) - (12*I*d^2*cos(6*b*x + 6*a) - 12*I*d^2*cos(4*b
*x + 4*a) - 12*I*d^2*cos(2*b*x + 2*a) - 12*d^2*sin(6*b*x + 6*a) + 12*d^2*si
n(4*b*x + 4*a) + 12*d^2*sin(2*b*x + 2*a) + 12*I*d^2)*polylog(3, e^(I*b*x +
I*a)) + (12*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I*a*d^2 + (24*b*c*d - (24*a + 8
*I)*d^2)*(b*x + a))*sin(5*b*x + 5*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^
2)*(b*x + a))*sin(3*b*x + 3*a) + (12*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^
2 + (24*b*c*d - (24*a - 8*I)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^2*cos(6*
b*x + 6*a) + 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*si
n(6*b*x + 6*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
)/b
```

Fricas [C] time = 0.958072, size = 4454, normalized size = 14.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a)
 - ((-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a)^3 + (6*I*b*d^2*x + 6*I*b*c*d)*c
os(b*x + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) - ((6*I*b*d^2*x + 6*I*b*c
*d)*cos(b*x + a)^3 + (-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a))*dilog(cos(b*x
 + a) - I*sin(b*x + a)) - (4*I*d^2*cos(b*x + a)^3 - 4*I*d^2*cos(b*x + a))*d
ilog(I*cos(b*x + a) + sin(b*x + a)) - (4*I*d^2*cos(b*x + a)^3 - 4*I*d^2*cos
(b*x + a))*dilog(I*cos(b*x + a) - sin(b*x + a)) - (-4*I*d^2*cos(b*x + a)^3
+ 4*I*d^2*cos(b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (-4*I*d^2*c
os(b*x + a)^3 + 4*I*d^2*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a))
 - ((-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a)^3 + (6*I*b*d^2*x + 6*I*b*c*d)*c
os(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((6*I*b*d^2*x + 6*I*b*
c*d)*cos(b*x + a)^3 + (-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a))*dilog(-cos(b
*x + a) - I*sin(b*x + a)) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d
^2)*cos(b*x + a)^3 - (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(
b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b
*x + a)^3 - (b*c*d - a*d^2)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a)
 + I) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a)^3 -
 (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a))*log(cos(b*
x + a) - I*sin(b*x + a) + 1) - 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d -
 a*d^2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) + 4*((b*d^2*x
+ a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^2)*cos(b*x + a))*log(I*cos(b*x + a
) + sin(b*x + a) + 1) - 4*((b*d^2*x + a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*
d^2)*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 4*((b*d^2*x + a
*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^2)*cos(b*x + a))*log(-I*cos(b*x + a)
+ sin(b*x + a) + 1) - 4*((b*d^2*x + a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^
2)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - ((3*b^2*c^2 - 6*
a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^3 - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2
 + 2)*d^2)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)
- ((3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^3 - (3*b^2*c^2 -
6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*si
n(b*x + a) + 1/2) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b*x +
a)^3 - (b*c*d - a*d^2)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + I
) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^3 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a))*log(-cos(b*x
 + a) - I*sin(b*x + a) + 1) - 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d -
a*d^2)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(d^2*cos(b
*x + a)^3 - d^2*cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 6
```

```
*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3, cos(b*x + a) - I*sin(b*
x + a)) + 6*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3, -cos(b*x + a
) + I*sin(b*x + a)) + 6*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3,
-cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^3 - b^3*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

3.281 $\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=154

$$\frac{3idPolyLog\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3idPolyLog\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3(c+dx)\sec(a+bx)}{2b}$$

```
[Out] (-3*d*x*ArcTanh[E^(I*(a + b*x))])/b - (3*c*ArcTanh[Cos[a + b*x]])/(2*b) - (
d*ArcTanh[Sin[a + b*x]])/b^2 - (d*Csc[a + b*x])/(2*b^2) + (((3*I)/2)*d*Poly
Log[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*d*PolyLog[2, E^(I*(a + b*x))])/b
^2 + (3*(c + d*x)*Sec[a + b*x])/(2*b) - ((c + d*x)*Csc[a + b*x]^2*Sec[a + b
*x])/(2*b)
```

Rubi [A] time = 0.191378, antiderivative size = 174, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2622, 288, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770, 2621}

$$\frac{3idPolyLog\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3idPolyLog\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3(c+dx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*d*x*ArcTanh[E^(I*(a + b*x))])/b + (3*d*x*ArcTanh[Cos[a + b*x]])/(2*b) -
 (3*(c + d*x)*ArcTanh[Cos[a + b*x]])/(2*b) - (d*ArcTanh[Sin[a + b*x]])/b^2
- (d*Csc[a + b*x])/(2*b^2) + (((3*I)/2)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2
 - (((3*I)/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + (3*(c + d*x)*Sec[a + b*x
])/(2*b) - ((c + d*x)*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc^2(a+}{2b}$$

$$= -\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc^2(a+}{2b}$$

$$= \frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3d\tanh^{-1}(\sin(}{2b^2}$$

$$= \frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3d\tanh^{-1}(\sin(}{2b^2}$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a}{2b}$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a}{2b}$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a}{2b}$$

Mathematica [B] time = 5.32919, size = 520, normalized size = 3.38

$$\frac{3d\left(i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{2b^2}-\frac{d\tan\left(\frac{1}{2}(a+bx\right.}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (d*x)/b - (d*Cot[(a + b*x)/2])/(4*b^2) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*
x*Csc[(a + b*x)/2]^2)/(8*b) - (3*c*Log[Cos[(a + b*x)/2]])/(2*b) + (d*Log[Co
s[(a + b*x)/2] - Sin[(a + b*x)/2]])/b^2 + (3*c*Log[Sin[(a + b*x)/2]])/(2*b)
 - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])/b^2 - (3*a*d*Log[Tan[(a + b
*x)/2]])/(2*b^2) + (3*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I
*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x)
)])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*x*Sec[(a + b*x)/2]^2)/(8*
b) + (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (c*Si
n[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*(a*Sin[(a +
b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Cos[(a + b*x)/2] + Sin[(a + b*
x)/2])) + (d*(-(a*Sin[(a + b*x)/2]) + (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Co
s[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

Maple [A] time = 0.352, size = 267, normalized size = 1.7

$$\frac{3dxbe^{5i(bx+a)}+3bce^{5i(bx+a)}-2dxbe^{3i(bx+a)}-2bce^{3i(bx+a)}-ide^{5i(bx+a)}+3dxbe^{i(bx+a)}+3bce^{i(bx+a)}+ide^{i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2\left(e^{2i(bx+a)}+1\right)}-\frac{3c\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*d*x*b*exp(5*I*(b*x+a))
+3*b*c*exp(5*I*(b*x+a))-2*d*x*b*exp(3*I*(b*x+a))-2*b*c*exp(3*I*(b*x+a))-I*d
*exp(5*I*(b*x+a))+3*d*x*b*exp(I*(b*x+a))+3*b*c*exp(I*(b*x+a))+I*d*exp(I*(b*
x+a)))-3/2/b*c*ln(exp(I*(b*x+a))+1)+3/2/b*c*ln(exp(I*(b*x+a))-1)-3/2/b^2*d*
a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+3/2*I/b^2*d*dilog(e
xp(I*(b*x+a))+1)-3/2/b*d*ln(exp(I*(b*x+a))+1)*x+3/2*I/b^2*d*dilog(exp(I*(b*
x+a)))
```

Maxima [B] time = 2.80594, size = 2029, normalized size = 13.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((4*d*cos(6*b*x + 6*a) - 4*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 4*I
*d*sin(6*b*x + 6*a) - 4*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 4*d
)*arctan2(2*(cos(b*x + 2*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)
^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*
a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - si
n(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + (6*b*d*x + 6*b*c + 6*(b*d*x
 + b*c)*cos(6*b*x + 6*a) - 6*(b*d*x + b*c)*cos(4*b*x + 4*a) - 6*(b*d*x + b*
c)*cos(2*b*x + 2*a) - (-6*I*b*d*x - 6*I*b*c)*sin(6*b*x + 6*a) - (6*I*b*d*x
+ 6*I*b*c)*sin(4*b*x + 4*a) - (6*I*b*d*x + 6*I*b*c)*sin(2*b*x + 2*a))*arcta
n2(sin(b*x + a), cos(b*x + a) + 1) - (6*b*c*cos(6*b*x + 6*a) - 6*b*c*cos(4*
b*x + 4*a) - 6*b*c*cos(2*b*x + 2*a) + 6*I*b*c*sin(6*b*x + 6*a) - 6*I*b*c*si
n(4*b*x + 4*a) - 6*I*b*c*sin(2*b*x + 2*a) + 6*b*c)*arctan2(sin(b*x + a), co
s(b*x + a) - 1) + (6*b*d*x*cos(6*b*x + 6*a) - 6*b*d*x*cos(4*b*x + 4*a) - 6*
b*d*x*cos(2*b*x + 2*a) + 6*I*b*d*x*sin(6*b*x + 6*a) - 6*I*b*d*x*sin(4*b*x +
 4*a) - 6*I*b*d*x*sin(2*b*x + 2*a) + 6*b*d*x)*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) - (-12*I*b*d*x - 12*I*b*c - 4*d)*cos(5*b*x + 5*a) - (8*I*b*d*x
+ 8*I*b*c)*cos(3*b*x + 3*a) - (-12*I*b*d*x - 12*I*b*c + 4*d)*cos(b*x + a) -
 (6*d*cos(6*b*x + 6*a) - 6*d*cos(4*b*x + 4*a) - 6*d*cos(2*b*x + 2*a) + 6*I*
d*sin(6*b*x + 6*a) - 6*I*d*sin(4*b*x + 4*a) - 6*I*d*sin(2*b*x + 2*a) + 6*d)
*dilog(-e^(I*b*x + I*a)) + (6*d*cos(6*b*x + 6*a) - 6*d*cos(4*b*x + 4*a) - 6
*d*cos(2*b*x + 2*a) + 6*I*d*sin(6*b*x + 6*a) - 6*I*d*sin(4*b*x + 4*a) - 6*I
*d*sin(2*b*x + 2*a) + 6*d)*dilog(e^(I*b*x + I*a)) - (3*I*b*d*x + 3*I*b*c +
(3*I*b*d*x + 3*I*b*c)*cos(6*b*x + 6*a) + (-3*I*b*d*x - 3*I*b*c)*cos(4*b*x +
 4*a) + (-3*I*b*d*x - 3*I*b*c)*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(6*b*x
 + 6*a) + 3*(b*d*x + b*c)*sin(4*b*x + 4*a) + 3*(b*d*x + b*c)*sin(2*b*x + 2*
a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-3*I*b*d*x
 - 3*I*b*c + (-3*I*b*d*x - 3*I*b*c)*cos(6*b*x + 6*a) + (3*I*b*d*x + 3*I*b*c
)*cos(4*b*x + 4*a) + (3*I*b*d*x + 3*I*b*c)*cos(2*b*x + 2*a) + 3*(b*d*x + b*
c)*sin(6*b*x + 6*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) - 3*(b*d*x + b*c)*si
n(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) -
 (-2*I*d*cos(6*b*x + 6*a) + 2*I*d*cos(4*b*x + 4*a) + 2*I*d*cos(2*b*x + 2*a)
 + 2*d*sin(6*b*x + 6*a) - 2*d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a) - 2*I
*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 +
```

```
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)) - 4*(3*b*d*x + 3*b*c - I*d)*sin(5*b*x + 5*a) + 8*(b*d*x + b*c)*sin(
3*b*x + 3*a) - 4*(3*b*d*x + 3*b*c + I*d)*sin(b*x + a))/(-4*I*b^2*cos(6*b*x
+ 6*a) + 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(6*
b*x + 6*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

Fricas [B] time = 0.633759, size = 1652, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*d*x - 6*(b*d*x + b*c)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x +
 a) + 4*b*c - (-3*I*d*cos(b*x + a)^3 + 3*I*d*cos(b*x + a))*dilog(cos(b*x +
a) + I*sin(b*x + a)) - (3*I*d*cos(b*x + a)^3 - 3*I*d*cos(b*x + a))*dilog(co
s(b*x + a) - I*sin(b*x + a)) - (-3*I*d*cos(b*x + a)^3 + 3*I*d*cos(b*x + a))
*dilog(-cos(b*x + a) + I*sin(b*x + a)) - (3*I*d*cos(b*x + a)^3 - 3*I*d*cos(
b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) + 3*((b*d*x + b*c)*cos(b*x
+ a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1)
 + 3*((b*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*
x + a) - I*sin(b*x + a) + 1) - 3*((b*c - a*d)*cos(b*x + a)^3 - (b*c - a*d)*
cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*((b*c -
 a*d)*cos(b*x + a)^3 - (b*c - a*d)*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/
2*I*sin(b*x + a) + 1/2) - 3*((b*d*x + a*d)*cos(b*x + a)^3 - (b*d*x + a*d)*c
os(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*((b*d*x + a*d)*cos
(b*x + a)^3 - (b*d*x + a*d)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a
) + 1) + 2*(d*cos(b*x + a)^3 - d*cos(b*x + a))*log(sin(b*x + a) + 1) - 2*(d
*cos(b*x + a)^3 - d*cos(b*x + a))*log(-sin(b*x + a) + 1))/(b^2*cos(b*x + a)
^3 - b^2*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\csc(bx+a)^3\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

3.282 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi [A] time = 0.201175, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx=\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 23.0601, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]
```

Maple [A] time = 2.755, size = 0, normalized size = 0.

$$\int \frac{\left(\csc \left(bx+a\right)\right)^{3} \left(\sec \left(bx+a\right)\right)^{2}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc \left(bx+a\right)^{3} \sec \left(bx+a\right)^{2}}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{3}\left(a+bx\right)\sec^{2}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.283 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [A] time = 0.224429, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 31.8605, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A] time = 4.368, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx + a))^3 (\sec (bx + a))^2}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx + a)^3 \sec (bx + a)^2}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c)^2, x)
```

3.284 $\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(x^m \csc^3(a+bx)\sec^2(a+bx),x\right)$$

```
[Out] CannotIntegrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi [A] time = 0.923652, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Mathematica [A] time = 11.3338, size = 0, normalized size = 0.

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Maple [A] time = 0.102, size = 0, normalized size = 0.

$$\int x^{m} (\csc (bx + a))^{3} (\sec (bx + a))^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int x^{m} \csc (bx + a)^{3} \sec (bx + a)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(x^{m} \csc (bx + a)^{3} \sec (bx + a)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int x^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

3.285 $\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=387

$$\frac{9ix^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{9ix^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{6ix\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{9x\text{Po}}{}$$

```
[Out] ((6*I)*x^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*x*ArcTanh[E^(I*(a + b*x))])/b^
3 - (3*x^3*ArcTanh[E^(I*(a + b*x))])/b - (3*x^2*Csc[a + b*x])/(2*b^2) + ((3
*I)*PolyLog[2, -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*x^2*PolyLog[2, -E^(I*(a
+ b*x))])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*x*P
olyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*PolyLog[2, E^(I*(a + b*x))])/b^4
 - (((9*I)/2)*x^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (9*x*PolyLog[3, -E^(I*
(a + b*x))])/b^3 + (6*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*PolyLog[3,
 I*E^(I*(a + b*x))])/b^4 + (9*x*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I)*P
olyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*PolyLog[4, E^(I*(a + b*x))])/b^4
+ (3*x^3*Sec[a + b*x])/(2*b) - (x^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rubi [A] time = 0.960072, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.9, Rules used = {2622, 288, 321, 207, 4420, 14, 6273, 12, 4183, 2531, 6609, 2282, 6589, 6742, 4181, 2621, 2279, 2391}

$$\frac{9ix^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{9ix^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{6ix\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{9x\text{Po}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```



```
[Out] ((6*I)*x^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*x*ArcTanh[E^(I*(a + b*x))])/b^
3 - (3*x^3*ArcTanh[E^(I*(a + b*x))])/b - (3*x^2*Csc[a + b*x])/(2*b^2) + ((3
*I)*PolyLog[2, -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*x^2*PolyLog[2, -E^(I*(a
+ b*x))])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*x*P
olyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*PolyLog[2, E^(I*(a + b*x))])/b^4
 - (((9*I)/2)*x^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (9*x*PolyLog[3, -E^(I*
(a + b*x))])/b^3 + (6*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*PolyLog[3,
 I*E^(I*(a + b*x))])/b^4 + (9*x*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I)*P
olyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*PolyLog[4, E^(I*(a + b*x))])/b^4
+ (3*x^3*Sec[a + b*x])/(2*b) - (x^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
```

```
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} - 3\int$$

$$= -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} - 3\int$$

$$= -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\int}{}$$

$$= \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\int bx^3\csc(a+bx)\,dx}{2b} + \frac{3\int\left(-3x\right.}{}$$

$$= \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3}{2}\int x^3\csc(a+bx)\,dx + \frac{3\int x^2\,\mathrm{cs}}{}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{3x^2\csc(}{2b}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{3x^2\csc(}{2b}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{3x^2\csc(}{2b}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(a+b}{2b^2}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(a+b}{2b^2}$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(a+b}{2b^2}$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(a+b}{2b^2}$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(a+b}{2b^2}$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(a+b}{2b^2}$$

Mathematica [A] time = 7.19335, size = 672, normalized size = 1.74

$$\frac{3\left(i\left(3b^2x^2+2\right)\mathrm{PolyLog}(2,-\cos(a+bx)-i\sin(a+bx)) - i\left(3b^2x^2+2\right)\mathrm{PolyLog}(2,\cos(a+bx)+i\sin(a+bx)) - 6b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] -(x^3*Csc[a/2 + (b*x)/2]^2)/(8*b) + (6*(I*b^2*x^2*ArcTan[Cos[a + b*x] + I*S
in[a + b*x]] + I*b*x*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - I*b*x*Poly
Log[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] - PolyLog[3, I*Cos[a + b*x] - Sin[
a + b*x]] + PolyLog[3, (-I)*Cos[a + b*x] + Sin[a + b*x]]))/b^4 + (3*(2*b*x*
Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] + b^3*x^3*Log[1 - Cos[a + b*x] - I*S
in[a + b*x]] - 2*b*x*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] - b^3*x^3*Log[1
 + Cos[a + b*x] + I*Sin[a + b*x]] + I*(2 + 3*b^2*x^2)*PolyLog[2, -Cos[a + b
*x] - I*Sin[a + b*x]] - I*(2 + 3*b^2*x^2)*PolyLog[2, Cos[a + b*x] + I*Sin[a
 + b*x]] - 6*b*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] + 6*b*x*PolyLog
[3, Cos[a + b*x] + I*Sin[a + b*x]] - (6*I)*PolyLog[4, -Cos[a + b*x] - I*Sin
[a + b*x]] + (6*I)*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]))/(2*b^4) + (x
^3*Sec[a/2 + (b*x)/2]^2)/(8*b) + (x^2*Csc[a]*Sec[a]*(-3*Cos[a] + 2*b*x*Sin[
a]))/(2*b^2) + (3*x^2*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) - (
3*x^2*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) + (x^3*Sin[(b*x)/2]
)/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (x^
3*Sin[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b
*x)/2]))
```

Maple [F] time = 1.401, size = 0, normalized size = 0.

$$\int x^3 (\csc (bx + a))^3 (\sec (bx + a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Maxima [B] time = 3.74018, size = 5385, normalized size = 13.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 4*((12*(b*x + a)^2 - 24*(b*
x + a)*a + 12*a^2 + 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(6*b*x + 6*a)
 - 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) - 12*((b*x + a)^
2 - 2*(b*x + a)*a + a^2)*cos(2*b*x + 2*a) - (-12*I*(b*x + a)^2 + 24*I*(b*x
+ a)*a - 12*I*a^2)*sin(6*b*x + 6*a) - (12*I*(b*x + a)^2 - 24*I*(b*x + a)*a
+ 12*I*a^2)*sin(4*b*x + 4*a) - (12*I*(b*x + a)^2 - 24*I*(b*x + a)*a + 12*I*
a^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (12*(b*x +
 a)^2 - 24*(b*x + a)*a + 12*a^2 + 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*co
s(6*b*x + 6*a) - 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) -
12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(2*b*x + 2*a) - (-12*I*(b*x + a)^
2 + 24*I*(b*x + a)*a - 12*I*a^2)*sin(6*b*x + 6*a) - (12*I*(b*x + a)^2 - 24*
I*(b*x + a)*a + 12*I*a^2)*sin(4*b*x + 4*a) - (12*I*(b*x + a)^2 - 24*I*(b*x
+ a)*a + 12*I*a^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) +
1) - (6*(b*x + a)^3 - 18*(b*x + a)^2*a + 6*(3*a^2 + 2)*(b*x + a) + 6*((b*x
+ a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos(6*b*x + 6*a) -
6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos(4*b*x +
 4*a) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos
(2*b*x + 2*a) + (6*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + (18*I*a^2 + 12*I)*(
b*x + a) - 12*I*a)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3 + 18*I*(b*x + a)^2*
a + (-18*I*a^2 - 12*I)*(b*x + a) + 12*I*a)*sin(4*b*x + 4*a) + (-6*I*(b*x +
a)^3 + 18*I*(b*x + a)^2*a + (-18*I*a^2 - 12*I)*(b*x + a) + 12*I*a)*sin(2*b*
x + 2*a) - 12*a)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (12*a*cos(6*b*x
+ 6*a) - 12*a*cos(4*b*x + 4*a) - 12*a*cos(2*b*x + 2*a) + 12*I*a*sin(6*b*x +
 6*a) - 12*I*a*sin(4*b*x + 4*a) - 12*I*a*sin(2*b*x + 2*a) + 12*a)*arctan2(s
in(b*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^3 - 18*(b*x + a)^2*a + 6*(3*a
^2 + 2)*(b*x + a) + 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a
))*cos(6*b*x + 6*a) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x +
 a))*cos(4*b*x + 4*a) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x
 + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + (18*I*a^2
 + 12*I)*(b*x + a))*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3 + 18*I*(b*x + a)^2
*a + (-18*I*a^2 - 12*I)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^3 + 1
8*I*(b*x + a)^2*a + (-18*I*a^2 - 12*I)*(b*x + a))*sin(2*b*x + 2*a))*arctan2
(sin(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^3 - 12*(b*x + a)^2*(3*I
*a - 1) + (36*I*a^2 - 24*a)*(b*x + a) + 12*a^2)*cos(5*b*x + 5*a) - (-8*I*(b
*x + a)^3 + 24*I*(b*x + a)^2*a - 24*I*(b*x + a)*a^2)*cos(3*b*x + 3*a) - (12
*I*(b*x + a)^3 - 12*(b*x + a)^2*(3*I*a + 1) + (36*I*a^2 + 24*a)*(b*x + a) -
 12*a^2)*cos(b*x + a) + (24*b*x*cos(6*b*x + 6*a) - 24*b*x*cos(4*b*x + 4*a)
- 24*b*x*cos(2*b*x + 2*a) + 24*I*b*x*sin(6*b*x + 6*a) - 24*I*b*x*sin(4*b*x
+ 4*a) - 24*I*b*x*sin(2*b*x + 2*a) + 24*b*x)*dilog(I*e^(I*b*x + I*a)) - (24
*b*x*cos(6*b*x + 6*a) - 24*b*x*cos(4*b*x + 4*a) - 24*b*x*cos(2*b*x + 2*a) +
 24*I*b*x*sin(6*b*x + 6*a) - 24*I*b*x*sin(4*b*x + 4*a) - 24*I*b*x*sin(2*b*x
 + 2*a) + 24*b*x)*dilog(-I*e^(I*b*x + I*a)) + (18*(b*x + a)^2 - 36*(b*x + a
)*a + 18*a^2 + 6*(3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(6*b*x + 6*
a) - 6*(3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(4*b*x + 4*a) - 6*(3*
```

```
(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(2*b*x + 2*a) - (-18*I*(b*x + a
)^2 + 36*I*(b*x + a)*a - 18*I*a^2 - 12*I)*sin(6*b*x + 6*a) - (18*I*(b*x + a
)^2 - 36*I*(b*x + a)*a + 18*I*a^2 + 12*I)*sin(4*b*x + 4*a) - (18*I*(b*x + a
)^2 - 36*I*(b*x + a)*a + 18*I*a^2 + 12*I)*sin(2*b*x + 2*a) + 12)*dilog(-e^(
I*b*x + I*a)) - (18*(b*x + a)^2 - 36*(b*x + a)*a + 18*a^2 + 6*(3*(b*x + a)^
2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(6*b*x + 6*a) - 6*(3*(b*x + a)^2 - 6*(b*x
 + a)*a + 3*a^2 + 2)*cos(4*b*x + 4*a) - 6*(3*(b*x + a)^2 - 6*(b*x + a)*a +
3*a^2 + 2)*cos(2*b*x + 2*a) + (18*I*(b*x + a)^2 - 36*I*(b*x + a)*a + 18*I*a
^2 + 12*I)*sin(6*b*x + 6*a) + (-18*I*(b*x + a)^2 + 36*I*(b*x + a)*a - 18*I*
a^2 - 12*I)*sin(4*b*x + 4*a) + (-18*I*(b*x + a)^2 + 36*I*(b*x + a)*a - 18*I
*a^2 - 12*I)*sin(2*b*x + 2*a) + 12)*dilog(e^(I*b*x + I*a)) - (-3*I*(b*x + a
)^3 + 9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) + (-3*I*(b*x + a)^3 +
9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) + 6*I*a)*cos(6*b*x + 6*a) +
(3*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + (9*I*a^2 + 6*I)*(b*x + a) - 6*I*a)*c
os(4*b*x + 4*a) + (3*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + (9*I*a^2 + 6*I)*(b
*x + a) - 6*I*a)*cos(2*b*x + 2*a) + 3*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a
^2 + 2)*(b*x + a) - 2*a)*sin(6*b*x + 6*a) - 3*((b*x + a)^3 - 3*(b*x + a)^2*
a + (3*a^2 + 2)*(b*x + a) - 2*a)*sin(4*b*x + 4*a) - 3*((b*x + a)^3 - 3*(b*x
 + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*sin(2*b*x + 2*a) + 6*I*a)*log(cos(
b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b*x + a)^3 - 9*I*
(b*x + a)^2*a + (9*I*a^2 + 6*I)*(b*x + a) + (3*I*(b*x + a)^3 - 9*I*(b*x + a
)^2*a + (9*I*a^2 + 6*I)*(b*x + a) - 6*I*a)*cos(6*b*x + 6*a) + (-3*I*(b*x +
a)^3 + 9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) + 6*I*a)*cos(4*b*x +
4*a) + (-3*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) +
 6*I*a)*cos(2*b*x + 2*a) - 3*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(
b*x + a) - 2*a)*sin(6*b*x + 6*a) + 3*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^
2 + 2)*(b*x + a) - 2*a)*sin(4*b*x + 4*a) + 3*((b*x + a)^3 - 3*(b*x + a)^2*a
 + (3*a^2 + 2)*(b*x + a) - 2*a)*sin(2*b*x + 2*a) - 6*I*a)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-6*I*(b*x + a)^2 + 12*I*(b*x +
a)*a - 6*I*a^2 + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 6*I*a^2)*cos(6*b*x
+ 6*a) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a + 6*I*a^2)*cos(4*b*x + 4*a) +
(6*I*(b*x + a)^2 - 12*I*(b*x + a)*a + 6*I*a^2)*cos(2*b*x + 2*a) + 6*((b*x +
 a)^2 - 2*(b*x + a)*a + a^2)*sin(6*b*x + 6*a) - 6*((b*x + a)^2 - 2*(b*x + a
)*a + a^2)*sin(4*b*x + 4*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (6*I
*(b*x + a)^2 - 12*I*(b*x + a)*a + 6*I*a^2 + (6*I*(b*x + a)^2 - 12*I*(b*x +
a)*a + 6*I*a^2)*cos(6*b*x + 6*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 6
*I*a^2)*cos(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 6*I*a^2)*
cos(2*b*x + 2*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*sin(6*b*x + 6*a) +
 6*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*sin(4*b*x + 4*a) + 6*((b*x + a)^2 -
2*(b*x + a)*a + a^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*sin(b*x + a) + 1) - (36*cos(6*b*x + 6*a) - 36*cos(4*b*x + 4*a) - 36*cos
(2*b*x + 2*a) + 36*I*sin(6*b*x + 6*a) - 36*I*sin(4*b*x + 4*a) - 36*I*sin(2*
b*x + 2*a) + 36)*polylog(4, -e^(I*b*x + I*a)) + (36*cos(6*b*x + 6*a) - 36*c
os(4*b*x + 4*a) - 36*cos(2*b*x + 2*a) + 36*I*sin(6*b*x + 6*a) - 36*I*sin(4*
```

```
b*x + 4*a) - 36*I*sin(2*b*x + 2*a) + 36)*polylog(4, e^(I*b*x + I*a)) - (-24
*I*cos(6*b*x + 6*a) + 24*I*cos(4*b*x + 4*a) + 24*I*cos(2*b*x + 2*a) + 24*si
n(6*b*x + 6*a) - 24*sin(4*b*x + 4*a) - 24*sin(2*b*x + 2*a) - 24*I)*polylog(
3, I*e^(I*b*x + I*a)) - (24*I*cos(6*b*x + 6*a) - 24*I*cos(4*b*x + 4*a) - 24
*I*cos(2*b*x + 2*a) - 24*sin(6*b*x + 6*a) + 24*sin(4*b*x + 4*a) + 24*sin(2*
b*x + 2*a) + 24*I)*polylog(3, -I*e^(I*b*x + I*a)) - (-36*I*b*x*cos(6*b*x +
6*a) + 36*I*b*x*cos(4*b*x + 4*a) + 36*I*b*x*cos(2*b*x + 2*a) + 36*b*x*sin(6
*b*x + 6*a) - 36*b*x*sin(4*b*x + 4*a) - 36*b*x*sin(2*b*x + 2*a) - 36*I*b*x)
*polylog(3, -e^(I*b*x + I*a)) - (36*I*b*x*cos(6*b*x + 6*a) - 36*I*b*x*cos(4
*b*x + 4*a) - 36*I*b*x*cos(2*b*x + 2*a) - 36*b*x*sin(6*b*x + 6*a) + 36*b*x*
sin(4*b*x + 4*a) + 36*b*x*sin(2*b*x + 2*a) + 36*I*b*x)*polylog(3, e^(I*b*x
+ I*a)) + (12*(b*x + a)^3 - (b*x + a)^2*(36*a + 12*I) + 12*(3*a^2 + 2*I*a)*
(b*x + a) - 12*I*a^2)*sin(5*b*x + 5*a) - 8*((b*x + a)^3 - 3*(b*x + a)^2*a +
 3*(b*x + a)*a^2)*sin(3*b*x + 3*a) + (12*(b*x + a)^3 - (b*x + a)^2*(36*a -
12*I) + 12*(3*a^2 - 2*I*a)*(b*x + a) + 12*I*a^2)*sin(b*x + a))/(-4*I*cos(6*
b*x + 6*a) + 4*I*cos(4*b*x + 4*a) + 4*I*cos(2*b*x + 2*a) + 4*sin(6*b*x + 6*
a) - 4*sin(4*b*x + 4*a) - 4*sin(2*b*x + 2*a) - 4*I))/b^4
```

Fricas [C] time = 0.911414, size = 4625, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b^3*x^3*cos(b*x + a)^2 - 4*b^3*x^3 + 6*b^2*x^2*cos(b*x + a)*sin(b*x
+ a) + ((-9*I*b^2*x^2 - 6*I)*cos(b*x + a)^3 + (9*I*b^2*x^2 + 6*I)*cos(b*x +
 a))*dilog(cos(b*x + a) + I*sin(b*x + a)) + ((9*I*b^2*x^2 + 6*I)*cos(b*x +
a)^3 + (-9*I*b^2*x^2 - 6*I)*cos(b*x + a))*dilog(cos(b*x + a) - I*sin(b*x +
a)) + (12*I*b*x*cos(b*x + a)^3 - 12*I*b*x*cos(b*x + a))*dilog(I*cos(b*x + a
) + sin(b*x + a)) + (12*I*b*x*cos(b*x + a)^3 - 12*I*b*x*cos(b*x + a))*dilog
(I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b*x*cos(b*x + a)^3 + 12*I*b*x*cos(
b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-12*I*b*x*cos(b*x + a)^3
 + 12*I*b*x*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + ((-9*I*b^
2*x^2 - 6*I)*cos(b*x + a)^3 + (9*I*b^2*x^2 + 6*I)*cos(b*x + a))*dilog(-cos(
b*x + a) + I*sin(b*x + a)) + ((9*I*b^2*x^2 + 6*I)*cos(b*x + a)^3 + (-9*I*b^
2*x^2 - 6*I)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*((b^3*
x^3 + 2*b*x)*cos(b*x + a)^3 - (b^3*x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x +
 a) + I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*log(c
os(b*x + a) + I*sin(b*x + a) + I) - 3*((b^3*x^3 + 2*b*x)*cos(b*x + a)^3 - (
b^3*x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(
```

```
a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) +
I) - 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(
I*cos(b*x + a) + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b
^2*x^2 - a^2)*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*((b^
2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a
^2)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*((a^3 + 2*a)*
cos(b*x + a)^3 - (a^3 + 2*a)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2) - 3*((a^3 + 2*a)*cos(b*x + a)^3 - (a^3 + 2*a)*cos(b*x + a
))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*((b^3*x^3 + a^3 +
2*b*x + 2*a)*cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*cos(b*x + a))*l
og(-cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*
x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*((b^3*x^3 + a^3 + 2*b*x
 + 2*a)*cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*cos(b*x + a))*log(-c
os(b*x + a) - I*sin(b*x + a) + 1) + 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a
))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (18*I*cos(b*x + a)^3 - 18*I*co
s(b*x + a))*polylog(4, cos(b*x + a) + I*sin(b*x + a)) + (-18*I*cos(b*x + a)
^3 + 18*I*cos(b*x + a))*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (18*I*c
os(b*x + a)^3 - 18*I*cos(b*x + a))*polylog(4, -cos(b*x + a) + I*sin(b*x + a
)) + (-18*I*cos(b*x + a)^3 + 18*I*cos(b*x + a))*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) + 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, cos
(b*x + a) + I*sin(b*x + a)) + 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*po
lylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(b*x + a)
)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 - cos(b*x
+ a))*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(
b*x + a))*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 -
 cos(b*x + a))*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 18*(b*x*cos(b*x
 + a)^3 - b*x*cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 18
*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b
*x + a)))/(b^4*cos(b*x + a)^3 - b^4*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \csc(bx+a)^3 \sec(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

3.286 $\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=235

$$\frac{3ix\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3ix\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2i\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2i\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{3\text{PolyLog}}{}$$

```
[Out] ((4*I)*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*x^2*ArcTanh[E^(I*(a + b*x))])/b
- ArcTanh[Cos[a + b*x]]/b^3 - (x*Csc[a + b*x])/b^2 + ((3*I)*x*PolyLog[2, -E
^(I*(a + b*x))])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I
)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*x*PolyLog[2, E^(I*(a + b*x))]
)/b^2 - (3*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (3*PolyLog[3, E^(I*(a + b*x)
)])/b^3 + (3*x^2*Sec[a + b*x])/(2*b) - (x^2*Csc[a + b*x]^2*Sec[a + b*x])/(2
*b)
```

Rubi [A] time = 0.53745, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.95, Rules used = {2622, 288, 321, 207, 4420, 14, 6273, 12, 4183, 2531, 2282, 6589, 6742, 4181, 2279, 2391, 2621, 6271, 3770}

$$\frac{3ix\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3ix\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2i\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2i\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{3\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```


```
[Out] ((4*I)*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*x^2*ArcTanh[E^(I*(a + b*x))])/b
- ArcTanh[Cos[a + b*x]]/b^3 - (x*Csc[a + b*x])/b^2 + ((3*I)*x*PolyLog[2, -E
^(I*(a + b*x))])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I
)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*x*PolyLog[2, E^(I*(a + b*x))]
)/b^2 - (3*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (3*PolyLog[3, E^(I*(a + b*x)
)])/b^3 + (3*x^2*Sec[a + b*x])/(2*b) - (x^2*Csc[a + b*x]^2*Sec[a + b*x])/(2
*b)
```


Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
```

```
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} - 2\int x$$

$$= -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} - 2\int\Big($$

$$= -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{\int x\,(-$$

$$= \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{\int(-3x\sec(a+bx) + x\csc^2(a+bx)}{b}$$

$$= \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3}{2}\int x^2\csc(a+bx)\,dx + \frac{\int x\csc^2(a}{}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{x\tanh^{-1}(\sin(a+bx))}{b^2} - \frac{x\csc(a+bx)}{b^2}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{x\tanh^{-1}(\sin(a+bx))}{b^2} - \frac{x\csc(a+bx)}{b^2}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{x\csc(a+bx)}{b^2} +$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{x\csc(a+bx)}{b^2} +$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{x\csc(a+bx)}{b^2} +$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{x\csc(a+bx)}{b^2} +$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{x\csc(a+bx)}{b^2} +$$

Mathematica [B] time = 6.66329, size = 613, normalized size = 2.61

$$\frac{6ibx\text{PolyLog}(2,-\cos(a+bx)-i\sin(a+bx)) - 6ibx\text{PolyLog}(2,\cos(a+bx)+i\sin(a+bx)) - 6\text{PolyLog}(3,-\cos(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] -(x^2*Csc[a/2 + (b*x)/2]^2)/(8*b) - (2*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a
 + Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[
(-a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2
, E^(I*(-a + Pi/2 - b*x))])))/b^3 + (2*Log[1 - Cos[a + b*x] - I*Sin[a + b*x
]] + 3*b^2*x^2*Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] - 2*Log[1 + Cos[a + b
*x] + I*Sin[a + b*x]] - 3*b^2*x^2*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] +
(6*I)*b*x*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] - (6*I)*b*x*PolyLog[2,
 Cos[a + b*x] + I*Sin[a + b*x]] - 6*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*
x]] + 6*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]])/(2*b^3) + (x^2*Sec[a/2 +
 (b*x)/2]^2)/(8*b) + (x*Csc[a]*Sec[a]*(-Cos[a] + b*x*Sin[a]))/b^2 + (x*Csc[
a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b^2) - (x*Sec[a/2]*Sec[a/2 + (b*x)
/2]*Sin[(b*x)/2])/(2*b^2) + (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Co
s[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] +
 Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]))
```

Maple [B] time = 0.402, size = 429, normalized size = 1.8

$$\frac{x\left(3\,bx\mathrm{e}^{5\,i(bx+a)}-2\,bx\mathrm{e}^{3\,i(bx+a)}-2\,i\mathrm{e}^{5\,i(bx+a)}+3\,bx\mathrm{e}^{i(bx+a)}+2\,i\mathrm{e}^{i(bx+a)}\right)}{b^2\left(\mathrm{e}^{2\,i(bx+a)}-1\right)^2\left(\mathrm{e}^{2\,i(bx+a)}+1\right)}+\frac{3\,a^2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{2\,b^3}-\frac{2\,i\mathrm{dilog}\left(1+i\mathrm{e}^{i(bx+a)}\right)}{b^3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] x/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*b*x*exp(5*I*(b*x+a))-2
*b*x*exp(3*I*(b*x+a))-2*I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+2*I*exp(I*(
b*x+a)))+3/2/b^3*a^2*ln(exp(I*(b*x+a))-1)-2*I/b^3*dilog(1+I*exp(I*(b*x+a)))
-3/2/b^3*a^2*ln(1-exp(I*(b*x+a)))+2*I/b^3*dilog(1-I*exp(I*(b*x+a)))+3*I*x*p
olylog(2,-exp(I*(b*x+a)))/b^2+3*polylog(3,exp(I*(b*x+a)))/b^3+2/b^3*ln(1+I*
exp(I*(b*x+a)))*a-2/b^3*ln(1-I*exp(I*(b*x+a)))*a-3*I*x*polylog(2,exp(I*(b*x
+a)))/b^2+2/b^2*ln(1+I*exp(I*(b*x+a)))*x-2/b^2*ln(1-I*exp(I*(b*x+a)))*x-4*I
/b^3*a*arctan(exp(I*(b*x+a)))+3/2/b*ln(1-exp(I*(b*x+a)))*x^2-3/2/b*ln(exp(I
*(b*x+a))+1)*x^2+1/b^3*ln(exp(I*(b*x+a))-1)-1/b^3*ln(exp(I*(b*x+a))+1)-3*po
lylog(3,-exp(I*(b*x+a)))/b^3
```

Maxima [B] time = 2.52132, size = 2996, normalized size = 12.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")

[Out] 1/4*(a^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) + 4*((8*b*x*cos(6*b*x + 6*a) -
 8*b*x*cos(4*b*x + 4*a) - 8*b*x*cos(2*b*x + 2*a) + 8*I*b*x*sin(6*b*x + 6*a)
 - 8*I*b*x*sin(4*b*x + 4*a) - 8*I*b*x*sin(2*b*x + 2*a) + 8*b*x)*arctan2(cos
(b*x + a), sin(b*x + a) + 1) + (8*b*x*cos(6*b*x + 6*a) - 8*b*x*cos(4*b*x +
4*a) - 8*b*x*cos(2*b*x + 2*a) + 8*I*b*x*sin(6*b*x + 6*a) - 8*I*b*x*sin(4*b*
x + 4*a) - 8*I*b*x*sin(2*b*x + 2*a) + 8*b*x)*arctan2(cos(b*x + a), -sin(b*x
 + a) + 1) - (6*(b*x + a)^2 - 12*(b*x + a)*a + 2*(3*(b*x + a)^2 - 6*(b*x +
a)*a + 2)*cos(6*b*x + 6*a) - 2*(3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(4*b*
x + 4*a) - 2*(3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(2*b*x + 2*a) + (6*I*(b
*x + a)^2 - 12*I*(b*x + a)*a + 4*I)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2 +
12*I*(b*x + a)*a - 4*I)*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x +
a)*a - 4*I)*sin(2*b*x + 2*a) + 4)*arctan2(sin(b*x + a), cos(b*x + a) + 1) +
 (4*cos(6*b*x + 6*a) - 4*cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a) + 4*I*sin(6*
b*x + 6*a) - 4*I*sin(4*b*x + 4*a) - 4*I*sin(2*b*x + 2*a) + 4)*arctan2(sin(b
*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^2 - 12*(b*x + a)*a + 6*((b*x + a)
^2 - 2*(b*x + a)*a)*cos(6*b*x + 6*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a)*cos(
4*b*x + 4*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) + (6*I*(b*x
 + a)^2 - 12*I*(b*x + a)*a)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2 + 12*I*(b*
x + a)*a)*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*sin(2*b*
x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^2 - 8*
(b*x + a)*(3*I*a - 1) - 8*a)*cos(5*b*x + 5*a) - (-8*I*(b*x + a)^2 + 16*I*(b
*x + a)*a)*cos(3*b*x + 3*a) - (12*I*(b*x + a)^2 - 8*(b*x + a)*(3*I*a + 1) +
 8*a)*cos(b*x + a) + (8*cos(6*b*x + 6*a) - 8*cos(4*b*x + 4*a) - 8*cos(2*b*x
 + 2*a) + 8*I*sin(6*b*x + 6*a) - 8*I*sin(4*b*x + 4*a) - 8*I*sin(2*b*x + 2*a
) + 8)*dilog(I*e^(I*b*x + I*a)) - (8*cos(6*b*x + 6*a) - 8*cos(4*b*x + 4*a)
- 8*cos(2*b*x + 2*a) + 8*I*sin(6*b*x + 6*a) - 8*I*sin(4*b*x + 4*a) - 8*I*si
n(2*b*x + 2*a) + 8)*dilog(-I*e^(I*b*x + I*a)) + (12*b*x*cos(6*b*x + 6*a) -
12*b*x*cos(4*b*x + 4*a) - 12*b*x*cos(2*b*x + 2*a) + 12*I*b*x*sin(6*b*x + 6*
a) - 12*I*b*x*sin(4*b*x + 4*a) - 12*I*b*x*sin(2*b*x + 2*a) + 12*b*x)*dilog(
-e^(I*b*x + I*a)) - (12*b*x*cos(6*b*x + 6*a) - 12*b*x*cos(4*b*x + 4*a) - 12
*b*x*cos(2*b*x + 2*a) + 12*I*b*x*sin(6*b*x + 6*a) - 12*I*b*x*sin(4*b*x + 4*
a) - 12*I*b*x*sin(2*b*x + 2*a) + 12*b*x)*dilog(e^(I*b*x + I*a)) - (-3*I*(b*
x + a)^2 + 6*I*(b*x + a)*a + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos
(6*b*x + 6*a) + (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos(4*b*x + 4*a)
+ (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos(2*b*x + 2*a) + (3*(b*x + a)
^2 - 6*(b*x + a)*a + 2)*sin(6*b*x + 6*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a +
 2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(2*b*x + 2*a)
 - 2*I)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b
*x + a)^2 - 6*I*(b*x + a)*a + (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos
(6*b*x + 6*a) + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos(4*b*x + 4*a)
 + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos(2*b*x + 2*a) - (3*(b*x +
```

```
a)^2 - 6*(b*x + a)*a + 2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2 - 6*(b*x + a)*a
 + 2)*sin(4*b*x + 4*a) + (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(2*b*x + 2*
a) + 2*I)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-4*I
*b*x*cos(6*b*x + 6*a) + 4*I*b*x*cos(4*b*x + 4*a) + 4*I*b*x*cos(2*b*x + 2*a)
 + 4*b*x*sin(6*b*x + 6*a) - 4*b*x*sin(4*b*x + 4*a) - 4*b*x*sin(2*b*x + 2*a)
 - 4*I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (4*
I*b*x*cos(6*b*x + 6*a) - 4*I*b*x*cos(4*b*x + 4*a) - 4*I*b*x*cos(2*b*x + 2*a
) - 4*b*x*sin(6*b*x + 6*a) + 4*b*x*sin(4*b*x + 4*a) + 4*b*x*sin(2*b*x + 2*a
) + 4*I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-
12*I*cos(6*b*x + 6*a) + 12*I*cos(4*b*x + 4*a) + 12*I*cos(2*b*x + 2*a) + 12*
sin(6*b*x + 6*a) - 12*sin(4*b*x + 4*a) - 12*sin(2*b*x + 2*a) - 12*I)*polylo
g(3, -e^(I*b*x + I*a)) - (12*I*cos(6*b*x + 6*a) - 12*I*cos(4*b*x + 4*a) - 1
2*I*cos(2*b*x + 2*a) - 12*sin(6*b*x + 6*a) + 12*sin(4*b*x + 4*a) + 12*sin(2
*b*x + 2*a) + 12*I)*polylog(3, e^(I*b*x + I*a)) + (12*(b*x + a)^2 - (b*x +
a)*(24*a + 8*I) + 8*I*a)*sin(5*b*x + 5*a) - 8*((b*x + a)^2 - 2*(b*x + a)*a)
*sin(3*b*x + 3*a) + (12*(b*x + a)^2 - (b*x + a)*(24*a - 8*I) - 8*I*a)*sin(b
*x + a))/(-4*I*cos(6*b*x + 6*a) + 4*I*cos(4*b*x + 4*a) + 4*I*cos(2*b*x + 2*
a) + 4*sin(6*b*x + 6*a) - 4*sin(4*b*x + 4*a) - 4*sin(2*b*x + 2*a) - 4*I))/b
^3
```

Fricas [C] time = 0.770386, size = 3302, normalized size = 14.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b^2*x^2*cos(b*x + a)^2 - 4*b^2*x^2 + 4*b*x*cos(b*x + a)*sin(b*x + a)
 + (-6*I*b*x*cos(b*x + a)^3 + 6*I*b*x*cos(b*x + a))*dilog(cos(b*x + a) + I*
sin(b*x + a)) + (6*I*b*x*cos(b*x + a)^3 - 6*I*b*x*cos(b*x + a))*dilog(cos(b
*x + a) - I*sin(b*x + a)) + (4*I*cos(b*x + a)^3 - 4*I*cos(b*x + a))*dilog(I
*cos(b*x + a) + sin(b*x + a)) + (4*I*cos(b*x + a)^3 - 4*I*cos(b*x + a))*dil
og(I*cos(b*x + a) - sin(b*x + a)) + (-4*I*cos(b*x + a)^3 + 4*I*cos(b*x + a)
)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-4*I*cos(b*x + a)^3 + 4*I*cos(b*
x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*x*cos(b*x + a)^3 +
6*I*b*x*cos(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (6*I*b*x*cos(
b*x + a)^3 - 6*I*b*x*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
((3*b^2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*x^2 + 2)*cos(b*x + a))*log(cos(b*x
 + a) + I*sin(b*x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(cos
(b*x + a) + I*sin(b*x + a) + I) - ((3*b^2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*
x^2 + 2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*
```

```
x + a)^3 - a*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) - 4*((b*x
 + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*cos(b*x + a) + sin(b*x
 + a) + 1) + 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*co
s(b*x + a) - sin(b*x + a) + 1) - 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*co
s(b*x + a))*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 4*((b*x + a)*cos(b*x
+ a)^3 - (b*x + a)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) +
((3*a^2 + 2)*cos(b*x + a)^3 - (3*a^2 + 2)*cos(b*x + a))*log(-1/2*cos(b*x +
a) + 1/2*I*sin(b*x + a) + 1/2) + ((3*a^2 + 2)*cos(b*x + a)^3 - (3*a^2 + 2)*
cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*((b^2*x
^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(-cos(b*x + a)
+ I*sin(b*x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-cos(b*x
+ a) + I*sin(b*x + a) + I) + 3*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 -
 a^2)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*x
+ a)^3 - a*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 6*(cos(b
*x + a)^3 - cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(co
s(b*x + a)^3 - cos(b*x + a))*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*
(cos(b*x + a)^3 - cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a))
- 6*(cos(b*x + a)^3 - cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b*x +
a)))/(b^3*cos(b*x + a)^3 - b^3*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(b*x+a)**3*sec(b*x+a)**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate(x^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

3.287 $\int x \csc^3(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=126

$$\frac{3i\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{\csc(a+bx)}{2b^2}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{3x\tanh^{-1}}{b}$$

```
[Out] (-3*x*ArcTanh[E^(I*(a + b*x))])/b - ArcTanh[Sin[a + b*x]]/b^2 - Csc[a + b*x
]/(2*b^2) + (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*PolyL
og[2, E^(I*(a + b*x))])/b^2 + (3*x*Sec[a + b*x])/(2*b) - (x*Csc[a + b*x]^2*
Sec[a + b*x])/(2*b)
```

Rubi [A] time = 0.1677, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2622, 288, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770, 2621}

$$\frac{3i\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{\csc(a+bx)}{2b^2}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{3x\tanh^{-1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*x*ArcTanh[E^(I*(a + b*x))])/b - ArcTanh[Sin[a + b*x]]/b^2 - Csc[a + b*x
]/(2*b^2) + (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*PolyL
og[2, E^(I*(a + b*x))])/b^2 + (3*x*Sec[a + b*x])/(2*b) - (x*Csc[a + b*x]^2*
Sec[a + b*x])/(2*b)
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int x\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}-\int\Big(-\frac{3\,\mathrm{ta}}{}$$

$$= -\frac{3x\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}+\frac{\int\csc^2(a}{}$$

$$= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}-\frac{\mathrm{Subst}\big(\int}{}$$

$$= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx}{2b}$$

$$= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\,\mathrm{cs}}{}$$

$$= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\,\mathrm{cs}}{}$$

$$= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3i\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}-\frac{3i}{}$$

Mathematica [B] time = 2.56348, size = 282, normalized size = 2.24

$$12i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)+12(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-2\tan\left(\frac{1}{2}(a+bx)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (8*b*x - 2*Cot[(a + b*x)/2] - b*x*Csc[(a + b*x)/2]^2 + 12*(a + b*x)*(Log[1
- E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + 8*Log[Cos[(a + b*x)/2] - S
in[(a + b*x)/2]] - 8*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]] - 12*a*Log[Ta
n[(a + b*x)/2]] + (12*I)*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a
 + b*x))]) + b*x*Sec[(a + b*x)/2]^2 + (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*
x)/2] - Sin[(a + b*x)/2]) - (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*x)/2] + Si
n[(a + b*x)/2]) - 2*Tan[(a + b*x)/2])/(8*b^2)
```

Maple [A] time = 0.299, size = 182, normalized size = 1.4

$$\frac{3\,bx\mathrm{e}^{5\,i(bx+a)}-2\,bx\mathrm{e}^{3\,i(bx+a)}-i\mathrm{e}^{5\,i(bx+a)}+3\,bx\mathrm{e}^{i(bx+a)}+i\mathrm{e}^{i(bx+a)}}{b^2\left(\mathrm{e}^{2\,i(bx+a)}-1\right)^2\left(\mathrm{e}^{2\,i(bx+a)}+1\right)}-\frac{3\,a\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{2\,b^2}+\frac{2\,i\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{\frac{3\,i}{2}\mathrm{dilog}\left(\mathrm{e}^{i\right.}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*b*x*exp(5*I*(b*x+a))-2
*b*x*exp(3*I*(b*x+a))-I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+I*exp(I*(b*x+
a)))-3/2/b^2*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*arctan(exp(I*(b*x+a)))+3/2*I/b^
2*dilog(exp(I*(b*x+a)))+3/2*I/b^2*dilog(exp(I*(b*x+a))+1)-3/2/b*ln(exp(I*(b
*x+a))+1)*x
```

Maxima [B] time = 2.34442, size = 1598, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (8*I*b*x*cos(3*b*x + 3*a) - 8*b*x*sin(3*b*x + 3*a) - (4*cos(6*b*x + 6*a) -
4*cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a) + 4*I*sin(6*b*x + 6*a) - 4*I*sin(4*
b*x + 4*a) - 4*I*sin(2*b*x + 2*a) + 4)*arctan2(2*(cos(b*x + 2*a)*cos(a) + s
in(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a
) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)
^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 +
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)) - (6*b*x*cos(6*b*x + 6*a) - 6*b*x*cos(4*b*x + 4*a) - 6*b*x*cos(2*b*
x + 2*a) + 6*I*b*x*sin(6*b*x + 6*a) - 6*I*b*x*sin(4*b*x + 4*a) - 6*I*b*x*si
n(2*b*x + 2*a) + 6*b*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*b*x*co
s(6*b*x + 6*a) - 6*b*x*cos(4*b*x + 4*a) - 6*b*x*cos(2*b*x + 2*a) + 6*I*b*x*
sin(6*b*x + 6*a) - 6*I*b*x*sin(4*b*x + 4*a) - 6*I*b*x*sin(2*b*x + 2*a) + 6*
b*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(3*I*b*x + 1)*cos(5*b*x +
 5*a) - 4*(3*I*b*x - 1)*cos(b*x + a) + (6*cos(6*b*x + 6*a) - 6*cos(4*b*x +
4*a) - 6*cos(2*b*x + 2*a) + 6*I*sin(6*b*x + 6*a) - 6*I*sin(4*b*x + 4*a) - 6
*I*sin(2*b*x + 2*a) + 6)*dilog(-e^(I*b*x + I*a)) - (6*cos(6*b*x + 6*a) - 6*
cos(4*b*x + 4*a) - 6*cos(2*b*x + 2*a) + 6*I*sin(6*b*x + 6*a) - 6*I*sin(4*b*
x + 4*a) - 6*I*sin(2*b*x + 2*a) + 6)*dilog(e^(I*b*x + I*a)) + (3*I*b*x*cos(
6*b*x + 6*a) - 3*I*b*x*cos(4*b*x + 4*a) - 3*I*b*x*cos(2*b*x + 2*a) - 3*b*x*
sin(6*b*x + 6*a) + 3*b*x*sin(4*b*x + 4*a) + 3*b*x*sin(2*b*x + 2*a) + 3*I*b*
x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (-3*I*b*x*co
s(6*b*x + 6*a) + 3*I*b*x*cos(4*b*x + 4*a) + 3*I*b*x*cos(2*b*x + 2*a) + 3*b*
x*sin(6*b*x + 6*a) - 3*b*x*sin(4*b*x + 4*a) - 3*b*x*sin(2*b*x + 2*a) - 3*I*
b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-2*I*cos(
6*b*x + 6*a) + 2*I*cos(4*b*x + 4*a) + 2*I*cos(2*b*x + 2*a) + 2*sin(6*b*x +
6*a) - 2*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a) - 2*I)*log((cos(b*x + 2*a)^2
 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)
*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a)
+ sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + (12*b*x - 4*I)*
sin(5*b*x + 5*a) + (12*b*x + 4*I)*sin(b*x + a))/(-4*I*b^2*cos(6*b*x + 6*a)
+ 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(6*b*x + 6
*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

Fricas [B] time = 0.607694, size = 1459, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*x*cos(b*x + a)^2 - 4*b*x + (-3*I*cos(b*x + a)^3 + 3*I*cos(b*x + a)
)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*cos(b*x + a)^3 - 3*I*cos(b*x
+ a))*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*cos(b*x + a)^3 + 3*I*cos
(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*cos(b*x + a)^3 - 3*
I*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*x*cos(b*x + a)
^3 - b*x*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*x*cos(
b*x + a)^3 - b*x*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 3*(
a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x +
a) + 1/2) - 3*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) + 3*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x
+ a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*((b*x + a)*cos(b*x + a)^3
 - (b*x + a)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(cos
(b*x + a)^3 - cos(b*x + a))*log(sin(b*x + a) + 1) + 2*(cos(b*x + a)^3 - cos
(b*x + a))*log(-sin(b*x + a) + 1) + 2*cos(b*x + a)*sin(b*x + a))/(b^2*cos(b
*x + a)^3 - b^2*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

3.288 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

Rubi [A] time = 0.499808, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Mathematica [A] time = 49.5159, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

Maple [A] time = 0.784, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx+a))^3 (\sec (bx+a))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx+a)^3 \sec (bx+a)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

3.289 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=22

$$\text{CannotIntegrate}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

Rubi [A] time = 0.513283, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 20.9802, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

Maple [A] time = 0.884, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx+a))^{3} (\sec (bx+a))^{2}}{x^{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc (bx+a)^{3} \sec (bx+a)^{2}}{x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(bx+a\right)^{3}\sec\left(bx+a\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

3.290 $\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\tan(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

Rubi [A] time = 0.152441, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Mathematica [A] time = 2.41648, size = 0, normalized size = 0.

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

Maple [A] time = 0.17, size = 0, normalized size = 0.

$$\int (dx+c)^m (\sec (bx+a))^2 \tan (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec (bx+a)^2 \tan (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \sec (bx+a)^2 \tan (bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec (bx+a)^2 \tan (bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

3.291 $\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=139

$$\frac{6id^3(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4}-\frac{3d^4\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^5}-\frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\tan(a+}{b^2}$$

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))]
)/b^3 + ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 - (3*d^4
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b)
 - (2*d*(c + d*x)^3*Tan[a + b*x])/b^2
```

Rubi [A] time = 0.257889, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4409, 4184, 3719, 2190, 2531, 2282, 6589}

$$\frac{6id^3(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4}-\frac{3d^4\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^5}-\frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\tan(a+}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```



```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))]
)/b^3 + ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 - (3*d^4
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b)
 - (2*d*(c + d*x)^3*Tan[a + b*x])/b^2
```


Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3\sec^2(a+bx)\,dx}{b}$$
$$= \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{\left(6d^2\right)\int (c+dx)^2\tan(a+b}{b^2}$$
$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} - \frac{\left(12id^2\right)\int \frac{e}{}}{}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d(c}{}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^4} +$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^4} +$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^4} -$$

Mathematica [B] time = 6.58545, size = 418, normalized size = 3.01

$$-\frac{6cd^3\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\mathrm{PolyLog}\left(2,e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\mathrm{t}\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^4\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((-I/2)*d^4*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(
a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (
3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^5*E^(I
*a)) + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (6*c^2*d^2*Sec[a]*(Cos[a]*Log[C
os[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)
) - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi -
 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*
Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot
[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[C
ot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[
a]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3
```

```
*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2
```

Maple [B] time = 0.193, size = 489, normalized size = 3.5

$$2\,\frac{bd^4x^4e^{2\,i(bx+a)}+4\,bcd^3x^3e^{2\,i(bx+a)}+6\,bc^2d^2x^2e^{2\,i(bx+a)}+4\,bc^3dxe^{2\,i(bx+a)}-2\,id^4x^3e^{2\,i(bx+a)}+bc^4e^{2\,i(bx+a)}-6\,icd^3x^2e^{2\,i(b}}{b^2\left(e^{2\,i(bx+a)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] 2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^
2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a)
)+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2
*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^2*d^
2*x-2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))+1)^2-6*d^2/b^3*c^2*ln(exp(2*I*(b*x+a))
+1)+12*d^2/b^3*c^2*ln(exp(I*(b*x+a)))+12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))+24*
I*d^3/b^3*c*a*x+12*I*d^3/b^4*c*a^2-8*I*d^4/b^5*a^3-6*d^4/b^3*ln(exp(2*I*(b*
x+a))+1)*x^2+6*I*d^4/b^4*polylog(2,-exp(2*I*(b*x+a)))*x-3*d^4*polylog(3,-ex
p(2*I*(b*x+a)))/b^5-24*d^3/b^4*c*a*ln(exp(I*(b*x+a)))-12*d^3/b^3*c*ln(exp(2
*I*(b*x+a))+1)*x+12*I*d^3/b^2*c*x^2+6*I*d^3/b^4*c*polylog(2,-exp(2*I*(b*x+a
)))+4*I*d^4/b^2*x^3-12*I*d^4/b^4*a^2*x
```

Maxima [B] time = 2.31788, size = 4641, normalized size = 33.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*tan(b*x + a)^2 - 4*a*c^3*d*tan(b*x + a)^2/b + 6*a^2*c^2*d^2*tan(b*
x + a)^2/b^2 - 4*a^3*c*d^3*tan(b*x + a)^2/b^3 + a^4*d^4*tan(b*x + a)^2/b^4
+ 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(
b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a
)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*
sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos
(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a
)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*
```

```
b*x + 2*a) + 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2
*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x
 + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*c^2*d^2/((2*(2*c
os(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x +
2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2
*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + 24*(4*(b*x + a)*cos(2*b*x +
2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + s
in(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x
+ a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x
+ 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x
 + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^3) - 8*
(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x
+ a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*co
s(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(
4*b*x + 4*a) - sin(2*b*x + 2*a))*a^3*d^4/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4
*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^
2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*
x + 2*a) + 1)*b^4) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*si
n(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*
x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x +
 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + s
in(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a
)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a
)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^
2*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 +
4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) - 12*(8*(b*x + a
)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*c
os(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2
*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(
4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x
 + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x
+ 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)
*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x +
 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*co
s(2*b*x + 2*a) + 1)*b^3) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a
)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*co
s(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2
*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)
^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x
```

```
 + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b
*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*
a))*a^2*d^4/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a
)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*
b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^4) - 2*((6*(b
*x + a)^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b*x + a)^2*d^4 + 2*(b*
c*d^3 - a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a)^2*d^4 + 2*(b*c*d
^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*
d^3 - 12*I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*(b*x + a)^2*d^4 + (24
*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2
*a), cos(2*b*x + 2*a) + 1) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x
+ a)^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 4*(2*I*a +
 1)*d^4)*(b*x + a)^3 - 12*(b*c*d^3 - a*d^4)*(b*x + a)^2)*cos(2*b*x + 2*a) -
 (6*b*c*d^3 + 6*(b*x + a)*d^4 - 6*a*d^4 + 6*(b*c*d^3 + (b*x + a)*d^4 - a*d^
4)*cos(4*b*x + 4*a) + 12*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a)
 - (-6*I*b*c*d^3 - 6*I*(b*x + a)*d^4 + 6*I*a*d^4)*sin(4*b*x + 4*a) - (-12*I
*b*c*d^3 - 12*I*(b*x + a)*d^4 + 12*I*a*d^4)*sin(2*b*x + 2*a))*dilog(-e^(2*I
*b*x + 2*I*a)) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x +
a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a))*cos(4*b*
x + 4*a) + (-6*I*(b*x + a)^2*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a))*
cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(
4*b*x + 4*a) + 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
+ 1) + (-3*I*d^4*cos(4*b*x + 4*a) - 6*I*d^4*cos(2*b*x + 2*a) + 3*d^4*sin(4*
b*x + 4*a) + 6*d^4*sin(2*b*x + 2*a) - 3*I*d^4)*polylog(3, -e^(2*I*b*x + 2*I
*a)) + (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*si
n(4*b*x + 4*a) - (2*(b*x + a)^4*d^4 + (8*b*c*d^3 - (8*a - 4*I)*d^4)*(b*x +
a)^3 - (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-I*b^4*
cos(4*b*x + 4*a) - 2*I*b^4*cos(2*b*x + 2*a) + b^4*sin(4*b*x + 4*a) + 2*b^4*
sin(2*b*x + 2*a) - I*b^4))/b
```

Fricas [C] time = 0.776583, size = 2211, normalized size = 15.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 - 12*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 1
```

```
2*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*d^4*cos
(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*d^4*cos(b*x + a
)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b*d^4*x - 12*I*b*c*
d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (12*I*b*d^4*x +
12*I*b*c*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (12*I*b
*d^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))
 + (-12*I*b*d^4*x - 12*I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(co
s(b*x + a) + I*sin(b*x + a) + I) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(I*cos(b*x + a) + si
n(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*c
os(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(-I*cos(b*x + a) + si
n(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*c
os(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I)
 - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
 + b^3*c^3*d)*cos(b*x + a)*sin(b*x + a))/(b^5*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^4 \tan(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \sec(bx+a)^2 \tan(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)^2*tan(b*x + a), x)
```

3.292 $\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=115

$$\frac{3id^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}+\frac{3id(c+\cdots}{2\cdots}$$

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b^3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((c + d*x)
^3*Sec[a + b*x]^2)/(2*b) - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.173672, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4409, 4184, 3719, 2190, 2279, 2391}

$$\frac{3id^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}+\frac{3id(c+\cdots}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```


```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b^3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((c + d*x)
^3*Sec[a + b*x]^2)/(2*b) - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
```


Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^3\sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2\sec^2(a+bx)\,dx}{2b}$$

$$= \frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{\left(3d^2\right)\int (c+dx)\tan(a+$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} - \frac{\left(6id^2\right)\int$$

$$= \frac{3id(c+dx)^2}{2b^2} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{3d(c$$

$$= \frac{3id(c+dx)^2}{2b^2} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{3d(c$$

$$= \frac{3id(c+dx)^2}{2b^2} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4} + \frac{(c+dx$$

Mathematica [B] time = 6.39472, size = 286, normalized size = 2.49

$$-\frac{3d^3\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))}-\frac{\cot(a)\left(i\text{PolyLog}\left(2,e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\text{ta}\right.}\right.}{\sqrt{\cot^2(a)+1}}\right.}{2b^4\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos
[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*d^
3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[C
ot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^(
(2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Si
n[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))
/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) -
 (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[
b*x]))/(2*b^2)
```

Maple [B] time = 0.158, size = 301, normalized size = 2.6

$$\frac{2\,bd^3x^3e^{2\,i(bx+a)}-3\,id^3x^2e^{2\,i(bx+a)}+6\,bcd^2x^2e^{2\,i(bx+a)}-6\,icd^2xe^{2\,i(bx+a)}+6\,bc^2dxe^{2\,i(bx+a)}-3\,ic^2de^{2\,i(bx+a)}-3\,id^3x^2+2\,bc^3}{b^2\left(e^{2\,i(bx+a)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] (2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*ex
p(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*
I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3
*I*c^2*d)/b^2/(exp(2*I*(b*x+a))+1)^2+6/b^3*d^2*c*ln(exp(I*(b*x+a)))-3*d^2/b
^3*c*ln(exp(2*I*(b*x+a))+1)+3*I*d^3/b^2*x^2+6*I*d^3/b^3*a*x+3*I*d^3/b^4*a^2
-3*d^3/b^3*ln(exp(2*I*(b*x+a))+1)*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/
b^4-6/b^4*d^3*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 2.24113, size = 900, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -(6*b^2*c^2*d + (6*b*d^3*x + 6*b*c*d^2 + 6*(b*d^3*x + b*c*d^2)*cos(4*b*x +
4*a) + 12*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) + (6*I*b*d^3*x + 6*I*b*c*d^2
)*sin(4*b*x + 4*a) + (12*I*b*d^3*x + 12*I*b*c*d^2)*sin(2*b*x + 2*a))*arctan
2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)
*cos(4*b*x + 4*a) + (4*I*b^3*d^3*x^3 + 4*I*b^3*c^3 + 6*b^2*c^2*d + (12*I*b^
3*c*d^2 - 6*b^2*d^3)*x^2 - 12*(-I*b^3*c^2*d + b^2*c*d^2)*x)*cos(2*b*x + 2*a
) - (3*d^3*cos(4*b*x + 4*a) + 6*d^3*cos(2*b*x + 2*a) + 3*I*d^3*sin(4*b*x +
4*a) + 6*I*d^3*sin(2*b*x + 2*a) + 3*d^3)*dilog(-e^(2*I*b*x + 2*I*a)) + (-3*
I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x - 3*I*b*c*d^2)*cos(4*b*x + 4*a) + (
-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(2*b*x + 2*a) + 3*(b*d^3*x + b*c*d^2)*sin(4*
b*x + 4*a) + 6*(b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*
b^2*c*d^2*x)*sin(4*b*x + 4*a) - (4*b^3*d^3*x^3 + 4*b^3*c^3 - 6*I*b^2*c^2*d
+ 6*(2*b^3*c*d^2 + I*b^2*d^3)*x^2 + (12*b^3*c^2*d + 12*I*b^2*c*d^2)*x)*sin(
2*b*x + 2*a))/(-2*I*b^4*cos(4*b*x + 4*a) - 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4
*sin(4*b*x + 4*a) + 4*b^4*sin(2*b*x + 2*a) - 2*I*b^4)
```

Fricas [B] time = 0.690775, size = 1374, normalized size = 11.95

$$\frac{b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 3i\,d^3\cos\left(bx+a\right)^2\operatorname{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right) + 3i\,d^3\cos\left(bx+a\right)^2\operatorname{Li}_2\left(i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 3*I*d^3*cos(
b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*di
log(I*cos(b*x + a) - sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*
x + a) + sin(b*x + a)) - 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin
(b*x + a)) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*
x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b
*x + a) + I) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(
```

```
b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin
(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x + a) + s
in(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x + a) -
 sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) +
 I*sin(b*x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*
x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^3 \tan (a+bx) \sec^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \sec (bx+a)^2 \tan (bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2*tan(b*x + a), x)
```

3.293 $\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{d(c+dx)\tan(a+bx)}{b^2}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{(c+dx)^2\sec^2(a+bx)}{2b}$$

```
[Out] -((d^2*Log[Cos[a + b*x]])/b^3) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (d*(c
 + d*x)*Tan[a + b*x])/b^2
```

Rubi [A] time = 0.0615893, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4409, 4184, 3475}

$$-\frac{d(c+dx)\tan(a+bx)}{b^2}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{(c+dx)^2\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] -((d^2*Log[Cos[a + b*x]])/b^3) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (d*(c
 + d*x)*Tan[a + b*x])/b^2
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\sec^2(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)^2\sec^2(a+bx)}{2b}-\frac{d\int (c+dx)\sec^2(a+bx)\,dx}{b}\\
&= \frac{(c+dx)^2\sec^2(a+bx)}{2b}-\frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{d^2\int \tan(a+bx)\,dx}{b^2}\\
&= -\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{(c+dx)^2\sec^2(a+bx)}{2b}-\frac{d(c+dx)\tan(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.524199, size = 66, normalized size = 1.2

$$\frac{b^2(c+dx)^2\sec^2(a+bx)-2bd\sec(a)\sin(bx)(c+dx)\sec(a+bx)-2d^2(bx\tan(a)+\log(\cos(a+bx)))}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (b^2*(c + d*x)^2*Sec[a + b*x]^2 - 2*b*d*(c + d*x)*Sec[a]*Sec[a + b*x]*Sin[b
*x] - 2*d^2*(Log[Cos[a + b*x]] + b*x*Tan[a]))/(2*b^3)
```

Maple [A] time = 0.029, size = 95, normalized size = 1.7

$$\frac{d^2x^2}{2\,b\left(\cos\left(bx+a\right)\right)^2}-\frac{d^2\tan\left(bx+a\right)x}{b^2}-\frac{d^2\ln\left(\cos\left(bx+a\right)\right)}{b^3}+\frac{cdx}{b\left(\cos\left(bx+a\right)\right)^2}-\frac{cd\tan\left(bx+a\right)}{b^2}+\frac{c^2}{2\,b\left(\cos\left(bx+a\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] 1/2/b*d^2/cos(b*x+a)^2*x^2-1/b^2*d^2*tan(b*x+a)*x-d^2*ln(cos(b*x+a))/b^3+1/
b*c*d/cos(b*x+a)^2*x-1/b^2*c*d*tan(b*x+a)+1/2/b*c^2/cos(b*x+a)^2
```

Maxima [B] time = 1.57501, size = 1334, normalized size = 24.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*tan(b*x + a)^2 - 2*a*c*d*tan(b*x + a)^2/b + a^2*d^2*tan(b*x + a)^2
/b^2 + 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 +
 (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*
x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
- 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) + 1)*
cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) + 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin
(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b
*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) -
cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*d^2/((2*(2*cos
(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*
a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a
)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4
*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*
a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*
cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2
+ sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x +
 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a
)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos
(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x +
 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2
))/b
```

Fricas [A] time = 0.508424, size = 208, normalized size = 3.78

$$\frac{b^2d^2x^2 + 2\,b^2cdx - 2\,d^2\cos\left(bx+a\right)^2\log\left(-\cos\left(bx+a\right)\right) + b^2c^2 - 2\left(bd^2x + bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b^3\cos\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2*cos(b*x + a)^2*log(-cos(b*x + a)) +
b^2*c^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^
```

```
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \tan (a+bx) \sec^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [B] time = 2.83337, size = 6040, normalized size = 109.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*t
an(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*tan
(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c*d*
x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b
^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^
4 - d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a
)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1
/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 -
8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b
*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/
2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2
*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)
*tan(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^2*d^2*x^
2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c^2*ta
n(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d^2*x
^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*b
*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 - 4*b*d^2*x*tan(1/2*b*x)^4*
tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*d^2*x*tan(1/2*b
*x)^3*tan(1/2*a)^2 + 2*d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1
```

/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*t
an(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x
)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2
*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*ta
n(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a)
+ 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)
^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/
2*a)^2 - 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 8*d^2*log(4*(tan(1/2*a)^4
 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(
1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^
7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)
^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1
/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 -
 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b
*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^
2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 2*b^2*c*d*x*tan(1/2*a)^4 - 4*b*d^2*x*
tan(1/2*b*x)*tan(1/2*a)^4 + 2*d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)
/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b
*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(
1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2
 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(
1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan
(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2
*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^2 + b^2*c^2*tan(1/2*b*x)^4 - 4*b
*c*d*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^2*d^2*x^2*tan(1/2*a)^2 + 4*b^2*c^2*tan
(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 24*b*c*d*
tan(1/2*b*x)^2*tan(1/2*a)^3 + b^2*c^2*tan(1/2*a)^4 - 4*b*c*d*tan(1/2*b*x)*t
an(1/2*a)^4 + 4*b^2*c*d*x*tan(1/2*b*x)^2 + 4*b*d^2*x*tan(1/2*b*x)^3 - d^2*l
og(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*t
an(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8
 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2
*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan
(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 -
2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*
a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*
a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/
2*a) - 8*d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(
1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 +
tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)
^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(
1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*t
an(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b
*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2
*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*b^2*c

*d*x*tan(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 - 20*d^2*log(4*(ta
n(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b
*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan
(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*
tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a)
+ 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/
2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8
*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*t
an(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*a)^3 - 8*
d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4
- 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b
*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*ta
n(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^
5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)
^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan
(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan
(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 - d^2*log(4*(tan(1
/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)
^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/
2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan
(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 3
6*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b
*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*ta
n(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(
1/2*a)^2 + 1))*tan(1/2*a)^4 + b^2*d^2*x^2 + 2*b^2*c^2*tan(1/2*b*x)^2 + 4*b*
c*d*tan(1/2*b*x)^3 + 24*b*c*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c^2*tan(1/2
*a)^2 + 24*b*c*d*tan(1/2*b*x)*tan(1/2*a)^2 + 4*b*c*d*tan(1/2*a)^3 + 2*b^2*c
*d*x - 4*b*d^2*x*tan(1/2*b*x) + 2*d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2
+ 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1
/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*
tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*
a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*
tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 +
 tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*
x)^2 - 4*b*d^2*x*tan(1/2*a) + 8*d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 +
1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2
*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*ta
n(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^
4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)
^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*ta
n(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + t
an(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)
*tan(1/2*a) + 2*d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)
^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*

```
a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan
(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b
*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*ta
n(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*
tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1))*tan(1/2*a)^2 + b^2*c^2 - 4*b
*c*d*tan(1/2*b*x) - 4*b*c*d*tan(1/2*a) - d^2*log(4*(tan(1/2*a)^4 + 2*tan(1/
2*a)^2 + 1)/(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 -
8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a
) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(
1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*t
an(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*
b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2
*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)))/(b
^3*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*b^3*
tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^3*tan(1
/2*b*x)^4 + 8*b^3*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^3*tan(1/2*b*x)^2*tan(1/2
*a)^2 + 8*b^3*tan(1/2*b*x)*tan(1/2*a)^3 + b^3*tan(1/2*a)^4 - 2*b^3*tan(1/2*
b*x)^2 - 8*b^3*tan(1/2*b*x)*tan(1/2*a) - 2*b^3*tan(1/2*a)^2 + b^3)
```

3.294 $\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

```
[Out] ((c + d*x)*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)
```

Rubi [A] time = 0.0316128, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4409, 3767, 8}

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((c + d*x)*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)\sec^2(a+bx)}{2b} - \frac{d\int \sec^2(a+bx)\,dx}{2b}\\ &= \frac{(c+dx)\sec^2(a+bx)}{2b} + \frac{d\,\mathrm{Subst}(\int 1\,dx, x, -\tan(a+bx))}{2b^2}\\ &= \frac{(c+dx)\sec^2(a+bx)}{2b} - \frac{d\tan(a+bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.0621442, size = 48, normalized size = 1.37

$$-\frac{d\tan(a+bx)}{2b^2} + \frac{c\sec^2(a+bx)}{2b} + \frac{dx\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (c*Sec[a + b*x]^2)/(2*b) + (d*x*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2
*b^2)
```

Maple [A] time = 0.027, size = 61, normalized size = 1.7

$$\frac{1}{b}\left(\frac{d}{b}\left(\frac{bx+a}{2\,(\cos(bx+a))^2} - \frac{\tan(bx+a)}{2}\right) - \frac{ad}{2\,b\,(\cos(bx+a))^2} + \frac{c}{2\,(\cos(bx+a))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] 1/b*(d/b*(1/2*(b*x+a)/cos(b*x+a)^2-1/2*tan(b*x+a))-1/2/b*d*a/cos(b*x+a)^2+1
/2*c/cos(b*x+a)^2)
```

Maxima [B] time = 0.976842, size = 382, normalized size = 10.91

$$\frac{c\tan(bx+a)^2 - \frac{ad\tan(bx+a)^2}{b} + \frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2 + 4\,(bx+a)\sin(2\,bx+2\,a)^2 + (2\,(bx+a)\cos(2\,bx+2\,a) + \sin(2\,bx+2\,a))\cos(4\,bx+4\,a) + 2\,(bx+a)\,c}{\left(2\,(2\,\cos(2\,bx+2\,a)+1)\cos(4\,bx+4\,a) + \cos(4\,bx+4\,a)^2 + 4\,\cos(2\,bx+2\,a)^2 + \sin(4\,bx+4\,a)^2 + 4\,\sin}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c*tan(b*x + a)^2 - a*d*tan(b*x + a)^2/b + 2*(4*(b*x + a)*cos(2*b*x + 2
*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + si
n(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x +
 a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x +
 2*a))*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b))/b
```

Fricas [A] time = 0.464679, size = 95, normalized size = 2.71

$$\frac{bdx - d\cos(bx+a)\sin(bx+a) + bc}{2\,b^2\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x - d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\tan(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)*sec(a + b*x)**2, x)
```

Giac [B] time = 1.23716, size = 771, normalized size = 22.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 +
 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b*
c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan
(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b*
c*tan(1/2*b*x)^4 - 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*tan
(1/2*a)^2 - 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)^2*tan(1/2*
a)^3 + b*c*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*b
*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 + 2*d*tan(1/2*b*x)^3 +
12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*d*tan(1/2*a)^3 + b*d*x + b*c - 2*d*tan(1/2*b*x) - 2*d*tan(1/2
*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 -
 8*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^
2*tan(1/2*b*x)^4 + 8*b^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^2*tan(1/2*b*x)^2*
tan(1/2*a)^2 + 8*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/2*a)^4 - 2*b^2*t
an(1/2*b*x)^2 - 8*b^2*tan(1/2*b*x)*tan(1/2*a) - 2*b^2*tan(1/2*a)^2 + b^2)
```

3.295 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Rubi [A] time = 0.106442, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 7.37083, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Maple [A] time = 0.367, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx+a))^2 \tan(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)^2 \tan(bx+a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (bx+a)^2 \tan (bx+a)}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

3.296 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi [A] time = 0.128311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 10.436, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.382, size = 0, normalized size = 0.

$$\int \frac{\left(\sec \left(bx+a\right)\right)^{2} \tan \left(bx+a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec \left(bx+a\right)^{2} \tan \left(bx+a\right)}{d^{2}x^{2}+2\,cdx+c^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan \left(a+bx\right) \sec^{2}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^2\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c)^2, x)
```

3.297 $\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\sec^3(a+bx)(c+dx)^m,x\right)-\text{Unintegrable}\left(\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] -Unintegrable[(c + d*x)^m*Sec[a + b*x], x] + Unintegrable[(c + d*x)^m*Sec[a
 + b*x]^3, x]
```

Rubi [A] time = 0.0839489, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -Defer[Int][(c + d*x)^m*Sec[a + b*x], x] + Defer[Int][(c + d*x)^m*Sec[a + b
*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx=-\int (c+dx)^m \sec(a+bx)\,dx+\int (c+dx)^m \sec^3(a+bx)\,dx$$

Mathematica [A] time = 8.98918, size = 0, normalized size = 0.

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2, x]
```

Maple [A] time = 0.174, size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a) (\tan(bx+a))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a) \tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \sec(bx+a) \tan(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

3.298 $\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx) \, dx$

Optimal. Leaf size=337

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{3id(c+$$

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 + (I*(c + d*x)^3*ArcTan[
E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 - ((
(3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*P
olyLog[2, I*E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E
^(I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyL
og[4, (-I)*E^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
)/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*
Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.408332, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4413, 4181, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{3id(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```


```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 + (I*(c + d*x)^3*ArcTan[
E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 - ((
(3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*P
olyLog[2, I*E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E
^(I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyL
og[4, (-I)*E^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
)/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*
Tan[a + b*x])/(2*b)
```


Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
```

```
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
```

```
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^3 \sec(a+bx)\,dx + \int (c+dx)^3 \sec^3(a+bx)\,dx$$

$$= \frac{2i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \sec(a+bx)}{2b^2} + \frac{(c+dx)^3 \sec(a+bx)}{2b}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2}{b^2}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2}{2b^2}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id^3\mathrm{Li}_2\left(-ie^{i(a+b}\right.}{b^4}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id^3\mathrm{Li}_2\left(-ie^{i(a+b}\right.}{b^4}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id^3\mathrm{Li}_2\left(-ie^{i(a+b}\right.}{b^4}$$

Mathematica [A] time = 3.6347, size = 530, normalized size = 1.57

$$\frac{-3id\left(b^2(c+dx)^2-2d^2\right)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)+3id\left(b^2(c+dx)^2-2d^2\right)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)+6bcd^2\mathrm{PolyLog}\left(3,-i\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] - (12*I)*b*c*d^2*ArcTan[E^(I*(a + b*
x))] - 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 - I*E^(I*
(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[
1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 6*b*d^3
*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))]
+ b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*d*(-2*d^2 + b^2*(c + d*x)^
2)*PolyLog[2, (-I)*E^(I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*Po
lyLog[2, I*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] +
6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a
 + b*x))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4,
(-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))] + b^2*(c +
d*x)^2*Sec[a + b*x]*(-3*d + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

Maple [B] time = 0.355, size = 1127, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 3*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a
+3/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x-3/b^3*d^3*ln(1+I*exp(I*(b*x+a)))*
x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a+1/2/b^4*d^3*a^3*ln(1+I*exp(I*(b*x+a)))
-3/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x+1/2/b*d^3*ln(1+I*exp(I*(b*x+a)))*x
^3-1/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3-6*I/b^3*d^2*c*arctan(exp(I*(b*x+a))
)+3/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))-I/b^4*d^3*a^3*arctan(exp(I*(b
*x+a)))-3*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))+3*I/b^3*c*d^2*a^2*arctan(exp
(I*(b*x+a)))-3*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x+3*I/b^2*d^2*c*pol
ylog(2,I*exp(I*(b*x+a)))*x-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3*I*d^3*
polylog(4,I*exp(I*(b*x+a)))/b^4+I/b*c^3*arctan(exp(I*(b*x+a)))+3/b^3*d^2*c*
polylog(3,-I*exp(I*(b*x+a)))-3/b^3*d^2*c*polylog(3,I*exp(I*(b*x+a)))-1/2/b^
4*d^3*a^3*ln(1-I*exp(I*(b*x+a)))+3/b^3*d^3*ln(1-I*exp(I*(b*x+a)))*x-I/b^2/(
exp(2*I*(b*x+a))+1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*
x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))-d^3*x^3*b*exp(I*(b*x+a))+c^3*b*exp(3*I*(
b*x+a))-3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(3*I*(b*x+a))-3*c^2*d*x
*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a))-c^3*b*exp(I*(b*x+a))-3*I*c^2
*d*exp(3*I*(b*x+a))-3*I*d^3*x^2*exp(I*(b*x+a))-6*I*c*d^2*x*exp(I*(b*x+a))-3
*I*c^2*d*exp(I*(b*x+a)))+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4-3/2/b*d^2
*c*ln(1-I*exp(I*(b*x+a)))*x^2+3/2/b*d^2*c*ln(1+I*exp(I*(b*x+a)))*x^2+3/2/b*
```

```
c^2*d*ln(1+I*exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-3/2/b
*c^2*d*ln(1-I*exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3/2/
b^3*d^2*c*a^2*ln(1+I*exp(I*(b*x+a)))+3/2/b^3*d^2*c*a^2*ln(1-I*exp(I*(b*x+a)
))+6*I/b^4*d^3*a*arctan(exp(I*(b*x+a)))-3/2*I/b^2*c^2*d*polylog(2,-I*exp(I*
(b*x+a)))-3/2*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2+3/2*I/b^2*d^3*poly
log(2,I*exp(I*(b*x+a)))*x^2
```

Maxima [B] time = 6.80647, size = 5168, normalized size = 15.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x + a)
/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1))/b^3 - 4*((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 +
6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^
3)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos
(4*b*x + 4*a) + 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*co
s(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*
b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I
*a^2 + 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^3*d^3 + 24*
I*b*c*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^3*d^3 - 12*b*
c*d^2 + 12*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c
*d^2 + (a^2 - 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*
d^3)*(b*x + a))*cos(4*b*x + 4*a) + 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)
*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 -
12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*
I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-4*I*(b
*x + a)^3*d^3 + 24*I*b*c*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b
*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*
```

x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (4*(b*
x + a)^3*d^3 - 12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2
 - (12*a + 12*I)*d^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 +
 12*(a^2 + 2*I*a)*d^3)*(b*x + a))*cos(3*b*x + 3*a) + (4*(b*x + a)^3*d^3 + 1
2*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I)
d^3)(b*x + a)^2 + (12*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a
)*d^3)*(b*x + a))*cos(b*x + a) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^
2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2
*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2
 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*b^2*c^2*
d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 + (-6*I*a^2 + 12*I)*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-12*I*b^2*c^2*d + 24*I*
a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 24*I)*d^3 + (-24*I*b*c*d^2
 + 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (6*b^
2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2
 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 -
2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^
3 + (6*I*a^2 - 12*I)*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x
 + 4*a) + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + (12*I*a
^2 - 24*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*d
ilog(-I*e^(I*b*x + I*a)) - (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 +
(-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 +
(-3*I*a^2 + 6*I)*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a
*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c
*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^
3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2
 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*cos(
2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4
*b*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (
I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*
x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x + a)
 + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*
(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (
6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6
*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 - 6*b*c*
d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c
*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d

```
^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 - 2*sin(b*x + a) + 1) - (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b
*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)
*polylog(4, I*e^(I*b*x + I*a)) + (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b*
x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*
polylog(4, -I*e^(I*b*x + I*a)) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I
*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(4*b*x + 4*a)
 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(2*b*x + 2*a) + 12*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a
)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)) - (12*I*b*c*
d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
- 12*I*a*d^3)*cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*
a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x +
4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I
*e^(I*b*x + I*a)) - (4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12
*a^2*d^3 + (12*I*b*c*d^2 - 12*(I*a - 1)*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d
- 24*(I*a - 1)*b*c*d^2 + (12*I*a^2 - 24*a)*d^3)*(b*x + a))*sin(3*b*x + 3*a)
 - (-4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + (-12*
I*b*c*d^2 - 12*(-I*a - 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d - 24*(-I*a -
1)*b*c*d^2 + (-12*I*a^2 - 24*a)*d^3)*(b*x + a))*sin(b*x + a))/(-4*I*b^3*cos
(4*b*x + 4*a) - 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a) + 8*b^3*s
in(2*b*x + 2*a) - 4*I*b^3))/b
```

Fricas [C] time = 0.945466, size = 3214, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(-6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*
I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3*co
s(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x +
 a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b
^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) +
 sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d
^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b
*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2
*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3*c
^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*
```

```
log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2
- 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x
 + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*
x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a
)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3
)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^
3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1
) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b
*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*
d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*
cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog
(3, I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*p
olylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x +
 a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c
*d^2*x + b^2*c^2*d)*cos(b*x + a) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3
*c^2*d*x + b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \tan^2 (a + bx) \sec (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)**2*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \sec (bx + a) \tan (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a)^2, x)
```

3.299 $\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=193

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{PolyLog}\left(3,ie^{i(a+}\right.}{b^3}$$

```
[Out] (I*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/b^3
 - (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x])/b
^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.271143, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4413, 4181, 2531, 2282, 6589, 4186, 3770}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{PolyLog}\left(3,ie^{i(a+}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```


```
[Out] (I*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/b^3
 - (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x])/b
^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```


Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^2 \sec(a+bx)\,dx + \int (c+dx)^2 \sec^3(a+bx)\,dx$$
$$= \frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2\sec(a+bx)\tan}{2b}$$
$$= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx}\right.}{b^2}$$
$$= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right.}{b^2}$$
$$= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right.}{b^2}$$
$$= \frac{i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right.}{b^2}$$

<u>Mathematica</u> [B] time = 7.22457, size = 526, normalized size = 2.73

$$\frac{-id(c+dx)\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right) + id(c+dx)\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right) + \frac{d^2\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b} - \frac{d^2\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b} + ibc^2\tan}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*b*c^2*ArcTan[E^(I*(a + b*x))] - ((2*I)*d^2*ArcTan[E^(I*(a + b*x))])/b -
b*c*d*x*Log[1 - I*E^(I*(a + b*x))] - (b*d^2*x^2*Log[1 - I*E^(I*(a + b*x))])
/2 + b*c*d*x*Log[1 + I*E^(I*(a + b*x))] + (b*d^2*x^2*Log[1 + I*E^(I*(a + b*
x))])/2 - I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] + I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))] + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b - (d
^2*PolyLog[3, I*E^(I*(a + b*x))])/b)/b^2 - (d*(c + d*x)*Sec[a])/b^2 + (c^2
+ 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])^2) + (-
(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - Sin[a/2])*(Cos[a/
2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2*x^2)/(4*b*(Cos[
a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*
x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])
)
```

Maple [B] time = 0.31, size = 584, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))
*x+I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))-1/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x
^2+1/b*c*d*ln(1+I*exp(I*(b*x+a)))*x-d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-1/2
/b^3*d^2*a^2*ln(1+I*exp(I*(b*x+a)))+1/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a+1/2/
b^3*d^2*a^2*ln(1-I*exp(I*(b*x+a)))+I/b*c^2*arctan(exp(I*(b*x+a)))-2*I/b^3*d
^2*arctan(exp(I*(b*x+a)))-2*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))+1/2/b*d^2*ln
(1+I*exp(I*(b*x+a)))*x^2-1/b*c*d*ln(1-I*exp(I*(b*x+a)))*x+I/b^2*d^2*polylog
(2,I*exp(I*(b*x+a)))*x-1/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+d^2*polylog(3,-I*
exp(I*(b*x+a)))/b^3-I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-I/b^2/(exp(2*I*(
b*x+a))+1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+a))+c^2*b*e
xp(3*I*(b*x+a))-d^2*x^2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(b*x+a))-2*I*d^2*x
*exp(3*I*(b*x+a))-c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x+a))-2*I*d^2*x*e
xp(I*(b*x+a))-2*I*d*c*exp(I*(b*x+a)))
```

Maxima [B] time = 2.89778, size = 2556, normalized size = 13.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(s
in(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(sin(
b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2 - 4*((
2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^
2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x +
a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*a) -
 (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(
2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^2*d^2
+ 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a
```

```
*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^2*d^2 + (-4*I*b
*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^2
*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(2*b*x + 2*a))*arct
an2(cos(b*x + a), -sin(b*x + a) + 1) - (4*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I
*a*d^2 + (8*b*c*d - (8*a + 8*I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) + (4*(b*x
+ a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (8*b*c*d - (8*a - 8*I)*d^2)*(b*x + a))
*cos(b*x + a) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)
*d^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*
x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(4*b*x + 4*a) -
(-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I
*b*x + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*
d^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
 + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8
*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b
*x + I*a)) - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + 2*I
*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + 2*I*d^2)*
cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x +
a) + 4*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) - 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*s
in(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) -
 2*I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) - 2*I*d^2
)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x +
a) - 4*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) - 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*s
in(b*x + a) + 1) - (-4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) +
4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, I*e^(
I*b*x + I*a)) - (4*I*d^2*cos(4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^
2*sin(4*b*x + 4*a) - 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -I*e^(I*b
*x + I*a)) - (4*I*(b*x + a)^2*d^2 + 8*b*c*d - 8*a*d^2 + (8*I*b*c*d - 8*(I*a
 - 1)*d^2)*(b*x + a))*sin(3*b*x + 3*a) - (-4*I*(b*x + a)^2*d^2 + 8*b*c*d -
8*a*d^2 + (-8*I*b*c*d - 8*(-I*a - 1)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^
2*cos(4*b*x + 4*a) - 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) + 8*
b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b
```

Fricas [C] time = 0.747209, size = 2033, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2
*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x +
 a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*pol
ylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x
 + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*co
s(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c
*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x -
2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2
- 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x
+ a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(
b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) +
1) - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a)
 + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 4*(b*d^2*x + b*c*d)*cos(b*x +
a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a
)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2 \tan^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**2*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \sec(bx+a)\tan(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a)^2, x)
```

3.300 $\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=117

$$-\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)\tan(a+bx}{2b}$$

```
[Out] (I*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - ((I/2)*d*PolyLog[2, (-I)*E^(I*(a
+ b*x))])/b^2 + ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a + b*
x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rubi [A] time = 0.12925, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4413, 4181, 2279, 2391, 4185}

$$-\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)\tan(a+bx}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```


```
[Out] (I*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - ((I/2)*d*PolyLog[2, (-I)*E^(I*(a
+ b*x))])/b^2 + ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a + b*
x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```


Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx &= -\int (c+dx)\sec(a+bx)\,dx + \int (c+dx)\sec^3(a+bx)\,dx \\
&= \frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b} + \\
&= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b} - \\
&= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} + \frac{id\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{d\sec(a+bx)}{2b^2} \\
&= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2} + \frac{id\mathrm{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a+bx)}{2b^2}
\end{aligned}
$$

Mathematica [B] time = 6.5224, size = 555, normalized size = 4.74

$$
\frac{dx\left(-i\mathrm{PolyLog}\left(2,\frac{1}{2}\left((1+i)-(1-i)\tan\left(\frac{1}{2}(a+bx)\right)\right)\right)+i\mathrm{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(\tan\left(\frac{1}{2}(a+bx)\right)+i\right)\right)-i\mathrm{PolyLog}\left(2,\right.\right.}{}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -(c*ArcTanh[Sin[a + b*x]])/(2*b) + (d*x*(a*Log[1 - Tan[(a + b*x)/2]] + I*Lo
g[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] - I*Log
[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2])] - I*Log[
1 + I*Tan[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + I*Log[1 -
 I*Tan[(a + b*x)/2]]*Log[(1/2 + I/2)*(1 + Tan[(a + b*x)/2])] - a*Log[1 + Ta
n[(a + b*x)/2]] - I*PolyLog[2, ((1 + I) - (1 - I)*Tan[(a + b*x)/2])/2] + I*
PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2])] - I*PolyLog[2, ((1 + I) + (
1 - I)*Tan[(a + b*x)/2])/2] + I*PolyLog[2, ((1 - I) + (1 + I)*Tan[(a + b*x)
/2])/2]))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*
x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])^2) - (d*Sin[(a
+ b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*x)/(4*b*(Cos[
(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a +
 b*x)/2] + Sin[(a + b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Maple [B] time = 0.162, size = 267, normalized size = 2.3

$$\frac{-i\left(dxbe^{3i(bx+a)} - ide^{3i(bx+a)} + bce^{3i(bx+a)} - dxbe^{i(bx+a)} - ide^{i(bx+a)} - bce^{i(bx+a)}\right)}{b^2\left(e^{2i(bx+a)}+1\right)^2} + \frac{ic\arctan\left(e^{i(bx+a)}\right)}{b} + \frac{d\ln\left(1 + ie^{i(bx+a}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] -I/b^2/(exp(2*I*(b*x+a))+1)^2*(d*x*b*exp(3*I*(b*x+a))-I*d*exp(3*I*(b*x+a))+
b*c*exp(3*I*(b*x+a))-d*x*b*exp(I*(b*x+a))-I*d*exp(I*(b*x+a))-b*c*exp(I*(b*x
+a)))+I/b*c*arctan(exp(I*(b*x+a)))+1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x+1/2/b^2
*d*ln(1+I*exp(I*(b*x+a)))*a-1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x-1/2/b^2*d*ln(1
-I*exp(I*(b*x+a)))*a-1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))+1/2*I/b^2*d*dilo
g(1-I*exp(I*(b*x+a)))-I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.624285, size = 1169, normalized size = 9.99

$$\frac{i\,d\cos\left(bx+a\right)^{2}\,\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+i\,d\cos\left(bx+a\right)^{2}\,\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)-i\,d\cos\left(bx+a\right)^{2}\,\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x
+ a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a)
+ I) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) -
(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*
x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a
*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*
cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*cos(b*
x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) + 2*(b*d*x +
b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\tan^2{(a+bx)}\sec{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)**2*sec(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\sec\left(bx+a\right)\tan\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*tan(b*x + a)^2, x)
```

3.301 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\sec^3(a+bx)}{c+dx},x\right)-\text{Unintegrable}\left(\frac{\sec(a+bx)}{c+dx},x\right)$$

```
[Out] -Unintegrable[Sec[a + b*x]/(c + d*x), x] + Unintegrable[Sec[a + b*x]^3/(c +
 d*x), x]
```

Rubi [A] time = 0.0911012, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -Defer[Int][Sec[a + b*x]/(c + d*x), x] + Defer[Int][Sec[a + b*x]^3/(c + d*x
), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx=-\int \frac{\sec(a+bx)}{c+dx}\,dx+\int \frac{\sec^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 26.6999, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

Maple [A] time = 1.455, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)(\tan(bx+a))^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec(bx+a)\tan(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\tan(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

3.302 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=37

$$\text{Unintegrable}\left(\frac{\sec^3(a+bx)}{(c+dx)^2},x\right)-\text{Unintegrable}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -Unintegrable[Sec[a + b*x]/(c + d*x)^2, x] + Unintegrable[Sec[a + b*x]^3/(c
 + d*x)^2, x]
```

Rubi [A] time = 0.0888488, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Defer[Int][Sec[a + b*x]/(c + d*x)^2, x] + Defer[Int][Sec[a + b*x]^3/(c + d
*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx=-\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 29.522, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

Maple [A] time = 2.21, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)(\tan(bx+a))^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec(bx+a)\tan(bx+a)^2}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

3.303 $\int (c+dx)^m \tan^3(a+bx)\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\tan^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable[(c + d*x)^m*Tan[a + b*x]^3, x]
```

Rubi [A] time = 0.0348048, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \tan^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x]^3,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \tan^3(a+bx)\,dx=\int (c+dx)^m \tan^3(a+bx)\,dx$$

Mathematica [A] time = 5.54264, size = 0, normalized size = 0.

$$\int (c+dx)^m \tan^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x]^3,x]
```

```
[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^3, x]
```

Maple [A] time = 0.197, size = 0, normalized size = 0.

$$\int (dx+c)^m (\tan (bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*tan(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*tan(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \tan (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx+c)^m \tan (bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*tan(b*x + a)^3, x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^m \tan^3 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)**3, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \tan(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

3.304 $\int (c+dx)^3 \tan^3(a+bx)\,dx$

Optimal. Leaf size=259

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{PolyLo}}{}$$

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - ((I/4)*(c + d*x)^4)/d -
 (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^3*Log[1 +
E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/
b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d
^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*Pol
yLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
 + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.355707, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3720, 3719, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - ((I/4)*(c + d*x)^4)/d -
 (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^3*Log[1 +
E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/
b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d
^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*Pol
yLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
 + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \tan^3(a+bx)\,dx = \frac{(c+dx)^3 \tan^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2 \tan^2(a+bx)\,dx}{2b} - \int (c+dx)^3 \tan(a+bx)\,dx$$

$$= -\frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2 \tan(a+bx)}{2b^2} + \frac{(c+dx)^3 \tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \tan(a}{2b^2}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3 \log\left(1\right.}{b}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3 \log\left(1\right.}{b}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3 \log\left(1\right.}{b}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3 \log\left(1\right.}{b}$$

Mathematica [B] time = 6.9231, size = 803, normalized size = 3.1

$$\frac{\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))c^3}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{3d\csc(a)\left(b^2 e^{-i\tan^{-1}(\cot(a))}x^2 - \frac{\cot(a)\left(ibx\left(-2\tan^{-1}(\cot(a\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x]^3,x]
```

```
[Out] ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
(3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(
I*a)) + (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*
x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*Poly
Log[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))]
 - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + ((c + d*
x)^3*Sec[a + b*x]^2)/(2*b) + (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[
a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Co
s[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 +
 Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*
b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan
[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*
ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos
[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[
a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x -
ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]
] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)
*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^
2*(Cos[a]^2 + Sin[a]^2)]) - (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^
2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2
*x^2 + d^3*x^3)*Tan[a])/4
```

Maple [B] time = 0.368, size = 720, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tan(b*x+a)^3,x)
```

```
[Out] 3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+1/b*c^3*ln(exp(2*I*(b*x+a))+1)+3
/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+3/2/b^3*d^3*polylog(3,-exp(2*I*(b
*x+a)))*x+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*
d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(
b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I
*c*d^2*x-3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))+1)^2-3*d^2/b^3*c*ln(exp(2*I*(b*x+
```

```
a))+1)-3*d^3/b^3*ln(exp(2*I*(b*x+a))+1)*x+3*I*d^3/b^2*x^2+3*I*d^3/b^4*a^2-3
/2*I/b^4*d^3*a^4+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4-3/2*I/b^2*c^2*d
*polylog(2,-exp(2*I*(b*x+a)))-3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^
2+4*I/b^3*a^3*c*d^2-3*I/b^2*a^2*c^2*d+3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x+3/
b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-2*I/b
^3*d^3*a^3*x-6/b^4*d^3*a*ln(exp(I*(b*x+a)))+6/b^3*d^2*c*ln(exp(I*(b*x+a)))+
6*I*d^3/b^3*a*x-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))+6/b^2*c^2*d*a*ln(exp(I*(
b*x+a)))+I*c^3*x-I*c*d^2*x^3-3/2*I*c^2*d*x^2+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)
))-3*I/b^2*polylog(2,-exp(2*I*(b*x+a)))*c*d^2*x-6*I/b*a*c^2*d*x+6*I/b^2*a^2
*c*d^2*x-2/b*c^3*ln(exp(I*(b*x+a)))-1/4*I*d^3*x^4
```

Maxima [B] time = 4.48058, size = 3247, normalized size = 12.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 3*a*c^2*d*(1
/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + 3*a^2*c*d^2*(1/(sin(b*
x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 -
 1) - log(sin(b*x + a)^2 - 1))/b^3 + 2*(3*(b*x + a)^4*d^3 + 36*b^2*c^2*d -
72*a*b*c*d^2 + 36*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 18*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - (16*(b*x + a)^3*d^3 - 36*b*c*d^2 +
 36*a*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 36*(b^2*c^2*d - 2*a*b*c*d^2
+ (a^2 - 1)*d^3)*(b*x + a) + 4*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9
*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3
)*(b*x + a))*cos(4*b*x + 4*a) + 8*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3
+ 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*
d^3)*(b*x + a))*cos(2*b*x + 2*a) + (16*I*(b*x + a)^3*d^3 - 36*I*b*c*d^2 + 3
6*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*
I*a*b*c*d^2 + (36*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (32*I*(b
*x + a)^3*d^3 - 72*I*b*c*d^2 + 72*I*a*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*
x + a)^2 + (72*I*b^2*c^2*d - 144*I*a*b*c*d^2 + (72*I*a^2 - 72*I)*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 3
*((b*x + a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*
c*d^2 + (a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*
b*x + 4*a) + (6*(b*x + a)^4*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3
+ (24*b*c*d^2 - (24*a - 24*I)*d^3)*(b*x + a)^3 + (36*b^2*c^2*d - (72*a - 72
*I)*b*c*d^2 + 36*(a^2 - 2*I*a - 1)*d^3)*(b*x + a)^2 - (-72*I*b^2*c^2*d - 72
*(-2*I*a - 1)*b*c*d^2 + (-72*I*a^2 - 72*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a)
```

```
 + (18*b^2*c^2*d - 36*a*b*c*d^2 + 24*(b*x + a)^2*d^3 + 18*(a^2 - 1)*d^3 + 3
6*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^
2*d^3 + 3*(a^2 - 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) +
 12*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 - 1)*d^3 + 6*(b
*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c
*d^2 - 24*I*(b*x + a)^2*d^3 + (-18*I*a^2 + 18*I)*d^3 + (-36*I*b*c*d^2 + 36*
I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 -
48*I*(b*x + a)^2*d^3 + (-36*I*a^2 + 36*I)*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - (-8*I*(b*x + a
)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a
)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 + 18*I)*d^3)*(b*x + a)
 + (-8*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*
I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 + 18*
I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-16*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2
 - 36*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d
 + 72*I*a*b*c*d^2 + (-36*I*a^2 + 36*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 2
*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a)
+ 4*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a
)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*
a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) -
 (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x +
4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a))
 - (-18*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3 + (-18*I*b*c*d^2 - 24*I
*(b*x + a)*d^3 + 18*I*a*d^3)*cos(4*b*x + 4*a) + (-36*I*b*c*d^2 - 48*I*(b*x
+ a)*d^3 + 36*I*a*d^3)*cos(2*b*x + 2*a) + 6*(3*b*c*d^2 + 4*(b*x + a)*d^3 -
3*a*d^3)*sin(4*b*x + 4*a) + 12*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(
2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) - (-3*I*(b*x + a)^4*d^3 + (-
12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^3 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2
+ (-18*I*a^2 + 36*I)*d^3)*(b*x + a)^2 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x +
a))*sin(4*b*x + 4*a) - (-6*I*(b*x + a)^4*d^3 - 36*I*b^2*c^2*d + 72*I*a*b*c*
d^2 - 36*I*a^2*d^3 + (-24*I*b*c*d^2 - 24*(-I*a - 1)*d^3)*(b*x + a)^3 + (-36
*I*b^2*c^2*d - 72*(-I*a - 1)*b*c*d^2 + (-36*I*a^2 - 72*a + 36*I)*d^3)*(b*x
+ a)^2 + (72*b^2*c^2*d - (144*a - 72*I)*b*c*d^2 + 72*(a^2 - I*a)*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))/(-12*I*b^3*cos(4*b*x + 4*a) - 24*I*b^3*cos(2*b*x +
2*a) + 12*b^3*sin(4*b*x + 4*a) + 24*b^3*sin(2*b*x + 2*a) - 12*I*b^3))/b
```

Fricas [C] time = 0.537343, size = 1443, normalized size = 5.57

$$4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 12\,b^3c^2dx - 3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 3*I*d^3*polylog(4,
 (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 3*I*d^3*po
lylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 4*
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a)^2 +
(6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 6*I*d^3)*dilog(2*(I*t
an(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c
*d^2*x - 6*I*b^2*c^2*d + 6*I*d^3)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x +
a)^2 + 1) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3
*(b^3*c^2*d - b*d^3)*x)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) +
 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(b^3*c^2*d - b*
d^3)*x)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*(b*d^3*x + b
*c*d^2)*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2
+ 1)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1)) - 12*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*
tan(b*x + a))/b^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \tan^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \tan(bx + a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tan(b*x + a)^3, x)
```

3.305 $\int (c+dx)^2 \tan^3(a+bx)\,dx$

Optimal. Leaf size=169

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\tan(a+bx)}{b^2}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{(c+dx}{}$$

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 +
E^((2*I)*(a + b*x))])/b - (d^2*Log[Cos[a + b*x]])/b^3 - (I*d*(c + d*x)*Poly
Log[2, -E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/
(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2)/(2*
b)
```

Rubi [A] time = 0.221749, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3475, 3719, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\tan(a+bx)}{b^2}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x]^3,x]
```

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 +
E^((2*I)*(a + b*x))])/b - (d^2*Log[Cos[a + b*x]])/b^3 - (I*d*(c + d*x)*Poly
Log[2, -E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/
(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2)/(2*
b)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \tan^3(a+bx)\,dx = \frac{(c+dx)^2 \tan^2(a+bx)}{2b} - \frac{d\int (c+dx)\tan^2(a+bx)\,dx}{b} - \int (c+dx)^2 \tan(a+bx)\,dx$$

$$= -\frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{d(c+dx)\,\mathrm{t}}{b}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{id(c+dx)\mathrm{I}}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{id(c+dx)\mathrm{I}}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{id(c+dx)\mathrm{I}}{}$$

<u>Mathematica</u> [B] time = 6.64045, size = 454, normalized size = 2.69

$$\frac{cd\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\text{PolyLog}\left(2,e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x]^3,x]
```

```
[Out] ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(
a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (
3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I
*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*
Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (d^2
*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*
(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot
[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x -
 ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x
]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I
)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2
*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Si
n[b*x]))/b^2 - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

Maple [B] time = 0.321, size = 400, normalized size = 2.4

$$\frac{-id^2\mathrm{polylog}\left(2,-\mathrm{e}^{2\,i(bx+a)}\right)x}{b^2}-icdx^2+ic^2x+2\,\frac{bd^2x^2\mathrm{e}^{2\,i(bx+a)}-id^2x\mathrm{e}^{2\,i(bx+a)}+2\,bcdx\mathrm{e}^{2\,i(bx+a)}-icd\mathrm{e}^{2\,i(bx+a)}+bc^2\mathrm{e}^{2\,i(bx}}{b^2\left(\mathrm{e}^{2\,i(bx+a)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tan(b*x+a)^3,x)
```

```
[Out] -I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-I*c*d*x^2+I*c^2*x+2*(b*d^2*x^2*ex
p(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))-I*c*d*ex
p(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x-I*d*c)/b^2/(exp(2*I*(b*x+a))+
1)^2-4*I/b*a*c*d*x-I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-1/3*I*d^2*x^3+2*I
/b^2*a^2*d^2*x+1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-2*I/b^2*a^2*c*d-2/b*c^2*l
n(exp(I*(b*x+a)))+1/b*c^2*ln(exp(2*I*(b*x+a))+1)-2/b^3*d^2*a^2*ln(exp(I*(b*
x+a)))+4/3*I/b^3*a^3*d^2+2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*d^2*polylog(3
,-exp(2*I*(b*x+a)))/b^3+2/b^3*d^2*ln(exp(I*(b*x+a)))-1/b^3*d^2*ln(exp(2*I*(
b*x+a))+1)+4/b^2*c*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 2.44249, size = 1655, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 2*a*c*d*(1/(
sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + a^2*d^2*(1/(sin(b*x + a)
^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 + 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d -
a*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - (6*(b*x + a)^2*d^2 + 12*(b*c*d -
 a*d^2)*(b*x + a) - 6*d^2 + 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a
) - d^2)*cos(4*b*x + 4*a) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
a) - d^2)*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^
2)*(b*x + a) - 6*I*d^2)*sin(4*b*x + 4*a) + (12*I*(b*x + a)^2*d^2 + (24*I*b*
c*d - 24*I*a*d^2)*(b*x + a) - 12*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x
 + 2*a), cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*
x + a)^2 - 6*(b*x + a)*d^2)*cos(4*b*x + 4*a) + (4*(b*x + a)^3*d^2 + (12*b*c
*d - (12*a - 12*I)*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - (-24*I*b*c*d -
```

```
12*(-2*I*a - 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (6*b*c*d + 6*(b*x + a)*d
^2 - 6*a*d^2 + 6*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 12*(b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-6*I*b*c*d - 6*I*(b*x + a)*
d^2 + 6*I*a*d^2)*sin(4*b*x + 4*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*
I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - (-3*I*(b*x + a)^2*
d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*d^2
+ (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*cos(4*b*x + 4*a) + (-6*I*(b
*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) + 6*I*d^2)*cos(2*b*x +
 2*a) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x +
 4*a) + 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x +
 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
) - (-3*I*d^2*cos(4*b*x + 4*a) - 6*I*d^2*cos(2*b*x + 2*a) + 3*d^2*sin(4*b*x
 + 4*a) + 6*d^2*sin(2*b*x + 2*a) - 3*I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)
) - (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a)^2 + 12*I*(b*
x + a)*d^2)*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^3*d^2 + (-12*I*b*c*d - 12*(-
I*a - 1)*d^2)*(b*x + a)^2 - 12*I*b*c*d + 12*I*a*d^2 + (24*b*c*d - (24*a - 1
2*I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^2*cos(4*b*x + 4*a) - 12*I*b^
2*cos(2*b*x + 2*a) + 6*b^2*sin(4*b*x + 4*a) + 12*b^2*sin(2*b*x + 2*a) - 6*I
*b^2))/b
```

Fricas [C] time = 0.518016, size = 883, normalized size = 5.22

$$\frac{2\,b^2d^2x^2+4\,b^2cdx+d^2\text{polylog}\left(3,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+d^2\text{polylog}\left(3,\frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+2\left(b^2d^2x^2+2\,b^2cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + d^2*polylog(3, (tan(b*x + a)^2 + 2*I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + d^2*polylog(3, (tan(b*x + a)^2 - 2*I
*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*tan(b*x + a)^2 + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(2*(I*tan(b*x + a)
- 1)/(tan(b*x + a)^2 + 1) + 1) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(2*(-I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2 - d^2)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2 - d^2)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x
+ a)^2 + 1)) - 4*(b*d^2*x + b*c*d)*tan(b*x + a))/b^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^2\tan^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2\tan(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tan(b*x + a)^3, x)
```

3.306 $\int (c+dx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=108

$$-\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{dx}{2b}-\frac{i(c+dx)^2}{2d}$$

```
[Out] (d*x)/(2*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(
2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.116911, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3473, 8, 3719, 2190, 2279, 2391}

$$-\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{dx}{2b}-\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x]^3,x]
```

```
[Out] (d*x)/(2*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(
2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\tan^3(a+bx)\,dx = \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\int \tan^2(a+bx)\,dx}{2b} - \int (c+dx)\tan(a+bx)\,dx$$

$$= -\frac{i(c+dx)^2}{2d} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx + \frac{d\int 1\,d}{2b}$$

$$= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+bx)}{2b} - $$

$$= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+bx)}{2b} + $$

$$= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id\,\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{d\tan(a+bx)}{2b^2} + $$

Mathematica [B] time = 6.15395, size = 240, normalized size = 2.22

$$\frac{d\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))}-\frac{\cot(a)\left(i\text{PolyLog}\left(2,e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}(c\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{2b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Tan[a + b*x]^3,x]
```

```
[Out] (d*x*Sec[a + b*x]^2)/(2*b) + (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (C
ot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x
 - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b
*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2
*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[
a]^2*(Cos[a]^2 + Sin[a]^2)]) - (d*Sec[a]*Sec[a + b*x]*Sin[b*x])/(2*b^2) - (
d*x^2*Tan[a])/2 + (c*(2*Log[Cos[a + b*x]] + Tan[a + b*x]^2))/(2*b)
```

Maple [A] time = 0.159, size = 183, normalized size = 1.7

$$-\frac{i}{2}dx^2+icx+\frac{2\,bdx\mathrm{e}^{2\,i(bx+a)}+2\,bc\mathrm{e}^{2\,i(bx+a)}-id\mathrm{e}^{2\,i(bx+a)}-id}{b^2\left(\mathrm{e}^{2\,i(bx+a)}+1\right)^2}-2\,\frac{c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b}+\frac{c\ln\left(\mathrm{e}^{2\,i(bx+a)}+1\right)}{b}-\frac{2\,idax}{b}-\frac{ida^2}{b^2}+\frac{d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tan(b*x+a)^3,x)
```

```
[Out] -1/2*I*d*x^2+I*c*x+(2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp
(2*I*(b*x+a))-I*d)/b^2/(exp(2*I*(b*x+a))+1)^2-2/b*c*ln(exp(I*(b*x+a)))+1/b*
c*ln(exp(2*I*(b*x+a))+1)-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(exp(2*I*(b*x+a))+
1)*x-1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [B] time = 2.04237, size = 701, normalized size = 6.49

$$-\frac{b^2dx^2+2\,b^2cx-(2\,bdx+2\,bc+2\,(bdx+bc)\cos(4\,bx+4\,a)+4\,(bdx+bc)\cos(2\,bx+2\,a)+(2i\,bdx+2i\,bc)\sin(4\,bx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(b^2*d*x^2 + 2*b^2*c*x - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*
a) + 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4
*a) + (4*I*b*d*x + 4*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos
(2*b*x + 2*a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) + (2*b^2*d*x^
2 + 4*I*b*c + (4*b^2*c + 4*I*b*d)*x + 2*d)*cos(2*b*x + 2*a) + (d*cos(4*b*x
+ 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*
a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c
)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c
)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2
*c*x)*sin(4*b*x + 4*a) - (-2*I*b^2*d*x^2 + 4*b*c - 4*(I*b^2*c - b*d)*x - 2*
I*d)*sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x
 + 2*a) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

Fricas [A] time = 0.492585, size = 447, normalized size = 4.14

$$\frac{2\,bdx+2\,(bdx+bc)\tan\left(bx+a\right)^{2}+i\,d\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^{2}+1}+1\right)-i\,d\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^{2}+1}+1\right)+2\,(bdx+bc)\log\left(-\frac{2\,(i\,\tan\ldots}{\tan(\ldots}\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d*x + 2*(b*d*x + b*c)*tan(b*x + a)^2 + I*d*dilog(2*(I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1) + 1) - I*d*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*
x + a)^2 + 1) + 1) + 2*(b*d*x + b*c)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x +
 a)^2 + 1)) + 2*(b*d*x + b*c)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2
+ 1)) - 2*d*tan(b*x + a))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (c+dx)\tan^{3}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\tan(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*tan(b*x + a)^3, x)
```

3.307 $\int \frac{\tan^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\tan^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable[Tan[a + b*x]^3/(c + d*x), x]
```

Rubi [A] time = 0.0385794, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/(c + d*x),x]
```

```
[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 6.45732, size = 0, normalized size = 0.

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/(c + d*x),x]
```

```
[Out] Integrate[Tan[a + b*x]^3/(c + d*x), x]
```

Maple [A] time = 1.853, size = 0, normalized size = 0.

$$\int \frac{(\tan (bx+a))^3}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(tan(b*x+a)^3/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
(2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*
b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 - d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) + (d*cos(2*b*x + 2*a) + 2*(b*d*x + b*
c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*sin(2*b*x + 2*a))/(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x
 + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
```

```
b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b
*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\tan\left(bx+a\right)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] integral(tan(b*x + a)^3/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/(d*x+c),x)

[Out] Integral(tan(a + b*x)**3/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(bx+a\right)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] integrate(tan(b*x + a)^3/(d*x + c), x)
```

3.308 $\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\tan^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable[Tan[a + b*x]^3/(c + d*x)^2, x]
```

Rubi [A] time = 0.0366005, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx = \int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 7.03406, size = 0, normalized size = 0.

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Integrate[Tan[a + b*x]^3/(c + d*x)^2, x]
```

Maple [A] time = 2.576, size = 0, normalized size = 0.

$$\int \frac{(\tan (bx + a))^3}{(dx + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(tan(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*inte
grate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*sin(2*b*x + 2*a)/(b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 +
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^
3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)), x)
+ 2*(d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4
*a) + 2*d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4
```

```
*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*
c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan\left(bx+a\right)^{3}}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan^{3}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(bx+a\right)^{3}}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^3/(d*x + c)^2, x)
```

3.309 $\int (c+dx)^m \csc(a+bx) \sec^3(a+bx)\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\csc(a+bx)\sec^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

Rubi [A] time = 0.268662, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$$

Mathematica [A] time = 9.56064, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

Maple [A] time = 0.185, size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a) (\sec(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a) \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \csc(bx+a) \sec(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a) \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

3.310 $\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=399

$$\frac{6id^3(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c}{}$$

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 + (c + d*x)^4/(2*b) - (2*(c + d*x)^4*ArcTanh[E^((
2*I)*(a + b*x))])/b - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^3
+ ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((2*I)*d*(c
+ d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyL
og[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^4*PolyLog[3, -E^((2*I)*(a + b*x))])/
b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c
+ d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLo
g[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*
(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (3*d^
4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) - (2*d*(c + d*x)^3*Tan[a + b*x])
/b^2 + ((c + d*x)^4*Tan[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.971034, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 32}

$$\frac{6id^3(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```


```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 + (c + d*x)^4/(2*b) - (2*(c + d*x)^4*ArcTanh[E^((
2*I)*(a + b*x))])/b - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^3
+ ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((2*I)*d*(c
+ d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyL
og[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^4*PolyLog[3, -E^((2*I)*(a + b*x))])/
b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c
+ d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLo
g[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*
(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (3*d^
4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) - (2*d*(c + d*x)^3*Tan[a + b*x])
```

```
/b^2 + ((c + d*x)^4*Tan[a + b*x]^2)/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - (4d)\int (c+dx)^3 \left(\log(\mathrm{t}\ldots\right.$$

$$= \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - (4d)\int \frac{(c+dx)^3\left(2\log\ldots\right.}{\ldots}$$

$$= \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3\left(2\log\ldots\right.}{\ldots}$$

$$= \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - \frac{(2d)\int \left(2(c+dx)^3\log\ldots\right.}{\ldots}$$

$$= \frac{(c+dx)^4\log(\tan(a+bx))}{b} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3\tan^2(a\ldots}{b}$$

$$= -\frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{(c+dx)^4\tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^4\csc(2a+\ldots}{b}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{(c+dx)^4\tan^2(a+bx)}{2b}\ldots$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2\log\left(1\ldots\right.}{b^3}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2\log\left(1\ldots\right.}{b^3}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2\log\left(1\ldots\right.}{b^3}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2\log\left(1\ldots\right.}{b^3}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2\log\left(1\ldots\right.}{b^3}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2\log\left(1\ldots\right.}{b^3}$$

Mathematica [B] time = 7.50809, size = 2090, normalized size = 5.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*L
```

og[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)
*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E
^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a +
 b*x))]))/E^((2*I)*a)))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a)
 + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3
*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))
*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(
a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^(
(2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^
((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^4 -
(d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a))
*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 +
 E^((-I)*(a + b*x))] - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, -E^((-I)*
(a + b*x))] - (3*I)*b^2*x^2*PolyLog[3, -E^((-I)*(a + b*x))] - 6*b*x*PolyLog
[4, -E^((-I)*(a + b*x))] + (6*I)*PolyLog[5, -E^((-I)*(a + b*x))]))/E^((2*I)
a) - (20(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, E^((-I)*(a + b*x))] - (3*I
)*b^2*x^2*PolyLog[3, E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, E^((-I)*(a + b*
x))] + (6*I)*PolyLog[5, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(10*b^5) + (x*(
5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Csc[a]*Sec[a])
/5 - ((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^(
(-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*
x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(
b^3*E^(I*a)) - ((I/2)*d^4*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1
 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(
a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec
[a])/(b^5*E^(I*a)) - (I/2)*c*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 +
 E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*
(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2
I)(a + b*x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Se
c[a] + (d^4*((-4*I)*x^5 - (10*(1 + E^((2*I)*a))*x^4*Log[1 + E^((-2*I)*(a +
b*x))])/b + (5*(1 + E^((2*I)*a))*((-4*I)*b^3*x^3*PolyLog[2, -E^((-2*I)*(a +
 b*x))] - 6*b^2*x^2*PolyLog[3, -E^((-2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[4
, -E^((-2*I)*(a + b*x))] + 3*PolyLog[5, -E^((-2*I)*(a + b*x))]))/b^5)*Sec[a
])/(20*E^(I*a)) + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (c^4*Sec[a]*(Cos[a]*
Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]
^2)) - (6*c^2*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b
*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c^4*Csc[a]*(-(b*x*Cos[a]) + Log[
Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (2*
c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcT
an[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 -
 E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Lo
g[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])
)]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)])
 - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi -
2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*L

```
og[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[
a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Co
t[a]]))]]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a
]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*
c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 - (2*c^3*d*Csc[a]*Sec[a]*(b^2*E
^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^(
(-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[
a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
 + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^
2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.483, size = 1729, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] 24*I*d^3/b^3*c*a*x-4/b*c*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-6*d^2/b^3*c^2*ln(ex
p(2*I*(b*x+a))+1)-6*d^4/b^3*ln(exp(2*I*(b*x+a))+1)*x^2+4*I*d^4/b^2*x^3-8*I*
d^4/b^5*a^3+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-4/b*ln(exp(2*I*(b*x+a)
)+1)*c^3*d*x-6/b*ln(exp(2*I*(b*x+a))+1)*c^2*d^2*x^2+2*I/b^2*c^3*d*polylog(2
,-exp(2*I*(b*x+a)))-3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a)))+2*I/b^2*d^4*
polylog(2,-exp(2*I*(b*x+a)))*x^3-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x
+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4-12*d^3
/b^3*c*ln(exp(2*I*(b*x+a))+1)*x-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b^5+4/b*
c*d^3*ln(1-exp(I*(b*x+a)))*x^3+6*I*d^3/b^4*c*polylog(2,-exp(2*I*(b*x+a)))+1
2*d^4/b^5*a^2*ln(exp(I*(b*x+a)))+12*d^2/b^3*c^2*ln(exp(I*(b*x+a)))+4/b^4*c*
d^3*ln(1-exp(I*(b*x+a)))*a^3-12*I*d^4/b^4*a^2*x+12*I*d^3/b^2*c*x^2+6*I*d^4/
b^4*polylog(2,-exp(2*I*(b*x+a)))*x+6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+a)
))*x^2-6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-1/b*d^4*ln(exp(2*I*(b*x+a
))+1)*x^4+12*I*d^3/b^4*c*a^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+6*I/b^2*pol
ylog(2,-exp(2*I*(b*x+a)))*c^2*d^2*x-1/b*c^4*ln(exp(2*I*(b*x+a))+1)+12/b^3*d
^4*polylog(3,exp(I*(b*x+a)))*x^2+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))+1
2/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,-exp(I*(b*x+a
)))*x^2+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2+1/b*d^4*ln(exp(I*(b*x+a))+1)*x
^4-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+
4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+24/b^3*
c*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/
b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x-24*d^3/b^4*c*a*ln(exp(I*(b*x+a)))+1/b
^5*d^4*a^4*ln(exp(I*(b*x+a))-1)+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)-4/b^
```

```
4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)-4*I/b^2
*c^3*d*polylog(2,exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+2
4*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)
))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-e
xp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+24*I/b^4*c*d^3*po
lylog(4,-exp(I*(b*x+a)))-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polyl
og(5,exp(I*(b*x+a)))/b^5+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*
I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*
I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*
x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))
-6*I*c*d^3*x^2-6*I*c^2*d^2*x-2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))+1)^2-12*I/b^2
*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x
+a)))*x-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^4*ln(exp(I*(b*x
+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-3/b^3*c^2*d^2*polylog(3,-exp(2*I*(b*x+
a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2-12*I/b^2*c*d^3*polylog(2,ex
p(I*(b*x+a)))*x^2
```

Maxima [B] time = 23.4574, size = 11840, normalized size = 29.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^4*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 4*a*c^3*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x +
 a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/(sin(b*x + a)^2 - 1)
 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + a^4*d^4*(1/(sin(b*x
 + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^4 + 2*(24*b
^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*a^3*d^4 + (12*(b*x + a)^4
*d^4 + 36*b^2*c^2*d^2 - 72*a*b*c*d^3 + 36*a^2*d^4 + 32*(b*c*d^3 - a*d^4)*(b
*x + a)^3 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 24
*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x
 + a) + 4*(3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8
*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d
^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a
^3 + 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 8*(3*(b*x + a)^4*d^4 + 9*b^2*c
^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b
^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*cos(2*b*x +
```

```
2*a) + (12*I*(b*x + a)^4*d^4 + 36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2
*d^4 + (32*I*b*c*d^3 - 32*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a
*b*c*d^3 + (36*I*a^2 + 36*I)*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^
2*c^2*d^2 + (72*I*a^2 + 72*I)*b*c*d^3 + (-24*I*a^3 - 72*I*a)*d^4)*(b*x + a)
)*sin(4*b*x + 4*a) + (24*I*(b*x + a)^4*d^4 + 72*I*b^2*c^2*d^2 - 144*I*a*b*c
*d^3 + 72*I*a^2*d^4 + (64*I*b*c*d^3 - 64*I*a*d^4)*(b*x + a)^3 + (72*I*b^2*c
^2*d^2 - 144*I*a*b*c*d^3 + (72*I*a^2 + 72*I)*d^4)*(b*x + a)^2 + (48*I*b^3*c
^3*d - 144*I*a*b^2*c^2*d^2 + (144*I*a^2 + 144*I)*b*c*d^3 + (-48*I*a^3 - 144
*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) - (6*(b*x + a)^4*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 36*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 24*(b^3*c^3*d - 3*a*b^2*
c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + 6*((b*x + a)^4*d^4 + 4*(b*c*
d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a
)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*
cos(4*b*x + 4*a) + 12*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 +
6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^
2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b
*x + a)^4*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d
^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a
*b^2*c^2*d^2 - 72*I*a^2*b*c*d^3 + 24*I*a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a)
 - (-12*I*(b*x + a)^4*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a)^3 + (-72
*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 - 72*I*a^2*d^4)*(b*x + a)^2 + (-48*I*b^3*c
^3*d + 144*I*a*b^2*c^2*d^2 - 144*I*a^2*b*c*d^3 + 48*I*a^3*d^4)*(b*x + a))*s
in(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*(b*x + a)^4*d
^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4)*(b*x + a)^2 + 24*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a) + 6*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a
)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3
*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 -
24*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4
)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 2
4*I*a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*(b*x + a)^4*d^4 + (48*I*b*
c*d^3 - 48*I*a*d^4)*(b*x + a)^3 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*
I*a^2*d^4)*(b*x + a)^2 + (48*I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I*a^2*
b*c*d^3 - 48*I*a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a),
-cos(b*x + a) + 1) - 24*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2
+ 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (12
*I*(b*x + a)^4*d^4 + 24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*
a^3*d^4 + (48*I*b*c*d^3 - 24*(2*I*a + 1)*d^4)*(b*x + a)^3 + (72*I*b^2*c^2*d
^2 - 72*(2*I*a + 1)*b*c*d^3 + (72*I*a^2 + 72*a)*d^4)*(b*x + a)^2 + (48*I*b^
3*c^3*d - 72*(2*I*a + 1)*b^2*c^2*d^2 + (144*I*a^2 + 144*a)*b*c*d^3 + (-48*I
*a^3 - 72*a^2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (12*b^3*c^3*d - 36*a*b^2*
```

```
c^2*d^2 + 24*(b*x + a)^3*d^4 + 36*(a^2 + 1)*b*c*d^3 - 12*(a^3 + 3*a)*d^4 +
48*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1
)*d^4)*(b*x + a) + 12*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*
(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 24
*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(a^2 + 1)*b*c*d^3 - (
a^3 + 3*a)*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c
*d^3 + (a^2 + 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-12*I*b^3*c^3*d + 36*I
*a*b^2*c^2*d^2 - 24*I*(b*x + a)^3*d^4 + (-36*I*a^2 - 36*I)*b*c*d^3 + (12*I*
a^3 + 36*I*a)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a)^2 + (-36*I*b^2*c
^2*d^2 + 72*I*a*b*c*d^3 + (-36*I*a^2 - 36*I)*d^4)*(b*x + a))*sin(4*b*x + 4*
a) - (-24*I*b^3*c^3*d + 72*I*a*b^2*c^2*d^2 - 48*I*(b*x + a)^3*d^4 + (-72*I*
a^2 - 72*I)*b*c*d^3 + (24*I*a^3 + 72*I*a)*d^4 + (-96*I*b*c*d^3 + 96*I*a*d^4
)*(b*x + a)^2 + (-72*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 + (-72*I*a^2 - 72*I)*d
^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (24*b^3*c^3*
d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 + 24*(b*x + a)^3*d^4 - 24*a^3*d^4 + 7
2*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
(b*x + a) + 24*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d
^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d
^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 48*(b^3*c^3*d - 3*a*b^2*c^2*d^2
 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a
)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) +
 (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3
*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c
^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (48*
I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I*a^2*b*c*d^3 + 48*I*(b*x + a)^3*d^
4 - 48*I*a^3*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a)^2 + (144*I*b^2*c
^2*d^2 - 288*I*a*b*c*d^3 + 144*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilo
g(-e^(I*b*x + I*a)) + (24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 + 2
4*(b*x + a)^3*d^4 - 24*a^3*d^4 + 72*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 72*(b^2
*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 24*(b^3*c^3*d - 3*a*b^2*c^2*d
^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x +
 a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a)
 + 48*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*
d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*
d^4)*(b*x + a))*cos(2*b*x + 2*a) + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 7
2*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*
I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*
(b*x + a))*sin(4*b*x + 4*a) + (48*I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I
*a^2*b*c*d^3 + 48*I*(b*x + a)^3*d^4 - 48*I*a^3*d^4 + (144*I*b*c*d^3 - 144*I
*a*d^4)*(b*x + a)^2 + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*a^2*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-6*I*(b*x + a)^4*d^
4 - 18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 18*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*
I*a*d^4)*(b*x + a)^3 + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 + (-18*I*a^2 - 1
8*I)*d^4)*(b*x + a)^2 + (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 + (-36*I*a^2
```

- 36*I)*b*c*d^3 + (12*I*a^3 + 36*I*a)*d^4)*(b*x + a) + (-6*I*(b*x + a)^4*d^
4 - 18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 18*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*
I*a*d^4)*(b*x + a)^3 + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 + (-18*I*a^2 - 1
8*I)*d^4)*(b*x + a)^2 + (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 + (-36*I*a^2
- 36*I)*b*c*d^3 + (12*I*a^3 + 36*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-
12*I*(b*x + a)^4*d^4 - 36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4 + (
-32*I*b*c*d^3 + 32*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d
^3 + (-36*I*a^2 - 36*I)*d^4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a*b^2*c^
2*d^2 + (-72*I*a^2 - 72*I)*b*c*d^3 + (24*I*a^3 + 72*I*a)*d^4)*(b*x + a))*co
s(2*b*x + 2*a) + 2*(3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^
2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a
^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c
*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + 4*(3*(b*x + a)^4*d^4
+ 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^
3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*
d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*sin
(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3
+ (18*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^
3*c^3*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a) +
(3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 + (18*I*b^2
*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^3*c^3*d - 3
6*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a))*cos(4*b*x +
4*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^3 + (3
6*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^
3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*cos(
2*b*x + 2*a) - 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2
*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^
4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x +
a)^3 + (18*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (1
2*I*b^3*c^3*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x
+ a) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 + (18
*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^3*c^3
*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a))*cos(4
*b*x + 4*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^
3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*
b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a)
)*cos(2*b*x + 2*a) - 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 +
6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b
^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x
+ a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^

```
3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*cos(b*x + a) + 1) + (18*I*d^4*cos(4*b*x + 4*a) + 36*I*d^4*cos(2*b*x + 2
*a) - 18*d^4*sin(4*b*x + 4*a) - 36*d^4*sin(2*b*x + 2*a) + 18*I*d^4)*polylog
(5, -e^(2*I*b*x + 2*I*a)) + (-144*I*d^4*cos(4*b*x + 4*a) - 288*I*d^4*cos(2*
b*x + 2*a) + 144*d^4*sin(4*b*x + 4*a) + 288*d^4*sin(2*b*x + 2*a) - 144*I*d^
4)*polylog(5, -e^(I*b*x + I*a)) + (-144*I*d^4*cos(4*b*x + 4*a) - 288*I*d^4*
cos(2*b*x + 2*a) + 144*d^4*sin(4*b*x + 4*a) + 288*d^4*sin(2*b*x + 2*a) - 14
4*I*d^4)*polylog(5, e^(I*b*x + I*a)) + (24*b*c*d^3 + 36*(b*x + a)*d^4 - 24*
a*d^4 + 12*(2*b*c*d^3 + 3*(b*x + a)*d^4 - 2*a*d^4)*cos(4*b*x + 4*a) + 24*(2
*b*c*d^3 + 3*(b*x + a)*d^4 - 2*a*d^4)*cos(2*b*x + 2*a) + (24*I*b*c*d^3 + 36
*I*(b*x + a)*d^4 - 24*I*a*d^4)*sin(4*b*x + 4*a) + (48*I*b*c*d^3 + 72*I*(b*x
 + a)*d^4 - 48*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(2*I*b*x + 2*I*a))
- (144*b*c*d^3 + 144*(b*x + a)*d^4 - 144*a*d^4 + 144*(b*c*d^3 + (b*x + a)*d
^4 - a*d^4)*cos(4*b*x + 4*a) + 288*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*
b*x + 2*a) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*sin(4*b*x
 + 4*a) - (-288*I*b*c*d^3 - 288*I*(b*x + a)*d^4 + 288*I*a*d^4)*sin(2*b*x +
2*a))*polylog(4, -e^(I*b*x + I*a)) - (144*b*c*d^3 + 144*(b*x + a)*d^4 - 144
*a*d^4 + 144*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) + 288*(b*c*
d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-144*I*b*c*d^3 - 144*I*(b*
x + a)*d^4 + 144*I*a*d^4)*sin(4*b*x + 4*a) - (-288*I*b*c*d^3 - 288*I*(b*x +
 a)*d^4 + 288*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) + (-18
*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 36*I*(b*x + a)^2*d^4 + (-18*I*a^2 - 18*I)
*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-18*I*b^2*c^2*d^2 + 36*I*a
*b*c*d^3 - 36*I*(b*x + a)^2*d^4 + (-18*I*a^2 - 18*I)*d^4 + (-48*I*b*c*d^3 +
 48*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*
d^3 - 72*I*(b*x + a)^2*d^4 + (-36*I*a^2 - 36*I)*d^4 + (-96*I*b*c*d^3 + 96*I
*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 6*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b
*x + a)^2*d^4 + 3*(a^2 + 1)*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x
+ 4*a) + 12*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*(a^2 + 1)*
d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(2*I*b
*x + 2*I*a)) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 +
 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a) + (72*I*b^2*c^2*d^2
 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 -
 144*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (144*I*b^2*c^2*d^2 - 288*I*a*b*
c*d^3 + 144*I*(b*x + a)^2*d^4 + 144*I*a^2*d^4 + (288*I*b*c*d^3 - 288*I*a*d^
4)*(b*x + a))*cos(2*b*x + 2*a) - 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^
2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 144*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(
b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) + (72*I*b^2*c^2*d^
2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3
- 144*I*a*d^4)*(b*x + a) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x
+ a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a))*cos(4*
b*x + 4*a) + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*(b*x + a)^2*d^4 +
 144*I*a^2*d^4 + (288*I*b*c*d^3 - 288*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a)
```

```
- 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 -
a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 144*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x
+ a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*pol
ylog(3, e^(I*b*x + I*a)) + (-24*I*(b*x + a)^3*d^4 + (-72*I*b*c*d^3 + 72*I*a
*d^4)*(b*x + a)^2 + (-72*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 - 72*I*a^2*d^4)*(b
*x + a))*sin(4*b*x + 4*a) - (12*(b*x + a)^4*d^4 - 24*I*b^3*c^3*d + 72*I*a*b
^2*c^2*d^2 - 72*I*a^2*b*c*d^3 + 24*I*a^3*d^4 + (48*b*c*d^3 - (48*a - 24*I)*
d^4)*(b*x + a)^3 + (72*b^2*c^2*d^2 - (144*a - 72*I)*b*c*d^3 + 72*(a^2 - I*a
)*d^4)*(b*x + a)^2 + (48*b^3*c^3*d - (144*a - 72*I)*b^2*c^2*d^2 + 144*(a^2
- I*a)*b*c*d^3 - 24*(2*a^3 - 3*I*a^2)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/(-6
*I*b^4*cos(4*b*x + 4*a) - 12*I*b^4*cos(2*b*x + 2*a) + 6*b^4*sin(4*b*x + 4*a
) + 12*b^4*sin(2*b*x + 2*a) - 6*I*b^4))/b
```

Fricas [C] time = 1.96768, size = 8008, normalized size = 20.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 - 24*d^4*cos(b*x + a)^2*polylog(5, cos(b*x + a) + I*sin(b*x + a)) - 2
4*d^4*cos(b*x + a)^2*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*cos
(b*x + a)^2*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*cos(b*x + a)
^2*polylog(5, I*cos(b*x + a) - sin(b*x + a)) + 24*d^4*cos(b*x + a)^2*polylo
g(5, -I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*cos(b*x + a)^2*polylog(5, -I*
cos(b*x + a) - sin(b*x + a)) - 24*d^4*cos(b*x + a)^2*polylog(5, -cos(b*x +
a) + I*sin(b*x + a)) - 24*d^4*cos(b*x + a)^2*polylog(5, -cos(b*x + a) - I*s
in(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x
- 4*I*b^3*c^3*d)*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + (4*I
*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*cos
(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I
*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d - 12*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 + b*d^4)*
x)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 +
 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d + 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 + b*
d^4)*x)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (4*I*b^3*d^4*
x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d + 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2
 + b*d^4)*x)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-4*I*b
^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d - 12*I*b*c*d^3 - 12*I*(b^3*
c^2*d^2 + b*d^4)*x)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
(4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)
```

```
*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-4*I*b^3*d^4*x^3 -
 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*cos(b*x + a)^2*di
log(-cos(b*x + a) - I*sin(b*x + a)) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^
4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x + a)^2*log(cos(b*x + a) +
I*sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 -
4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(cos(b*x + a)
+ I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2
+ 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a)
 + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*
b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x +
a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^
2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2*d^4)*x
^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin
(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2
*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2
*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a
) - sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*
a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^
2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(-I*cos
(b*x + a) + sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^
3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^
4*c^2*d^2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*lo
g(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) +
 1/2*I*sin(b*x + a) + 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(
b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^
4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*co
s(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)
*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*
b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b
^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) - I*
sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*
(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) -
 I*sin(b*x + a) + I) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)^2*polylog
(4, cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x
 + a)^2*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*
c*d^3)*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + (-24*I*b*
d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x +
a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2*polylog(4, -I*cos(b*x +
 a) + sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)^2*polylog(
4, -I*cos(b*x + a) - sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x
 + a)^2*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b
*c*d^3)*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2
```

```
*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*polylog(3, cos(b*x +
 a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(
b*x + a)^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) +
 sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b
*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b
*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(
b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*p
olylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^
2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*cos(b*x + a)*sin(b*x + a))/(b^5*cos(b*x +
a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^4 \csc(bx+a)\sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^3, x)
```

3.311 $\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=325

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c+$$

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh[
E^((2*I)*(a + b*x))])/b - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^
3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/2)*d*(c
+ d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*P
olyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)
*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(
2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*
d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(
2*b^2) + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.640141, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 2279, 2391, 32}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```


```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh[
E^((2*I)*(a + b*x))])/b - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^
3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/2)*d*(c
+ d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*P
olyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)
*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(
2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*
d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(
2*b^2) + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```


Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^3\csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^3\log(\tan(a+bx))}{b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-(3d)\int (c+dx)^2\left(\frac{\log(\mathrm{ta}\right.$$

$$=\frac{(c+dx)^3\log(\tan(a+bx))}{b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-(3d)\int\frac{(c+dx)^2\left(2\log(\right.}{}$$

$$=\frac{(c+dx)^3\log(\tan(a+bx))}{b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{(3d)\int (c+dx)^2\left(2\log(\mathrm{t}\right.}{}$$

$$=\frac{(c+dx)^3\log(\tan(a+bx))}{b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{(3d)\int\left(2(c+dx)^2\log(\mathrm{t}\right.}{}$$

$$=\frac{(c+dx)^3\log(\tan(a+bx))}{b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{(3d)\int (c+dx)^2\tan^2(a}{2b}$$

$$=-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}+\frac{\int 2b(c+dx)^3\csc(2a+2}{b}$$

$$=\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}+$$

$$=\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{3d^2(c+dx)\log\left(1+\right.}{b^3}$$

$$=\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{3d^2(c+dx)\log\left(1+\right.}{b^3}$$

$$=\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{3d^2(c+dx)\log\left(1+\right.}{b^3}$$

$$=\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{3d^2(c+dx)\log\left(1+\right.}{b^3}$$

$$=\frac{3id(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{2b}-\frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{3d^2(c+dx)\log\left(1+\right.}{b^3}$$

Mathematica [B] time = 6.92562, size = 1486, normalized size = 4.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*
a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[
1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a
 + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((
```

```
2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*
x))]))/E^((2*I)*a)))/(2*b^3) - (d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) +
 (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(
1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(
b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a
+ b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2
*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((
-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(4*b^4)
+ (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)*
c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*
x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(
1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) -
 (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log
[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*PolyLog[2,
-E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))] - Poly
Log[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + ((c + d*x)^3*Se
c[a + b*x]^2)/(2*b) - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[
b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Cos[a]*Lo
g[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]
^2)) + (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*
Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan
[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*
b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))]
+ Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*Po
lyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2
*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*((b^2*x^2)/E^(I*
ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a
]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[
a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*Sec[a]*Sec[a + b*x]*(
c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (3*c^2*d
*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[
a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*
I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b
*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan
[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.4, size = 1115, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] 3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b*c
^3*ln(exp(2*I*(b*x+a))+1)-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^
3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*
x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*
x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2
*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))+1)^2-3
*d^2/b^3*c*ln(exp(2*I*(b*x+a))+1)-3*d^3/b^3*ln(exp(2*I*(b*x+a))+1)*x+3*I*d^
3/b^2*x^2+3*I*d^3/b^4*a^2+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4-6*I/b^
2*polylog(2,-exp(I*(b*x+a)))*c*d^2*x-6*I/b^2*d^2*c*polylog(2,exp(I*(b*x+a))
)*x-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,-exp(I*(
b*x+a)))*x^2+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+3/2*I/b^2*c^2*d*polylog(
2,-exp(2*I*(b*x+a)))-3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*
I*(b*x+a))+1)*x^2-1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3+3/2*I/b^2*d^3*polylog(
2,-exp(2*I*(b*x+a)))*x^2-6/b^4*d^3*a*ln(exp(I*(b*x+a)))+6/b^3*d^2*c*ln(exp(
I*(b*x+a)))+6*I*d^3/b^3*a*x+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1
-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/b^3*c*d^2*a^2*ln(ex
p(I*(b*x+a))-1)-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-3/b^3*c*d^2*a^2*ln(1-exp
(I*(b*x+a)))+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a))
)*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)
))-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3*I/b^2*c*d^2*polylog(2,-exp(
2*I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-
exp(I*(b*x+a)))*x+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*c*d^2*polylog
(3,-exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)-3*I/b^2*d^3*polylog(
2,exp(I*(b*x+a)))*x^2-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+1/b*c^3*ln(ex
p(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)
```

Maxima [B] time = 7.39027, size = 6639, normalized size = 20.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 3*a*c^2*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a
)^2 - 1) - log(sin(b*x + a)^2))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + 2*(18*b^2*c^2*d - 36*a*b*
c*d^2 + 18*a^2*d^3 + (8*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d
```

```
^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x
 + a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*
x + 4*a) + 4*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2
*b*x + 2*a) + (8*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (18*I*b*c*
d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^
2 + 18*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (16*I*(b*x + a)^3*d^3 + 36*I*b
*c*d^2 - 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c
^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 36*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (6*(b*x + a)^3*d^3 + 18*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x
 + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a)^3*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I
*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3)*(b*
x + a))*sin(4*b*x + 4*a) - (-12*I*(b*x + a)^3*d^3 + (-36*I*b*c*d^2 + 36*I*a
*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*a^2*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*(b*x +
 a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(
(b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b
*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*
a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*(b*x + a)^3*d^3 + (36*I*b*c*d^
2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*a^2*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) -
 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (1
2*I*(b*x + a)^3*d^3 + 18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 + (36*I*b*c*
d^2 - 18*(2*I*a + 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 36*(2*I*a + 1)*b*
c*d^2 + (36*I*a^2 + 36*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (9*b^2*c^2*d -
 18*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 9*(a^2 + 1)*d^3 + 18*(b*c*d^2 - a*d^3)
*(b*x + a) + 3*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)
*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6
*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x
 + a))*cos(2*b*x + 2*a) - (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 12*I*(b*x + a)
^2*d^3 + (-9*I*a^2 - 9*I)*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a))*sin
(4*b*x + 4*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 +
(-18*I*a^2 - 18*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(2*b*x
+ 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*
x + a)^2*d^3 + 18*a^2*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(4*b*x + 4*a) + 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
```

+ 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I*
a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^
3)*(b*x + a))*sin(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b
*x + a)^2*d^3 + 36*I*a^2*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b
*x + a)^2*d^3 + 18*a^2*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*cos(4*b*x + 4*a) + 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I
*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d
^3)*(b*x + a))*sin(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(
b*x + a)^2*d^3 + 36*I*a^2*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-4*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2
+ 9*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18
*I*a*b*c*d^2 + (-9*I*a^2 - 9*I)*d^3)*(b*x + a) + (-4*I*(b*x + a)^3*d^3 - 9*
I*b*c*d^2 + 9*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*
c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 9*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a)
+ (-8*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I
*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 18*I
)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d
^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 +
1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*
a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
 + 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (3*I*(b*x + a)^3*d^3 + (9*I*b*c*d^2 -
9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*
x + a) + (3*I*(b*x + a)^3*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*
I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
6*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c
^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x
 + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3
I(b*x + a)^3*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d
 - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + (9*I*b*
c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*
d^3)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 1
8*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(
b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a)
- 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 - 2*cos(b*x + a) + 1) + (6*d^3*cos(4*b*x + 4*a) + 12*d^3*cos(2*b*
x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) + 12*I*d^3*sin(2*b*x + 2*a) + 6*d^3)*po

```
lylog(4, -e^(2*I*b*x + 2*I*a)) - (36*d^3*cos(4*b*x + 4*a) + 72*d^3*cos(2*b*
x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) + 72*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*
polylog(4, -e^(I*b*x + I*a)) - (36*d^3*cos(4*b*x + 4*a) + 72*d^3*cos(2*b*x
+ 2*a) + 36*I*d^3*sin(4*b*x + 4*a) + 72*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*po
lylog(4, e^(I*b*x + I*a)) + (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3
+ (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3)*cos(4*b*x + 4*a) + (-18*I
*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3)*cos(2*b*x + 2*a) + 3*(3*b*c*d^2
 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(4*b*x + 4*a) + 6*(3*b*c*d^2 + 4*(b*x + a)
*d^3 - 3*a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + (36*I*
b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*
d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 7
2*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*
x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3
, -e^(I*b*x + I*a)) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36
*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (72*I*b*c*
d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*
x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)
*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (-18*I*(b*x + a)^2*d^3 + (
-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (12*(b*x + a)^3*d
^3 - 18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3 + (36*b*c*d^2 - (36*a -
 18*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a - 36*I)*b*c*d^2 + 36*(a^2 -
 I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^3*cos(4*b*x + 4*a) - 12*I*b
^3*cos(2*b*x + 2*a) + 6*b^3*sin(4*b*x + 4*a) + 12*b^3*sin(2*b*x + 2*a) - 6*
I*b^3))/b
```

Fricas [C] time = 1.41763, size = 5605, normalized size = 17.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 6*I*d^3*cos(
b*x + a)^2*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)
^2*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polyl
og(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, I*
cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x
 + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) -
 sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) + I*sin(b*
x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b*x + a))
 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)^2*dilo
```

g(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I
*b^2*c^2*d)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2
*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2*dilog(
I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b
^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (
3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2
*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*
x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x
 + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x + a)^2
*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*
x - 3*I*b^2*c^2*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2*log
(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1
)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x
 + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d +
 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I
*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2*log(I*
cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x
 + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d
^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(
b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1
) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2*log(
-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b
*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*
x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*
d^3)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b
^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(-cos(b
*x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylo
g(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*
polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x +
 a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos
(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^
2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x +
 b*c*d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*
d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a))
+ 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) - I*sin(b*x

```
+ a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x +
a))/(b^4*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \csc(bx+a)\sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^3, x)
```

3.312 $\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=201

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+b}\right.}{2b^3}$$

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/
b - (d^2*Log[Cos[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Po
lyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2
)/(2*b)
```

Rubi [A] time = 0.413906, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 2282, 6589, 3720, 3475}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+b}\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```


```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/
b - (d^2*Log[Cos[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Po
lyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2
)/(2*b)
```


Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2\csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - (2d)\int (c+dx)\Big(\frac{\log(\mathrm{ta}}{}$$

$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - (2d)\int \frac{(c+dx)\big(2\log(}{}$$

$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx)\big(2\log(\tan}{}$$

$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int \big(2(c+dx)\log(\tan}{}$$

$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx)\tan^2(a+b}{b}$$

$$= -\frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^2\csc(2a+2b}{b}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(}{2b}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{d(c+dx)}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} + \frac{id(c+dx}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} + \frac{id(c+dx}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} + \frac{id(c+dx}{}$$

Mathematica [B] time = 6.74202, size = 875, normalized size = 4.35

$$-\frac{\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))c^2}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{\csc(a)(\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a)}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -(d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a)
)*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1
+ E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a +
```

```
 b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]]))/E^((2*I)*a) - (6*(-1 + E^((2*
I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x)
)]))/E^((2*I)*a)))/(6*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/
3 - ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*
I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
 - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*
E^(I*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (c^2*Sec[a]*(Cos[a]*Log[Cos
[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) -
(d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(
b^3*(Cos[a]^2 + Sin[a]^2)) + (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[
a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^
2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*
Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x -
ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTa
n[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot
[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a
+ b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/b^2 - (c*d*Csc[a]*Sec[a]*(b^2*E^
(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a
]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2
]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Maple [B] time = 0.339, size = 614, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] -1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^
2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d
^2*ln(1-exp(I*(b*x+a)))*x^2-1/b*c^2*ln(exp(2*I*(b*x+a))+1)+I/b^2*c*d*polylo
g(2,-exp(2*I*(b*x+a)))-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*d^2*
polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-2*I/b^2*
c*d*polylog(2,-exp(I*(b*x+a)))+2/b^3*d^2*ln(exp(I*(b*x+a)))+2/b*c*d*ln(exp(
I*(b*x+a))+1)*x+1/b*c^2*ln(exp(I*(b*x+a))+1)+1/b*c^2*ln(exp(I*(b*x+a))-1)+2
*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-1/b
^3*d^2*ln(exp(2*I*(b*x+a))+1)+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-2/b*
c*d*ln(exp(2*I*(b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-
exp(I*(b*x+a)))*a+2*(b*d^2*x^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))+2*
```

```
b*c*d*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^
2*x-I*d*c)/b^2/(exp(2*I*(b*x+a))+1)^2+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-2/
b^2*c*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 2.8488, size = 3297, normalized size = 16.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 2*a*c*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(s
in(b*x + a)^2))/b + a^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 -
1) - log(sin(b*x + a)^2))/b^2 - 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(
b*x + a)*d^2*sin(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 - (2*(b*x + a)^2*d^2 + 4*
(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a) + d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a) + d^2)*cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I
*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I
*b*c*d - 8*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b
*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a
) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-
2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a)
- (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(2*b*x + 2
*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*
d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*co
s(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*
x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(4*
b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (4*I*(b*x + a)^2*d
^2 + 4*b*c*d - 4*a*d^2 + (8*I*b*c*d - 4*(2*I*a + 1)*d^2)*(b*x + a))*cos(2*b
*x + 2*a) + (2*b*c*d + 2*(b*x + a)*d^2 - 2*a*d^2 + 2*(b*c*d + (b*x + a)*d^2
 - a*d^2)*cos(4*b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x +
2*a) - (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*sin(4*b*x + 4*a) - (-4*
I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*
x + 2*I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d
^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
+ 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8*
I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x
```

```
+ I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 -
 a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*
a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8*I*b*
c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a
)) - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - I*d^2 + (-I
*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - I*d^2)*cos(4*b*x +
4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) - 2*I*d^2
)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*
sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*
sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + (I*
(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*cos(4*b*x + 4*a) + (2*
I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (
(b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x +
 a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d
 - 2*I*a*d^2)*(b*x + a) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x
 + a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b
*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))
*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-
I*d^2*cos(4*b*x + 4*a) - 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) +
2*d^2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) - (4*I*d^2
*cos(4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) - 8*d
^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -e^(I*b*x + I*a)) - (4*I*d^2*cos(
4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) - 8*d^2*si
n(2*b*x + 2*a) + 4*I*d^2)*polylog(3, e^(I*b*x + I*a)) + (4*(b*x + a)^2*d^2
- 4*I*b*c*d + 4*I*a*d^2 + (8*b*c*d - (8*a - 4*I)*d^2)*(b*x + a))*sin(2*b*x
+ 2*a))/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4
*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2))/b
```

Fricas [C] time = 0.963077, size = 3629, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x +
a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin
(b*x + a)) - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
```

```
 - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*c
os(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x +
a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*poly
log(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, -c
os(b*x + a) - I*sin(b*x + a)) + b^2*c^2 + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*
x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*c
os(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*
c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2
*I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*
x + 2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I
*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))
+ (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x
+ a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*s
in(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos
(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos
(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^
2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(
b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*
cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
 a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*
x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2*log(-1/2*cos
(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*
c*d - a^2*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^
2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin
(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x
+ a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^
2 + 1)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*d
^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc (bx+a) \sec (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^3, x)
```

3.313 $\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=139

$$\frac{id\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{c\tan^2(a+bx)}{2b}+\frac{c\log(\tan(a+bx))}{b}+\frac{dx\tan^2(}{2l}$$

```
[Out] (d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*a + (2*I)*b*x)])/b + (c*Log[Tan[a + b
*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog
[2, E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(2*b^2) + (c*Tan[a + b*x]^
2)/(2*b) + (d*x*Tan[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.135266, antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2620, 14, 4420, 2548, 12, 4183, 2279, 2391, 3473, 8}

$$\frac{id\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*x*Log[Tan[a + b*x
]])/b + ((c + d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a
+ b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a +
b*x])/(2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)\log(\tan(a+bx))}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}-d\int\left(\frac{\log(\tan(a+bx))}{b}+\right.$$

$$=\frac{(c+dx)\log(\tan(a+bx))}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}-\frac{d\int\tan^2(a+bx)\,dx}{2b}-\frac{d\int}{}$$

$$=-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)}{}$$

$$=\frac{dx}{2b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+}{}$$

$$=\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b}$$

$$=\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b}$$

$$=\frac{dx}{2b}-\frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b}$$

Mathematica [A] time = 0.565265, size = 212, normalized size = 1.53

$$\frac{d\left(\frac{1}{2}i\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)+\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)\right)}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (a*d*Log[Cos[a + b*x]])/b^2 - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a +
b*x)^2 - (a + b*x)*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I
)*(a + b*x))]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((
a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (d*x*Sec[a + b*x]^2)/
(2*b) - (c*(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2))/(2
*b) - (d*Tan[a + b*x])/(2*b^2)
```

Maple [B] time = 0.158, size = 270, normalized size = 1.9

$$\frac{2\,bdx\mathrm{e}^{2\,i(bx+a)}+2\,bc\mathrm{e}^{2\,i(bx+a)}-id\mathrm{e}^{2\,i(bx+a)}-id}{b^2\left(\mathrm{e}^{2\,i(bx+a)}+1\right)^2}-\frac{c\ln\left(\mathrm{e}^{2\,i(bx+a)}+1\right)}{b}+\frac{c\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b}+\frac{d\ln\left(1-\mathrm{e}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))-I*d)/
b^2/(exp(2*I*(b*x+a))+1)^2-1/b*c*ln(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*x
+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(
1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(2*I*(b*x
+a))+1)*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+
1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 2.22434, size = 1397, normalized size = 10.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) + 4*(b*d*x + b*c)*cos
(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (4*I*b*d*x + 4*I*b
*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2*
b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) + 4*(b*d*x + b*c)*cos(2*b*
x + 2*a) - (-2*I*b*d*x - 2*I*b*c)*sin(4*b*x + 4*a) - (-4*I*b*d*x - 4*I*b*c)
*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(4*b
*x + 4*a) + 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) + 4*I*b*c*sin
(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(4*b*x + 4*a) + 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a) + 4
*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) + 2
*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*di
log(-e^(2*I*b*x + 2*I*a)) + (2*d*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) +
2*I*d*sin(4*b*x + 4*a) + 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*
a)) + (2*d*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a)
 + 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c
 + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x +
 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*
log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*
b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*c
```

```
os(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 2*(2*
b*d*x + 2*b*c - I*d)*sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4
*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) -
 2*I*b^2)
```

Fricas [B] time = 0.751633, size = 2072, normalized size = 14.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x
 + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(I*c
os(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin
(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d
*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x + a)^2*
dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-cos(b*x +
 a) - I*sin(b*x + a)) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) + I*s
in(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a)
+ 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) -
(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*
x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a
*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*
cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*cos(b*
x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*co
s(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a
*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*co
s(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x + a*d)*cos(b*
x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) + b*d*x - d*cos(b*x + a)*sin(b*x
 + a) + b*c)/(b^2*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\csc(bx+a)\sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^3, x)
```

3.314 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi [A] time = 0.188899, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
```

```
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 9.06757, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

Maple [A] time = 3.113, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)(\sec(bx+a))^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
(2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*
b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 + d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*si
n(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x
```

```
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/((d*x
+ c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x
+ a) + c), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x +
4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d
*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (d*cos(2
*b*x + 2*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*si
n(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x +
4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a))
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^{3}}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(a+bx\right)\sec^{3}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)\sec(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

3.315 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=24

$$\text{CannotIntegrate}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi [A] time = 0.208693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 6.70905, size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Maple [A] time = 4.859, size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)(\sec(bx+a))^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^3}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)\sec(bx+a)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)^3/(d*x + c)^2, x)
```

3.316 $\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\csc^2(a+bx)\sec^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

Rubi [A] time = 0.232894, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

Mathematica [A] time = 11.9623, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

Maple [A] time = 0.215, size = 0, normalized size = 0.

$$\int (dx+c)^m (\csc(bx+a))^2 (\sec(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc (bx+a)^2 \sec (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

3.317 $\int (c+dx)^3 \csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=486

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{9d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{9d^2(c+$$

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - ((3*I)*(c + d*x)^3*Arc
Tan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 -
(3*(c + d*x)^3*Csc[a + b*x])/(2*b) + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*
(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + (((9*
I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*Poly
Log[2, I*E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I
*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 -
 (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 - (9*d^2*(c + d*x)*PolyLog[3, (-I
)*E^(I*(a + b*x))])/b^3 + (9*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b
^3 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((9*I)*d^3*PolyLog[4, (-I)*E
^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d
*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]
^2)/(2*b)
```

Rubi [A] time = 1.20635, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {2621, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4181, 2531, 6609, 2282, 6589, 4183, 2622, 6741, 2279, 2391}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{9d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{9d^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```


```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - ((3*I)*(c + d*x)^3*Arc
Tan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 -
(3*(c + d*x)^3*Csc[a + b*x])/(2*b) + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*
(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + (((9*
I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*Poly
Log[2, I*E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I
*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 -
 (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 - (9*d^2*(c + d*x)*PolyLog[3, (-I
)*E^(I*(a + b*x))])/b^3 + (9*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b
```

```
^3 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((9*I)*d^3*PolyLog[4, (-I)*E
^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d
*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]
^2)/(2*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3\csc(a+}{2}$$

$$= \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3\csc(a+}{2}$$

$$= \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3\csc(a+}{2}$$

$$= \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3\csc(a+}{2}$$

$$= \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3\csc(a+}{2}$$

$$= \frac{3d(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b^2} + \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - 3(c+$$

$$= \frac{3d(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b^2} + \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - 3(c+$$

$$= -\frac{9d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b^2} - 3(c+d$$

$$= -\frac{9d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b^2} - 3(c+d$$

$$= -\frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + 3d(c+dx)^2\,\mathrm{ta}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+dx)^2\,\mathrm{ta}}{b}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+dx)^2\,\mathrm{ta}}{b}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\,\mathrm{ta}}{b}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\,\mathrm{ta}}{b}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\,\mathrm{ta}}{b}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\,\mathrm{ta}}{b}$$

Mathematica [A] time = 7.9131, size = 819, normalized size = 1.69

$$-\frac{\csc(a+bx)\left(bc^3+3b\cos(2a+2bx)c^3+3bdxc^2+9bdx\cos(2a+2bx)c^2+3d\sin(2a+2bx)c^2+3bd^2x^2c+9bd^2x^2\cos(2\right.}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c + d*x)^2*Log[1 + E^(I*(a +
b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3
, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x
))] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2))/b^2 - (3*((2*I)*b^3*c^3*ArcTan[
E^(I*(a + b*x))] + (4*I)*b*c*d^2*ArcTan[E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Lo
g[1 - I*E^(I*(a + b*x))] - 2*b*d^3*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d
^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))]
+ 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] + 2*b*d^3*x*Log[1 + I*E^(I*(a +
b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 + I
*E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, (-I)*E^(I*(a
 + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, I*E^(I*(a + b*x))] +
 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(
I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog
[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)
*d^3*PolyLog[4, I*E^(I*(a + b*x))]))/(2*b^4) - (Csc[a + b*x]*Sec[a + b*x]^2
*(b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 + b*d^3*x^3 + 3*b*c^3*Cos[2*a + 2*b*x
] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*Cos[2*a + 2*b*x] + 3*b*d^3
*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin[2*a + 2*b*
x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

Maple [B] time = 0.738, size = 1629, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] -6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+3
*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/
b^2*ln(1-exp(I*(b*x+a)))*x^2+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a-9/b^3*d^3*p
olylog(3,-I*exp(I*(b*x+a)))*x-3/b^3*d^3*ln(1+I*exp(I*(b*x+a)))*x-3/b^4*d^3*
```

```
ln(1+I*exp(I*(b*x+a)))*a-3/2/b^4*d^3*a^3*ln(1+I*exp(I*(b*x+a)))+9/b^3*d^3*p
olylog(3,I*exp(I*(b*x+a)))*x-3/2/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+3/2/b*d^3
*ln(1-I*exp(I*(b*x+a)))*x^3+3/b^2*c^2*d*ln(exp(I*(b*x+a))-1)-3/b^2*c^2*d*ln
(exp(I*(b*x+a))+1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)-3*I/b*c^3*arctan(exp(
I*(b*x+a)))-6*I/b^3*d^2*c*arctan(exp(I*(b*x+a)))-9*I*d^3*polylog(4,-I*exp(I
*(b*x+a)))/b^4+6/b^3*d^3*ln(1-exp(I*(b*x+a)))*a*x-6/b^3*d^2*c*a*ln(exp(I*(b
*x+a))-1)-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-6*I*d^3/b^3*polylog(2,exp(I*(b
*x+a)))*x-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-I/b^2/(exp(2*I*(b*x+a))+1
)^2/(exp(2*I*(b*x+a))-1)*(3*d^3*x^3*b*exp(5*I*(b*x+a))+9*c*d^2*x^2*b*exp(5*
I*(b*x+a))+9*c^2*d*x*b*exp(5*I*(b*x+a))+2*d^3*x^3*b*exp(3*I*(b*x+a))+3*c^3*
b*exp(5*I*(b*x+a))+6*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a)
)+6*c^2*d*x*b*exp(3*I*(b*x+a))+3*d^3*x^3*b*exp(I*(b*x+a))+6*I*c*d^2*x*exp(I
*(b*x+a))+2*c^3*b*exp(3*I*(b*x+a))+9*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*c^2*d*e
xp(5*I*(b*x+a))+9*c^2*d*x*b*exp(I*(b*x+a))+3*c^3*b*exp(I*(b*x+a))-6*I*c*d^2
*x*exp(5*I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))-
9/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))+9/b^3*d^2*c*polylog(3,I*exp(I*(b*x
+a)))+3/2/b^4*d^3*a^3*ln(1-I*exp(I*(b*x+a)))+3/b^3*d^3*ln(1-I*exp(I*(b*x+a)
))*x-9*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x+9*I/b^2*c*d^2*polylog(2,-I
*exp(I*(b*x+a)))*x-9*I/b^3*d^2*c*a^2*arctan(exp(I*(b*x+a)))+9*I/b^2*c^2*d*a
*arctan(exp(I*(b*x+a)))+6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x+3*I*d^3*po
lylog(2,-I*exp(I*(b*x+a)))/b^4+9/2/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-9/2/b
*d^2*c*ln(1+I*exp(I*(b*x+a)))*x^2-9/2/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x-9/2/
b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+9/2/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+9/2
/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a+9/2/b^3*d^2*c*a^2*ln(1+I*exp(I*(b*x+a))
)-9/2/b^3*d^2*c*a^2*ln(1-I*exp(I*(b*x+a)))+6*I/b^4*d^3*a*arctan(exp(I*(b*x+
a)))+9*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*I/b^4*d^3*polylog(2,exp(I*(b
*x+a)))*a-9/2*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+9/2*I/b^2*d^3*polyl
og(2,-I*exp(I*(b*x+a)))*x^2-6*I/b^4*d^3*a*dilog(exp(I*(b*x+a)))+9/2*I/b^2*c
^2*d*polylog(2,-I*exp(I*(b*x+a)))-9/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)
))+6*I/b^3*dilog(exp(I*(b*x+a))+1)*c*d^2-6*I/b^4*d^3*a*dilog(exp(I*(b*x+a))
+1)+6*I/b^3*d^2*c*dilog(exp(I*(b*x+a)))+3*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a
)))+6*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))*a
```

Maxima [B] time = 20.6871, size = 10843, normalized size = 22.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*sin(b*x + a
```

)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(
sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^
3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^3 - 4*((6*(b*x + a)^3*d^3 +
12*b*c*d^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d
- 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2
- 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 +
(3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*
d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^
2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*((b*x + a)^3*d^3 + 2*b
*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^3*d^3
- 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (
-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(6
*b*x + 6*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*
c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*
I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + 12*
I*b*c*d^2 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^
2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*
a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (6*(b*x + a)^3*d^3 + 12*b*c*d
^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 +
2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a
d^3 + 3(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^
2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*((b*x + a)^3*d^3 + 2*b*c*d^2 -
2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3
*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b
*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2
*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(6*b*x + 6*
a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 1
8*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 1
2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2
- 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d -
36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arcta
n2(cos(b*x + a), -sin(b*x + a) + 1) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*
x + a)^2*d^3 + 12*a^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
+ 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d - 2*a*b*c
*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x
+ 2*a) + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a
^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b

```
*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*b^2*c^2*d - 24*I*a
*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (1
2*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)*cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(4*b*x +
 4*a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(2*b*x + 2*a) - (12*I*b^2
*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*sin(6*b*x + 6*a) - (12*I*b^2*c^2*d
- 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*sin(4*b*x + 4*a) - (-12*I*b^2*c^2*d + 24*I
*a*b*c*d^2 - 12*I*a^2*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x
+ a) - 1) + (12*(b*x + a)^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*((b*x
 + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*((b*x +
a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a)^
2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*(b*x + a)^
2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-12*I*(
b*x + a)^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) +
 (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*(b*x + a)^3*d^3 - 12*
I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (36*b*c*d^2 - (36*a + 12*I)*d
^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a + 24*I)*b*c*d^2 + 12*(3*a^2 + 2*I*a
)*d^3)*(b*x + a))*cos(5*b*x + 5*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(3*b*x
 + 3*a) - (12*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*
d^3 + (36*b*c*d^2 - (36*a - 12*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a
- 24*I)*b*c*d^2 + 12*(3*a^2 - 2*I*a)*d^3)*(b*x + a))*cos(b*x + a) + (18*b^2
*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*(b*c*d^
2 - a*d^3)*(b*x + a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (
3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*(3*b^2
*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a
*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x +
 a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*
a) + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2
- 12*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-
18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I)
*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (18*I*b^2
*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 + (3
6*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I
*a)) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^
3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x
+ a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6
*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 +
6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6*a*b*c*
d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(2*b*x + 2*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3
+ (18*I*a^2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(6*b*x
+ 6*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^
```

2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (
-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I
)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*
e^(I*b*x + I*a)) - (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d^2
+ (b*x + a)*d^3 - a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*cos(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2
*a) - (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(6*b*x + 6*a) - (
24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(4*b*x + 4*a) - (-24*I*b
*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x
 + I*a)) + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d^2 + (b*x +
 a)*d^3 - a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*co
s(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-
24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(6*b*x + 6*a) + (-24*I*b
*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(4*b*x + 4*a) + (24*I*b*c*d^2
+ 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a))
 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (
-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6
I(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*c
os(6*b*x + 6*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6
*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-6*
I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d
^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x +
 6*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x +
 a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (6*I*b^2*c^2*d - 12
*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d
^3)*(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*
I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-6*
I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*b^2*c^2*d - 12*I*a*
b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 +
 a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + 6*(b^2*c^2*d -
 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*s
in(4*b*x + 4*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 +
2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 - 2*cos(b*x + a) + 1) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*
a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c
*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2
 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 1
8*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-3*I*(
b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x
+ a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a)

)*cos(4*b*x + 4*a) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*
b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a
^2 + 6*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2
 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 +
 (3*a^2 + 2)*d^3)*(b*x + a))*sin(6*b*x + 6*a) + 3*((b*x + a)^3*d^3 + 2*b*c*
d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^
2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 3*((b*x + a)^3*d^3 + 2*b
*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*sin(b*x + a) + 1) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 +
 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*
I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 6*I*
b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*
d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (3*
I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*
x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*
I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9
*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 2*b*c
*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d
^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(6*b*x + 6*a) - 3*((b*x + a)^3*d^3 + 2*
b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*
c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 3*((b*x + a)^3*d^3 +
 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a
*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (36*d^3*cos(6*b*x + 6*a) + 36*d^3
*cos(4*b*x + 4*a) - 36*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) + 3
6*I*d^3*sin(4*b*x + 4*a) - 36*I*d^3*sin(2*b*x + 2*a) - 36*d^3)*polylog(4, I
*e^(I*b*x + I*a)) - (36*d^3*cos(6*b*x + 6*a) + 36*d^3*cos(4*b*x + 4*a) - 36
*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) + 36*I*d^3*sin(4*b*x + 4*
a) - 36*I*d^3*sin(2*b*x + 2*a) - 36*d^3)*polylog(4, -I*e^(I*b*x + I*a)) + (
36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (-36*I*b*c*d^2 - 36*I*(b*x
 + a)*d^3 + 36*I*a*d^3)*cos(6*b*x + 6*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*
d^3 + 36*I*a*d^3)*cos(4*b*x + 4*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 3
6*I*a*d^3)*cos(2*b*x + 2*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(6*b*
x + 6*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 36*(b*c*
d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)
) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I
*(b*x + a)*d^3 - 36*I*a*d^3)*cos(6*b*x + 6*a) + (36*I*b*c*d^2 + 36*I*(b*x +
 a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^
3 + 36*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin
(6*b*x + 6*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 36*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x
 + I*a)) + (24*I*d^3*cos(6*b*x + 6*a) + 24*I*d^3*cos(4*b*x + 4*a) - 24*I*d^3
*cos(2*b*x + 2*a) - 24*d^3*sin(6*b*x + 6*a) - 24*d^3*sin(4*b*x + 4*a) + 24*

```
d^3*sin(2*b*x + 2*a) - 24*I*d^3)*polylog(3, -e^(I*b*x + I*a)) + (-24*I*d^3*
cos(6*b*x + 6*a) - 24*I*d^3*cos(4*b*x + 4*a) + 24*I*d^3*cos(2*b*x + 2*a) +
24*d^3*sin(6*b*x + 6*a) + 24*d^3*sin(4*b*x + 4*a) - 24*d^3*sin(2*b*x + 2*a)
 + 24*I*d^3)*polylog(3, e^(I*b*x + I*a)) + (-12*I*(b*x + a)^3*d^3 - 12*b^2*
c^2*d + 24*a*b*c*d^2 - 12*a^2*d^3 + (-36*I*b*c*d^2 - 12*(-3*I*a + 1)*d^3)*(
b*x + a)^2 + (-36*I*b^2*c^2*d - 24*(-3*I*a + 1)*b*c*d^2 + (-36*I*a^2 + 24*a
)*d^3)*(b*x + a))*sin(5*b*x + 5*a) + (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^2
 + 24*I*a*d^3)*(b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d
^3)*(b*x + a))*sin(3*b*x + 3*a) + (-12*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 2
4*a*b*c*d^2 + 12*a^2*d^3 + (-36*I*b*c*d^2 - 12*(-3*I*a - 1)*d^3)*(b*x + a)^
2 + (-36*I*b^2*c^2*d - 24*(-3*I*a - 1)*b*c*d^2 + (-36*I*a^2 - 24*a)*d^3)*(b
*x + a))*sin(b*x + a))/(-4*I*b^3*cos(6*b*x + 6*a) - 4*I*b^3*cos(4*b*x + 4*a
) + 4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(6*b*x + 6*a) + 4*b^3*sin(4*b*x + 4
*a) - 4*b^3*sin(2*b*x + 2*a) + 4*I*b^3))/b
```

Fricas [C] time = 1.45295, size = 5646, normalized size = 11.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 18*I*d^3*cos(b*x + a
)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 18*I*d^3*cos(b
*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*I*d^3
*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 1
8*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x +
 a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b
*x + a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin(b*x + a))*s
in(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a))*sin(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) - I*sin(b
*x + a))*sin(b*x + a) + 2*b^3*c^3 + (-12*I*b*d^3*x - 12*I*b*c*d^2)*cos(b*x
+ a)^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (12*I*b*d^3*x +
12*I*b*c*d^2)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + (-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b
*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-9*I*b^2*d^3
*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2*dilog(I*c
os(b*x + a) - sin(b*x + a))*sin(b*x + a) + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^
2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b
*x + a))*sin(b*x + a) + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d
 + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x +
```

```
a) + (-12*I*b*d^3*x - 12*I*b*c*d^2)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*
sin(b*x + a))*sin(b*x + a) + (12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a)^2*d
ilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 6*(b^2*d^3*x^2 + 2*b^2*
c*d^2*x + b^2*c^2*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*
sin(b*x + a) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*
a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a)
- 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)^2*log(cos(b*x +
a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2
 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*
x + a) + I)*sin(b*x + a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a
)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*
c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)*s
in(b*x + a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*co
s(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*
d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a)
 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x +
a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*s
in(b*x + a) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b
*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^3*c^3
 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*lo
g(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 6*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(
b*x + a) + 1)*sin(b*x + a) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d
^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I
)*sin(b*x + a) - 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos(b*x
 + a) + sin(b*x + a))*sin(b*x + a) + 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*
polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*(b*d^3*x + b*c*
d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a)
 + 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b
*x + a))*sin(b*x + a) - 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*cos(b*x + a)^2 - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b
*x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.318 $\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=341

$$\frac{3id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,\right.}{b^3}$$

```
[Out] ((-3*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d^2*x*ArcTanh[E^(I*(a +
 b*x))])/b^2 - (6*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (d^2*x*ArcTan
h[Cos[a + b*x]])/b^2 + (d*(c + d*x)*ArcTanh[Cos[a + b*x]])/b^2 + (d^2*ArcTa
nh[Sin[a + b*x]])/b^3 - (3*(c + d*x)^2*Csc[a + b*x])/(2*b) + ((2*I)*d^2*Pol
yLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 - ((3*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((
2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (3*d^2*PolyLog[3, (-I)*E^(I*(a
+ b*x))])/b^3 + (3*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Se
c[a + b*x])/b^2 + ((c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.64765, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {2621, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4181, 2531, 2282, 6589, 4183, 2279, 2391, 2622, 6271, 3770}

$$\frac{3id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```


```
[Out] ((-3*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d^2*x*ArcTanh[E^(I*(a +
 b*x))])/b^2 - (6*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (d^2*x*ArcTan
h[Cos[a + b*x]])/b^2 + (d*(c + d*x)*ArcTanh[Cos[a + b*x]])/b^2 + (d^2*ArcTa
nh[Sin[a + b*x]])/b^3 - (3*(c + d*x)^2*Csc[a + b*x])/(2*b) + ((2*I)*d^2*Pol
yLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 - ((3*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((
2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (3*d^2*PolyLog[3, (-I)*E^(I*(a
+ b*x))])/b^3 + (3*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Se
c[a + b*x])/b^2 + ((c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```


Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\,dx = \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} + \frac{(c+dx)^2 \csc(a+ \ldots}{2 \ldots}$$

$$= \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} + \frac{(c+dx)^2 \csc(a+ \ldots}{2 \ldots}$$

$$= \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} + \frac{(c+dx)^2 \csc(a+ \ldots}{2 \ldots}$$

$$= \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} + \frac{(c+dx)^2 \csc(a+ \ldots}{2 \ldots}$$

$$= \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} + \frac{(c+dx)^2 \csc(a+ \ldots}{2 \ldots}$$

$$= \frac{d(c+dx) \tanh^{-1}(\cos(a+bx))}{b^2} + \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx \ldots}{\ldots}$$

$$= \frac{d(c+dx) \tanh^{-1}(\cos(a+bx))}{b^2} + \frac{3(c+dx)^2 \tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx \ldots}{\ldots}$$

$$= -\frac{6d(c+dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2 x \tanh^{-1}(\cos(a+bx))}{b^2} + \frac{d(c+dx) \tanh^{- \ldots}}{b^2}$$

$$= -\frac{6d(c+dx) \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2 x \tanh^{-1}(\cos(a+bx))}{b^2} + \frac{d(c+dx) \tanh^{- \ldots}}{b^2}$$

$$= -\frac{3i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2 x \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx) \tanh^{-1}\left(e \ldots\right.}{b^2}$$

$$= -\frac{3i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2 x \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx) \tanh^{-1}\left(e \ldots\right.}{b^2}$$

$$= -\frac{3i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2 x \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx) \tanh^{-1}\left(e \ldots\right.}{b^2}$$

$$= -\frac{3i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2 x \tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx) \tanh^{-1}\left(e \ldots\right.}{b^2}$$

Mathematica [B] time = 7.56174, size = 889, normalized size = 2.61

$$\frac{2\left(\frac{\left(\left(bx+\tan^{-1}(\tan(a))\right)\left(\log\left(1-e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)-\log\left(1+e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)-\text{PolyLog}\left(2,e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)\right)\text{s}\ldots}{\sqrt{\tan^2(a)+1}} \ldots\right.}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] -((6*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + (4*I)*d^2*ArcTan[E^(I*(a + b*x))]
 - 6*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 - I*E^(I*(a
 + b*x))] + 6*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 +
I*E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] +
 (6*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^2*PolyLog[3, (-I)*
E^(I*(a + b*x))] - 6*d^2*PolyLog[3, I*E^(I*(a + b*x))])/(2*b^3) - ((c + d*x
)*Csc[a]*Sec[a]*(b*c*Cos[a] + b*d*x*Cos[a] + d*Sin[a]))/b^2 + ((4*I)*c*d*Ar
cTan[(I*Cos[a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sq
rt[Cos[a]^2 + Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]
) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2
 + (b*x)/2]*(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]
))/(2*b) + (c^2 + 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (
b*x)/2])^2) + (-(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - S
in[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2
*x^2)/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2]
 + d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin
[a/2 + (b*x)/2])) + (2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Ta
n[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2] + (((b*x
+ ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*
x + ArcTan[Tan[a]]))]) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - Pol
yLog[2, E^(I*(b*x + ArcTan[Tan[a]]))]))*Sec[a])/Sqrt[1 + Tan[a]^2]))/b^3
```

Maple [B] time = 0.499, size = 770, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] 2*d/b^2*c*ln(exp(I*(b*x+a))-1)-2*d/b^2*c*ln(exp(I*(b*x+a))+1)-2*d^2/b^2*ln(
exp(I*(b*x+a))+1)*x-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)-3/b*c*d*ln(1+I*exp(I*(
b*x+a)))*x+3/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+3/2/b^3*d^2*a^2*ln(1+I*exp(
I*(b*x+a)))-3/2/b^3*d^2*a^2*ln(1-I*exp(I*(b*x+a)))-3/2/b*d^2*ln(1+I*exp(I*(
b*x+a)))*x^2-2*I/b^3*d^2*arctan(exp(I*(b*x+a)))-I/b^2/(exp(2*I*(b*x+a))+1)^
2/(exp(2*I*(b*x+a))-1)*(3*d^2*x^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*x
+a))+3*c^2*b*exp(5*I*(b*x+a))+2*d^2*x^2*b*exp(3*I*(b*x+a))+4*c*d*x*b*exp(3*
I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x+a))+2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b*
exp(I*(b*x+a))-2*I*d*c*exp(5*I*(b*x+a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*ex
p(I*(b*x+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))-3*d^2*polylog
```

```
(3,-I*exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-3/b^2*c*d*l
n(1+I*exp(I*(b*x+a)))*a+3/b*c*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c*d*ln(1-I*e
xp(I*(b*x+a)))*a+6*I/b^2*a*c*d*arctan(exp(I*(b*x+a)))+2*I/b^3*dilog(exp(I*(
b*x+a))+1)*d^2-3*I/b*c^2*arctan(exp(I*(b*x+a)))+2*I/b^3*d^2*dilog(exp(I*(b*
x+a)))-3*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x-3*I/b^3*d^2*a^2*arctan(exp
(I*(b*x+a)))+3*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x+3*I/b^2*c*d*polylog
(2,-I*exp(I*(b*x+a)))-3*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))
```

Maxima [B] time = 5.44259, size = 5154, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 2*a*c*d*(2*(3*sin(b*x + a)^
2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(si
n(b*x + a) - 1))/b + a^2*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - si
n(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2 - 4*((
6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 - 2*(3*(b*x + a)^2
*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - 2*(3*(b*x +
a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + 2*(3*(b*
x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) + (-6*
I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(6*b
*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) -
4*I*d^2)*sin(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2
)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a)
 + 1) + (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 - 2*(3*(b
*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - 2*(
3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) +
 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*
a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2
)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*
x + a) - 4*I*d^2)*sin(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 1
2*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -si
n(b*x + a) + 1) + (8*b*c*d + 8*(b*x + a)*d^2 - 8*a*d^2 - 8*(b*c*d + (b*x +
a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*
b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-8*I*b*c
*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(6*b*x + 6*a) + (-8*I*b*c*d - 8*I*(b
*x + a)*d^2 + 8*I*a*d^2)*sin(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*d^2
```

```
- 8*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (8
*b*c*d - 8*a*d^2 - 8*(b*c*d - a*d^2)*cos(6*b*x + 6*a) - 8*(b*c*d - a*d^2)*c
os(4*b*x + 4*a) + 8*(b*c*d - a*d^2)*cos(2*b*x + 2*a) - (8*I*b*c*d - 8*I*a*d
^2)*sin(6*b*x + 6*a) - (8*I*b*c*d - 8*I*a*d^2)*sin(4*b*x + 4*a) - (-8*I*b*c
*d + 8*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) -
 (8*(b*x + a)*d^2*cos(6*b*x + 6*a) + 8*(b*x + a)*d^2*cos(4*b*x + 4*a) - 8*(
b*x + a)*d^2*cos(2*b*x + 2*a) + 8*I*(b*x + a)*d^2*sin(6*b*x + 6*a) + 8*I*(b
*x + a)*d^2*sin(4*b*x + 4*a) - 8*I*(b*x + a)*d^2*sin(2*b*x + 2*a) - 8*(b*x
+ a)*d^2)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*(b*x + a)^2*d^2 -
8*I*b*c*d + 8*I*a*d^2 + (24*b*c*d - (24*a + 8*I)*d^2)*(b*x + a))*cos(5*b*x
+ 5*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(3*b*x + 3*a)
 - (12*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (24*b*c*d - (24*a - 8*I)*d
^2)*(b*x + a))*cos(b*x + a) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 - 12*
(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^
2 - a*d^2)*cos(4*b*x + 4*a) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
+ 2*a) + (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(6*b*x + 6*a) +
 (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(4*b*x + 4*a) + (12*I*b
*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x
+ I*a)) - (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 - 12*(b*c*d + (b*x + a)*d
^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x
 + 4*a) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (12*I*b*c*d
 + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(6*b*x + 6*a) - (12*I*b*c*d + 12*I*(
b*x + a)*d^2 - 12*I*a*d^2)*sin(4*b*x + 4*a) - (-12*I*b*c*d - 12*I*(b*x + a)
*d^2 + 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (8*d^2*cos
(6*b*x + 6*a) + 8*d^2*cos(4*b*x + 4*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I*d^2*s
in(6*b*x + 6*a) + 8*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x + 2*a) - 8*d
^2)*dilog(-e^(I*b*x + I*a)) - (8*d^2*cos(6*b*x + 6*a) + 8*d^2*cos(4*b*x + 4
*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I*d^2*sin(6*b*x + 6*a) + 8*I*d^2*sin(4*b*x
 + 4*a) - 8*I*d^2*sin(2*b*x + 2*a) - 8*d^2)*dilog(e^(I*b*x + I*a)) + (-4*I*
b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2 + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*
I*a*d^2)*cos(6*b*x + 6*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos
(4*b*x + 4*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(2*b*x + 2*
a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(6*b*x + 6*a) - 4*(b*c*d + (b*x +
 a)*d^2 - a*d^2)*sin(4*b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (4
*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2 + (-4*I*b*c*d - 4*I*(b*x + a)*d^2
+ 4*I*a*d^2)*cos(6*b*x + 6*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2
)*cos(4*b*x + 4*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(2*b*x
+ 2*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(6*b*x + 6*a) + 4*(b*c*d + (b
*x + a)*d^2 - a*d^2)*sin(4*b*x + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*s
in(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
+ (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2 + (-3*
I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(6*b*x
 + 6*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*
d^2)*cos(4*b*x + 4*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x
```

```
 + a) + 2*I*d^2)*cos(2*b*x + 2*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*
(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2
)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d
^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*
(b*x + a) - 2*I*d^2 + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x +
 a) + 2*I*d^2)*cos(6*b*x + 6*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a
*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I
*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - (3*(b*x + a)^2*
d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) - (3*(b*x + a)^
2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) + (3*(b*x + a
)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (-12*I*d^2*cos(6*b*x + 6*
a) - 12*I*d^2*cos(4*b*x + 4*a) + 12*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(6*b
*x + 6*a) + 12*d^2*sin(4*b*x + 4*a) - 12*d^2*sin(2*b*x + 2*a) + 12*I*d^2)*p
olylog(3, I*e^(I*b*x + I*a)) + (12*I*d^2*cos(6*b*x + 6*a) + 12*I*d^2*cos(4*
b*x + 4*a) - 12*I*d^2*cos(2*b*x + 2*a) - 12*d^2*sin(6*b*x + 6*a) - 12*d^2*s
in(4*b*x + 4*a) + 12*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*polylog(3, -I*e^(I*b*
x + I*a)) + (-12*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (-24*I*b*c*d - 8*(
-3*I*a + 1)*d^2)*(b*x + a))*sin(5*b*x + 5*a) + (-8*I*(b*x + a)^2*d^2 + (-16
*I*b*c*d + 16*I*a*d^2)*(b*x + a))*sin(3*b*x + 3*a) + (-12*I*(b*x + a)^2*d^2
 + 8*b*c*d - 8*a*d^2 + (-24*I*b*c*d - 8*(-3*I*a - 1)*d^2)*(b*x + a))*sin(b*
x + a))/(-4*I*b^2*cos(6*b*x + 6*a) - 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos
(2*b*x + 2*a) + 4*b^2*sin(6*b*x + 6*a) + 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin
(2*b*x + 2*a) + 4*I*b^2))/b
```

Fricas [C] time = 0.972988, size = 3596, normalized size = 10.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^2*x^2 - 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x
+ a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) - 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x
 + a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*
x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin
(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) -
sin(b*x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a
) + sin(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x
```

```
 + a) - sin(b*x + a))*sin(b*x + a) + 4*b^2*c*d*x + (-6*I*b*d^2*x - 6*I*b*c*
d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-6*I
*b*d^2*x - 6*I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*s
in(b*x + a) + (6*I*b*d^2*x + 6*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a
) + sin(b*x + a))*sin(b*x + a) + (6*I*b*d^2*x + 6*I*b*c*d)*cos(b*x + a)^2*d
ilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos
(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (3*b^2*c^
2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*
x + a) + I)*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos(b*x + a)^2*log(cos(b*x +
 a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 +
2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a)
+ 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*
cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) +
 1)*sin(b*x + a) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(
b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x +
a) - sin(b*x + a) + 1)*sin(b*x + a) + 4*(b*c*d - a*d^2)*cos(b*x + a)^2*log(
-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 4*(b*c*d - a*d
^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*
x + a) + 4*(b*d^2*x + a*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x +
 a) + 1)*sin(b*x + a) + (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x +
 a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 4*(b*d^2*x + a
*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) -
 (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x +
a))/(b^3*cos(b*x + a)^2*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc (bx+a)^2 \sec (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

3.319 $\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=162

$$\frac{3id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{3(c+dx)\csc(a+bx)}{2b}$$

```
[Out] ((-3*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 + (3
*c*ArcTanh[Sin[a + b*x]])/(2*b) - (3*(c + d*x)*Csc[a + b*x])/(2*b) + (((3*I
)/2)*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (((3*I)/2)*d*PolyLog[2, I*E^
(I*(a + b*x))])/b^2 - (d*Sec[a + b*x])/(2*b^2) + ((c + d*x)*Csc[a + b*x]*Se
c[a + b*x]^2)/(2*b)
```

Rubi [A] time = 0.195867, antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2621, 288, 321, 207, 4420, 6271, 12, 4181, 2279, 2391, 3770, 2622}

$$\frac{3id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{3(c+dx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-3*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (3
*d*x*ArcTanh[Sin[a + b*x]])/(2*b) + (3*(c + d*x)*ArcTanh[Sin[a + b*x]])/(2*
b) - (3*(c + d*x)*Csc[a + b*x])/(2*b) + (((3*I)/2)*d*PolyLog[2, (-I)*E^(I*(
a + b*x))])/b^2 - (((3*I)/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[
a + b*x])/(2*b^2) + ((c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3(c+dx)\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)\csc(a+bx)}{2b} + \frac{(c+dx)\csc(a+bx)\,\mathrm{s}}{2b}$$

$$= \frac{3(c+dx)\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)\csc(a+bx)}{2b} + \frac{(c+dx)\csc(a+bx)\,\mathrm{s}}{2b}$$

$$= -\frac{3d\tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3(c+dx)\tanh^{-1}(\sin(a}{2b}$$

$$= -\frac{3d\tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3(c+dx)\tanh^{-1}(\sin(a}{2b}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3}{}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3}{}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3}{}$$

Mathematica [C] time = 6.57654, size = 669, normalized size = 4.13

$$-\frac{c\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\sin^2(a+bx)\right)}{b}-\frac{3dx\left(-i\left(\text{PolyLog}\left(2,\frac{1}{2}\left((1+i)-(1-i)\tan\left(\frac{1}{2}(a+bx)\right.\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b
^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b -
(d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (3*d*x*(a*L
og[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*Tan[(
a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1 + I)
- (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 +
 I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/
2])]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)
/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 + I*
Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + PolyLog[2, ((
1 - I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x
)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Si
n[(a + b*x)/2])^2) - (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a
 + b*x)/2])) - (d*x)/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin
[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*Sec[(a +
b*x)/2]*(a*Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)
```

Maple [B] time = 0.355, size = 344, normalized size = 2.1

$$\frac{-i\left(3\,dxb\mathrm{e}^{5\,i(bx+a)}+3\,bc\mathrm{e}^{5\,i(bx+a)}+2\,dxb\mathrm{e}^{3\,i(bx+a)}+2\,bc\mathrm{e}^{3\,i(bx+a)}-id\mathrm{e}^{5\,i(bx+a)}+3\,dxb\mathrm{e}^{i(bx+a)}+3\,bc\mathrm{e}^{i(bx+a)}+id\mathrm{e}^{i(bx+a)}\right)}{b^2\left(\mathrm{e}^{2\,i(bx+a)}+1\right)^2\left(\mathrm{e}^{2\,i(bx+a)}-1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] -I/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x+a))-1)*(3*d*x*b*exp(5*I*(b*x+a)
)+3*b*c*exp(5*I*(b*x+a))+2*d*x*b*exp(3*I*(b*x+a))+2*b*c*exp(3*I*(b*x+a))-I*
d*exp(5*I*(b*x+a))+3*d*x*b*exp(I*(b*x+a))+3*b*c*exp(I*(b*x+a))+I*d*exp(I*(b
*x+a)))-3*I/b*c*arctan(exp(I*(b*x+a)))+3*I/b^2*d*a*arctan(exp(I*(b*x+a)))+d
```

```
/b^2*ln(exp(I*(b*x+a))-1)-d/b^2*ln(exp(I*(b*x+a))+1)+3/2*I/b^2*d*dilog(1+I*
exp(I*(b*x+a)))-3/2/b*d*ln(1+I*exp(I*(b*x+a)))*x-3/2/b^2*d*ln(1+I*exp(I*(b*
x+a)))*a+3/2/b*d*ln(1-I*exp(I*(b*x+a)))*x+3/2/b^2*d*ln(1-I*exp(I*(b*x+a)))*
a-3/2*I/b^2*d*dilog(1-I*exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.696318, size = 1644, normalized size = 10.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a
) - 3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a)
+ 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) +
 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
 + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)
*sin(b*x + a) - 2*d*cos(b*x + a)^2*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a)
 + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1)*si
n(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x +
a) + 1)*sin(b*x + a) + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1)*sin(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos
(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 2*d*cos(b*x + a)^2*log(-1/2*co
s(b*x + a) + 1/2)*sin(b*x + a) + 3*(b*c - a*d)*cos(b*x + a)^2*log(-cos(b*x
+ a) + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(
-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*d*x - 6*(b*d*x + b*c
)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + 2*b*c)/(b^2*cos(b*x + a)
```

```
^2*sin(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)\csc (bx + a)^2 \sec (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

3.320 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi [A] time = 0.18662, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]

[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx=\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 21.1897, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]

[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

Maple [A] time = 2.754, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx+a))^2 (\sec (bx+a))^3}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc (bx+a)^2 \sec (bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.321 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi [A] time = 0.197133, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 26.4328, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Maple [A] time = 4.312, size = 0, normalized size = 0.

$$\int \frac{(\csc (bx+a))^2 (\sec (bx+a))^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc (bx+a)^2 \sec (bx+a)^3}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2 \sec(bx+a)^3}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c)^2, x)
```

3.322 $\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\text{CannotIntegrate}\left(\csc^3(a+bx)\sec^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

Rubi [A] time = 0.264578, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Mathematica [A] time = 10.558, size = 0, normalized size = 0.

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

Maple [A] time = 0.187, size = 0, normalized size = 0.

$$\int (dx+c)^m (\csc(bx+a))^3 (\sec(bx+a))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^m \csc (bx+a)^3 \sec (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

3.323 $\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=318

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{3id(c}{}$$

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b^3 - (4*(c + d*x)^3*ArcTan
h[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[2*a + 2*b*x])/b^2 - (2*(c
+ d*x)^3*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (((3*I)/2)*d^3*PolyLog[2, -
E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - ((3*I)*
d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyL
og[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a
+ b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3
*I)/2)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rubi [A] time = 0.32147, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4419, 4186, 4183, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{3id(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```


```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b^3 - (4*(c + d*x)^3*ArcTan
h[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[2*a + 2*b*x])/b^2 - (2*(c
+ d*x)^3*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (((3*I)/2)*d^3*PolyLog[2, -
E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - ((3*I)*
d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyL
og[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a
+ b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3
*I)/2)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```


Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
```

```
x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)^3\csc^3(2a+2bx)\,dx$$
$$= -\frac{3d(c+dx)^2\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^3\cot(2a+2bx)\csc(2a+2bx)}{b} + 4\int$$
$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{}$$
$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{}$$
$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{}$$
$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{}$$
$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{}$$

Mathematica [A] time = 9.18225, size = 582, normalized size = 1.83

$$8\left(-\frac{\csc^2(2a+2bx)\left(3c^2d\sin(2a+2bx)+6bc^2dx\cos(2a+2bx)+2bc^3\cos(2a+2bx)+6bcd^2x^2\cos(2a+2bx)+6cd^2x\right.}{8b^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] 8*(-(8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b*c*d^2*ArcTanh[E^((2*I)*(
a + b*x))] - 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(a + b*x))] - 6*b*d^3*x*Log[1
- E^((2*I)*(a + b*x))] - 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] - 4*
b^3*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 + E^((2*I)*
(a + b*x))] + 6*b*d^3*x*Log[1 + E^((2*I)*(a + b*x))] + 12*b^3*c*d^2*x^2*Log
[1 + E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] - (3
*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, -E^((2*I)*(a + b*x))] + (3*I)*d*
(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, E^((2*I)*(a + b*x))] + 6*b*c*d^2*PolyL
og[3, -E^((2*I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^((2*I)*(a + b*x))] -
6*b*c*d^2*PolyLog[3, E^((2*I)*(a + b*x))] - 6*b*d^3*x*PolyLog[3, E^((2*I)*(
a + b*x))] + (3*I)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (3*I)*d^3*PolyLog
[4, E^((2*I)*(a + b*x))])/(16*b^4) - (Csc[2*a + 2*b*x]^2*(2*b*c^3*Cos[2*a +
 2*b*x] + 6*b*c^2*d*x*Cos[2*a + 2*b*x] + 6*b*c*d^2*x^2*Cos[2*a + 2*b*x] + 2
*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin[2*a
+ 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(8*b^2))
```

Maple [B] time = 0.295, size = 1329, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] 6/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-2/b*c
^3*ln(exp(2*I*(b*x+a))+1)-3/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/b^3*d^
3*polylog(3,-exp(2*I*(b*x+a)))*x+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3
*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-3*I/b^4*d^3*polylo
g(2,-exp(I*(b*x+a)))-3*d^2/b^3*c*ln(exp(2*I*(b*x+a))+1)-3*d^3/b^3*ln(exp(2*
I*(b*x+a))+1)*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4+12*I/b^4*d^3*pol
ylog(4,exp(I*(b*x+a)))+12*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-3/2*I*d^3*po
lylog(4,-exp(2*I*(b*x+a)))/b^4-6/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-6/b*c*d^2
*ln(exp(2*I*(b*x+a))+1)*x^2-2/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-3/b^4*d^3*a*
ln(exp(I*(b*x+a))-1)+3/b^3*d^2*c*ln(exp(I*(b*x+a))+1)+3/b^3*d^2*c*ln(exp(I*
(b*x+a))-1)+2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+2/b^4*d^3*ln(1-exp(I*(b*x+a)))
*a^3+2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-
6/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-6/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b
*c^2*d*ln(exp(I*(b*x+a))+1)*x+6/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+6/b^2*c^2*d*
ln(1-exp(I*(b*x+a)))*a+2/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x+a))-1)^2*
(2*d^3*x^3*b*exp(6*I*(b*x+a))+6*c*d^2*x^2*b*exp(6*I*(b*x+a))+6*c^2*d*x*b*ex
```

```
p(6*I*(b*x+a))+2*c^3*b*exp(6*I*(b*x+a))-3*I*d^3*x^2*exp(6*I*(b*x+a))+2*b*d^
3*x^3*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(6*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(
b*x+a))-3*I*c^2*d*exp(6*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))+2*b*c^3*exp
(2*I*(b*x+a))+3*I*d^3*x^2*exp(2*I*(b*x+a))+6*I*c*d^2*x*exp(2*I*(b*x+a))+3*I
*c^2*d*exp(2*I*(b*x+a)))+12/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+12/b^3*d^3*
polylog(3,-exp(I*(b*x+a)))*x+12/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+12/b^3*
c*d^2*polylog(3,-exp(I*(b*x+a)))-2/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+2/b*c^3
*ln(exp(I*(b*x+a))-1)+2/b*c^3*ln(exp(I*(b*x+a))+1)+3*I/b^2*c^2*d*polylog(2,
-exp(2*I*(b*x+a)))-6*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+3*I/b^2*d^3*pol
ylog(2,-exp(2*I*(b*x+a)))*x^2-6*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I*d
^3*polylog(2,exp(I*(b*x+a)))/b^4-12*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2*
x-12*I/b^2*polylog(2,exp(I*(b*x+a)))*c*d^2*x+6*I/b^2*polylog(2,-exp(2*I*(b*
x+a)))*c*d^2*x-6*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-6*I/b^2*d^3*polyl
og(2,exp(I*(b*x+a)))*x^2
```

Maxima [B] time = 11.6935, size = 7574, normalized size = 23.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 3*a*c^2*d*((2*sin(b*x + a)^
2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*lo
g(sin(b*x + a)^2))/b + 3*a^2*c*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4
- sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2
- a^3*d^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^3 + 2*((16*(b*x + a)^3*d^3
+ 18*b*c*d^2 - 18*a*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(2*b^2*c^2*
d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + 2*(8*(b*x + a)^3*d^3 + 9*b*c
*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*
c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 4*(8*(b*x + a)^3*d^3
 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d
- 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (16*I*(b*x +
 a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x +
a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3)*(b*x + a))
*sin(8*b*x + 8*a) + (-32*I*(b*x + a)^3*d^3 - 36*I*b*c*d^2 + 36*I*a*d^3 + (-
72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a)^2 + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2
 + (-72*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) - (12*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3
```

+ 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 12*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-12*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3)*(b*x + a))*sin(8*b*x + 8*a) - (24*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2 - 36*I*a*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 144*I*a*b*c*d^2 + (72*I*a^2 + 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^2 - a*d^3)*cos(8*b*x + 8*a) - 36*(b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) - (-18*I*b*c*d^2 + 18*I*a*d^3)*sin(8*b*x + 8*a) - (36*I*b*c*d^2 - 36*I*a*d^3)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (12*(b*x + a)^3*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + 6*(2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 12*(2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (12*I*(b*x + a)^3*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3)*(b*x + a))*sin(8*b*x + 8*a) + (-24*I*(b*x + a)^3*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a)^2 + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 + (-72*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (24*I*(b*x + a)^3*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3 + (72*I*b*c*d^2 - 36*(2*I*a - 1)*d^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 72*(2*I*a - 1)*b*c*d^2 + (72*I*a^2 - 72*a)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (24*I*(b*x + a)^3*d^3 - 36*b^2*c^2*d + 72*a*b*c*d^2 - 36*a^2*d^3 + (72*I*b*c*d^2 - 36*(2*I*a + 1)*d^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 72*(2*I*a + 1)*b*c*d^2 + (72*I*a^2 + 72*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 24*(b*x + a)^2*d^3 + 9*(2*a^2 + 1)*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 3*(6*b^2*c^2*d - 12*a*b*c*d^2 + 8*(b*x + a)^2*d^3 + 3*(2*a^2 + 1)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 6*(6*b^2*c^2*d - 12*a*b*c*d^2 + 8*(b*x + a)^2*d^3 + 3*(2*a^2 + 1)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 9*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(8*b*x + 8*a) - (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 48*I*(b*x + a)^2*d^3 + (36*I*a^2 + 18*I)*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (36*b^2*c^2*d - 72*a*b*c*d^2 + 36*(b*x + a)^2*d^3 + 18*(2*a^2 + 1)*d^3 + 72*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 36*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b*x + a)^2*d^3 + (36*I*a^2 + 18*I)*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(8*b*x +

8*a) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 72*I*(b*x + a)^2*d^3 + (-72*I*a
^2 - 36*I)*d^3 + (-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a)
)*dilog(-e^(I*b*x + I*a)) + (36*b^2*c^2*d - 72*a*b*c*d^2 + 36*(b*x + a)^2*d
^3 + 18*(2*a^2 + 1)*d^3 + 72*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(2*b^2*c^2*d
- 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(8*b*x + 8*a) - 36*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*
d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b*x + a)^2*d^3 + (36*I*a^2 + 18*I)*
d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(8*b*x + 8*a) + (-72*I*b^2*
c^2*d + 144*I*a*b*c*d^2 - 72*I*(b*x + a)^2*d^3 + (-72*I*a^2 - 36*I)*d^3 + (
-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*dilog(e^(I*b*x +
I*a)) + (-8*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + (-18*I*b*c*d^2 +
18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 -
9*I)*d^3)*(b*x + a) + (-8*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + (-
18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2
+ (-18*I*a^2 - 9*I)*d^3)*(b*x + a))*cos(8*b*x + 8*a) + (16*I*(b*x + a)^3*d^
3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (
36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3)*(b*x + a))*cos(4*b
*x + 4*a) + (8*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*s
in(8*b*x + 8*a) - 2*(8*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2
- a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x
+ a))*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*co
s(2*b*x + 2*a) + 1) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (18*
I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (1
8*I*a^2 + 9*I)*d^3)*(b*x + a) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*
d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*
c*d^2 + (18*I*a^2 + 9*I)*d^3)*(b*x + a))*cos(8*b*x + 8*a) + (-12*I*(b*x + a
)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a
)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) - 3*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b
*x + a))*sin(8*b*x + 8*a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*
d^3)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*c
os(b*x + a) + 1) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (18*I*b
*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I
*a^2 + 9*I)*d^3)*(b*x + a) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3
+ (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d
^2 + (18*I*a^2 + 9*I)*d^3)*(b*x + a))*cos(8*b*x + 8*a) + (-12*I*(b*x + a)^3
*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2
+ (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3)*(b*x + a))*c
os(4*b*x + 4*a) - 3*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x
+ a))*sin(8*b*x + 8*a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*

```
c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3
)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) + (12*d^3*cos(8*b*x + 8*a) - 24*d^3*cos(4*b*x + 4*a) + 12*I*d
^3*sin(8*b*x + 8*a) - 24*I*d^3*sin(4*b*x + 4*a) + 12*d^3)*polylog(4, -e^(2*
I*b*x + 2*I*a)) - (72*d^3*cos(8*b*x + 8*a) - 144*d^3*cos(4*b*x + 4*a) + 72*
I*d^3*sin(8*b*x + 8*a) - 144*I*d^3*sin(4*b*x + 4*a) + 72*d^3)*polylog(4, -e
^(I*b*x + I*a)) - (72*d^3*cos(8*b*x + 8*a) - 144*d^3*cos(4*b*x + 4*a) + 72*
I*d^3*sin(8*b*x + 8*a) - 144*I*d^3*sin(4*b*x + 4*a) + 72*d^3)*polylog(4, e^
(I*b*x + I*a)) + (-18*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3 + (-18*I*
b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3)*cos(8*b*x + 8*a) + (36*I*b*c*d^2
 + 48*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + 6*(3*b*c*d^2 + 4*(b*
x + a)*d^3 - 3*a*d^3)*sin(8*b*x + 8*a) - 12*(3*b*c*d^2 + 4*(b*x + a)*d^3 -
3*a*d^3)*sin(4*b*x + 4*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + (72*I*b*c*d^2
 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3 + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 7
2*I*a*d^3)*cos(8*b*x + 8*a) + (-144*I*b*c*d^2 - 144*I*(b*x + a)*d^3 + 144*I
*a*d^3)*cos(4*b*x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(8*b*x +
 8*a) + 144*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a))*polylog(3,
-e^(I*b*x + I*a)) + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3 + (72*I
*b*c*d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3)*cos(8*b*x + 8*a) + (-144*I*b*c*
d^2 - 144*I*(b*x + a)*d^3 + 144*I*a*d^3)*cos(4*b*x + 4*a) - 72*(b*c*d^2 + (
b*x + a)*d^3 - a*d^3)*sin(8*b*x + 8*a) + 144*(b*c*d^2 + (b*x + a)*d^3 - a*d
^3)*sin(4*b*x + 4*a))*polylog(3, e^(I*b*x + I*a)) - (24*(b*x + a)^3*d^3 - 3
6*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*a^2*d^3 + (72*b*c*d^2 - (72*a + 36*I)
*d^3)*(b*x + a)^2 + (72*b^2*c^2*d - (144*a + 72*I)*b*c*d^2 + 72*(a^2 + I*a)
*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (24*(b*x + a)^3*d^3 + 36*I*b^2*c^2*d -
72*I*a*b*c*d^2 + 36*I*a^2*d^3 + (72*b*c*d^2 - (72*a - 36*I)*d^3)*(b*x + a)^
2 + (72*b^2*c^2*d - (144*a - 72*I)*b*c*d^2 + 72*(a^2 - I*a)*d^3)*(b*x + a))
*sin(2*b*x + 2*a))/(-6*I*b^3*cos(8*b*x + 8*a) + 12*I*b^3*cos(4*b*x + 4*a) +
 6*b^3*sin(8*b*x + 8*a) - 12*b^3*sin(4*b*x + 4*a) - 6*I*b^3))/b
```

Fricas [C] time = 1.74066, size = 9802, normalized size = 30.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 2*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 3*(b^2*d^
3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - ((-6*I*b^2*d
^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^4 + (6*I*
```

b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2)*d
ilog(cos(b*x + a) + I*sin(b*x + a)) - ((6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x
+ 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*
d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin
(b*x + a)) - ((-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^
3)*cos(b*x + a)^4 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3
*I*d^3)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) - ((6*I*b^2*d^
3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*I*
b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*d
ilog(I*cos(b*x + a) - sin(b*x + a)) - ((6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x
+ 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*
d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + si
n(b*x + a)) - ((-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d
^3)*cos(b*x + a)^4 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d +
3*I*d^3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - ((6*I*b^2*
d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*
I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)
*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^
2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^4 + (6*I*b^2*d^3*x^2 + 12*I*b^2
*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I
*sin(b*x + a)) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2
+ 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*
x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*lo
g(cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a
^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*
c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(cos(
b*x + a) + I*sin(b*x + a) + I) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*
c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^
3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*co
s(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + ((2*b^3*c^3 - 6*a*b^
2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^
3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x
+ a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + ((2*b^3*d^3*x^3 + 6*b^3*c*
d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*
d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c
^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b
*x + a)^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + ((2*b^3*d^3*x^3 + 6*b^3
*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c
^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^
2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*co
s(b*x + a)^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + ((2*b^3*d^3*x^3 + 6*
b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^
3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a
*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)
*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + ((2*b^3*d^3*x^3

```
+ 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(
2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 +
 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3
)*x)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - ((2*b^3*c^3
- 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4
 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*
cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((2*b^3
*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x +
 a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*
d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - ((
2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 +
3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b
^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3
*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)
+ ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*
cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^
3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - ((2
*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3
*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^
3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*
c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) +
 ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*c
os(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3
 + 3*a)*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - (12*
I*d^3*cos(b*x + a)^4 - 12*I*d^3*cos(b*x + a)^2)*polylog(4, cos(b*x + a) + I
*sin(b*x + a)) - (-12*I*d^3*cos(b*x + a)^4 + 12*I*d^3*cos(b*x + a)^2)*polyl
og(4, cos(b*x + a) - I*sin(b*x + a)) - (12*I*d^3*cos(b*x + a)^4 - 12*I*d^3*
cos(b*x + a)^2)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - (-12*I*d^3*cos(
b*x + a)^4 + 12*I*d^3*cos(b*x + a)^2)*polylog(4, I*cos(b*x + a) - sin(b*x +
 a)) - (-12*I*d^3*cos(b*x + a)^4 + 12*I*d^3*cos(b*x + a)^2)*polylog(4, -I*c
os(b*x + a) + sin(b*x + a)) - (12*I*d^3*cos(b*x + a)^4 - 12*I*d^3*cos(b*x +
 a)^2)*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - (-12*I*d^3*cos(b*x + a)
^4 + 12*I*d^3*cos(b*x + a)^2)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) -
(12*I*d^3*cos(b*x + a)^4 - 12*I*d^3*cos(b*x + a)^2)*polylog(4, -cos(b*x + a
) - I*sin(b*x + a)) - 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b
*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 12*((b*
d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polyl
og(3, cos(b*x + a) - I*sin(b*x + a)) + 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)
^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) + sin(b*
x + a)) + 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(
b*x + a)^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*((b*d^3*x + b*c*
d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3
*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) -
12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2
```

```
)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*((b*d^3*x + b*c*d^2)*cos(
b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)))/(b^4*cos(b*x + a)^4 - b^4*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

3.324 $\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\, dx$

Optimal. Leaf size=190

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a}\right.}{b^3}$$

```
[Out] (-4*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b - (d^2*ArcTanh[Cos[2*a + 2*
b*x]])/b^3 - (2*d*(c + d*x)*Csc[2*a + 2*b*x])/b^2 - (2*(c + d*x)^2*Cot[2*a
+ 2*b*x]*Csc[2*a + 2*b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
 b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d
^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (d^2*PolyLog[3, E^((2*I)*(a + b*
x))])/b^3
```

Rubi [A] time = 0.212836, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4419, 4186, 3770, 4183, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```


```
[Out] (-4*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b - (d^2*ArcTanh[Cos[2*a + 2*
b*x]])/b^3 - (2*d*(c + d*x)*Csc[2*a + 2*b*x])/b^2 - (2*(c + d*x)^2*Cot[2*a
+ 2*b*x]*Csc[2*a + 2*b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
 b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d
^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (d^2*PolyLog[3, E^((2*I)*(a + b*
x))])/b^3
```


Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
```

```
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)^2 \csc^3(2a+2bx)\,dx$$

$$= -\frac{2d(c+dx)\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^2\cot(2a+2bx)\csc(2a+2bx)}{b} + 4\int (c$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+dx)\csc(2}{b^2}$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+dx)\csc(2}{b^2}$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+dx)\csc(2}{b^2}$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+dx)\csc(2}{b^2}$$

Mathematica [B] time = 8.16677, size = 381, normalized size = 2.01

$$8\left(-\frac{-2ibd(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)+2ibd(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)+d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)-d^2\text{PolyLog}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] 8*(-(d*(c + d*x)*Csc[2*a])/(4*b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a + b*
x]^2)/(16*b) - (4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 2*d^2*ArcTanh[E^((
2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 - E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2
*Log[1 - E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 + E^((2*I)*(a + b*x))] +
2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2,
 -E^((2*I)*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
] + d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(8*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a + b*x]^2)/(16*b) + (Sec[a]*
Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/(8*b^2) + (Csc[a]*Csc[a +
b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/(8*b^2))
```

Maple [B] time = 0.239, size = 716, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] -d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+4/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I
*(b*x+a))-1)^2*(d^2*x^2*b*exp(6*I*(b*x+a))+2*c*d*x*b*exp(6*I*(b*x+a))+c^2*b
*exp(6*I*(b*x+a))-I*d^2*x*exp(6*I*(b*x+a))+b*d^2*x^2*exp(2*I*(b*x+a))-I*d*c
*exp(6*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))+I*d^2*x
*exp(2*I*(b*x+a))+I*d*c*exp(2*I*(b*x+a)))-2/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^
2-2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+2/b*d
^2*ln(1-exp(I*(b*x+a)))*x^2-2/b*c^2*ln(exp(2*I*(b*x+a))+1)+1/b^3*d^2*ln(exp
(I*(b*x+a))+1)+1/b^3*d^2*ln(exp(I*(b*x+a))-1)+4/b*c*d*ln(exp(I*(b*x+a))+1)*
x+2/b*c^2*ln(exp(I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a))-1)-4*I/b^2*polylog
(2,exp(I*(b*x+a)))*d^2*x-4*I/b^2*polylog(2,-exp(I*(b*x+a)))*d^2*x-4*I/b^2*c
*d*polylog(2,-exp(I*(b*x+a)))+2*I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+2*
I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-4*I/b^2*c*d*polylog(2,exp(I*(b*x+a))
)+4*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+4*d^2*polylog(3,exp(I*(b*x+a)))/b^3-
1/b^3*d^2*ln(exp(2*I*(b*x+a))+1)-4/b*c*d*ln(exp(2*I*(b*x+a))+1)*x+4/b*c*d*l
n(1-exp(I*(b*x+a)))*x+4/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2/b^3*d^2*a^2*ln(exp
(I*(b*x+a))-1)-4/b^2*c*d*a*ln(exp(I*(b*x+a))-1)
```

Maxima [B] time = 3.66754, size = 3676, normalized size = 19.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 2*a*c*d*((2*sin(b*x + a)^2
- 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(
sin(b*x + a)^2))/b + a^2*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(
b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2 + 2*((
4*(b*x + a)^2*d^2 + 8*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*(2*(b*x + a)^2*
d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 4*(2*(b*x + a)^
2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) + (4*I*(b*x + a
)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(8*b*x + 8*a) + (
-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(
4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (4*(b*x + a
)^2*d^2 + 8*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*(2*(b*x + a)^2*d^2 + 4*(b
*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 4*(2*(b*x + a)^2*d^2 + 4*
(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) - (-4*I*(b*x + a)^2*d^2 +
 (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(8*b*x + 8*a) - (8*I*(b*x
```

```
 + a)^2*d^2 + (16*I*b*c*d - 16*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*
a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*d^2*cos(8*b*x + 8*a) - 4*d
^2*cos(4*b*x + 4*a) + 2*I*d^2*sin(8*b*x + 8*a) - 4*I*d^2*sin(4*b*x + 4*a) +
 2*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (4*(b*x + a)^2*d^2 + 8*(b
*c*d - a*d^2)*(b*x + a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))
*cos(8*b*x + 8*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4
*b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*sin
(8*b*x + 8*a) + (-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a
))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (8*I*(b*x +
 a)^2*d^2 + 8*b*c*d - 8*a*d^2 + (16*I*b*c*d - 8*(2*I*a - 1)*d^2)*(b*x + a))
*cos(6*b*x + 6*a) + (8*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (16*I*b*c*d
 - 8*(2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (4*b*c*d + 4*(b*x + a)*d
^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 8*(b*c*
d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d
^2 + 4*I*a*d^2)*sin(8*b*x + 8*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d
^2)*sin(4*b*x + 4*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (8*b*c*d + 8*(b*x + a)*
d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 16*(b*
c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*
d^2 - 8*I*a*d^2)*sin(8*b*x + 8*a) + (-16*I*b*c*d - 16*I*(b*x + a)*d^2 + 16*
I*a*d^2)*sin(4*b*x + 4*a))*dilog(-e^(I*b*x + I*a)) + (8*b*c*d + 8*(b*x + a)
*d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 16*(b
*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)
*d^2 - 8*I*a*d^2)*sin(8*b*x + 8*a) + (-16*I*b*c*d - 16*I*(b*x + a)*d^2 + 16
*I*a*d^2)*sin(4*b*x + 4*a))*dilog(e^(I*b*x + I*a)) + (-2*I*(b*x + a)^2*d^2
 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) - I*d^2 + (-2*I*(b*x + a)^2*d^2 + (-4*
I*b*c*d + 4*I*a*d^2)*(b*x + a) - I*d^2)*cos(8*b*x + 8*a) + (4*I*(b*x + a)^2
*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (2*(
b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(8*b*x + 8*a) - 2*(2
*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a))*log
(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (2*I*(
b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2 + (2*I*(b*x + a)
^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2)*cos(8*b*x + 8*a) + (-4*
I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x
 + 4*a) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(8*b*x
 + 8*a) + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b
*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*I
*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2 + (2*I*(b*x +
a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2)*cos(8*b*x + 8*a) + (-
4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b
*x + 4*a) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(8*b
*x + 8*a) + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4
*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-
2*I*d^2*cos(8*b*x + 8*a) + 4*I*d^2*cos(4*b*x + 4*a) + 2*d^2*sin(8*b*x + 8*a
) - 4*d^2*sin(4*b*x + 4*a) - 2*I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + (8
*I*d^2*cos(8*b*x + 8*a) - 16*I*d^2*cos(4*b*x + 4*a) - 8*d^2*sin(8*b*x + 8*a
```

```
) + 16*d^2*sin(4*b*x + 4*a) + 8*I*d^2)*polylog(3, -e^(I*b*x + I*a)) + (8*I*
d^2*cos(8*b*x + 8*a) - 16*I*d^2*cos(4*b*x + 4*a) - 8*d^2*sin(8*b*x + 8*a) +
 16*d^2*sin(4*b*x + 4*a) + 8*I*d^2)*polylog(3, e^(I*b*x + I*a)) - (8*(b*x +
 a)^2*d^2 - 8*I*b*c*d + 8*I*a*d^2 + (16*b*c*d - (16*a + 8*I)*d^2)*(b*x + a)
)*sin(6*b*x + 6*a) - (8*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (16*b*c*d
 - (16*a - 8*I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-2*I*b^2*cos(8*b*x + 8*a
) + 4*I*b^2*cos(4*b*x + 4*a) + 2*b^2*sin(8*b*x + 8*a) - 4*b^2*sin(4*b*x + 4
*a) - 2*I*b^2))/b
```

Fricas [C] time = 1.11671, size = 5801, normalized size = 30.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (
(-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^4 + (4*I*b*d^2*x + 4*I*b*c*d)*cos(b
*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) - ((4*I*b*d^2*x + 4*I*b*c*d
)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) - ((-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^4 + (4*I
*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a))
- ((4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*co
s(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - ((4*I*b*d^2*x + 4*I*b*
c*d)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - ((-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^4 +
(4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x +
 a)) - ((4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*
d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((-4*I*b*d^2*x -
 4*I*b*c*d)*cos(b*x + a)^4 + (4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^2)*dilo
g(-cos(b*x + a) - I*sin(b*x + a)) - ((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c
^2 + d^2)*cos(b*x + a)^4 - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*
cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*
b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 +
 1)*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) - ((2*b^2*d
^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^4 - (2*b^2*d^2*x^2 + 4
*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x
+ a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*
b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I
*sin(b*x + a) + I) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*c
```

```
os(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*b^2*c*d
*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a
*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) +
2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-I*cos(b*x
 + a) + sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2
*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - ((2*b^2*c^2 - 4*a*b
*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 +
1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) -
 ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4
*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*s
in(b*x + a) + 1/2) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*c
os(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d +
(2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)
*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - 2*((b^2*d^2*x^2
+ 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*
b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) -
I*sin(b*x + a) + I) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3
, cos(b*x + a) + I*sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)
^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 4*(d^2*cos(b*x + a)^4 - d^2
*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 4*(d^2*cos(b*x
 + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 4
*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, -I*cos(b*x + a) + sin
(b*x + a)) + 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, -I*cos(
b*x + a) - sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*poly
log(3, -cos(b*x + a) + I*sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*
x + a)^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^4 -
 b^3*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc (bx+a)^3 \sec (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

3.325 $\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=110

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d\csc(2a+2bx)}{b^2}-\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{2(c+dx)\cot(2a}{}$$

```
[Out] (-4*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Csc[2*a + 2*b*x])/b^2 -
(2*(c + d*x)*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (I*d*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rubi [A] time = 0.106173, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4419, 4185, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d\csc(2a+2bx)}{b^2}-\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{2(c+dx)\cot(2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```



```
[Out] (-4*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Csc[2*a + 2*b*x])/b^2 -
(2*(c + d*x)*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (I*d*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```


Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)\csc^3(2a+2bx)\,dx$$
$$= -\frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{b} + 4\int (c+dx)\,\mathrm{csc}$$
$$= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2bx)\,\mathrm{csc}}{b}$$
$$= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2bx)\,\mathrm{csc}}{b}$$
$$= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2bx)\,\mathrm{csc}}{b}$$

Mathematica [B] time = 2.21126, size = 236, normalized size = 2.15

$$\frac{d\left(i\left(\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)-\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)+2(a+bx)\left(\log\left(1-e^{2i(a+bx)}\right)-\log\left(1+e^{2i(a+bx)}\right)\right)\right)}{b^2} - \frac{d\tan(a+}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] -(d*Cot[a + b*x])/(2*b^2) - (c*Csc[a + b*x]^2)/(2*b) + (d*(2*a - 2*(a + b*x
))*Csc[a + b*x]^2)/(4*b^2) - (2*c*Log[Cos[a + b*x]])/b + (2*c*Log[Sin[a + b
```

```
*x]])/b - (2*a*d*Log[Tan[a + b*x]])/b^2 + (d*(2*(a + b*x)*(Log[1 - E^((2*I)
*(a + b*x))] - Log[1 + E^((2*I)*(a + b*x))]) + I*(PolyLog[2, -E^((2*I)*(a +
 b*x))] - PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (c*Sec[a + b*x]^2)/(2*b)
 + (d*(-2*a + 2*(a + b*x))*Sec[a + b*x]^2)/(4*b^2) - (d*Tan[a + b*x])/(2*b^
2)
```

Maple [B] time = 0.209, size = 325, normalized size = 3.

$$2\,\frac{2\,dxbe^{6\,i(bx+a)}+2\,bce^{6\,i(bx+a)}-ide^{6\,i(bx+a)}+2\,bdxe^{2\,i(bx+a)}+2\,bce^{2\,i(bx+a)}+ide^{2\,i(bx+a)}}{b^2\left(e^{2\,i(bx+a)}-1\right)^2\left(e^{2\,i(bx+a)}+1\right)^2}-2\,\frac{c\ln\left(e^{2\,i(bx+a)}+1\right)}{b}+2\,\frac{c\ln\left(e^{i(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] 2/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)^2*(2*d*x*b*exp(6*I*(b*x+a
))+2*b*c*exp(6*I*(b*x+a))-I*d*exp(6*I*(b*x+a))+2*b*d*x*exp(2*I*(b*x+a))+2*b
*c*exp(2*I*(b*x+a))+I*d*exp(2*I*(b*x+a)))-2/b*c*ln(exp(2*I*(b*x+a))+1)+2/b*
c*ln(exp(I*(b*x+a))-1)+2/b*c*ln(exp(I*(b*x+a))+1)+2/b*d*ln(1-exp(I*(b*x+a))
)*x+2/b^2*d*ln(1-exp(I*(b*x+a)))*a-2*I/b^2*d*polylog(2,exp(I*(b*x+a)))-2/b*
d*ln(exp(2*I*(b*x+a))+1)*x+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b*d*ln(ex
p(I*(b*x+a))+1)*x-2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(
b*x+a))-1)
```

Maxima [B] time = 2.46755, size = 1455, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(8*b*x + 8*a) - 4*(b*d*x + b*c)*cos
(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*sin(8*b*x + 8*a) + (-4*I*b*d*x - 4*I*
b*c)*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2
*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(8*b*x + 8*a) - 4*(b*d*x + b*c)*cos(4*b
*x + 4*a) - (-2*I*b*d*x - 2*I*b*c)*sin(8*b*x + 8*a) - (4*I*b*d*x + 4*I*b*c)
*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(8*b
*x + 8*a) - 4*b*c*cos(4*b*x + 4*a) + 2*I*b*c*sin(8*b*x + 8*a) - 4*I*b*c*sin
```

```
(4*b*x + 4*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(8*b*x + 8*a) - 4*b*d*x*cos(4*b*x + 4*a) + 2*I*b*d*x*sin(8*b*x + 8*a) - 4
*I*b*d*x*sin(4*b*x + 4*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(6*b*x + 6*a) + (4*I*b*d*x + 4*I*b*c -
2*d)*cos(2*b*x + 2*a) - (d*cos(8*b*x + 8*a) - 2*d*cos(4*b*x + 4*a) + I*d*si
n(8*b*x + 8*a) - 2*I*d*sin(4*b*x + 4*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) +
(2*d*cos(8*b*x + 8*a) - 4*d*cos(4*b*x + 4*a) + 2*I*d*sin(8*b*x + 8*a) - 4*I
*d*sin(4*b*x + 4*a) + 2*d)*dilog(-e^(I*b*x + I*a)) + (2*d*cos(8*b*x + 8*a)
- 4*d*cos(4*b*x + 4*a) + 2*I*d*sin(8*b*x + 8*a) - 4*I*d*sin(4*b*x + 4*a) +
2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(8*
b*x + 8*a) + (2*I*b*d*x + 2*I*b*c)*cos(4*b*x + 4*a) + (b*d*x + b*c)*sin(8*b
*x + 8*a) - 2*(b*d*x + b*c)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*
b*c)*cos(8*b*x + 8*a) + (-2*I*b*d*x - 2*I*b*c)*cos(4*b*x + 4*a) - (b*d*x +
b*c)*sin(8*b*x + 8*a) + 2*(b*d*x + b*c)*sin(4*b*x + 4*a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*
b*c)*cos(8*b*x + 8*a) + (-2*I*b*d*x - 2*I*b*c)*cos(4*b*x + 4*a) - (b*d*x +
b*c)*sin(8*b*x + 8*a) + 2*(b*d*x + b*c)*sin(4*b*x + 4*a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 2*(2*b*d*x + 2*b*c - I*d)*sin(6*
b*x + 6*a) - 2*(2*b*d*x + 2*b*c + I*d)*sin(2*b*x + 2*a))/(-I*b^2*cos(8*b*x
+ 8*a) + 2*I*b^2*cos(4*b*x + 4*a) + b^2*sin(8*b*x + 8*a) - 2*b^2*sin(4*b*x
+ 4*a) - I*b^2)
```

Fricas [B] time = 0.797302, size = 3087, normalized size = 28.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*sin(b*x + a)
+ b*c - (-2*I*d*cos(b*x + a)^4 + 2*I*d*cos(b*x + a)^2)*dilog(cos(b*x + a) +
 I*sin(b*x + a)) - (2*I*d*cos(b*x + a)^4 - 2*I*d*cos(b*x + a)^2)*dilog(cos(
b*x + a) - I*sin(b*x + a)) - (-2*I*d*cos(b*x + a)^4 + 2*I*d*cos(b*x + a)^2)
*dilog(I*cos(b*x + a) + sin(b*x + a)) - (2*I*d*cos(b*x + a)^4 - 2*I*d*cos(b
*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (2*I*d*cos(b*x + a)^4 - 2
*I*d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (-2*I*d*cos(b*
x + a)^4 + 2*I*d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (2
*I*d*cos(b*x + a)^4 - 2*I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x
 + a)) - (-2*I*d*cos(b*x + a)^4 + 2*I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a)
 - I*sin(b*x + a)) - 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*
```

```
x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x +
 a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I)
- 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a)^2)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)
*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*((b*d*x + a*d)*
cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x
 + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4
- (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2
*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b
*x + a) - sin(b*x + a) + 1) - 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)*c
os(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*((b*c
- a*d)*cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) - 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d
)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*
cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x
+ a) + I) - 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)
*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 -
(b*c - a*d)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^2*c
os(b*x + a)^4 - b^2*cos(b*x + a)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)\csc(bx+a)^3\sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

3.326 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=23

$$8\text{Unintegrable}\left(\frac{\csc^3(2a+2bx)}{c+dx},x\right)$$

```
[Out] 8*Unintegrable[Csc[2*a + 2*b*x]^3/(c + d*x), x]
```

Rubi [A] time = 0.0871065, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
```

```
[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = 8\int \frac{\csc^3(2a+2bx)}{c+dx}\,dx$$

Mathematica [A] time = 29.3312, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x), x]
```

Maple [A] time = 2.741, size = 0, normalized size = 0.

$$\int \frac{\left(\csc \left(bx+a\right)\right)^{3} \left(\sec \left(bx+a\right)\right)^{3}}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc \left(bx+a\right)^{3} \sec \left(bx+a\right)^{3}}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc (bx+a)^3 \sec (bx+a)^3}{dx+c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

3.327 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$8\text{Unintegrable}\left(\frac{\csc^3(2a+2bx)}{(c+dx)^2},x\right)$$

```
[Out] 8*Unintegrable[Csc[2*a + 2*b*x]^3/(c + d*x)^2, x]
```

Rubi [A] time = 0.088088, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = 8\int \frac{\csc^3(2a+2bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 33.482, size = 0, normalized size = 0.

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Maple [A] time = 4.84, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^3 (\sec(bx+a))^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^3 \sec(bx+a)^3}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc{(bx+a)}^3 \sec{(bx+a)}^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c)^2, x)
```

3.328 $\int x \cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=83

$$\frac{20\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{147b^2}+\frac{4\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{20\sin(a+bx)\sqrt{\cos(a+bx)}}{147b^2}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] (-2*x*Cos[a + b*x]^(7/2))/(7*b) + (20*EllipticF[(a + b*x)/2, 2])/(147*b^2)
+ (20*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(147*b^2) + (4*Cos[a + b*x]^(5/2)*Si
n[a + b*x])/(49*b^2)
```

Rubi [A] time = 0.0535435, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2635, 2641}

$$\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{4\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{20\sin(a+bx)\sqrt{\cos(a+bx)}}{147b^2}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```


```
[Out] (-2*x*Cos[a + b*x]^(7/2))/(7*b) + (20*EllipticF[(a + b*x)/2, 2])/(147*b^2)
+ (20*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(147*b^2) + (4*Cos[a + b*x]^(5/2)*Si
n[a + b*x])/(49*b^2)
```


Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int\cos^{\frac{7}{2}}(a+bx)\,dx}{7b}\\ &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{49b^2}+\frac{10\int\cos^{\frac{3}{2}}(a+bx)\,dx}{49b}\\ &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{20\sqrt{\cos(a+bx)}\sin(a+bx)}{147b^2}+\frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{49b^2}+\frac{10}{}\\ &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{20\sqrt{\cos(a+bx)}\sin(a+bx)}{147b^2}+\frac{4\cos^{\frac{5}{2}}(a+}{}\end{aligned}$$

Mathematica [A] time = 0.33476, size = 73, normalized size = 0.88

$$\frac{40\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sqrt{\cos(a+bx)}(46\sin(a+bx)+6\sin(3(a+bx))-63bx\cos(a+bx)-21bx\cos(3(a+bx)))}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (40*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(-63*b*x*Cos[a + b*x] -
21*b*x*Cos[3*(a + b*x)] + 46*Sin[a + b*x] + 6*Sin[3*(a + b*x)]))/(294*b^2)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x\left(\cos\left(bx+a\right)\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

```
[Out] int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos{(bx+a)}^{\frac{5}{2}}\sin{(bx+a)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(5/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.329 $\int x \cos^{\frac{3}{2}}(a+bx) \sin(a+bx) \, dx$

Optimal. Leaf size=60

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{25b^2}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] (-2*x*Cos[a + b*x]^(5/2))/(5*b) + (12*EllipticE[(a + b*x)/2, 2])/(25*b^2) +
 (4*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(25*b^2)
```

Rubi [A] time = 0.0397231, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2635, 2639}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{25b^2}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(5/2))/(5*b) + (12*EllipticE[(a + b*x)/2, 2])/(25*b^2) +
 (4*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(25*b^2)
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\cos^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}+\frac{6\int\sqrt{\cos(a+bx)}\,dx}{25b}$$
$$= -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}$$

Mathematica [A] time = 0.390156, size = 51, normalized size = 0.85

$$-\frac{2\left(\cos^{\frac{3}{2}}(a+bx)(5bx\cos(a+bx)-2\sin(a+bx))-6E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{25b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*(-6*EllipticE[(a + b*x)/2, 2] + Cos[a + b*x]^(3/2)*(5*b*x*Cos[a + b*x]
- 2*Sin[a + b*x])))/(25*b^2)
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x\left(\cos\left(bx+a\right)\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

```
[Out] int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(3/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

3.330 $\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx$

Optimal. Leaf size=60

$$\frac{4\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{9b^2}+\frac{4\sin(a+bx)\sqrt{\cos(a+bx)}}{9b^2}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] (-2*x*Cos[a + b*x]^(3/2))/(3*b) + (4*EllipticF[(a + b*x)/2, 2])/(9*b^2) + (
4*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(9*b^2)
```

Rubi [A] time = 0.0388699, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2635, 2641}

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}+\frac{4\sin(a+bx)\sqrt{\cos(a+bx)}}{9b^2}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(3/2))/(3*b) + (4*EllipticF[(a + b*x)/2, 2])/(9*b^2) + (
4*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(9*b^2)
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx &= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int\cos^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2}+\frac{2\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{9b}\\ &= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}+\frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.152188, size = 52, normalized size = 0.87

$$\frac{4\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+2\sqrt{\cos(a+bx)}(2\sin(a+bx)-3bx\cos(a+bx))}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (4*EllipticF[(a + b*x)/2, 2] + 2*Sqrt[Cos[a + b*x]]*(-3*b*x*Cos[a + b*x] +
2*Sin[a + b*x]))/(9*b^2)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x\sin(bx+a)\sqrt{\cos(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
```

```
[Out] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos(bx+a)}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\cos(bx+a)}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

3.331 $\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=33

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

```
[Out] (-2*x*Sqrt[Cos[a + b*x]])/b + (4*EllipticE[(a + b*x)/2, 2])/b^2
```

Rubi [A] time = 0.0262149, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3444, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (-2*x*Sqrt[Cos[a + b*x]])/b + (4*EllipticE[(a + b*x)/2, 2])/b^2
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = -\frac{2x\sqrt{\cos(a+bx)}}{b} + \frac{2\int \sqrt{\cos(a+bx)}\,dx}{b}$$
$$= -\frac{2x\sqrt{\cos(a+bx)}}{b} + \frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Mathematica [B] time = 1.7706, size = 181, normalized size = 5.48

$$\frac{4\cos^2\left(\frac{1}{2}(a+bx)\right)^{3/2}\sqrt{\frac{\cos(a+bx)}{(\cos(a+bx)+1)^2}}\sqrt{\frac{1}{\cos(a+bx)+1}}\left(-2\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right),-1\right)+\left(2\,\text{ta}\right.\right.}{b^2\sqrt{\frac{\cos(a+bx}{\cos(a+bx)-}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (4*(Cos[(a + b*x)/2]^2)^(3/2)*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])^2]*Sqrt[
(1 + Cos[a + b*x])^(-1)]*(2*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Se
c[(a + b*x)/2]^2] - 2*EllipticF[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Sec[(a +
 b*x)/2]^2] + Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^2]*(-(b*x) + 2*Tan[(a + b*
x)/2])))/(b^2*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])])
```

Maple [C] time = 0.212, size = 310, normalized size = 9.4

$$-\frac{(bx+2\,i)\left(\left(e^{i(bx+a)}\right)^2+1\right)\sqrt{2}}{b^2e^{i(bx+a)}}\frac{1}{\sqrt{\frac{\left(e^{i(bx+a)}\right)^2+1}{e^{i(bx+a)}}}}-\frac{2\,i\sqrt{2}}{b^2e^{i(bx+a)}}\left(-2\frac{\left(e^{i(bx+a)}\right)^2+1}{\sqrt{\left(\left(e^{i(bx+a)}\right)^2+1\right)e^{i(bx+a)}}}+i\sqrt{2}\sqrt{-i\left(e^{i(bx+a)}+i\right)}\sqrt{i\left(e^{i(bx\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(1/2),x)
```

```
[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)/((exp(I*(b*x+a))^2+1)/exp(I*(b*
x+a)))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a
))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(e
xp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b
```

```
*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))+I*
EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/((exp(I*(b*x
+a))^2+1)/exp(I*(b*x+a)))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)
/exp(I*(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

3.332 $\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=33

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

```
[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

Rubi [A] time = 0.0256239, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3444, 2641}

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{b}$$
$$= \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Mathematica [A] time = 0.164701, size = 33, normalized size = 1.

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int x\sin(bx+a)\,(\cos(bx+a))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin (bx+a)}{\cos (bx+a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

3.333 $\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=60

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}+\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}$$

```
[Out] (2*x)/(3*b*Cos[a + b*x]^(3/2)) + (4*EllipticE[(a + b*x)/2, 2])/(3*b^2) - (4
*Sin[a + b*x])/(3*b^2*Sqrt[Cos[a + b*x]])
```

Rubi [A] time = 0.0360545, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2636, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}+\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```


```
[Out] (2*x)/(3*b*Cos[a + b*x]^(3/2)) + (4*EllipticE[(a + b*x)/2, 2])/(3*b^2) - (4
*Sin[a + b*x])/(3*b^2*Sqrt[Cos[a + b*x]])
```


Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx &= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}+\frac{2\int \sqrt{\cos(a+bx)}\,dx}{3b}\\
&= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.198494, size = 54, normalized size = 0.9

$$\frac{2\left(-\sin(2(a+bx))+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+bx\right)}{3b^2\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

```
[Out] (2*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] - Sin[2*(a + b*x)]
))/(3*b^2*Cos[a + b*x]^(3/2))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int x\sin(bx+a)\left(\cos(bx+a)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

3.334 $\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=60

$$-\frac{4\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}+\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}$$

```
[Out] (2*x)/(5*b*Cos[a + b*x]^(5/2)) - (4*EllipticF[(a + b*x)/2, 2])/(15*b^2) - (
4*Sin[a + b*x])/(15*b^2*Cos[a + b*x]^(3/2))
```

Rubi [A] time = 0.0379388, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2636, 2641}

$$-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}+\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

```
[Out] (2*x)/(5*b*Cos[a + b*x]^(5/2)) - (4*EllipticF[(a + b*x)/2, 2])/(15*b^2) - (
4*Sin[a + b*x])/(15*b^2*Cos[a + b*x]^(3/2))
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx &= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\ &= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{15b}\\ &= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.239905, size = 53, normalized size = 0.88

$$-\frac{2\left(2\cos^{\frac{5}{2}}(a+bx)\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sin(2(a+bx))-3bx\right)}{15b^2\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

```
[Out] (-2*(-3*b*x + 2*Cos[a + b*x]^(5/2)*EllipticF[(a + b*x)/2, 2] + Sin[2*(a + b
*x)]))/(15*b^2*Cos[a + b*x]^(5/2))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int x\sin(bx+a)\,(\cos(bx+a))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

3.335 $\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx$

Optimal. Leaf size=83

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}-\frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)}-\frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}+\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}$$

```
[Out] (2*x)/(7*b*Cos[a + b*x]^(7/2)) + (12*EllipticE[(a + b*x)/2, 2])/(35*b^2) -
(4*Sin[a + b*x])/(35*b^2*Cos[a + b*x]^(5/2)) - (12*Sin[a + b*x])/(35*b^2*Sq
rt[Cos[a + b*x]])
```

Rubi [A] time = 0.0484516, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2636, 2639}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}-\frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)}-\frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}+\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

```
[Out] (2*x)/(7*b*Cos[a + b*x]^(7/2)) + (12*EllipticE[(a + b*x)/2, 2])/(35*b^2) -
(4*Sin[a + b*x])/(35*b^2*Cos[a + b*x]^(5/2)) - (12*Sin[a + b*x])/(35*b^2*Sq
rt[Cos[a + b*x]])
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx &= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx}{7b} \\
&= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{6\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx}{35b} \\
&= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}} + \frac{6\int \sqrt{\cos(a+bx)}\,dx}{35b} \\
&= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} + \frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.275733, size = 65, normalized size = 0.78

$$\frac{-10\sin(2(a+bx)) - 3\sin(4(a+bx)) + 24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right) + 20bx}{70b^2\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

```
[Out] (20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2, 2] - 10*Sin[2*(a + b
*x)] - 3*Sin[4*(a + b*x)])/(70*b^2*Cos[a + b*x]^(7/2))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int x\sin(bx+a)\left(\cos(bx+a)\right)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

3.336 $\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=103

$$-\frac{4\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\sin(a+bx)\sqrt{\sec(a+bx)}}{35b^2}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}+\frac{2x\sec^{\frac{7}{2}}(a}{7b}$$

```
[Out] (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(35*b^
2) + (2*x*Sec[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/
(35*b^2) - (4*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(35*b^2)
```

Rubi [A] time = 0.0620826, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3768, 3771, 2639}

$$-\frac{4\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\sin(a+bx)\sqrt{\sec(a+bx)}}{35b^2}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}+\frac{2x\sec^{\frac{7}{2}}(a}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

```
[Out] (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(35*b^
2) + (2*x*Sec[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/
(35*b^2) - (4*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(35*b^2)
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int\sec^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2} - \frac{6\int\sec^{\frac{3}{2}}(a+bx)\,dx}{35b}$$
$$= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)}\sin(a+bx)}{35b^2} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2} + \frac{6\int\frac{1}{\sqrt{\text{se}}}\cdots}{3\cdots}$$
$$= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)}\sin(a+bx)}{35b^2} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2} + \frac{\left(6\sqrt{\cos}\cdots\right.}{\cdots}$$
$$= \frac{12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{35b^2} + \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)}\text{si}\cdots}{35b^2}$$

Mathematica [A] time = 0.286994, size = 65, normalized size = 0.63

$$\frac{\sec^{\frac{7}{2}}(a+bx)\left(-10\sin(2(a+bx))-3\sin(4(a+bx))+24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+20bx\right)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

```
[Out] (Sec[a + b*x]^(7/2)*(20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2,
2] - 10*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)]))/(70*b^2)
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int x(\sec(bx+a))^{\frac{9}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sec(bx+a)^{\frac{9}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(9/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sec(bx+a)^{\frac{9}{2}}\sin(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

3.337 $\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{15b^2}-\frac{4\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(15*b^
2) + (2*x*Sec[a + b*x]^(5/2))/(5*b) - (4*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(
15*b^2)
```

Rubi [A] time = 0.0488525, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3768, 3771, 2641}

$$-\frac{4\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```


```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(15*b^
2) + (2*x*Sec[a + b*x]^(5/2))/(5*b) - (4*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(
15*b^2)
```


Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx=\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int\sec^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$=\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2}-\frac{2\int\sqrt{\sec(a+bx)}\,dx}{15b}$$
$$=\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2}-\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int\frac{1}{\sqrt{\cos(}}}{15b}$$
$$=-\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\sec^{\frac{3}{2}}(a+bx)\sin}{15b^2}$$

Mathematica [A] time = 0.242782, size = 61, normalized size = 0.76

$$\frac{2\sqrt{\sec(a+bx)}\left(-2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)-2\tan(a+bx)+3bx\sec^2(a+bx)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

```
[Out] (2*Sqrt[Sec[a + b*x]]*(-2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 3*
b*x*Sec[a + b*x]^2 - 2*Tan[a + b*x]))/(15*b^2)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x\left(\sec\left(bx+a\right)\right)^{\frac{7}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sec\left(bx+a\right)^{\frac{7}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(7/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec(bx+a)^{\frac{7}{2}} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

3.338 $\int x \sec^{\frac{5}{2}}(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=80

$$-\frac{4\sin(a+bx)\sqrt{\sec(a+bx)}}{3b^2}+\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b^2)
 + (2*x*Sec[a + b*x]^(3/2))/(3*b) - (4*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(3*
b^2)
```

Rubi [A] time = 0.0525987, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3768, 3771, 2639}

$$-\frac{4\sin(a+bx)\sqrt{\sec(a+bx)}}{3b^2}+\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b^2)
 + (2*x*Sec[a + b*x]^(3/2))/(3*b) - (4*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(3*
b^2)
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx &= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int\sec^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\sec(a+bx)}\sin(a+bx)}{3b^2}+\frac{2\int\frac{1}{\sqrt{\sec(a+bx)}}\,dx}{3b}\\ &= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\sec(a+bx)}\sin(a+bx)}{3b^2}+\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int\sqrt{\cos}}{3b}\\ &= \frac{4\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\sec(a+bx)}\sin(}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.195177, size = 54, normalized size = 0.68

$$\frac{2\sec^{\frac{3}{2}}(a+bx)\left(-\sin(2(a+bx))+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+bx\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (2*Sec[a + b*x]^(3/2)*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2]
 - Sin[2*(a + b*x)]))/(3*b^2)
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int x\left(\sec\left(bx+a\right)\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec(bx+a)^{\frac{5}{2}} \sin(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(5/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec (bx + a)^{\frac{5}{2}} \sin (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

3.339 $\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

```
[Out] (2*x*Sqrt[Sec[a + b*x]])/b - (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2
]*Sqrt[Sec[a + b*x]])/b^2
```

Rubi [A] time = 0.0358781, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4212, 3771, 2641}

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (2*x*Sqrt[Sec[a + b*x]])/b - (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2
]*Sqrt[Sec[a + b*x]])/b^2
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx &= \frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{2\int\sqrt{\sec(a+bx)}\,dx}{b}\\ &= \frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{b}\\ &= \frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.140606, size = 42, normalized size = 0.79

$$\frac{2\sqrt{\sec(a+bx)}\left(bx-2\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (2*(b*x - 2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])*Sqrt[Sec[a + b*x]
])/b^2
```

Maple [F] time = 0.101, size = 0, normalized size = 0.

$$\int x\left(\sec(bx+a)\right)^{\frac{3}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec(bx+a)^{\frac{3}{2}} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(3/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \sec(bx+a)^{\frac{3}{2}} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")

[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

3.340 $\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

```
[Out] (-2*x)/(b*Sqrt[Sec[a + b*x]]) + (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2
, 2]*Sqrt[Sec[a + b*x]])/b^2
```

Rubi [A] time = 0.0366521, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4212, 3771, 2639}

$$\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```


```
[Out] (-2*x)/(b*Sqrt[Sec[a + b*x]]) + (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2
, 2]*Sqrt[Sec[a + b*x]])/b^2
```


Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = -\frac{2x}{b\sqrt{\sec(a+bx)}}+\frac{2\int\frac{1}{\sqrt{\sec(a+bx)}}\,dx}{b}$$
$$= -\frac{2x}{b\sqrt{\sec(a+bx)}}+\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{b}$$
$$= -\frac{2x}{b\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}$$

Mathematica [B] time = 2.21082, size = 132, normalized size = 2.49

$$\frac{2\left(-\frac{2\sec^2\left(\frac{1}{2}(a+bx)\right)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right),-1\right)}{\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}}+2\tan\left(\frac{1}{2}(a+bx)\right)+\frac{2\sec^2\left(\frac{1}{2}(a+bx)\right)E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\middle|-1\right)}{\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}}-bx\right)}{b^2\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (2*(-(b*x) + (2*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sec[(a + b*x)/2]^2)
/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] - (2*EllipticF[ArcSin[Tan[(a + b*x)/
2]], -1]*Sec[(a + b*x)/2]^2)/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] + 2*Tan[
(a + b*x)/2]))/(b^2*Sqrt[Sec[a + b*x]])
```

Maple [C] time = 0.184, size = 310, normalized size = 5.9

$$-\frac{(bx+2i)\left(\left(e^{i(bx+a)}\right)^2+1\right)\sqrt{2}}{b^2e^{i(bx+a)}}\sqrt{\frac{e^{i(bx+a)}}{\left(e^{i(bx+a)}\right)^2+1}}-\frac{2i\sqrt{2}}{b^2e^{i(bx+a)}}\left(-2\frac{\left(e^{i(bx+a)}\right)^2+1}{\sqrt{\left(\left(e^{i(bx+a)}\right)^2+1\right)e^{i(bx+a)}}}+i\sqrt{2}\sqrt{-i\left(e^{i(bx+a)}+i\right)}\sqrt{i\left(e^{i(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)*sec(b*x+a)^(1/2),x)
```

```
[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)*(exp(I*(b*x+a))/(exp(I*(b*x+a))
^2+1))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a
))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(e
xp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b
*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))+I*
EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))))*2^(1/2)*(exp(I*(b*x+
a))/(exp(I*(b*x+a))^2+1))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)
/exp(I*(b*x+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sec(bx+a)}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sin(a+bx)\sqrt{\sec(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)*sqrt(sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sec(bx+a)}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

3.341 $\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$

Optimal. Leaf size=80

$$\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{9b^2}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

```
[Out] (-2*x)/(3*b*Sec[a + b*x]^(3/2)) + (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)
/2, 2]*Sqrt[Sec[a + b*x]])/(9*b^2) + (4*Sin[a + b*x])/(9*b^2*Sqrt[Sec[a + b
*x]])
```

Rubi [A] time = 0.0456498, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3769, 3771, 2641}

$$\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```



```
[Out] (-2*x)/(3*b*Sec[a + b*x]^(3/2)) + (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)
/2, 2]*Sqrt[Sec[a + b*x]])/(9*b^2) + (4*Sin[a + b*x])/(9*b^2*Sqrt[Sec[a + b
*x]])
```


Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx &= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{2\int \sqrt{\sec(a+bx)}\,dx}{9b}\\
&= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{9b}\\
&= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{9b^2}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.247597, size = 63, normalized size = 0.79

$$\frac{\sqrt{\sec(a+bx)}\left(4\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+2\sin(2(a+bx))-6bx\cos^2(a+bx)\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-6*b*x*Cos[a + b*x]^2 + 4*Sqrt[Cos[a + b*x]]*EllipticF
[(a + b*x)/2, 2] + 2*Sin[2*(a + b*x)]))/(9*b^2)
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int x \sin(bx + a) \frac{1}{\sqrt{\sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\sqrt{\sec(bx + a)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(a + bx)}{\sqrt{\sec(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sqrt(sec(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

3.342 $\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

```
[Out] (-2*x)/(5*b*Sec[a + b*x]^(5/2)) + (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(25*b^2) + (4*Sin[a + b*x])/(25*b^2*Sec[a + b*x
]^(3/2))
```

Rubi [A] time = 0.0465426, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3769, 3771, 2639}

$$\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(5*b*Sec[a + b*x]^(5/2)) + (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(25*b^2) + (4*Sin[a + b*x])/(25*b^2*Sec[a + b*x
]^(3/2))
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx &= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}+\frac{2\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\ &= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}+\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}+\frac{6\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx}{25b}\\ &= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}+\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}+\frac{\left(6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{25b}\\ &= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}+\frac{12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{25b^2}+\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [B] time = 8.09831, size = 212, normalized size = 2.65

$$\frac{\cos^2\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec(a+bx)}\left(-12\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right),-1\right)+\left(5(a+bx)\right.\right.}{25b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-(x*Cos[a + b*x])/10 - (x*Cos[3*(a + b*x)])/10 + Sin[a
 + b*x]/(25*b) + Sin[3*(a + b*x)]/(25*b)))/b + (Cos[(a + b*x)/2]^2*Sqrt[Sec
```

```
[a + b*x]]*(12*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Cos[a + b*x]*Se
c[(a + b*x)/2]^4] - 12*EllipticF[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Cos[a +
 b*x]*Sec[(a + b*x)/2]^4] + (-5*a + 5*(a + b*x) - 12*Tan[(a + b*x)/2])*(-1
+ Tan[(a + b*x)/2]^2)))/(25*b^2)
```

Maple [F] time = 0.097, size = 0, normalized size = 0.

$$\int x \sin(bx + a) (\sec(bx + a))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\sec(bx + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(a + bx)}{\sec^{\frac{3}{2}}(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sec(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\sec(bx + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

3.343 $\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=103

$$\frac{20\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{147b^2}+\frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}+\frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

```
[Out] (-2*x)/(7*b*Sec[a + b*x]^(7/2)) + (20*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(147*b^2) + (4*Sin[a + b*x])/(49*b^2*Sec[a + b*
x]^(5/2)) + (20*Sin[a + b*x])/(147*b^2*Sqrt[Sec[a + b*x]])
```

Rubi [A] time = 0.0600873, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3769, 3771, 2641}

$$\frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)}+\frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}+\frac{20\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(7*b*Sec[a + b*x]^(7/2)) + (20*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(147*b^2) + (4*Sin[a + b*x])/(49*b^2*Sec[a + b*
x]^(5/2)) + (20*Sin[a + b*x])/(147*b^2*Sqrt[Sec[a + b*x]])
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{10\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}} + \frac{10\int \sqrt{\sec(a+bx)}\,dx}{147b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}} + \frac{\left(10\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\cdots}}{147b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{20\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{147b^2} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{20\sin\cdots}{147b^2\sqrt{\cdots}}$$

Mathematica [A] time = 0.339256, size = 89, normalized size = 0.86

$$\frac{\sqrt{\sec(a+bx)}\left(80\sqrt{\cos(a+bx)}\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) + 52\sin(2(a+bx)) + 6\sin(4(a+bx)) - 84bx\cos(2(a+bx)) - 21\cdots\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-63*b*x - 84*b*x*Cos[2*(a + b*x)] - 21*b*x*Cos[4*(a +
b*x)] + 80*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 52*Sin[2*(a + b*x
)] + 6*Sin[4*(a + b*x)]))/(588*b^2)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x\sin\left(bx+a\right)\left(\sec\left(bx+a\right)\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin\left(bx+a\right)}{\sec\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(bx + a)}{\sec(bx + a)^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

3.344 $\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{20\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{147b^2}+\frac{4\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{49b^2}+\frac{20\sqrt{\sin(a+bx)}\cos(a+bx)}{147b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] (-20*EllipticF[(a - Pi/2 + b*x)/2, 2])/(147*b^2) + (20*Cos[a + b*x]*Sqrt[Si
n[a + b*x]])/(147*b^2) + (4*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(49*b^2) + (2*
x*Sin[a + b*x]^(7/2))/(7*b)
```

Rubi [A] time = 0.0442303, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2635, 2641}

$$-\frac{20F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{4\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{49b^2}+\frac{20\sqrt{\sin(a+bx)}\cos(a+bx)}{147b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

```
[Out] (-20*EllipticF[(a - Pi/2 + b*x)/2, 2])/(147*b^2) + (20*Cos[a + b*x]*Sqrt[Si
n[a + b*x]])/(147*b^2) + (4*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(49*b^2) + (2*
x*Sin[a + b*x]^(7/2))/(7*b)
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx=\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int\sin^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$=\frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{10\int\sin^{\frac{3}{2}}(a+bx)\,dx}{49b}$$
$$=\frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2}+\frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{10\int\ldots}{\ldots}$$
$$=-\frac{20F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{147b^2}+\frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2}+\frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2}\ldots$$

Mathematica [A] time = 0.54583, size = 67, normalized size = 0.76

$$\frac{40\text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi),2\right)+\sqrt{\sin(a+bx)}\left(84bx\sin^3(a+bx)+46\cos(a+bx)-6\cos(3(a+bx))\right)}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

```
[Out] (40*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*(46*Cos[a + b*
x] - 6*Cos[3*(a + b*x)] + 84*b*x*Sin[a + b*x]^3))/(294*b^2)
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\left(\sin(bx+a)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

3.345 $\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{4\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(25*b^2) + (4*Cos[a + b*x]*Sin[a + b
*x]^(3/2))/(25*b^2) + (2*x*Sin[a + b*x]^(5/2))/(5*b)
```

Rubi [A] time = 0.0323581, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2635, 2639}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{4\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(25*b^2) + (4*Cos[a + b*x]*Sin[a + b
*x]^(3/2))/(25*b^2) + (2*x*Sin[a + b*x]^(5/2))/(5*b)
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx &= \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int\sin^{\frac{5}{2}}(a+bx)\,dx}{5b}\\ &= \frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}-\frac{6\int\sqrt{\sin(a+bx)}\,dx}{25b}\\ &= -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{25b^2}+\frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}\end{aligned}$$

Mathematica [C] time = 0.925856, size = 108, normalized size = 1.66

$$\frac{\sqrt{\sin(a+bx)}\left(4\tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2\sin(2(a+bx)\right.}{25b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Sin[a + b*x]]*(5*b*x - 5*b*x*Cos[2*(a + b*x)] + 2*Sin[2*(a + b*x)] -
12*Tan[(a + b*x)/2] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^
2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(25*b^2)
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\left(\sin(bx+a)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

3.346 $\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx$

Optimal. Leaf size=65

$$-\frac{4\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{9b^2}+\frac{4\sqrt{\sin(a+bx)}\cos(a+bx)}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] (-4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(9*b^2) + (4*Cos[a + b*x]*Sqrt[Sin[a
+ b*x]])/(9*b^2) + (2*x*Sin[a + b*x]^(3/2))/(3*b)
```

Rubi [A] time = 0.0315743, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2635, 2641}

$$-\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{4\sqrt{\sin(a+bx)}\cos(a+bx)}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(9*b^2) + (4*Cos[a + b*x]*Sqrt[Sin[a
+ b*x]])/(9*b^2) + (2*x*Sin[a + b*x]^(3/2))/(3*b)
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx &= \frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int\sin^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= \frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx}{9b}\\ &= -\frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{9b^2}+\frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.188767, size = 56, normalized size = 0.86

$$\frac{4\text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi),2\right)+2\sqrt{\sin(a+bx)}(3bx\sin(a+bx)+2\cos(a+bx))}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

```
[Out] (4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + 2*Sqrt[Sin[a + b*x]]*(2*Cos[a + b*
x] + 3*b*x*Sin[a + b*x]))/(9*b^2)
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\sqrt{\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

```
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\sqrt{\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{\sin(a+bx)}\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(sin(a + b*x))*cos(a + b*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\sqrt{\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

3.347 $\int \frac{x \cos(a+bx)}{\sqrt{\sin(a+bx)}} dx$

Optimal. Leaf size=38

$$\frac{2x\sqrt{\sin(a+bx)}}{b} - \frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/b^2 + (2*x*Sqrt[Sin[a + b*x]])/b
```

Rubi [A] time = 0.0222333, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3443, 2639}

$$\frac{2x\sqrt{\sin(a+bx)}}{b} - \frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/b^2 + (2*x*Sqrt[Sin[a + b*x]])/b
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \frac{2x\sqrt{\sin(a+bx)}}{b} - \frac{2\int \sqrt{\sin(a+bx)}\,dx}{b}$$
$$= -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2} + \frac{2x\sqrt{\sin(a+bx)}}{b}$$

Mathematica [C] time = 1.18553, size = 86, normalized size = 2.26

$$\frac{2\sqrt{\sin(a+bx)}\left(2\tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-6\tan\left(\frac{1}{2}(a+b\right.\right.}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*Sqrt[Sin[a + b*x]]*(3*b*x - 6*Tan[(a + b*x)/2] + 2*Hypergeometric2F1[1/2
, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2])
)/(3*b^2)
```

Maple [C] time = 0.204, size = 308, normalized size = 8.1

$$\frac{-i\,(bx+2\,i)\left(\left(e^{i(bx+a)}\right)^2-1\right)\sqrt{2}}{b^2e^{i(bx+a)}}\frac{1}{\sqrt{\frac{-i\left(\left(e^{i(bx+a)}\right)^2-1\right)}{e^{i(bx+a)}}}}-2\frac{\sqrt{2}\sqrt{-i\left(\left(e^{i(bx+a)}\right)^2-1\right)e^{i(bx+a)}}}{b^2e^{i(bx+a)}}\left(\frac{2\,i\left(i-i\left(e^{i(bx+a)}\right)^2\right)}{\sqrt{e^{i(bx+a)}\left(i-i\left(e^{i(bx+a)}\right)^2\right)}}-\frac{\sqrt{e^{i(bx}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(1/2),x)
```

```
[Out] -I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(
I*(b*x+a)))^(1/2)/exp(I*(b*x+a))-2/b^2*(2*I*(I-I*exp(I*(b*x+a))^2)/(exp(I*(
b*x+a))*(I-I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*(b
*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(-I*exp(I*(b*x+a))^3+I*exp(I*(b*x+a
)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+EllipticF((ex
p(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(I
*(b*x+a)))^(1/2)*(-I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+
```

a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sqrt(sin(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

3.348 $\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{4\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/b^2 - (2*x)/(b*Sqrt[Sin[a + b*x]])
```

Rubi [A] time = 0.0229804, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3443, 2641}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/b^2 - (2*x)/(b*Sqrt[Sin[a + b*x]])
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{b\sqrt{\sin(a+bx)}} + \frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{b}$$
$$= \frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2} - \frac{2x}{b\sqrt{\sin(a+bx)}}$$

Mathematica [A] time = 0.181046, size = 37, normalized size = 0.97

$$\frac{2\left(-2\text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi),2\right)-\frac{bx}{\sqrt{\sin(a+bx)}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

```
[Out] (2*(-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] - (b*x)/Sqrt[Sin[a + b*x]]))/b^2
```

Maple [F] time = 0.098, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\,(\sin(bx+a))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos{(a+bx)}}{\sin^{\frac{3}{2}}{(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sin(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos{(bx+a)}}{\sin{(bx+a)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

3.349 $\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=65

$$-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/(3*b^2) - (2*x)/(3*b*Sin[a + b*x]^(3/
2)) - (4*Cos[a + b*x])/(3*b^2*Sqrt[Sin[a + b*x]])
```

Rubi [A] time = 0.0319102, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2636, 2639}

$$-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```


```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/(3*b^2) - (2*x)/(3*b*Sin[a + b*x]^(3/
2)) - (4*Cos[a + b*x])/(3*b^2*Sqrt[Sin[a + b*x]])
```


Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= -\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2\int \sqrt{\sin(a+bx)}\,dx}{3b}\\
&= -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b^2}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.184205, size = 56, normalized size = 0.86

$$
-\frac{2\left(\sin(2(a+bx))-2\sin^{\frac{3}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+bx\right)}{3b^2\sin^{\frac{3}{2}}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

```
[Out] (-2*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2) + Sin[2
*(a + b*x)]))/(3*b^2*Sin[a + b*x]^(3/2))
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$
\int x\cos(bx+a)\left(\sin(bx+a)\right)^{-\frac{5}{2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

[Out] int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

3.350 $\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=65

$$\frac{4\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{15b^2}-\frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(15*b^2) - (2*x)/(5*b*Sin[a + b*x]^(5/
2)) - (4*Cos[a + b*x])/(15*b^2*Sin[a + b*x]^(3/2))
```

Rubi [A] time = 0.0339171, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2636, 2641}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(15*b^2) - (2*x)/(5*b*Sin[a + b*x]^(5/
2)) - (4*Cos[a + b*x])/(15*b^2*Sin[a + b*x]^(3/2))
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$

$$= -\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)} + \frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{15b}$$

$$= \frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{15b^2} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}$$

Mathematica [A] time = 0.232829, size = 57, normalized size = 0.88

$$-\frac{2\left(2\sin^{\frac{5}{2}}(a+bx)\text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi),2\right)+\sin(2(a+bx))+3bx\right)}{15b^2\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

```
[Out] (-2*(3*b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(5/2) + Sin
[2*(a + b*x)]))/(15*b^2*Sin[a + b*x]^(5/2))
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\left(\sin(bx+a)\right)^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

```
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

3.351 $\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$

Optimal. Leaf size=88

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(35*b^2) - (2*x)/(7*b*Sin[a + b*x]^(
7/2)) - (4*Cos[a + b*x])/(35*b^2*Sin[a + b*x]^(5/2)) - (12*Cos[a + b*x])/(3
5*b^2*Sqrt[Sin[a + b*x]])
```

Rubi [A] time = 0.0443534, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2636, 2639}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(35*b^2) - (2*x)/(7*b*Sin[a + b*x]^(
7/2)) - (4*Cos[a + b*x])/(35*b^2*Sin[a + b*x]^(5/2)) - (12*Cos[a + b*x])/(3
5*b^2*Sqrt[Sin[a + b*x]])
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx &= -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}+\frac{2\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx}{7b}\\ &= -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}+\frac{6\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{35b}\\ &= -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}-\frac{6\int \sqrt{\sin(a+bx)}\,dx}{35b}\\ &= -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{35b^2}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.299497, size = 73, normalized size = 0.83

$$-\frac{2\left(\sin(2(a+bx))+6\sin^3(a+bx)\cos(a+bx)-6\sin^{\frac{7}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+5bx\right)}{35b^2\sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

```
[Out] (-2*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE[(-2*a + Pi - 2*b*x
)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2*Sin[a + b*x]^(7/2))
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\left(\sin(bx+a)\right)^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

```
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

3.352 $\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx$

Optimal. Leaf size=108

$$-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{2x\csc$$

```
[Out] (-12*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(35*b^2) - (4*Cos[a + b*x]*Csc[a + b*
x]^(5/2))/(35*b^2) - (2*x*Csc[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Csc[a + b*x]
]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(35*b^2)
```

Rubi [A] time = 0.0576411, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3768, 3771, 2639}

$$-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{2x\csc$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

```
[Out] (-12*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(35*b^2) - (4*Cos[a + b*x]*Csc[a + b*
x]^(5/2))/(35*b^2) - (2*x*Csc[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Csc[a + b*x]
]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(35*b^2)
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int\csc^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}+\frac{6\int\csc^{\frac{3}{2}}(a+bx)\,dx}{35b}$$
$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}-\frac{6\int}{}$$
$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}-\frac{(6}{}$$
$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}-\frac{12}{}$$

Mathematica [A] time = 0.272145, size = 73, normalized size = 0.68

$$-\frac{2\csc^{\frac{7}{2}}(a+bx)\left(\sin(2(a+bx))+6\sin^3(a+bx)\cos(a+bx)-6\sin^{\frac{7}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+5bx\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

```
[Out] (-2*Csc[a + b*x]^(7/2)*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE
[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2)
```

Maple [F] time = 0.116, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\,(\csc(bx+a))^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

```
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx + a) \csc(bx + a)^{\frac{9}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

3.353 $\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{15b^2}-\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] (-4*Cos[a + b*x]*Csc[a + b*x]^(3/2))/(15*b^2) - (2*x*Csc[a + b*x]^(5/2))/(5
*b) + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(15*b^2)
```

Rubi [A] time = 0.0443634, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3768, 3771, 2641}

$$-\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

```
[Out] (-4*Cos[a + b*x]*Csc[a + b*x]^(3/2))/(15*b^2) - (2*x*Csc[a + b*x]^(5/2))/(5
*b) + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(15*b^2)
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\csc^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\sqrt{\csc(a+bx)}\,dx}{15b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\frac{}{\sqrt{}}}{15b}$$
$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right.}{15b^2}$$

Mathematica [A] time = 0.297869, size = 65, normalized size = 0.76

$$-\frac{2\sqrt{\csc(a+bx)}\left(2\sqrt{\sin(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi),2\right)+2\cot(a+bx)+3bx\csc^2(a+bx)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

```
[Out] (-2*Sqrt[Csc[a + b*x]]*(2*Cot[a + b*x] + 3*b*x*Csc[a + b*x]^2 + 2*EllipticF
[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]))/(15*b^2)
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int x\cos(bx+a)(\csc(bx+a))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

```
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos\left(bx+a\right)\csc\left(bx+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

3.354 $\int x \cos(a+bx) \csc^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=85

$$-\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] (-4*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(3*b^2) - (2*x*Csc[a + b*x]^(3/2))/(3*
b) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*
x]])/(3*b^2)
```

Rubi [A] time = 0.0401547, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3768, 3771, 2639}

$$-\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

```
[Out] (-4*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(3*b^2) - (2*x*Csc[a + b*x]^(3/2))/(3*
b) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*
x]])/(3*b^2)
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int\csc^{\frac{3}{2}}(a+bx)\,dx}{3b}$$

$$= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int\frac{1}{\sqrt{\csc(a+bx)}}\,dx}{3b}$$

$$= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\sqrt{}}{3b}$$

$$= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\right|}{3b^2}$$

Mathematica [A] time = 0.184442, size = 56, normalized size = 0.66

$$-\frac{2\csc^{\frac{3}{2}}(a+bx)\left(\sin(2(a+bx))-2\sin^{\frac{3}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+bx\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

```
[Out] (-2*Csc[a + b*x]^(3/2)*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a +
 b*x]^(3/2) + Sin[2*(a + b*x)]))/(3*b^2)
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int x\cos(bx+a)(\csc(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

3.355 $\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=58

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b^2}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

```
[Out] (-2*x*Sqrt[Csc[a + b*x]])/b + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b
*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rubi [A] time = 0.0320025, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4213, 3771, 2641}

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[Csc[a + b*x]])/b + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b
*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx &= -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{2\int\sqrt{\csc(a+bx)}\,dx}{b}\\ &= -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx}{b}\\ &= -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{4\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.148038, size = 46, normalized size = 0.79

$$-\frac{2\sqrt{\csc(a+bx)}\left(2\sqrt{\sin(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi),2\right)+bx\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Csc[a + b*x]]*(b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Si
n[a + b*x]]))/b^2
```

Maple [F] time = 0.109, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\left(\csc(bx+a)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx+a)\csc(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \cos(bx+a)\csc(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

3.356 $\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=58

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

```
[Out] (2*x)/(b*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 +
b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rubi [A] time = 0.0319711, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4213, 3771, 2639}

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*x)/(b*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 +
b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx &= \frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{2\int\frac{1}{\sqrt{\csc(a+bx)}}\,dx}{b}\\
&= \frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\sqrt{\sin(a+bx)}\,dx}{b}\\
&= \frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}
\end{aligned}$$

Mathematica [C] time = 0.729526, size = 106, normalized size = 1.83

$$\frac{4\sin\left(\frac{1}{2}(a+bx)\right)\cos\left(\frac{1}{2}(a+bx)\right)\sqrt{\csc(a+bx)}\left(2\tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\text{t}\right.\right.}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (4*Cos[(a + b*x)/2]*Sqrt[Csc[a + b*x]]*Sin[(a + b*x)/2]*(3*b*x - 6*Tan[(a +
 b*x)/2] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec
[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(3*b^2)
```

Maple [C] time = 0.212, size = 308, normalized size = 5.3

$$\frac{-i\,(bx+2\,i)\left(\left(e^{i(bx+a)}\right)^2-1\right)\sqrt{2}}{b^2e^{i(bx+a)}}\sqrt{\frac{ie^{i(bx+a)}}{\left(e^{i(bx+a)}\right)^2-1}}-2\,\frac{\sqrt{2}\sqrt{i\left(\left(e^{i(bx+a)}\right)^2-1\right)e^{i(bx+a)}}}{b^2e^{i(bx+a)}}\left(\frac{-2\,i\left(-i+i\left(e^{i(bx+a)}\right)^2\right)}{\sqrt{e^{i(bx+a)}\left(-i+i\left(e^{i(bx+a)}\right)^2\right)}}-\frac{\sqrt{e^{i}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(1/2),x)
```

```
[Out] -I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*x
+a))^2-1))^(1/2)/exp(I*(b*x+a))-2/b^2*(-2*I*(-I+I*exp(I*(b*x+a))^2)/(exp(I*
(b*x+a))*(-I+I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*
(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(I*exp(I*(b*x+a))^3-I*exp(I*(b*x+
a)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+EllipticF((e
xp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*x
+a))^2-1))^(1/2)*(I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\sqrt{\csc(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cos(a + b*x)*sqrt(csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(bx+a)\sqrt{\csc(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

3.357 $\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=85

$$-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{9b^2}+\frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}}+\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}$$

```
[Out] (2*x)/(3*b*Csc[a + b*x]^(3/2)) + (4*Cos[a + b*x])/(9*b^2*Sqrt[Csc[a + b*x]]
) - (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x
]])/(9*b^2)
```

Rubi [A] time = 0.0442773, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3769, 3771, 2641}

$$\frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```


```
[Out] (2*x)/(3*b*Csc[a + b*x]^(3/2)) + (4*Cos[a + b*x])/(9*b^2*Sqrt[Csc[a + b*x]]
) - (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x
]])/(9*b^2)
```


Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}\,dx}{3b} \\
&= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{2\int \sqrt{\csc(a+bx)}\,dx}{9b} \\
&= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{9b} \\
&= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.244726, size = 65, normalized size = 0.76

$$\frac{2\sqrt{\csc(a+bx)}\left(2\sqrt{\sin(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi),2\right)+3bx\sin^2(a+bx)+\sin(2(a+bx))\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*Sqrt[Csc[a + b*x]]*(2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b
*x]] + 3*b*x*Sin[a + b*x]^2 + Sin[2*(a + b*x)]))/(9*b^2)
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\,\frac{1}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sqrt(csc(a + b*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

3.358 $\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=85

$$\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}$$

```
[Out] (2*x)/(5*b*Csc[a + b*x]^(5/2)) + (4*Cos[a + b*x])/(25*b^2*Csc[a + b*x]^(3/2
)) - (12*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(25*b^2)
```

Rubi [A] time = 0.0437454, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3769, 3771, 2639}

$$\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Csc[a + b*x]^(5/2)) + (4*Cos[a + b*x])/(25*b^2*Csc[a + b*x]^(3/2
)) - (12*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(25*b^2)
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{5}{2}}(a+bx)}\,dx}{5b} \\
&= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{6\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx}{25b} \\
&= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{\left(6\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx}{25b} \\
&= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{12\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{25b^2}
\end{aligned}
$$

Mathematica [C] time = 0.997851, size = 114, normalized size = 1.34

$$
\frac{\tan\left(\frac{1}{2}(a+bx)\right)\left(4\sqrt{2}\sqrt{\frac{1}{\cos(a+bx)+1}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+10bx\sin(a+bx)+5bx\sin(2(a
}{25b^2\sqrt{\csc(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

```
[Out] ((-10 + 4*Cos[a + b*x] + 2*Cos[2*(a + b*x)] + 4*Sqrt[2]*Sqrt[(1 + Cos[a + b
*x])^(-1)]*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2] + 10*b*x*S
```

```
in[a + b*x] + 5*b*x*Sin[2*(a + b*x)])*Tan[(a + b*x)/2])/(25*b^2*Sqrt[Csc[a
+ b*x]])
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int x\cos(bx+a)\left(\csc(bx+a)\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/csc(a + b*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

3.359 $\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=108

$$-\frac{20\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{147b^2}+\frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)}+\frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}}+\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}$$

```
[Out] (2*x)/(7*b*Csc[a + b*x]^(7/2)) + (4*Cos[a + b*x])/(49*b^2*Csc[a + b*x]^(5/2
)) + (20*Cos[a + b*x])/(147*b^2*Sqrt[Csc[a + b*x]]) - (20*Sqrt[Csc[a + b*x]
]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(147*b^2)
```

Rubi [A] time = 0.0596252, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3769, 3771, 2641}

$$\frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)}+\frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}}-\frac{20\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Csc[a + b*x]^(7/2)) + (4*Cos[a + b*x])/(49*b^2*Csc[a + b*x]^(5/2
)) + (20*Cos[a + b*x])/(147*b^2*Sqrt[Csc[a + b*x]]) - (20*Sqrt[Csc[a + b*x]
]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(147*b^2)
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} - \frac{10\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{10\int \sqrt{\csc(a+bx)}\,dx}{147b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{\left(10\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{}{}}{147b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{20\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\right|}{147b^2}$$

Mathematica [A] time = 0.420339, size = 93, normalized size = 0.86

$$\frac{\sqrt{\csc(a+bx)}\left(80\sqrt{\sin(a+bx)}\text{EllipticF}\left(\frac{1}{4}(-2a-2bx+\pi),2\right)+52\sin(2(a+bx))-6\sin(4(a+bx))-84bx\cos(2(a+\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(63*b*x - 84*b*x*Cos[2*(a + b*x)] + 21*b*x*Cos[4*(a + b
*x)] + 80*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]] + 52*Sin[2
*(a + b*x)] - 6*Sin[4*(a + b*x)]))/(588*b^2)
```

Maple [F] time = 0.099, size = 0, normalized size = 0.

$$\int x\cos\left(bx+a\right)\left(\csc\left(bx+a\right)\right)^{-\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos\left(bx+a\right)}{\csc\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

3.360 $\int x\csc(x)\sin(3x)\,dx$

Optimal. Leaf size=31

$$\frac{x^2}{2}-\frac{\sin^2(x)}{4}+\frac{3\cos^2(x)}{4}+2x\sin(x)\cos(x)$$

```
[Out] x^2/2 + (3*Cos[x]^2)/4 + 2*x*Cos[x]*Sin[x] - Sin[x]^2/4
```

Rubi [A] time = 0.0412594, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4431, 3310, 30}

$$\frac{x^2}{2}-\frac{\sin^2(x)}{4}+\frac{3\cos^2(x)}{4}+2x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sin[3*x],x]
```

```
[Out] x^2/2 + (3*Cos[x]^2)/4 + 2*x*Cos[x]*Sin[x] - Sin[x]^2/4
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\csc(x)\sin(3x)\,dx &= \int \left(3x\cos^2(x) - x\sin^2(x)\right)\,dx\\
&= 3\int x\cos^2(x)\,dx - \int x\sin^2(x)\,dx\\
&= \frac{3\cos^2(x)}{4} + 2x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} - \frac{\int x\,dx}{2} + \frac{3\int x\,dx}{2}\\
&= \frac{x^2}{2} + \frac{3\cos^2(x)}{4} + 2x\cos(x)\sin(x) - \frac{\sin^2(x)}{4}
\end{aligned}$$

Mathematica [A] time = 0.0152247, size = 22, normalized size = 0.71

$$\frac{x^2}{2} + x\sin(2x) + \frac{1}{2}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sin[3*x],x]
```

```
[Out] x^2/2 + Cos[2*x]/2 + x*Sin[2*x]
```

Maple [A] time = 0.048, size = 26, normalized size = 0.8

$$4\,x\,(1/2\,\cos(x)\sin(x) + x/2) - \frac{3\,x^2}{2} - (\sin(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sin(3*x),x)
```

```
[Out] 4*x*(1/2*cos(x)*sin(x)+1/2*x)-3/2*x^2-sin(x)^2
```

Maxima [A] time = 0.990269, size = 24, normalized size = 0.77

$$\frac{1}{2}\,x^2 + x\sin(2\,x) + \frac{1}{2}\,\cos(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

Fricas [A] time = 0.518457, size = 54, normalized size = 1.74

$$2\,x\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x^{2}+\cos\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 2*x*cos(x)*sin(x) + 1/2*x^2 + cos(x)^2
```

Sympy [A] time = 6.2343, size = 37, normalized size = 1.19

$$-x^{2}\sin^{2}\left(x\right)-x^{2}\cos^{2}\left(x\right)+\frac{3x^{2}}{2}+2x\sin\left(x\right)\cos\left(x\right)-\sin^{2}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x)
```

```
[Out] -x**2*sin(x)**2 - x**2*cos(x)**2 + 3*x**2/2 + 2*x*sin(x)*cos(x) - sin(x)**2
```

Giac [A] time = 1.14296, size = 24, normalized size = 0.77

$$\frac{1}{2}\,x^{2}+x\sin\left(2\,x\right)+\frac{1}{2}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

3.361 $\int (c+dx)^4 \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=131

$$\frac{3}{2}d^3\sin^2(x)(c+dx)-\frac{9}{2}d^3\cos^2(x)(c+dx)-6d^2\sin(x)\cos(x)(c+dx)^2+\frac{(c+dx)^5}{5d}-d(c+dx)^3-d\sin^2(x)(c+dx)^3+3$$

```
[Out] (3*d^4*x)/2 - d*(c + d*x)^3 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*Cos[x]^2
)/2 + 3*d*(c + d*x)^3*Cos[x]^2 + 3*d^4*Cos[x]*Sin[x] - 6*d^2*(c + d*x)^2*Co
s[x]*Sin[x] + 2*(c + d*x)^4*Cos[x]*Sin[x] + (3*d^3*(c + d*x)*Sin[x]^2)/2 -
d*(c + d*x)^3*Sin[x]^2
```

Rubi [A] time = 0.188099, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{3}{2}d^3\sin^2(x)(c+dx)-\frac{9}{2}d^3\cos^2(x)(c+dx)-6d^2\sin(x)\cos(x)(c+dx)^2+\frac{(c+dx)^5}{5d}-d(c+dx)^3-d\sin^2(x)(c+dx)^3+3$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```



```
[Out] (3*d^4*x)/2 - d*(c + d*x)^3 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*Cos[x]^2
)/2 + 3*d*(c + d*x)^3*Cos[x]^2 + 3*d^4*Cos[x]*Sin[x] - 6*d^2*(c + d*x)^2*Co
s[x]*Sin[x] + 2*(c + d*x)^4*Cos[x]*Sin[x] + (3*d^3*(c + d*x)*Sin[x]^2)/2 -
d*(c + d*x)^3*Sin[x]^2
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^4\csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)^4\cos^2(x)-(c+dx)^4\sin^2(x)\right)\,dx\\
&= 3\int (c+dx)^4\cos^2(x)\,dx-\int (c+dx)^4\sin^2(x)\,dx\\
&= 3d(c+dx)^3\cos^2(x)+2(c+dx)^4\cos(x)\sin(x)-d(c+dx)^3\sin^2(x)-\frac{1}{2}\int (c+dx)^4\,dx+\frac{3}{2}\\
&= \frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)-6d^2(c+dx)^2\cos(x)\sin(x)+2(c\\
&= -d(c+dx)^3+\frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)+3d^4\cos(x)\sin(x)\\
&= \frac{3d^4x}{2}-d(c+dx)^3+\frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)+3d^4\cos(x)\,\mathrm{s}
\end{aligned}
$$

Mathematica [A] time = 0.218675, size = 154, normalized size = 1.18

$$2c^2d^2x^3+\frac{1}{2}\sin(2x)\left(6c^2d^2\left(2x^2-1\right)+8c^3dx+2c^4+4cd^3x\left(2x^2-3\right)+d^4\left(2x^4-6x^2+3\right)\right)+d\cos(2x)\left(6c^2dx+2c^3+3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

```
[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + d*(2*c^3 +
6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d^2*(-1 + 2*x^2))*Cos[2*x] + ((2*c^4 +
 8*c^3*d*x + 4*c*d^3*x*(-3 + 2*x^2) + 6*c^2*d^2*(-1 + 2*x^2) + d^4*(3 - 6*x
^2 + 2*x^4))*Sin[2*x])/2
```

Maple [B] time = 0.066, size = 260, normalized size = 2.

$$4\,d^4\left(x^4\,(1/2\,\cos(x)\sin(x)+x/2)+x^3\,(\cos(x))^2-3\,x^2\,(1/2\,\cos(x)\sin(x)+x/2)-3/2\,x\,(\cos(x))^2+3/4\,\cos(x)\sin(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(x)*sin(3*x),x)
```

```
[Out] 4*d^4*(x^4*(1/2*cos(x)*sin(x)+1/2*x)+x^3*cos(x)^2-3*x^2*(1/2*cos(x)*sin(x)+
1/2*x)-3/2*x*cos(x)^2+3/4*cos(x)*sin(x)+3/4*x+x^3-2/5*x^5)+16*c*d^3*(x^3*(1
/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin(x)+1/2*x)+3/
8*x^2+3/8*sin(x)^2-3/8*x^4)+24*c^2*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x
*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)-1/5*d^4*x^5+16*c^3*d*(x*(1/2*cos
(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-c*d^3*x^4+4*c^4*(1/2*cos(x)*sin(x)+
1/2*x)-2*c^2*d^2*x^3-2*c^3*d*x^2-c^4*x
```

Maxima [A] time = 1.04852, size = 197, normalized size = 1.5

$$2\left(x^2+2\,x\sin(2\,x)+\cos(2\,x)\right)c^3d+\left(2\,x^3+6\,x\cos(2\,x)+3\left(2\,x^2-1\right)\sin(2\,x)\right)c^2d^2+\left(x^4+3\left(2\,x^2-1\right)\cos(2\,x)+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^3*d + (2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
- 1)*sin(2*x))*c^2*d^2 + (x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 - 3*x)*si
n(2*x))*c*d^3 + 1/10*(2*x^5 + 10*(2*x^3 - 3*x)*cos(2*x) + 5*(2*x^4 - 6*x^2
+ 3)*sin(2*x))*d^4 + c^4*(x + sin(2*x))
```

Fricas [A] time = 0.515347, size = 419, normalized size = 3.2

$$\frac{1}{5}\,d^4x^5+cd^3x^4+2\left(c^2d^2-d^4\right)x^3+2\left(c^3d-3\,cd^3\right)x^2+2\left(2\,d^4x^3+6\,cd^3x^2+2\,c^3d-3\,cd^3+3\left(2\,c^2d^2-d^4\right)x\right)\cos(x)^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*(c^2*d^2 - d^4)*x^3 + 2*(c^3*d - 3*c*d^3)*x^2 +
 2*(2*d^4*x^3 + 6*c*d^3*x^2 + 2*c^3*d - 3*c*d^3 + 3*(2*c^2*d^2 - d^4)*x)*co
s(x)^2 + (2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 - 6*c^2*d^2 + 3*d^4 + 6*(2*c^2*d^
2 - d^4)*x^2 + 4*(2*c^3*d - 3*c*d^3)*x)*cos(x)*sin(x) + (c^4 - 6*c^2*d^2 +
3*d^4)*x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(x)*sin(3*x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13595, size = 225, normalized size = 1.72

$$\frac{1}{5}d^4x^5 + cd^3x^4 + 2c^2d^2x^3 + 2c^3dx^2 + c^4x + \left(2d^4x^3 + 6cd^3x^2 + 6c^2d^2x - 3d^4x + 2c^3d - 3cd^3\right)\cos(2x) + \frac{1}{2}\left(2d^4x^4 + 8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*c^2*d^2*x^3 + 2*c^3*d*x^2 + c^4*x + (2*d^4*x^3
+ 6*c*d^3*x^2 + 6*c^2*d^2*x - 3*d^4*x + 2*c^3*d - 3*c*d^3)*cos(2*x) + 1/2*(
2*d^4*x^4 + 8*c*d^3*x^3 + 12*c^2*d^2*x^2 - 6*d^4*x^2 + 8*c^3*d*x - 12*c*d^3
*x + 2*c^4 - 6*c^2*d^2 + 3*d^4)*sin(2*x)
```

3.362 $\int (c+dx)^3 \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=115

$$-\frac{3}{2}cd^2x-3d^2\sin(x)\cos(x)(c+dx)+\frac{(c+dx)^4}{4d}-\frac{3}{4}d\sin^2(x)(c+dx)^2+\frac{9}{4}d\cos^2(x)(c+dx)^2+2\sin(x)\cos(x)(c+dx)^3$$

```
[Out] (-3*c*d^2*x)/2 - (3*d^3*x^2)/4 + (c + d*x)^4/(4*d) - (9*d^3*Cos[x]^2)/8 + (
9*d*(c + d*x)^2*Cos[x]^2)/4 - 3*d^2*(c + d*x)*Cos[x]*Sin[x] + 2*(c + d*x)^3
*Cos[x]*Sin[x] + (3*d^3*Sin[x]^2)/8 - (3*d*(c + d*x)^2*Sin[x]^2)/4
```

Rubi [A] time = 0.141352, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4431, 3311, 32, 3310}

$$-\frac{3}{2}cd^2x-3d^2\sin(x)\cos(x)(c+dx)+\frac{(c+dx)^4}{4d}-\frac{3}{4}d\sin^2(x)(c+dx)^2+\frac{9}{4}d\cos^2(x)(c+dx)^2+2\sin(x)\cos(x)(c+dx)^3$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```


```
[Out] (-3*c*d^2*x)/2 - (3*d^3*x^2)/4 + (c + d*x)^4/(4*d) - (9*d^3*Cos[x]^2)/8 + (
9*d*(c + d*x)^2*Cos[x]^2)/4 - 3*d^2*(c + d*x)*Cos[x]*Sin[x] + 2*(c + d*x)^3
*Cos[x]*Sin[x] + (3*d^3*Sin[x]^2)/8 - (3*d*(c + d*x)^2*Sin[x]^2)/4
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3 \csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)^3\cos^2(x)-(c+dx)^3\sin^2(x)\right)\,dx\\
&= 3\int (c+dx)^3\cos^2(x)\,dx-\int (c+dx)^3\sin^2(x)\,dx\\
&= \frac{9}{4}d(c+dx)^2\cos^2(x)+2(c+dx)^3\cos(x)\sin(x)-\frac{3}{4}d(c+dx)^2\sin^2(x)-\frac{1}{2}\int (c+dx)^3\,dx+\\
&= \frac{(c+dx)^4}{4d}-\frac{9}{8}d^3\cos^2(x)+\frac{9}{4}d(c+dx)^2\cos^2(x)-3d^2(c+dx)\cos(x)\sin(x)+2(c+dx)^3\mathrm{co}\\
&= -\frac{3}{2}cd^2x-\frac{3d^3x^2}{4}+\frac{(c+dx)^4}{4d}-\frac{9}{8}d^3\cos^2(x)+\frac{9}{4}d(c+dx)^2\cos^2(x)-3d^2(c+dx)\cos(x)\mathrm{si}
\end{aligned}
$$

Mathematica [A] time = 0.157289, size = 109, normalized size = 0.95

$$\frac{1}{4}\left(x\left(6c^2dx+4c^3+4cd^2x^2+d^3x^3\right)+2\sin(2x)\left(6c^2dx+2c^3+3cd^2\left(2x^2-1\right)+d^3x\left(2x^2-3\right)\right)+3d\cos(2x)\left(2c^2+4cdx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

```
[Out] (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(2*c^2 + 4*c*d*x + d^2
*(-1 + 2*x^2))*Cos[2*x] + 2*(2*c^3 + 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d
^2*(-1 + 2*x^2))*Sin[2*x])/4
```

Maple [A] time = 0.051, size = 179, normalized size = 1.6

$$4\,d^3\left(x^3\,(1/2\,\cos(x)\sin(x)+x/2)+3/4\,x^2\,(\cos(x))^2-3/2\,x\,(1/2\,\cos(x)\sin(x)+x/2)+3/8\,x^2+3/8\,(\sin(x))^2-3/8\,x^4\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(x)*sin(3*x),x)
```

```
[Out] 4*d^3*(x^3*(1/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin
(x)+1/2*x)+3/8*x^2+3/8*sin(x)^2-3/8*x^4)+12*d^2*c*(x^2*(1/2*cos(x)*sin(x)+1
/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)+12*c^2*d*(x*(1/2*cos(
x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-1/4*d^3*x^4+4*c^3*(1/2*cos(x)*sin(x)
+1/2*x)-c*d^2*x^3-3/2*c^2*d*x^2-c^3*x
```

Maxima [A] time = 1.02836, size = 136, normalized size = 1.18

$$\frac{3}{2}\left(x^2+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)c^2d+\frac{1}{2}\left(2\,x^3+6\,x\cos\left(2\,x\right)+3\left(2\,x^2-1\right)\sin\left(2\,x\right)\right)cd^2+\frac{1}{4}\left(x^4+3\left(2\,x^2-1\right)\cos\left(2\,x\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 3/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^2*d + 1/2*(2*x^3 + 6*x*cos(2*x) + 3*(
2*x^2 - 1)*sin(2*x))*c*d^2 + 1/4*(x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 -
 3*x)*sin(2*x))*d^3 + c^3*(x + sin(2*x))
```

Fricas [A] time = 0.496439, size = 278, normalized size = 2.42

$$\frac{1}{4}\,d^3x^4+cd^2x^3+\frac{3}{2}\left(c^2d-d^3\right)x^2+\frac{3}{2}\left(2\,d^3x^2+4\,cd^2x+2\,c^2d-d^3\right)\cos\left(x\right)^2+\left(2\,d^3x^3+6\,cd^2x^2+2\,c^3-3\,cd^2+3\left(2\,c^2d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*(c^2*d - d^3)*x^2 + 3/2*(2*d^3*x^2 + 4*c*d^2*
x + 2*c^2*d - d^3)*cos(x)^2 + (2*d^3*x^3 + 6*c*d^2*x^2 + 2*c^3 - 3*c*d^2 +
3*(2*c^2*d - d^3)*x)*cos(x)*sin(x) + (c^3 - 3*c*d^2)*x
```

Sympy [B] time = 146.651, size = 289, normalized size = 2.51

$$c^3x+c^3\sin\left(2x\right)-3c^2dx^2\sin^2\left(x\right)-3c^2dx^2\cos^2\left(x\right)+\frac{9c^2dx^2}{2}+6c^2dx\sin\left(x\right)\cos\left(x\right)-3c^2d\sin^2\left(x\right)-2cd^2x^3\sin^2\left(x\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(x)*sin(3*x),x)
```

```
[Out] c**3*x + c**3*sin(2*x) - 3*c**2*d*x**2*sin(x)**2 - 3*c**2*d*x**2*cos(x)**2
+ 9*c**2*d*x**2/2 + 6*c**2*d*x*sin(x)*cos(x) - 3*c**2*d*sin(x)**2 - 2*c*d**
2*x**3*sin(x)**2 - 2*c*d**2*x**3*cos(x)**2 + 3*c*d**2*x**3 + 6*c*d**2*x**2*
sin(x)*cos(x) - 3*c*d**2*x*sin(x)**2 + 3*c*d**2*x*cos(x)**2 - 3*c*d**2*sin(
x)*cos(x) - d**3*x**4*sin(x)**2/2 - d**3*x**4*cos(x)**2/2 + 3*d**3*x**4/4 +
 2*d**3*x**3*sin(x)*cos(x) - 3*d**3*x**2*sin(x)**2/2 + 3*d**3*x**2*cos(x)**
2/2 - 3*d**3*x*sin(x)*cos(x) + 3*d**3*sin(x)**2/2
```

Giac [A] time = 1.14367, size = 151, normalized size = 1.31

$$\frac{1}{4}\,d^3x^4 + cd^2x^3 + \frac{3}{2}\,c^2dx^2 + c^3x + \frac{3}{4}\left(2\,d^3x^2 + 4\,cd^2x + 2\,c^2d - d^3\right)\cos\left(2\,x\right) + \frac{1}{2}\left(2\,d^3x^3 + 6\,cd^2x^2 + 6\,c^2dx - 3\,d^3x + 2\,c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x + 3/4*(2*d^3*x^2 + 4*c*d^2*
x + 2*c^2*d - d^3)*cos(2*x) + 1/2*(2*d^3*x^3 + 6*c*d^2*x^2 + 6*c^2*d*x - 3*
d^3*x + 2*c^3 - 3*c*d^2)*sin(2*x)
```

3.363 $\int (c+dx)^2 \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=73

$$\frac{(c+dx)^3}{3d}-\frac{1}{2}d\sin^2(x)(c+dx)+\frac{3}{2}d\cos^2(x)(c+dx)+2\sin(x)\cos(x)(c+dx)^2-\frac{d^2x}{2}-d^2\sin(x)\cos(x)$$

```
[Out] -(d^2*x)/2 + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[x]^2)/2 - d^2*Cos[x]*Si
n[x] + 2*(c + d*x)^2*Cos[x]*Sin[x] - (d*(c + d*x)*Sin[x]^2)/2
```

Rubi [A] time = 0.102692, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{(c+dx)^3}{3d}-\frac{1}{2}d\sin^2(x)(c+dx)+\frac{3}{2}d\cos^2(x)(c+dx)+2\sin(x)\cos(x)(c+dx)^2-\frac{d^2x}{2}-d^2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

```
[Out] -(d^2*x)/2 + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[x]^2)/2 - d^2*Cos[x]*Si
n[x] + 2*(c + d*x)^2*Cos[x]*Sin[x] - (d*(c + d*x)*Sin[x]^2)/2
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)^2\cos^2(x)-(c+dx)^2\sin^2(x)\right)\,dx \\
&= 3\int (c+dx)^2\cos^2(x)\,dx-\int (c+dx)^2\sin^2(x)\,dx \\
&= \frac{3}{2}d(c+dx)\cos^2(x)+2(c+dx)^2\cos(x)\sin(x)-\frac{1}{2}d(c+dx)\sin^2(x)-\frac{1}{2}\int (c+dx)^2\,dx+\frac{3}{2} \\
&= \frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(x)\sin(x)-\frac{1}{2}d(c+dx) \\
&= -\frac{d^2x}{2}+\frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(x)\sin(x)-\frac{1}{2}
\end{aligned}$$

Mathematica [A] time = 0.110072, size = 60, normalized size = 0.82

$$\sin(x)\cos(x)\left(2c^2+4cdx+d^2\left(2x^2-1\right)\right)+c^2x+cdx^2+d\cos(2x)(c+dx)+\frac{d^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

```
[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + d*(c + d*x)*Cos[2*x] + (2*c^2 + 4*c*d*x + d
^2*(-1 + 2*x^2))*Cos[x]*Sin[x]
```

Maple [A] time = 0.054, size = 107, normalized size = 1.5

$$4\,d^2\left(x^2\,(1/2\,\cos(x)\sin(x)+x/2)+1/2\,x\,(\cos(x))^2-1/4\,\cos(x)\sin(x)-x/4-1/3\,x^3\right)+8\,cd\left(x\,(1/2\,\cos(x)\sin(x)+x/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(x)*sin(3*x),x)
```

```
[Out] 4*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x
-1/3*x^3)+8*c*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c^2*(1
/2*cos(x)*sin(x)+1/2*x)-1/3*d^2*x^3-c*d*x^2-c^2*x
```

Maxima [A] time = 0.995402, size = 81, normalized size = 1.11

$$\left(x^2+2\,x\sin(2\,x)+\cos(2\,x)\right)cd+\frac{1}{6}\left(2\,x^3+6\,x\cos(2\,x)+3\left(2\,x^2-1\right)\sin(2\,x)\right)d^2+c^2(x+\sin(2\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] (x^2 + 2*x*sin(2*x) + cos(2*x))*c*d + 1/6*(2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
- 1)*sin(2*x))*d^2 + c^2*(x + sin(2*x))
```

Fricas [A] time = 0.512624, size = 159, normalized size = 2.18

$$\frac{1}{3}\,d^2x^3+cdx^2+2\left(d^2x+cd\right)\cos(x)^2+\left(2\,d^2x^2+4\,cdx+2\,c^2-d^2\right)\cos(x)\sin(x)+\left(c^2-d^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + 2*(d^2*x + c*d)*cos(x)^2 + (2*d^2*x^2 + 4*c*d*x + 2
*c^2 - d^2)*cos(x)*sin(x) + (c^2 - d^2)*x
```

Sympy [B] time = 21.0139, size = 155, normalized size = 2.12

$$c^2x + c^2\sin(2x) - 2cdx^2\sin^2(x) - 2cdx^2\cos^2(x) + 3cdx^2 + 4cdx\sin(x)\cos(x) - 2cd\sin^2(x) - \frac{2d^2x^3\sin^2(x)}{3} - \frac{2d^2x^3\cos^2(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(x)*sin(3*x),x)
```

```
[Out] c**2*x + c**2*sin(2*x) - 2*c*d*x**2*sin(x)**2 - 2*c*d*x**2*cos(x)**2 + 3*c*
d*x**2 + 4*c*d*x*sin(x)*cos(x) - 2*c*d*sin(x)**2 - 2*d**2*x**3*sin(x)**2/3
- 2*d**2*x**3*cos(x)**2/3 + d**2*x**3 + 2*d**2*x**2*sin(x)*cos(x) - d**2*x*
sin(x)**2 + d**2*x*cos(x)**2 - d**2*sin(x)*cos(x)
```

Giac [A] time = 1.12762, size = 86, normalized size = 1.18

$$\frac{1}{3}d^2x^3 + cdx^2 + c^2x + \left(d^2x + cd\right)\cos(2x) + \frac{1}{2}\left(2d^2x^2 + 4cdx + 2c^2 - d^2\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x + (d^2*x + c*d)*cos(2*x) + 1/2*(2*d^2*x^2 + 4
*c*d*x + 2*c^2 - d^2)*sin(2*x)
```

3.364 $\int (c+dx)\csc(x)\sin(3x)\,dx$

Optimal. Leaf size=41

$$2\sin(x)\cos(x)(c+dx)+cx+\frac{dx^2}{2}-\frac{1}{4}d\sin^2(x)+\frac{3}{4}d\cos^2(x)$$

```
[Out] c*x + (d*x^2)/2 + (3*d*Cos[x]^2)/4 + 2*(c + d*x)*Cos[x]*Sin[x] - (d*Sin[x]^
2)/4
```

Rubi [A] time = 0.0563706, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4431, 3310}

$$2\sin(x)\cos(x)(c+dx)+cx+\frac{dx^2}{2}-\frac{1}{4}d\sin^2(x)+\frac{3}{4}d\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[x]*Sin[3*x],x]
```

```
[Out] c*x + (d*x^2)/2 + (3*d*Cos[x]^2)/4 + 2*(c + d*x)*Cos[x]*Sin[x] - (d*Sin[x]^
2)/4
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)\csc(x)\sin(3x)\,dx = \int \left(3(c+dx)\cos^2(x)-(c+dx)\sin^2(x)\right)\,dx$$
$$= 3\int (c+dx)\cos^2(x)\,dx - \int (c+dx)\sin^2(x)\,dx$$
$$= \frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)-\frac{1}{2}\int (c+dx)\,dx+\frac{3}{2}\int (c+dx)\,dx$$
$$= cx+\frac{dx^2}{2}+\frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)$$

Mathematica [A] time = 0.0198871, size = 34, normalized size = 0.83

$$cx+c\sin(2x)+\frac{dx^2}{2}+dx\sin(2x)+\frac{1}{2}d\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[x]*Sin[3*x],x]
```

```
[Out] c*x + (d*x^2)/2 + (d*Cos[2*x])/2 + c*Sin[2*x] + d*x*Sin[2*x]
```

Maple [A] time = 0.048, size = 52, normalized size = 1.3

$$4\,d\left(x\,(1/2\;\cos(x)\sin(x)+x/2)-1/4\,x^2-1/4\;(\sin(x))^2\right)+4\,c\,(1/2\;\cos(x)\sin(x)+x/2)-\frac{dx^2}{2}-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(x)*sin(3*x),x)
```

```
[Out] 4*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c*(1/2*cos(x)*sin(
x)+1/2*x)-1/2*d*x^2-c*x
```

Maxima [A] time = 1.02177, size = 36, normalized size = 0.88

$$\frac{1}{2}\left(x^2+2\,x\sin(2\,x)+\cos(2\,x)\right)d+c(x+\sin(2\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*d + c*(x + sin(2*x))
```

Fricas [A] time = 0.492165, size = 78, normalized size = 1.9

$$\frac{1}{2}dx^2 + d\cos(x)^2 + 2(dx + c)\cos(x)\sin(x) + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/2*d*x^2 + d*cos(x)^2 + 2*(d*x + c)*cos(x)*sin(x) + c*x
```

Sympy [A] time = 14.3026, size = 56, normalized size = 1.37

$$cx + c\sin(2x) - dx^2\sin^2(x) - dx^2\cos^2(x) + \frac{3dx^2}{2} + 2dx\sin(x)\cos(x) - d\sin^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x)
```

```
[Out] c*x + c*sin(2*x) - d*x**2*sin(x)**2 - d*x**2*cos(x)**2 + 3*d*x**2/2 + 2*d*x
*sin(x)*cos(x) - d*sin(x)**2
```

Giac [A] time = 1.0992, size = 36, normalized size = 0.88

$$\frac{1}{2}dx^2 + cx + \frac{1}{2}d\cos(2x) + (dx + c)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/2*d*x^2 + c*x + 1/2*d*cos(2*x) + (d*x + c)*sin(2*x)
```

3.365 $\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx$

Optimal. Leaf size=57

$$\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}$$

```
[Out] (2*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x])/d + Log[c + d*x]/d + (2*Sin[(2*
c)/d]*SinIntegral[(2*c)/d + 2*x])/d
```

Rubi [A] time = 0.251852, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4431, 3312, 3303, 3299, 3302}

$$\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

```
[Out] (2*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x])/d + Log[c + d*x]/d + (2*Sin[(2*
c)/d]*SinIntegral[(2*c)/d + 2*x])/d
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx &= \int \left(\frac{3\cos^2(x)}{c+dx}-\frac{\sin^2(x)}{c+dx}\right)dx \\
&= 3\int \frac{\cos^2(x)}{c+dx}\,dx-\int \frac{\sin^2(x)}{c+dx}\,dx \\
&= 3\int \left(\frac{1}{2(c+dx)}+\frac{\cos(2x)}{2(c+dx)}\right)dx-\int \left(\frac{1}{2(c+dx)}-\frac{\cos(2x)}{2(c+dx)}\right)dx \\
&= \frac{\log(c+dx)}{d}+\frac{1}{2}\int \frac{\cos(2x)}{c+dx}\,dx+\frac{3}{2}\int \frac{\cos(2x)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{d}+\frac{1}{2}\cos\left(\frac{2c}{d}\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx+\frac{1}{2}\left(3\cos\left(\frac{2c}{d}\right)\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx+\frac{1}{2}\sin\left(\frac{2c}{d}\right. \\
&= \frac{2\cos\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0631687, size = 49, normalized size = 0.86

$$
\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right)+2\sin\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right)+\log(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

```
[Out] (2*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + Log[c + d*x] + 2*Sin[(2*c)/d]*Si
nIntegral[2*(c/d + x)])/d
```

Maple [A] time = 0.051, size = 58, normalized size = 1.

$$2\,\frac{1}{d}\mathrm{Ci}\left(2\,\frac{c}{d}+2\,x\right)\cos\left(2\,\frac{c}{d}\right)+\frac{\ln\left(dx+c\right)}{d}+2\,\frac{1}{d}\mathrm{Si}\left(2\,\frac{c}{d}+2\,x\right)\sin\left(2\,\frac{c}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c),x)
```

```
[Out] 2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d
```

Maxima [C] time = 1.23435, size = 128, normalized size = 2.25

$$-\frac{\left(E_1\left(\frac{2i\,dx+2i\,c}{d}\right)+E_1\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)-\left(-i\,E_1\left(\frac{2i\,dx+2i\,c}{d}\right)+i\,E_1\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)-\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(1, (2*I*d*x + 2*I*c)/d) + exp_integral_e(1, -(2*I*d*x + 2
*I*c)/d))*cos(2*c/d) - (-I*exp_integral_e(1, (2*I*d*x + 2*I*c)/d) + I*exp_i
ntegral_e(1, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d) - log(d*x + c))/d
```

Fricas [A] time = 0.504552, size = 182, normalized size = 3.19

$$\frac{\left(\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(dx+c)}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)+2\,\sin\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)+\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((cos_integral(2*(d*x + c)/d) + cos_integral(-2*(d*x + c)/d))*cos(2*c/d) +
2*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + log(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(3x)\csc(x)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x), x)
```

Giac [A] time = 1.12076, size = 69, normalized size = 1.21

$$\frac{2\cos\left(\frac{2c}{d}\right)\operatorname{Ci}\left(\frac{2(dx+c)}{d}\right)+2\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(\frac{2(dx+c)}{d}\right)+\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="giac")
```

```
[Out] (2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 2*sin(2*c/d)*sin_integral(2*(d*
x + c)/d) + log(d*x + c))/d
```

3.366 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$

Optimal. Leaf size=78

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^2}-\frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{3\cos^2(x)}{d(c+dx)}$$

```
[Out] (-3*Cos[x]^2)/(d*(c + d*x)) + (4*CosIntegral[(2*c)/d + 2*x]*Sin[(2*c)/d])/d
^2 + Sin[x]^2/(d*(c + d*x)) - (4*Cos[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d
^2
```

Rubi [A] time = 0.237926, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4431, 3313, 12, 3303, 3299, 3302}

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^2}-\frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{3\cos^2(x)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```


```
[Out] (-3*Cos[x]^2)/(d*(c + d*x)) + (4*CosIntegral[(2*c)/d + 2*x]*Sin[(2*c)/d])/d
^2 + Sin[x]^2/(d*(c + d*x)) - (4*Cos[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d
^2
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

```
LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = \int \left(\frac{3\cos^2(x)}{(c+dx)^2} - \frac{\sin^2(x)}{(c+dx)^2}\right)dx$$

$$= 3\int \frac{\cos^2(x)}{(c+dx)^2}\,dx - \int \frac{\sin^2(x)}{(c+dx)^2}\,dx$$

$$= -\frac{3\cos^2(x)}{d(c+dx)} + \frac{\sin^2(x)}{d(c+dx)} - \frac{2\int \frac{\sin(2x)}{2(c+dx)}\,dx}{d} + \frac{6\int -\frac{\sin(2x)}{2(c+dx)}\,dx}{d}$$

$$= -\frac{3\cos^2(x)}{d(c+dx)} + \frac{\sin^2(x)}{d(c+dx)} - \frac{\int \frac{\sin(2x)}{c+dx}\,dx}{d} - \frac{3\int \frac{\sin(2x)}{c+dx}\,dx}{d}$$

$$= -\frac{3\cos^2(x)}{d(c+dx)} + \frac{\sin^2(x)}{d(c+dx)} - \frac{\cos\left(\frac{2c}{d}\right)\int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d} - \frac{\left(3\cos\left(\frac{2c}{d}\right)\right)\int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d} + \frac{\sin\left(\frac{2c}{d}\right)\int \frac{\cos(}{c}}{d}$$

$$= -\frac{3\cos^2(x)}{d(c+dx)} + \frac{4\text{Ci}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d^2} + \frac{\sin^2(x)}{d(c+dx)} - \frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}$$

Mathematica [A] time = 0.138473, size = 61, normalized size = 0.78

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right) - 4\cos\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right) - \frac{d(2\cos(2x)+1)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

```
[Out] (-((d*(1 + 2*Cos[2*x]))/(c + d*x)) + 4*CosIntegral[2*(c/d + x)]*Sin[(2*c)/d
] - 4*Cos[(2*c)/d]*SinIntegral[2*(c/d + x)])/d^2
```

Maple [A] time = 0.054, size = 82, normalized size = 1.1

$$-2\,\frac{\cos(2\,x)}{(dx+c)\,d} - 2\,\frac{1}{d}\left(2\,\frac{1}{d}\text{Si}\left(2\,\frac{c}{d}+2\,x\right)\cos\left(2\,\frac{c}{d}\right) - 2\,\frac{1}{d}\text{Ci}\left(2\,\frac{c}{d}+2\,x\right)\sin\left(2\,\frac{c}{d}\right)\right) - \frac{1}{(dx+c)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c)^2,x)
```

```
[Out] -2*cos(2*x)/(d*x+c)/d-2*(2*Si(2*c/d+2*x)*cos(2*c/d)/d-2*Ci(2*c/d+2*x)*sin(2
*c/d)/d)/d-1/d/(d*x+c)
```

Maxima [C] time = 1.22548, size = 437, normalized size = 5.6

$$-\frac{\left(E_2\left(\frac{2i\,dx+2i\,c}{d}\right)+E_2\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)^3+\left(i\,E_2\left(\frac{2i\,dx+2i\,c}{d}\right)-i\,E_2\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)^3+\left(\left(E_2\left(\frac{2i\,dx+2i\,c}{d}\right)+E_2\left(-\frac{2i\,d}{}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((exp_integral_e(2, (2*I*d*x + 2*I*c)/d) + exp_integral_e(2, -(2*I*d*x
 + 2*I*c)/d))*cos(2*c/d)^3 + (I*exp_integral_e(2, (2*I*d*x + 2*I*c)/d) - I*
exp_integral_e(2, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d)^3 + ((exp_integral_e(2,
 (2*I*d*x + 2*I*c)/d) + exp_integral_e(2, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d)
 + 2)*sin(2*c/d)^2 + (exp_integral_e(2, (2*I*d*x + 2*I*c)/d) + exp_integral
_e(2, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d) + 2*cos(2*c/d)^2 + ((I*exp_integral
_e(2, (2*I*d*x + 2*I*c)/d) - I*exp_integral_e(2, -(2*I*d*x + 2*I*c)/d))*cos
(2*c/d)^2 + I*exp_integral_e(2, (2*I*d*x + 2*I*c)/d) - I*exp_integral_e(2,
-(2*I*d*x + 2*I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2*c/d)^2)*d^2*x + (
c*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d)
```

Fricas [A] time = 0.514176, size = 251, normalized size = 3.22

$$-\frac{4\,d\cos\left(x\right)^2+4\,(dx+c)\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)-2\left((dx+c)\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)+(dx+c)\,\mathrm{Ci}\left(-\frac{2\,(dx+c)}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)-d}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(4*d*cos(x)^2 + 4*(d*x + c)*cos(2*c/d)*sin_integral(2*(d*x + c)/d) - 2*((d
*x + c)*cos_integral(2*(d*x + c)/d) + (d*x + c)*cos_integral(-2*(d*x + c)/d
))*sin(2*c/d) - d)/(d^3*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11966, size = 150, normalized size = 1.92

$$\frac{4\,dx\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)-4\,dx\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)+4\,c\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)-4\,c\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)-2\,d\cos\left(2\,x\right)}{d^{3}x+cd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (4*d*x*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*d*x*cos(2*c/d)*sin_integr
al(2*(d*x + c)/d) + 4*c*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*c*cos(2*
c/d)*sin_integral(2*(d*x + c)/d) - 2*d*cos(2*x) - d)/(d^3*x + c*d^2)
```

3.367 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx$

Optimal. Leaf size=99

$$-\frac{4\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\sin(x)\cos(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{3\cos^2(x)}{2d(c+dx)^2}$$

```
[Out] (-3*Cos[x]^2)/(2*d*(c + d*x)^2) - (4*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x
])/d^3 + (4*Cos[x]*Sin[x])/(d^2*(c + d*x)) + Sin[x]^2/(2*d*(c + d*x)^2) - (
4*Sin[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d^3
```

Rubi [A] time = 0.328565, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4431, 3314, 31, 3312, 3303, 3299, 3302}

$$-\frac{4\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\sin(x)\cos(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{3\cos^2(x)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

```
[Out] (-3*Cos[x]^2)/(2*d*(c + d*x)^2) - (4*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x
])/d^3 + (4*Cos[x]*Sin[x])/(d^2*(c + d*x)) + Sin[x]^2/(2*d*(c + d*x)^2) - (
4*Sin[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d^3
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
```

```
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx &= \int\left(\frac{3\cos^2(x)}{(c+dx)^3}-\frac{\sin^2(x)}{(c+dx)^3}\right)dx\\
&= 3\int\frac{\cos^2(x)}{(c+dx)^3}\,dx-\int\frac{\sin^2(x)}{(c+dx)^3}\,dx\\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2}+\frac{4\cos(x)\sin(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{\int\frac{1}{c+dx}\,dx}{d^2}+\frac{2\int\frac{\sin^2(x)}{c+dx}\,dx}{d^2}+\frac{3\int\frac{1}{c+dx}\,dx}{d^2}-6\int\\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2}+\frac{2\log(c+dx)}{d^3}+\frac{4\cos(x)\sin(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}+\frac{2\int\left(\frac{1}{2(c+dx)}-\frac{\cos(2x)}{2(c+dx)}\right)dx}{d^2}-6\\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2}+\frac{4\cos(x)\sin(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{\int\frac{\cos(2x)}{c+dx}\,dx}{d^2}-\frac{3\int\frac{\cos(2x)}{c+dx}\,dx}{d^2}\\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2}+\frac{4\cos(x)\sin(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{\cos\left(\frac{2c}{d}\right)\int\frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d^2}-\frac{\left(3\cos\left(\frac{2c}{d}\right)\right)\int\frac{\cos\left(\frac{2c}{d}\right.}{c+d}}{d^2}\\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2}-\frac{4\cos\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\cos(x)\sin(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right.}{d^3}
\end{aligned}
$$

Mathematica [A] time = 0.233797, size = 77, normalized size = 0.78

$$\frac{-8\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right)-8\sin\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right)+\frac{d(4\sin(2x)(c+dx)-2d\cos(2x)-d)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

```
[Out] (-8*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + (d*(-d - 2*d*Cos[2*x] + 4*(c +
d*x)*Sin[2*x]))/(c + d*x)^2 - 8*Sin[(2*c)/d]*SinIntegral[2*(c/d + x)])/(2*d
^3)
```

Maple [A] time = 0.053, size = 104, normalized size = 1.1

$$-\frac{\cos(2x)}{(dx+c)^2 d}-\frac{1}{d}\left(-2\frac{\sin(2x)}{(dx+c)d}+2\frac{1}{d}\left(2\frac{1}{d}\text{Si}\left(2\frac{c}{d}+2x\right)\sin\left(2\frac{c}{d}\right)+2\frac{1}{d}\text{Ci}\left(2\frac{c}{d}+2x\right)\cos\left(2\frac{c}{d}\right)\right)\right)-\frac{1}{2(dx+c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c)^3,x)
```

```
[Out] -cos(2*x)/(d*x+c)^2/d-(-2*sin(2*x)/(d*x+c)/d+2*(2*Si(2*c/d+2*x)*sin(2*c/d)/
d+2*Ci(2*c/d+2*x)*cos(2*c/d)/d)/d)/d-1/2/d/(d*x+c)^2
```

Maxima [C] time = 1.37194, size = 489, normalized size = 4.94

$$-\frac{2\left(E_3\left(\frac{2i\,dx+2i\,c}{d}\right)+E_3\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)^3+\left(2i\,E_3\left(\frac{2i\,dx+2i\,c}{d}\right)-2i\,E_3\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)^3+2\left(\left(E_3\left(\frac{2i\,dx+2i\,c}{d}\right)+E_3\left(\right.\right.\right.}{4\left(\left(\cos\left(\frac{2\,c}{d}\right)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(exp_integral_e(3, (2*I*d*x + 2*I*c)/d) + exp_integral_e(3, -(2*I*d
*x + 2*I*c)/d))*cos(2*c/d)^3 + (2*I*exp_integral_e(3, (2*I*d*x + 2*I*c)/d)
- 2*I*exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d)^3 + 2*((exp_integ
ral_e(3, (2*I*d*x + 2*I*c)/d) + exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*co
s(2*c/d) + 1)*sin(2*c/d)^2 + 2*(exp_integral_e(3, (2*I*d*x + 2*I*c)/d) + ex
p_integral_e(3, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d) + 2*cos(2*c/d)^2 + ((2*I*
exp_integral_e(3, (2*I*d*x + 2*I*c)/d) - 2*I*exp_integral_e(3, -(2*I*d*x +
2*I*c)/d))*cos(2*c/d)^2 + 2*I*exp_integral_e(3, (2*I*d*x + 2*I*c)/d) - 2*I*
exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2
*c/d)^2)*d^3*x^2 + 2*(c*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d^2*x + (c^2*cos(2*c
/d)^2 + c^2*sin(2*c/d)^2)*d)
```

Fricas [A] time = 0.529877, size = 392, normalized size = 3.96

$$-\frac{4\,d^2\cos\left(x\right)^2-8\left(d^2x+cd\right)\cos\left(x\right)\sin\left(x\right)+8\left(d^2x^2+2\,cdx+c^2\right)\sin\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)-d^2+4\left(\left(d^2x^2+2\,cdx+c^2\right)\mathrm{Ci}\left(\right.\right.}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d^2*cos(x)^2 - 8*(d^2*x + c*d)*cos(x)*sin(x) + 8*(d^2*x^2 + 2*c*d*x
 + c^2)*sin(2*c/d)*sin_integral(2*(d*x + c)/d) - d^2 + 4*((d^2*x^2 + 2*c*d*
```

```
x + c^2)*cos_integral(2*(d*x + c)/d) + (d^2*x^2 + 2*c*d*x + c^2)*cos_integr
al(-2*(d*x + c)/d))*cos(2*c/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11449, size = 271, normalized size = 2.74

$$-\frac{8\,d^2x^2\cos\left(\frac{2\,c}{d}\right)\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)+8\,d^2x^2\sin\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\cos\left(\frac{2\,c}{d}\right)\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\sin\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)}{2\,\left(d^5x^2+2\,cd^4x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(8*d^2*x^2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 8*d^2*x^2*sin(2*c/
d)*sin_integral(2*(d*x + c)/d) + 16*c*d*x*cos(2*c/d)*cos_integral(2*(d*x +
c)/d) + 16*c*d*x*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + 8*c^2*cos(2*c/d)*
cos_integral(2*(d*x + c)/d) - 4*d^2*x*sin(2*x) + 8*c^2*sin(2*c/d)*sin_integ
ral(2*(d*x + c)/d) + 2*d^2*cos(2*x) - 4*c*d*sin(2*x) + d^2)/(d^5*x^2 + 2*c*
d^4*x + c^2*d^3)
```

3.368 $\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=198

$$\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{6d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}+$$

```
[Out] (3*d^4*x)/(2*b^4) - (d*(c + d*x)^3)/b^2 + (c + d*x)^5/(5*d) - (9*d^3*(c + d
*x)*Cos[a + b*x]^2)/(2*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*d^4
*Cos[a + b*x]*Sin[a + b*x])/b^5 - (6*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a + b
*x])/b^3 + (2*(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/b + (3*d^3*(c + d*x)*S
in[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```

Rubi [A] time = 0.251829, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{6d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```


```
[Out] (3*d^4*x)/(2*b^4) - (d*(c + d*x)^3)/b^2 + (c + d*x)^5/(5*d) - (9*d^3*(c + d
*x)*Cos[a + b*x]^2)/(2*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*d^4
*Cos[a + b*x]*Sin[a + b*x])/b^5 - (6*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a + b
*x])/b^3 + (2*(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/b + (3*d^3*(c + d*x)*S
in[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\csc(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)^4\cos^2(a+bx)-(c+dx)^4\sin^2(a+bx)\right)dx\\
&= 3\int (c+dx)^4\cos^2(a+bx)\,dx-\int (c+dx)^4\sin^2(a+bx)\,dx\\
&= \frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{2(c+dx)^4\cos(a+bx)\sin(a+bx)}{b}-\frac{d(c+dx)^3 \mathrm{s}}{b}\\
&= \frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}-\frac{6d^2(c+}{}\\
&= -\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d(c+dx)^3\cos^2(a+}{b^2}\\
&= \frac{3d^4x}{2b^4}-\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d(c+dx)^3\mathrm{co}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.692733, size = 128, normalized size = 0.65

$$\frac{\sin(2(a+bx))\left(-6b^2d^2(c+dx)^2+2b^4(c+dx)^4+3d^4\right)}{2b^5}+\frac{d(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)}{b^4}+2c^2d^2x^3+2c^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + (d*(c + d*x
)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])/b^4 + ((3*d^4 - 6*b^2*d^2*
(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(2*b^5)
```

Maple [B] time = 0.049, size = 1000, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c^4*x-1/5*d^4*x^5+4*c^4/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2*c^3*
d*x^2-2*c^2*d^2*x^3-c*d^3*x^4+4*d^4/b^5*((b*x+a)^4*(1/2*cos(b*x+a)*sin(b*x+
a)+1/2*b*x+1/2*a)+(b*x+a)^3*cos(b*x+a)^2-3*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*
x+a)+1/2*b*x+1/2*a)-3/2*(b*x+a)*cos(b*x+a)^2+3/4*cos(b*x+a)*sin(b*x+a)+3/4*
b*x+3/4*a+(b*x+a)^3-2/5*(b*x+a)^5-4*a*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)
+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(
b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+6*a^2*(
(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^
2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)-4*a^3*((b*x+a)*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^4*
(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+16*c^3*d/b^2*((b*x+a)*(1/2*cos(b
*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b
*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+24*c^2*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*
sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a
)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b
*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/
2*b*x+1/2*a))+16*c*d^3/b^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/
2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4-3*a*((b*x+a)^2*(1/2
*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+
a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3*a^2*((b*x+a)*(1/2*cos(b*x+a)*s
in(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-a^3*(1/2*cos(b*x+a
)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A] time = 1.19957, size = 329, normalized size = 1.66

$$\frac{(bx+\sin(2\,bx+2\,a))c^4}{b}+\frac{2\left(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)c^3d}{b^2}+\frac{\left(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+3\left(2\right.\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^4/b + 2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(
2*b*x + 2*a))*c^3*d/b^2 + (2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^
2 - 1)*sin(2*b*x + 2*a))*c^2*d^2/b^3 + (b^4*x^4 + 3*(2*b^2*x^2 - 1)*cos(2*b
*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*c*d^3/b^4 + 1/10*(2*b^5
*x^5 + 10*(2*b^3*x^3 - 3*b*x)*cos(2*b*x + 2*a) + 5*(2*b^4*x^4 - 6*b^2*x^2 +
 3)*sin(2*b*x + 2*a))*d^4/b^5
```

Fricas [A] time = 0.564808, size = 579, normalized size = 2.92

$$\frac{b^5d^4x^5+5\,b^5cd^3x^4+10\left(b^5c^2d^2-b^3d^4\right)x^3+10\left(b^5c^3d-3\,b^3cd^3\right)x^2+10\left(2\,b^3d^4x^3+6\,b^3cd^3x^2+2\,b^3c^3d-3\,bcd^3+3\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/5*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*(b^5*c^2*d^2 - b^3*d^4)*x^3 + 10*(b
^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c
^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(2*b^4*d
^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2
*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)*sin(b*x
 + a) + 5*(b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

[Out] Timed out

Giac [B] time = 1.7428, size = 6323, normalized size = 31.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")

```
[Out] 1/5*(b^5*d^4*x^5*tan(1/2*b*x)^4*tan(1/2*a)^4 + 5*b^5*c*d^3*x^4*tan(1/2*b*x)
^4*tan(1/2*a)^4 + 2*b^5*d^4*x^5*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^5*d^4*x^5
*tan(1/2*b*x)^2*tan(1/2*a)^4 + 10*b^5*c^2*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)
^4 + 10*b^5*c*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^2 - 20*b^4*d^4*x^4*tan(1/2*
b*x)^4*tan(1/2*a)^3 + 10*b^5*c*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^4 - 20*b^4
*d^4*x^4*tan(1/2*b*x)^3*tan(1/2*a)^4 + 10*b^5*c^3*d*x^2*tan(1/2*b*x)^4*tan(
1/2*a)^4 + b^5*d^4*x^5*tan(1/2*b*x)^4 + 4*b^5*d^4*x^5*tan(1/2*b*x)^2*tan(1/
2*a)^2 + 20*b^5*c^2*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 80*b^4*c*d^3*x^3*
tan(1/2*b*x)^4*tan(1/2*a)^3 + b^5*d^4*x^5*tan(1/2*a)^4 + 20*b^5*c^2*d^2*x^3
*tan(1/2*b*x)^2*tan(1/2*a)^4 - 80*b^4*c*d^3*x^3*tan(1/2*b*x)^3*tan(1/2*a)^4
 + 5*b^5*c^4*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b^3*d^4*x^3*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 5*b^5*c*d^3*x^4*tan(1/2*b*x)^4 + 20*b^4*d^4*x^4*tan(1/2*b*x)
^4*tan(1/2*a) + 20*b^5*c*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + 120*b^4*d^4*
x^4*tan(1/2*b*x)^3*tan(1/2*a)^2 + 20*b^5*c^3*d*x^2*tan(1/2*b*x)^4*tan(1/2*a
)^2 + 120*b^4*d^4*x^4*tan(1/2*b*x)^2*tan(1/2*a)^3 - 120*b^4*c^2*d^2*x^2*tan
(1/2*b*x)^4*tan(1/2*a)^3 + 5*b^5*c*d^3*x^4*tan(1/2*a)^4 + 20*b^4*d^4*x^4*ta
n(1/2*b*x)*tan(1/2*a)^4 + 20*b^5*c^3*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 12
0*b^4*c^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 30*b^3*c*d^3*x^2*tan(1/2*b*
x)^4*tan(1/2*a)^4 + 2*b^5*d^4*x^5*tan(1/2*b*x)^2 + 10*b^5*c^2*d^2*x^3*tan(1
/2*b*x)^4 + 80*b^4*c*d^3*x^3*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^5*d^4*x^5*tan(
1/2*a)^2 + 40*b^5*c^2*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 480*b^4*c*d^3*x
^3*tan(1/2*b*x)^3*tan(1/2*a)^2 + 10*b^5*c^4*x*tan(1/2*b*x)^4*tan(1/2*a)^2 -
 60*b^3*d^4*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 480*b^4*c*d^3*x^3*tan(1/2*b*x
)^2*tan(1/2*a)^3 - 160*b^3*d^4*x^3*tan(1/2*b*x)^3*tan(1/2*a)^3 - 80*b^4*c^3
*d*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 10*b^5*c^2*d^2*x^3*tan(1/2*a)^4 + 80*b^4
*c*d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^4 + 10*b^5*c^4*x*tan(1/2*b*x)^2*tan(1/2*
a)^4 - 60*b^3*d^4*x^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 80*b^4*c^3*d*x*tan(1/2*
b*x)^3*tan(1/2*a)^4 + 30*b^3*c^2*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b^5
*c*d^3*x^4*tan(1/2*b*x)^2 - 20*b^4*d^4*x^4*tan(1/2*b*x)^3 + 10*b^5*c^3*d*x^
2*tan(1/2*b*x)^4 - 120*b^4*d^4*x^4*tan(1/2*b*x)^2*tan(1/2*a) + 120*b^4*c^2*
d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 10*b^5*c*d^3*x^4*tan(1/2*a)^2 - 120*b^4
*d^4*x^4*tan(1/2*b*x)*tan(1/2*a)^2 + 40*b^5*c^3*d*x^2*tan(1/2*b*x)^2*tan(1/
```

2*a)^2 + 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 180*b^3*c*d^3*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 20*b^4*d^4*x^4*tan(1/2*a)^3 + 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 480*b^3*c*d^3*x^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 20*b^4*c^4*tan(1/2*b*x)^4*tan(1/2*a)^3 + 60*b^2*d^4*x^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 10*b^5*c^3*d*x^2*tan(1/2*a)^4 + 120*b^4*c^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 180*b^3*c*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 20*b^4*c^4*tan(1/2*b*x)^3*tan(1/2*a)^4 + 60*b^2*d^4*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 10*b^3*c^3*d*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^5*d^4*x^5 + 20*b^5*c^2*d^2*x^3*tan(1/2*b*x)^2 - 80*b^4*c*d^3*x^3*tan(1/2*b*x)^3 + 5*b^5*c^4*x*tan(1/2*b*x)^4 + 10*b^3*d^4*x^3*tan(1/2*b*x)^4 - 480*b^4*c*d^3*x^3*tan(1/2*b*x)^2*tan(1/2*a) + 160*b^3*d^4*x^3*tan(1/2*b*x)^3*tan(1/2*a) + 80*b^4*c^3*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 20*b^5*c^2*d^2*x^3*tan(1/2*a)^2 - 480*b^4*c*d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^2 + 20*b^5*c^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 360*b^3*d^4*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 480*b^4*c^3*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 180*b^3*c^2*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 80*b^4*c*d^3*x^3*tan(1/2*a)^3 + 160*b^3*d^4*x^3*tan(1/2*b*x)*tan(1/2*a)^3 + 480*b^4*c^3*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 480*b^3*c^2*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^3 + 120*b^2*c*d^3*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 5*b^5*c^4*x*tan(1/2*a)^4 + 10*b^3*d^4*x^3*tan(1/2*a)^4 + 80*b^4*c^3*d*x*tan(1/2*b*x)*tan(1/2*a)^4 - 180*b^3*c^2*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 120*b^2*c*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^4 - 15*b*d^4*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 5*b^5*c*d^3*x^4 + 20*b^4*d^4*x^4*tan(1/2*b*x) + 20*b^5*c^3*d*x^2*tan(1/2*b*x)^2 - 120*b^4*c^2*d^2*x^2*tan(1/2*b*x)^3 + 30*b^3*c*d^3*x^2*tan(1/2*b*x)^4 + 20*b^4*d^4*x^4*tan(1/2*a) - 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 480*b^3*c*d^3*x^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^4*c^4*tan(1/2*b*x)^4*tan(1/2*a) - 60*b^2*d^4*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 20*b^5*c^3*d*x^2*tan(1/2*a)^2 - 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^2 + 1080*b^3*c*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 120*b^4*c^4*tan(1/2*b*x)^3*tan(1/2*a)^2 - 360*b^2*d^4*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 60*b^3*c^3*d*tan(1/2*b*x)^4*tan(1/2*a)^2 - 120*b^4*c^2*d^2*x^2*tan(1/2*a)^3 + 480*b^3*c*d^3*x^2*tan(1/2*b*x)*tan(1/2*a)^3 + 120*b^4*c^4*tan(1/2*b*x)^2*tan(1/2*a)^3 - 360*b^2*d^4*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 160*b^3*c^3*d*tan(1/2*b*x)^3*tan(1/2*a)^3 + 60*b^2*c^2*d^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 30*b^3*c*d^3*x^2*tan(1/2*a)^4 + 20*b^4*c^4*tan(1/2*b*x)*tan(1/2*a)^4 - 60*b^2*d^4*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 60*b^3*c^3*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 60*b^2*c^2*d^2*tan(1/2*b*x)^3*tan(1/2*a)^4 - 15*b*c*d^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b^5*c^2*d^2*x^3 + 80*b^4*c*d^3*x^3*tan(1/2*b*x) + 10*b^5*c^4*x*tan(1/2*b*x)^2 - 60*b^3*d^4*x^3*tan(1/2*b*x)^2 - 80*b^4*c^3*d*x*tan(1/2*b*x)^3 + 30*b^3*c^2*d^2*x*tan(1/2*b*x)^4 + 80*b^4*c*d^3*x^3*tan(1/2*a) - 160*b^3*d^4*x^3*tan(1/2*b*x)*tan(1/2*a) - 480*b^4*c^3*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 480*b^3*c^2*d^2*x*tan(1/2*b*x)^3*tan(1/2*a) - 120*b^2*c*d^3*x*tan(1/2*b*x)^4*tan(1/2*a) + 10*b^5*c^4*x*tan(1/2*a)^2 - 60*b^3*d^4*x^3*tan(1/2*a)^2 - 480*b^4*c^3*d*x*tan(1/2*b*x)*tan(1/2*a)^2 + 1080*b^3*c^2*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 720*b^2*c*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^2 + 90*b*d^4*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 80*b^4*c^3*d*x*tan(1/2*a)^3 + 480*b^3*c^2*d^2*x*tan(1/2*b*x)*t an(1/2*a)^3 - 720*b^2*c*d^3*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 240*b*d^4*x*tan (1/2*b*x)^3*tan(1/2*a)^3 + 30*b^3*c^2*d^2*x*tan(1/2*a)^4 - 120*b^2*c*d^3*x* tan(1/2*b*x)*tan(1/2*a)^4 + 90*b*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 10*b^5 *c^3*d*x^2 + 120*b^4*c^2*d^2*x^2*tan(1/2*b*x) - 180*b^3*c*d^3*x^2*tan(1/2*b *x)^2 - 20*b^4*c^4*tan(1/2*b*x)^3 + 60*b^2*d^4*x^2*tan(1/2*b*x)^3 + 10*b^3* c^3*d*tan(1/2*b*x)^4 + 120*b^4*c^2*d^2*x^2*tan(1/2*a) - 480*b^3*c*d^3*x^2*t an(1/2*b*x)*tan(1/2*a) - 120*b^4*c^4*tan(1/2*b*x)^2*tan(1/2*a) + 360*b^2*d^ 4*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 160*b^3*c^3*d*tan(1/2*b*x)^3*tan(1/2*a) - 60*b^2*c^2*d^2*tan(1/2*b*x)^4*tan(1/2*a) - 180*b^3*c*d^3*x^2*tan(1/2*a)^2 - 120*b^4*c^4*tan(1/2*b*x)*tan(1/2*a)^2 + 360*b^2*d^4*x^2*tan(1/2*b*x)*tan(1/2*a)^2 + 360*b^3*c^3*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 360*b^2*c^2*d^2*tan(1/2*b*x)^3*tan(1/2*a)^2 + 90*b*c*d^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 20*b^4*c ^4*tan(1/2*a)^3 + 60*b^2*d^4*x^2*tan(1/2*a)^3 + 160*b^3*c^3*d*tan(1/2*b*x)* tan(1/2*a)^3 - 360*b^2*c^2*d^2*tan(1/2*b*x)^2*tan(1/2*a)^3 + 240*b*c*d^3*ta n(1/2*b*x)^3*tan(1/2*a)^3 - 30*d^4*tan(1/2*b*x)^4*tan(1/2*a)^3 + 10*b^3*c^3 *d*tan(1/2*a)^4 - 60*b^2*c^2*d^2*tan(1/2*b*x)*tan(1/2*a)^4 + 90*b*c*d^3*tan (1/2*b*x)^2*tan(1/2*a)^4 - 30*d^4*tan(1/2*b*x)^3*tan(1/2*a)^4 + 5*b^5*c^4*x + 10*b^3*d^4*x^3 + 80*b^4*c^3*d*x*tan(1/2*b*x) - 180*b^3*c^2*d^2*x*tan(1/2 *b*x)^2 + 120*b^2*c*d^3*x*tan(1/2*b*x)^3 - 15*b*d^4*x*tan(1/2*b*x)^4 + 80*b ^4*c^3*d*x*tan(1/2*a) - 480*b^3*c^2*d^2*x*tan(1/2*b*x)*tan(1/2*a) + 720*b^2 *c*d^3*x*tan(1/2*b*x)^2*tan(1/2*a) - 240*b*d^4*x*tan(1/2*b*x)^3*tan(1/2*a) - 180*b^3*c^2*d^2*x*tan(1/2*a)^2 + 720*b^2*c*d^3*x*tan(1/2*b*x)*tan(1/2*a)^ 2 - 540*b*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 120*b^2*c*d^3*x*tan(1/2*a)^3 - 240*b*d^4*x*tan(1/2*b*x)*tan(1/2*a)^3 - 15*b*d^4*x*tan(1/2*a)^4 + 30*b^3* c*d^3*x^2 + 20*b^4*c^4*tan(1/2*b*x) - 60*b^2*d^4*x^2*tan(1/2*b*x) - 60*b^3* c^3*d*tan(1/2*b*x)^2 + 60*b^2*c^2*d^2*tan(1/2*b*x)^3 - 15*b*c*d^3*tan(1/2*b *x)^4 + 20*b^4*c^4*tan(1/2*a) - 60*b^2*d^4*x^2*tan(1/2*a) - 160*b^3*c^3*d*t an(1/2*b*x)*tan(1/2*a) + 360*b^2*c^2*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 240*b* c*d^3*tan(1/2*b*x)^3*tan(1/2*a) + 30*d^4*tan(1/2*b*x)^4*tan(1/2*a) - 60*b^3 *c^3*d*tan(1/2*a)^2 + 360*b^2*c^2*d^2*tan(1/2*b*x)*tan(1/2*a)^2 - 540*b*c*d ^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 180*d^4*tan(1/2*b*x)^3*tan(1/2*a)^2 + 60*b ^2*c^2*d^2*tan(1/2*a)^3 - 240*b*c*d^3*tan(1/2*b*x)*tan(1/2*a)^3 + 180*d^4*t an(1/2*b*x)^2*tan(1/2*a)^3 - 15*b*c*d^3*tan(1/2*a)^4 + 30*d^4*tan(1/2*b*x)* tan(1/2*a)^4 + 30*b^3*c^2*d^2*x - 120*b^2*c*d^3*x*tan(1/2*b*x) + 90*b*d^4*x *tan(1/2*b*x)^2 - 120*b^2*c*d^3*x*tan(1/2*a) + 240*b*d^4*x*tan(1/2*b*x)*tan (1/2*a) + 90*b*d^4*x*tan(1/2*a)^2 + 10*b^3*c^3*d - 60*b^2*c^2*d^2*tan(1/2*b *x) + 90*b*c*d^3*tan(1/2*b*x)^2 - 30*d^4*tan(1/2*b*x)^3 - 60*b^2*c^2*d^2*ta n(1/2*a) + 240*b*c*d^3*tan(1/2*b*x)*tan(1/2*a) - 180*d^4*tan(1/2*b*x)^2*tan (1/2*a) + 90*b*c*d^3*tan(1/2*a)^2 - 180*d^4*tan(1/2*b*x)*tan(1/2*a)^2 - 30* d^4*tan(1/2*a)^3 - 15*b*d^4*x - 15*b*c*d^3 + 30*d^4*tan(1/2*b*x) + 30*d^4*t an(1/2*a))/(b^5*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^5*tan(1/2*b*x)^4*tan(1/2* a)^2 + 2*b^5*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^5*tan(1/2*b*x)^4 + 4*b^5*tan(1 /2*b*x)^2*tan(1/2*a)^2 + b^5*tan(1/2*a)^4 + 2*b^5*tan(1/2*b*x)^2 + 2*b^5*ta n(1/2*a)^2 + b^5)

3.369 $\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=171

$$-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{b^3}-\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{3d^3\sin^2(a+bx)}{8b^4}-\frac{9d^3 c}{}$$

```
[Out] (-3*c*d^2*x)/(2*b^2) - (3*d^3*x^2)/(4*b^2) + (c + d*x)^4/(4*d) - (9*d^3*Cos
[a + b*x]^2)/(8*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^3*Cos[a + b*x]*Sin[a
+ b*x])/b + (3*d^3*Sin[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin[a + b*x]^
2)/(4*b^2)
```

Rubi [A] time = 0.183662, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4431, 3311, 32, 3310}

$$-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{b^3}-\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{3d^3\sin^2(a+bx)}{8b^4}-\frac{9d^3 c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```


```
[Out] (-3*c*d^2*x)/(2*b^2) - (3*d^3*x^2)/(4*b^2) + (c + d*x)^4/(4*d) - (9*d^3*Cos
[a + b*x]^2)/(8*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^3*Cos[a + b*x]*Sin[a
+ b*x])/b + (3*d^3*Sin[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin[a + b*x]^
2)/(4*b^2)
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)^3\cos^2(a+bx)-(c+dx)^3\sin^2(a+bx)\right)dx\\ &= 3\int (c+dx)^3\cos^2(a+bx)\,dx-\int (c+dx)^3\sin^2(a+bx)\,dx\\ &= \frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)^3\cos(a+bx)\sin(a+bx)}{b}-\frac{3d(c+dx)^2 s}{4b}\\ &= \frac{(c+dx)^4}{4d}-\frac{9d^3\cos^2(a+bx)}{8b^4}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}-\frac{3d^2(c+dx)\cos(a}{b}\\ &= -\frac{3cd^2x}{2b^2}-\frac{3d^3x^2}{4b^2}+\frac{(c+dx)^4}{4d}-\frac{9d^3\cos^2(a+bx)}{8b^4}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.414968, size = 105, normalized size = 0.61

$$\frac{2b(c+dx)\sin(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+3d\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+b^4x\left(6c^2dx+4c^3+4cd^2x^2+d^3x\right.}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[
```

```
2*(a + b*x)])/(4*b^4)
```

Maple [B] time = 0.043, size = 580, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c^3*x-1/4*d^3*x^4+4*c^3/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/2*c^
2*d*x^2-c*d^2*x^3+4*d^3/b^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1
/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2
*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4-3*a*((b*x+a)^2*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x
+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3*a^2*((b*x+a)*(1/2*cos(b*x+a)*
sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-a^3*(1/2*cos(b*x+
a)*sin(b*x+a)+1/2*b*x+1/2*a))+12*c^2*d/b^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a))+12*d^2*c/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a
-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(
b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A] time = 1.12931, size = 234, normalized size = 1.37

$$\frac{(bx+\sin(2bx+2a))c^3}{b}+\frac{3\left(b^2x^2+2bx\sin(2bx+2a)+\cos(2bx+2a)\right)c^2d}{2b^2}+\frac{\left(2b^3x^3+6bx\cos(2bx+2a)+3\left(2\right.\right.}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^3/b + 3/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + co
s(2*b*x + 2*a))*c^2*d/b^2 + 1/2*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*
b^2*x^2 - 1)*sin(2*b*x + 2*a))*c*d^2/b^3 + 1/4*(b^4*x^4 + 3*(2*b^2*x^2 - 1)
*cos(2*b*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*d^3/b^4
```

Fricas [A] time = 0.504972, size = 389, normalized size = 2.27

$$\frac{b^4d^3x^4 + 4\,b^4cd^2x^3 + 6\left(b^4c^2d - b^2d^3\right)x^2 + 6\left(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3\right)\cos\left(bx + a\right)^2 + 4\left(2\,b^3d^3x^3 + 6\,b^3cd^2x^2\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*(b^4*c^2*d - b^2*d^3)*x^2 + 6*(2*b^2
*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(2*b^3*d^3
*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)
*cos(b*x + a)*sin(b*x + a) + 4*(b^4*c^3 - 3*b^2*c*d^2)*x)/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46399, size = 4238, normalized size = 24.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/4*(b^4*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^4*c*d^2*x^3*tan(1/2*b*x)
^4*tan(1/2*a)^4 + 2*b^4*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^4*d^3*x^4
*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*b^4*c^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4
+ 8*b^4*c*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*d^3*x^3*tan(1/2*b*x)
^4*tan(1/2*a)^3 + 8*b^4*c*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 16*b^3*d^3*
x^3*tan(1/2*b*x)^3*tan(1/2*a)^4 + 4*b^4*c^3*x*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 b^4*d^3*x^4*tan(1/2*b*x)^4 + 4*b^4*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + 1
```

2*b^4*c^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 48*b^3*c*d^2*x^2*tan(1/2*b*x)
^4*tan(1/2*a)^3 + b^4*d^3*x^4*tan(1/2*a)^4 + 12*b^4*c^2*d*x^2*tan(1/2*b*x)^
2*tan(1/2*a)^4 - 48*b^3*c*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*b^2*d^3*x
^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^4*c*d^2*x^3*tan(1/2*b*x)^4 + 16*b^3*d^
3*x^3*tan(1/2*b*x)^4*tan(1/2*a) + 16*b^4*c*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a
)^2 + 96*b^3*d^3*x^3*tan(1/2*b*x)^3*tan(1/2*a)^2 + 8*b^4*c^3*x*tan(1/2*b*x)
^4*tan(1/2*a)^2 + 96*b^3*d^3*x^3*tan(1/2*b*x)^2*tan(1/2*a)^3 - 48*b^3*c^2*d
*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b^4*c*d^2*x^3*tan(1/2*a)^4 + 16*b^3*d^3*
x^3*tan(1/2*b*x)*tan(1/2*a)^4 + 8*b^4*c^3*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4
8*b^3*c^2*d*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + 12*b^2*c*d^2*x*tan(1/2*b*x)^4*t
an(1/2*a)^4 + 2*b^4*d^3*x^4*tan(1/2*b*x)^2 + 6*b^4*c^2*d*x^2*tan(1/2*b*x)^4
 + 48*b^3*c*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^4*d^3*x^4*tan(1/2*a)^2
 + 24*b^4*c^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 288*b^3*c*d^2*x^2*tan(1/2*
b*x)^3*tan(1/2*a)^2 - 36*b^2*d^3*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 288*b^3*
c*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 96*b^2*d^3*x^2*tan(1/2*b*x)^3*tan(1
/2*a)^3 - 16*b^3*c^3*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b^4*c^2*d*x^2*tan(1/2*
a)^4 + 48*b^3*c*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 36*b^2*d^3*x^2*tan(1/2*
b*x)^2*tan(1/2*a)^4 - 16*b^3*c^3*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*b^2*c^2*d*
tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*b^4*c*d^2*x^3*tan(1/2*b*x)^2 - 16*b^3*d^3*x
^3*tan(1/2*b*x)^3 + 4*b^4*c^3*x*tan(1/2*b*x)^4 - 96*b^3*d^3*x^3*tan(1/2*b*x
)^2*tan(1/2*a) + 48*b^3*c^2*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^4*c*d^2*x^3
*tan(1/2*a)^2 - 96*b^3*d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^2 + 16*b^4*c^3*x*tan
(1/2*b*x)^2*tan(1/2*a)^2 + 288*b^3*c^2*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 72
*b^2*c*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*d^3*x^3*tan(1/2*a)^3 + 28
8*b^3*c^2*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 192*b^2*c*d^2*x*tan(1/2*b*x)^3*
tan(1/2*a)^3 + 24*b*d^3*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b^4*c^3*x*tan(1/2
*a)^4 + 48*b^3*c^2*d*x*tan(1/2*b*x)*tan(1/2*a)^4 - 72*b^2*c*d^2*x*tan(1/2*b
*x)^2*tan(1/2*a)^4 + 24*b*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^4*d^3*x^4 +
 12*b^4*c^2*d*x^2*tan(1/2*b*x)^2 - 48*b^3*c*d^2*x^2*tan(1/2*b*x)^3 + 6*b^2*
d^3*x^2*tan(1/2*b*x)^4 - 288*b^3*c*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 96*b
^2*d^3*x^2*tan(1/2*b*x)^3*tan(1/2*a) + 16*b^3*c^3*tan(1/2*b*x)^4*tan(1/2*a)
 + 12*b^4*c^2*d*x^2*tan(1/2*a)^2 - 288*b^3*c*d^2*x^2*tan(1/2*b*x)*tan(1/2*a
)^2 + 216*b^2*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 96*b^3*c^3*tan(1/2*b*x)
^3*tan(1/2*a)^2 - 36*b^2*c^2*d*tan(1/2*b*x)^4*tan(1/2*a)^2 - 48*b^3*c*d^2*x
^2*tan(1/2*a)^3 + 96*b^2*d^3*x^2*tan(1/2*b*x)*tan(1/2*a)^3 + 96*b^3*c^3*tan
(1/2*b*x)^2*tan(1/2*a)^3 - 96*b^2*c^2*d*tan(1/2*b*x)^3*tan(1/2*a)^3 + 24*b*
c*d^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b^2*d^3*x^2*tan(1/2*a)^4 + 16*b^3*c^3
*tan(1/2*b*x)*tan(1/2*a)^4 - 36*b^2*c^2*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 24*
b*c*d^2*tan(1/2*b*x)^3*tan(1/2*a)^4 - 3*d^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4
*b^4*c*d^2*x^3 + 16*b^3*d^3*x^3*tan(1/2*b*x) + 8*b^4*c^3*x*tan(1/2*b*x)^2 -
 48*b^3*c^2*d*x*tan(1/2*b*x)^3 + 12*b^2*c*d^2*x*tan(1/2*b*x)^4 + 16*b^3*d^3
*x^3*tan(1/2*a) - 288*b^3*c^2*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 192*b^2*c*d^2
*x*tan(1/2*b*x)^3*tan(1/2*a) - 24*b*d^3*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^4
*c^3*x*tan(1/2*a)^2 - 288*b^3*c^2*d*x*tan(1/2*b*x)*tan(1/2*a)^2 + 432*b^2*c
*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 144*b*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^

2 - 48*b^3*c^2*d*x*tan(1/2*a)^3 + 192*b^2*c*d^2*x*tan(1/2*b*x)*tan(1/2*a)^3 - 144*b*d^3*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 12*b^2*c*d^2*x*tan(1/2*a)^4 - 24*b*d^3*x*tan(1/2*b*x)*tan(1/2*a)^4 + 6*b^4*c^2*d*x^2 + 48*b^3*c*d^2*x^2*tan(1/2*b*x) - 36*b^2*d^3*x^2*tan(1/2*b*x)^2 - 16*b^3*c^3*tan(1/2*b*x)^3 + 6*b^2*c^2*d*tan(1/2*b*x)^4 + 48*b^3*c*d^2*x^2*tan(1/2*a) - 96*b^2*d^3*x^2*tan(1/2*b*x)*tan(1/2*a) - 96*b^3*c^3*tan(1/2*b*x)^2*tan(1/2*a) + 96*b^2*c^2*d*tan(1/2*b*x)^3*tan(1/2*a) - 24*b*c*d^2*tan(1/2*b*x)^4*tan(1/2*a) - 36*b^2*d^3*x^2*tan(1/2*a)^2 - 96*b^3*c^3*tan(1/2*b*x)*tan(1/2*a)^2 + 216*b^2*c^2*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 144*b*c*d^2*tan(1/2*b*x)^3*tan(1/2*a)^2 + 18*d^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*c^3*tan(1/2*a)^3 + 96*b^2*c^2*d*tan(1/2*b*x)*tan(1/2*a)^3 - 144*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a)^3 + 48*d^3*tan(1/2*b*x)^3*tan(1/2*a)^3 + 6*b^2*c^2*d*tan(1/2*a)^4 - 24*b*c*d^2*tan(1/2*b*x)*tan(1/2*a)^4 + 18*d^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b^4*c^3*x + 48*b^3*c^2*d*x*tan(1/2*b*x) - 72*b^2*c*d^2*x*tan(1/2*b*x)^2 + 24*b*d^3*x*tan(1/2*b*x)^3 + 48*b^3*c^2*d*x*tan(1/2*a) - 192*b^2*c*d^2*x*tan(1/2*b*x)*tan(1/2*a) + 144*b*d^3*x*tan(1/2*b*x)^2*tan(1/2*a) - 72*b^2*c*d^2*x*tan(1/2*a)^2 + 144*b*d^3*x*tan(1/2*b*x)*tan(1/2*a)^2 + 24*b*d^3*x*tan(1/2*a)^3 + 6*b^2*d^3*x^2 + 16*b^3*c^3*tan(1/2*b*x) - 36*b^2*c^2*d*tan(1/2*b*x)^2 + 24*b*c*d^2*tan(1/2*b*x)^3 - 3*d^3*tan(1/2*b*x)^4 + 16*b^3*c^3*tan(1/2*a) - 96*b^2*c^2*d*tan(1/2*b*x)*tan(1/2*a) + 144*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 48*d^3*tan(1/2*b*x)^3*tan(1/2*a) - 36*b^2*c^2*d*tan(1/2*a)^2 + 144*b*c*d^2*tan(1/2*b*x)*tan(1/2*a)^2 - 108*d^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*c*d^2*tan(1/2*a)^3 - 48*d^3*tan(1/2*b*x)*tan(1/2*a)^3 - 3*d^3*tan(1/2*a)^4 + 12*b^2*c*d^2*x - 24*b*d^3*x*tan(1/2*b*x) - 24*b*d^3*x*tan(1/2*a) + 6*b^2*c^2*d - 24*b*c*d^2*tan(1/2*b*x) + 18*d^3*tan(1/2*b*x)^2 - 24*b*c*d^2*tan(1/2*a) + 48*d^3*tan(1/2*b*x)*tan(1/2*a) + 18*d^3*tan(1/2*a)^2 - 3*d^3)/(b^4*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^4*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^4*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^4*tan(1/2*b*x)^4 + 4*b^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^4*tan(1/2*a)^4 + 2*b^4*tan(1/2*b*x)^2 + 2*b^4*tan(1/2*a)^2 + b^4)

3.370 $\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=112

$$-\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\sin(a+bx)\cos(a+bx)}{b^3}+\frac{2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b}-$$

```
[Out] -(d^2*x)/(2*b^2) + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[a + b*x]^2)/(2*b^
2) - (d^2*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^2*Cos[a + b*x]*Sin[
a + b*x])/b - (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2)
```

Rubi [A] time = 0.13595, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4431, 3311, 32, 2635, 8}

$$-\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\sin(a+bx)\cos(a+bx)}{b^3}+\frac{2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -(d^2*x)/(2*b^2) + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[a + b*x]^2)/(2*b^
2) - (d^2*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^2*Cos[a + b*x]*Sin[
a + b*x])/b - (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2)
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)^2\cos^2(a+bx)-(c+dx)^2\sin^2(a+bx)\right)dx \\
&= 3\int (c+dx)^2\cos^2(a+bx)\,dx-\int (c+dx)^2\sin^2(a+bx)\,dx \\
&= \frac{3d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{2(c+dx)^2\cos(a+bx)\sin(a+bx)}{b}-\frac{d(c+dx)\sin^2}{2b^2} \\
&= \frac{(c+dx)^3}{3d}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\cos(a+bx)\sin(a+bx)}{b^3}+\frac{2(c+dx)}{} \\
&= -\frac{d^2x}{2b^2}+\frac{(c+dx)^3}{3d}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\cos(a+bx)\sin(a+bx)}{b^3}+\frac{2}{}
\end{aligned}$$

Mathematica [A] time = 0.394428, size = 73, normalized size = 0.65

$$\frac{\sin(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)}{2b^3}+\frac{d(c+dx)\cos(2(a+bx))}{b^2}+c^2x+cdx^2+\frac{d^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + (d*(c + d*x)*Cos[2*(a + b*x)])/b^2 + ((-d^2
 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(2*b^3)
```

Maple [B] time = 0.041, size = 294, normalized size = 2.6

$$-c^2x - \frac{d^2x^3}{3} + 4\,\frac{c^2\,(1/2\,\cos(bx+a)\sin(bx+a) + 1/2\,bx + a/2)}{b} - cdx^2 + 4\,\frac{d^2\left((bx+a)^2\,(1/2\,\cos(bx+a)\sin(bx+a) + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c^2*x-1/3*d^2*x^3+4*c^2/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-c*d*x^
2+4*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a
)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((
b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+
a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+8*c*d/b^2*((b*x+a)*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A] time = 1.07337, size = 146, normalized size = 1.3

$$\frac{(bx + \sin(2\,bx + 2\,a))c^2}{b} + \frac{\left(b^2x^2 + 2\,bx\sin(2\,bx + 2\,a) + \cos(2\,bx + 2\,a)\right)cd}{b^2} + \frac{\left(2\,b^3x^3 + 6\,bx\cos(2\,bx + 2\,a) + 3\left(2\,b^2x\right.\right.}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^2/b + (b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(2*
b*x + 2*a))*c*d/b^2 + 1/6*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^
2 - 1)*sin(2*b*x + 2*a))*d^2/b^3
```

Fricas [A] time = 0.497992, size = 240, normalized size = 2.14

$$\frac{b^3d^2x^3 + 3\,b^3cdx^2 + 6\left(bd^2x + bcd\right)\cos(bx+a)^2 + 3\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos(bx+a)\sin(bx+a) + 3\left(b^3\right.}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(
2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a) +
3*(b^3*c^2 - b*d^2)*x)/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3095, size = 2538, normalized size = 22.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/3*(b^3*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 3*b^3*c*d*x^2*tan(1/2*b*x)^4
*tan(1/2*a)^4 + 2*b^3*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*d^2*x^3*t
an(1/2*b*x)^2*tan(1/2*a)^4 + 3*b^3*c^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 6*b^
3*c*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 12*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1
/2*a)^3 + 6*b^3*c*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 12*b^2*d^2*x^2*tan(1/
2*b*x)^3*tan(1/2*a)^4 + b^3*d^2*x^3*tan(1/2*b*x)^4 + 4*b^3*d^2*x^3*tan(1/2*
b*x)^2*tan(1/2*a)^2 + 6*b^3*c^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 24*b^2*c*d*
x*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^3*d^2*x^3*tan(1/2*a)^4 + 6*b^3*c^2*x*tan(
1/2*b*x)^2*tan(1/2*a)^4 - 24*b^2*c*d*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + 3*b*d^
2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 3*b^3*c*d*x^2*tan(1/2*b*x)^4 + 12*b^2*d^2
*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 12*b^3*c*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 72*b^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 + 72*b^2*d^2*x^2*tan(1/2*b*x)
^2*tan(1/2*a)^3 - 12*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 3*b^3*c*d*x^2*ta
```

n(1/2*a)^4 + 12*b^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 12*b^2*c^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 3*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^3*d^2*x^3*tan(1/2*b*x)^2 + 3*b^3*c^2*x*tan(1/2*b*x)^4 + 24*b^2*c*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^3*d^2*x^3*tan(1/2*a)^2 + 12*b^3*c^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 144*b^2*c*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 18*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 144*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 48*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^3 + 3*b^3*c^2*x*tan(1/2*a)^4 + 24*b^2*c*d*x*tan(1/2*b*x)*tan(1/2*a)^4 - 18*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*b^3*c*d*x^2*tan(1/2*b*x)^2 - 12*b^2*d^2*x^2*tan(1/2*b*x)^3 - 72*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 12*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a) + 6*b^3*c*d*x^2*tan(1/2*a)^2 - 72*b^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^2 + 72*b^2*c^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 18*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^2 - 12*b^2*d^2*x^2*tan(1/2*a)^3 + 72*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 48*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^3 + 6*d^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 12*b^2*c^2*tan(1/2*b*x)*tan(1/2*a)^4 - 18*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*d^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^3*d^2*x^3 + 6*b^3*c^2*x*tan(1/2*b*x)^2 - 24*b^2*c*d*x*tan(1/2*b*x)^3 + 3*b*d^2*x*tan(1/2*b*x)^4 - 144*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 48*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a) + 6*b^3*c^2*x*tan(1/2*a)^2 - 144*b^2*c*d*x*tan(1/2*b*x)*tan(1/2*a)^2 + 108*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*b^2*c*d*x*tan(1/2*a)^3 + 48*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^3 + 3*b*d^2*x*tan(1/2*a)^4 + 3*b^3*c*d*x^2 + 12*b^2*d^2*x^2*tan(1/2*b*x) - 12*b^2*c^2*tan(1/2*b*x)^3 + 3*b*c*d*tan(1/2*b*x)^4 + 12*b^2*d^2*x^2*tan(1/2*a) - 72*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a) + 48*b*c*d*tan(1/2*b*x)^3*tan(1/2*a) - 6*d^2*tan(1/2*b*x)^4*tan(1/2*a) - 72*b^2*c^2*tan(1/2*b*x)*tan(1/2*a)^2 + 108*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 36*d^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 12*b^2*c^2*tan(1/2*a)^3 + 48*b*c*d*tan(1/2*b*x)*tan(1/2*a)^3 - 36*d^2*tan(1/2*b*x)^2*tan(1/2*a)^3 + 3*b*c*d*tan(1/2*a)^4 - 6*d^2*tan(1/2*b*x)*tan(1/2*a)^4 + 3*b^3*c^2*x + 24*b^2*c*d*x*tan(1/2*b*x) - 18*b*d^2*x*tan(1/2*b*x)^2 + 24*b^2*c*d*x*tan(1/2*a) - 48*b*d^2*x*tan(1/2*b*x)*tan(1/2*a) - 18*b*d^2*x*tan(1/2*a)^2 + 12*b^2*c^2*tan(1/2*b*x) - 18*b*c*d*tan(1/2*b*x)^2 + 6*d^2*tan(1/2*b*x)^3 + 12*b^2*c^2*tan(1/2*a) - 48*b*c*d*tan(1/2*b*x)*tan(1/2*a) + 36*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 18*b*c*d*tan(1/2*a)^2 + 36*d^2*tan(1/2*b*x)*tan(1/2*a)^2 + 6*d^2*tan(1/2*a)^3 + 3*b*d^2*x + 3*b*c*d - 6*d^2*tan(1/2*b*x) - 6*d^2*tan(1/2*a))/(b^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^3*tan(1/2*b*x)^4 + 4*b^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^3*tan(1/2*a)^4 + 2*b^3*tan(1/2*b*x)^2 + 2*b^3*tan(1/2*a)^2 + b^3)

3.371 $\int (c + dx) \csc(a + bx) \sin(3a + 3bx) \, dx$

Optimal. Leaf size=66

$$-\frac{d \sin^2(a + bx)}{4b^2} + \frac{3d \cos^2(a + bx)}{4b^2} + \frac{2(c + dx) \sin(a + bx) \cos(a + bx)}{b} + cx + \frac{dx^2}{2}$$

```
[Out] c*x + (d*x^2)/2 + (3*d*Cos[a + b*x]^2)/(4*b^2) + (2*(c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/b - (d*Sin[a + b*x]^2)/(4*b^2)
```

Rubi [A] time = 0.0682019, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4431, 3310}

$$-\frac{d \sin^2(a + bx)}{4b^2} + \frac{3d \cos^2(a + bx)}{4b^2} + \frac{2(c + dx) \sin(a + bx) \cos(a + bx)}{b} + cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c*x + (d*x^2)/2 + (3*d*Cos[a + b*x]^2)/(4*b^2) + (2*(c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/b - (d*Sin[a + b*x]^2)/(4*b^2)
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)\cos^2(a+bx)-(c+dx)\sin^2(a+bx)\right)\,dx$$
$$= 3\int (c+dx)\cos^2(a+bx)\,dx - \int (c+dx)\sin^2(a+bx)\,dx$$
$$= \frac{3d\cos^2(a+bx)}{4b^2} + \frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b} - \frac{d\sin^2(a+bx)}{4b^2} - \frac{1}{2}\int($$
$$= cx + \frac{dx^2}{2} + \frac{3d\cos^2(a+bx)}{4b^2} + \frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b} - \frac{d\sin^2(a+}{4b^2}$$

Mathematica [A] time = 0.135017, size = 46, normalized size = 0.7

$$\frac{b(2(c+dx)\sin(2(a+bx))+bx(2c+dx))+d\cos(2(a+bx))}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*Cos[2*(a + b*x)] + b*(b*x*(2*c + d*x) + 2*(c + d*x)*Sin[2*(a + b*x)]))/(
2*b^2)
```

Maple [A] time = 0.037, size = 119, normalized size = 1.8

$$-cx - \frac{dx^2}{2} + 4\,\frac{c\,(1/2\,\cos(bx+a)\sin(bx+a)+1/2\,bx+a/2)}{b} + 4\,\frac{d\left((bx+a)\,(1/2\,\cos(bx+a)\sin(bx+a)+1/2\,bx+a/2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c*x-1/2*d*x^2+4*c/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+4*d/b^2*((b*
x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)
^2-a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A] time = 1.06442, size = 74, normalized size = 1.12

$$\frac{(bx+\sin(2\,bx+2\,a))c}{b} + \frac{\left(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)d}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c/b + 1/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(
2*b*x + 2*a))*d/b^2
```

Fricas [A] time = 0.484193, size = 132, normalized size = 2.

$$\frac{b^2dx^2 + 2\,b^2cx + 2\,d\cos\left(bx + a\right)^2 + 4\left(bdx + bc\right)\cos\left(bx + a\right)\sin\left(bx + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d*x^2 + 2*b^2*c*x + 2*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(b*x +
 a)*sin(b*x + a))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20591, size = 1242, normalized size = 18.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*x*tan(1/2*b*x)^4*tan(1
/2*a)^4 + 2*b^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d*x^2*tan(1/2*b*x
)^2*tan(1/2*a)^4 + 4*b^2*c*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*
b*x)^4*tan(1/2*a)^3 + 4*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 8*b*d*x*tan(1
/2*b*x)^3*tan(1/2*a)^4 + b^2*d*x^2*tan(1/2*b*x)^4 + 4*b^2*d*x^2*tan(1/2*b*x
)^2*tan(1/2*a)^2 - 8*b*c*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d*x^2*tan(1/2*a)
^4 - 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^4 + d*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*
b^2*c*x*tan(1/2*b*x)^4 + 8*b*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*x*tan(
1/2*b*x)^2*tan(1/2*a)^2 + 48*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 + 48*b*d*x*t
an(1/2*b*x)^2*tan(1/2*a)^3 + 2*b^2*c*x*tan(1/2*a)^4 + 8*b*d*x*tan(1/2*b*x)*
tan(1/2*a)^4 + 2*b^2*d*x^2*tan(1/2*b*x)^2 + 8*b*c*tan(1/2*b*x)^4*tan(1/2*a)
 + 2*b^2*d*x^2*tan(1/2*a)^2 + 48*b*c*tan(1/2*b*x)^3*tan(1/2*a)^2 - 6*d*tan(
1/2*b*x)^4*tan(1/2*a)^2 + 48*b*c*tan(1/2*b*x)^2*tan(1/2*a)^3 - 16*d*tan(1/2
*b*x)^3*tan(1/2*a)^3 + 8*b*c*tan(1/2*b*x)*tan(1/2*a)^4 - 6*d*tan(1/2*b*x)^2
*tan(1/2*a)^4 + 4*b^2*c*x*tan(1/2*b*x)^2 - 8*b*d*x*tan(1/2*b*x)^3 - 48*b*d*
x*tan(1/2*b*x)^2*tan(1/2*a) + 4*b^2*c*x*tan(1/2*a)^2 - 48*b*d*x*tan(1/2*b*x
)*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*a)^3 + b^2*d*x^2 - 8*b*c*tan(1/2*b*x)^3 +
d*tan(1/2*b*x)^4 - 48*b*c*tan(1/2*b*x)^2*tan(1/2*a) + 16*d*tan(1/2*b*x)^3*t
an(1/2*a) - 48*b*c*tan(1/2*b*x)*tan(1/2*a)^2 + 36*d*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 8*b*c*tan(1/2*a)^3 + 16*d*tan(1/2*b*x)*tan(1/2*a)^3 + d*tan(1/2*a)^4
 + 2*b^2*c*x + 8*b*d*x*tan(1/2*b*x) + 8*b*d*x*tan(1/2*a) + 8*b*c*tan(1/2*b*
x) - 6*d*tan(1/2*b*x)^2 + 8*b*c*tan(1/2*a) - 16*d*tan(1/2*b*x)*tan(1/2*a) -
 6*d*tan(1/2*a)^2 + d)/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*tan(1/2*b*x
)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^2*tan(1/2*b*x)^4 +
 4*b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^2*tan(1/2*a)^4 + 2*b^2*tan(1/2*b*x)^
2 + 2*b^2*tan(1/2*a)^2 + b^2)
```

3.372 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=71

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}$$

```
[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/d
- (2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
```

Rubi [A] time = 0.281018, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4431, 3312, 3303, 3299, 3302}

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/d
- (2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos^2(a+bx)}{c+dx} - \frac{\sin^2(a+bx)}{c+dx}\right) dx \\
&= 3\int \frac{\cos^2(a+bx)}{c+dx}\,dx - \int \frac{\sin^2(a+bx)}{c+dx}\,dx \\
&= 3\int \left(\frac{1}{2(c+dx)} + \frac{\cos(2a+2bx)}{2(c+dx)}\right) dx - \int \left(\frac{1}{2(c+dx)} - \frac{\cos(2a+2bx)}{2(c+dx)}\right) dx \\
&= \frac{\log(c+dx)}{d} + \frac{1}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx + \frac{3}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{d} + \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx + \frac{1}{2}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \text{co} \\
&= \frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d} + \frac{\log(c+dx)}{d} - \frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.15281, size = 63, normalized size = 0.89

$$
\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \log(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - 2*S
in[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d
```

Maple [A] time = 0.037, size = 116, normalized size = 1.6

$$-\frac{\ln(dx+c)}{d}+2\frac{1}{d}\mathrm{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\sin\left(2\frac{-ad+bc}{d}\right)+2\frac{1}{d}\mathrm{Ci}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\cos\left(2\frac{-ad+bc}{d}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] -ln(d*x+c)/d+2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*
x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d+2*ln((b*x+a)*d-a*d+b*c)/d
```

Maxima [C] time = 1.2865, size = 158, normalized size = 2.23

$$-\frac{\left(E_1\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-\left(i\,E_1\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-\log(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(1, -(2*I*b*d
*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (I*exp_integral_e(1, (2*I*b*d*x +
 2*I*b*c)/d) - I*exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) - log(d*x + c))/d
```

Fricas [A] time = 0.489241, size = 228, normalized size = 3.21

$$\frac{\left(\mathrm{Ci}\left(\frac{2(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-2\sin\left(-\frac{2(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2(bdx+bc)}{d}\right)+\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*cos(-
2*(b*c - a*d)/d) - 2*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d)
+ log(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.29773, size = 1509, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] (log(abs(d*x + c))*tan(1/2*a)^4*tan(b*c/d)^2 - real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 - real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(1/2*a)^4*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(1/2*a)^4*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan
(b*c/d) - 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d
)^2 + 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2
 - 8*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + log(abs(d*
x + c))*tan(1/2*a)^4 + real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^
4 + real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 8*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) - 8*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 2*log(abs(d*x + c))*tan(1
/2*a)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a
)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2
*tan(b*c/d)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 - 4
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 + 8*sin_integral(2*
(b*d*x + b*c)/d)*tan(1/2*a)^3 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))
```

```
*tan(1/2*a)^2*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(1/2*a)^2*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(
b*c/d) + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2
 - 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 8*
sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 + 2*log(abs(d*x + c
))*tan(1/2*a)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2 -
 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 + 8*real_part(cos
_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + 8*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + log(abs(d*x + c))*tan(b*c/d)^2
 - real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(1/2*a) + 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a
) - 8*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a) + 2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + log(abs(d*x +
c)) + real_part(cos_integral(2*b*x + 2*b*c/d)) + real_part(cos_integral(-2*
b*x - 2*b*c/d)))/(d*tan(1/2*a)^4*tan(b*c/d)^2 + d*tan(1/2*a)^4 + 2*d*tan(1/
2*a)^2*tan(b*c/d)^2 + 2*d*tan(1/2*a)^2 + d*tan(b*c/d)^2 + d)
```

3.373 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{3\cos^2(a+bx)}{d(c+dx)}$$

```
[Out] (-3*Cos[a + b*x]^2)/(d*(c + d*x)) - (4*b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin
[2*a - (2*b*c)/d])/d^2 + Sin[a + b*x]^2/(d*(c + d*x)) - (4*b*Cos[2*a - (2*b
*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2
```

Rubi [A] time = 0.276183, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4431, 3313, 12, 3303, 3299, 3302}

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{3\cos^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```


```
[Out] (-3*Cos[a + b*x]^2)/(d*(c + d*x)) - (4*b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin
[2*a - (2*b*c)/d])/d^2 + Sin[a + b*x]^2/(d*(c + d*x)) - (4*b*Cos[2*a - (2*b
*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

```
LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int\left(\frac{3\cos^2(a+bx)}{(c+dx)^2} - \frac{\sin^2(a+bx)}{(c+dx)^2}\right)dx$$
$$= 3\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= -\frac{3\cos^2(a+bx)}{d(c+dx)} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{(2b)\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d} + \frac{(6b)\int -\frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d}$$
$$= -\frac{3\cos^2(a+bx)}{d(c+dx)} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{b\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d} - \frac{(3b)\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d}$$
$$= -\frac{3\cos^2(a+bx)}{d(c+dx)} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} - \frac{\left(3b\cos\left(2a\right.\right.}{}$$
$$= -\frac{3\cos^2(a+bx)}{d(c+dx)} - \frac{4b\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{4b\cos\left(2a-\frac{2bc}{d}\right.}{}$$

Mathematica [A] time = 0.54519, size = 81, normalized size = 0.79

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + 4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(2\cos(2(a+bx))+1)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] -(((d*(1 + 2*Cos[2*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(2*b*(c + d*x))
/d]*Sin[2*a - (2*b*c)/d] + 4*b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d
*x))/d])/d^2)
```

Maple [A] time = 0.041, size = 169, normalized size = 1.7

$$\frac{1}{d(dx+c)} + 4\frac{1}{b}\left(1/4\,b^2\left(-2\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)d} - 2\frac{1}{d}\left(2\frac{1}{d}\operatorname{Si}\left(2bx+2a+2\frac{-ad+bc}{d}\right)\cos\left(2\frac{-ad+bc}{d}\right) - 2\frac{1}{d}\operatorname{Ci}\left(\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] 1/d/(d*x+c)+4/b*(1/4*b^2*(-2*cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d-2*(2*Si(2
*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/
d)*sin(2*(-a*d+b*c)/d)/d)/d)-1/2*b^2/((b*x+a)*d-a*d+b*c)/d)
```

Maxima [C] time = 1.32883, size = 159, normalized size = 1.56

$$-\frac{\left(E_2\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_2\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(i\,E_2\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_2\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+1}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(2, -(2*I*b*d
*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (I*exp_integral_e(2, (2*I*b*d*x +
 2*I*b*c)/d) - I*exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) + 1)/(d^2*x + c*d)
```

Fricas [A] time = 0.518525, size = 321, normalized size = 3.15

$$-\frac{4\,d\cos\left(bx+a\right)^2+4\,(bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+2\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\,\mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(4*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2
*(b*d*x + b*c)/d) + 2*((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d
*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - d)/(d^3
*x + c*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)\sin(3bx+3a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

3.374 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=136

$$-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}+\frac{\sin^2(a}{2d(c+}$$

```
[Out] (-3*Cos[a + b*x]^2)/(2*d*(c + d*x)^2) - (4*b^2*Cos[2*a - (2*b*c)/d]*CosInte
gral[(2*b*c)/d + 2*b*x])/d^3 + (4*b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*
x)) + Sin[a + b*x]^2/(2*d*(c + d*x)^2) + (4*b^2*Sin[2*a - (2*b*c)/d]*SinInt
egral[(2*b*c)/d + 2*b*x])/d^3
```

Rubi [A] time = 0.374217, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4431, 3314, 31, 3312, 3303, 3299, 3302}

$$-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}+\frac{\sin^2(a}{2d(c+}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-3*Cos[a + b*x]^2)/(2*d*(c + d*x)^2) - (4*b^2*Cos[2*a - (2*b*c)/d]*CosInte
gral[(2*b*c)/d + 2*b*x])/d^3 + (4*b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*
x)) + Sin[a + b*x]^2/(2*d*(c + d*x)^2) + (4*b^2*Sin[2*a - (2*b*c)/d]*SinInt
egral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
```

```
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx)}{(c+dx)^3}\right) dx$$

$$= 3\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\int \frac{1}{c+dx}\,dx}{d^2} + \frac{\left(2b^2\right)\int}{}$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{2b^2\log(c+dx)}{d^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{(2b}{}$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d^2} - \frac{(}{}$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int}{d^2}$$

$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} - \frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\mathrm{s}}{2}$$

Mathematica [A] time = 0.998589, size = 104, normalized size = 0.76

$$-\frac{8b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 8b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(-4b(c+dx)\sin(2(a+bx))+2d\cos(2(a+bx))+d)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] -(8*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d + 2*d*C
os[2*(a + b*x)] - 4*b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 - 8*b^2*Sin[
2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Maple [A] time = 0.038, size = 207, normalized size = 1.5

$$\frac{1}{2\,d\,(dx+c)^2} + 4\,\frac{1}{b}\Bigg(1/4\,b^3\Bigg(-\frac{\cos(2\,bx+2\,a)}{((bx+a)\,d-ad+bc)^2\,d} - \frac{1}{d}\Bigg(-2\,\frac{\sin(2\,bx+2\,a)}{((bx+a)\,d-ad+bc)\,d} + 2\,\frac{1}{d}\Bigg(2\,\frac{1}{d}\text{Si}\Bigg(2\,bx+2\,a+2\,\frac{-ad+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] 1/2/d/(d*x+c)^2+4/b*(1/4*b^3*(-cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d-(-2*s
in(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2
*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d
)-1/4*b^3/((b*x+a)*d-a*d+b*c)^2/d)
```

Maxima [C] time = 1.4501, size = 176, normalized size = 1.29

$$-\frac{2\left(E_3\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_3\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(2i\,E_3\left(\frac{2i\,bdx+2i\,bc}{d}\right)-2i\,E_3\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+1}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(exp_integral_e(3, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(3, -(2
*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (2*I*exp_integral_e(3, (2*I
*b*d*x + 2*I*b*c)/d) - 2*I*exp_integral_e(3, -(2*I*b*d*x + 2*I*b*c)/d))*sin
(-2*(b*c - a*d)/d) + 1)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [A] time = 0.530617, size = 516, normalized size = 3.79

$$-\frac{4\,d^2\cos\left(bx+a\right)^2-8\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-8\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{2\left(d^5x^2+2\,cd^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d^2*cos(b*x + a)^2 - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a)
- 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c - a*d)/d)*sin_integra
l(2*(b*d*x + b*c)/d) - d^2 + 4*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_i
ntegral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_inte
gral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2
*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [C] time = 2.1155, size = 12712, normalized size = 93.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
1/2*a)^4*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*d^2*x^2*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d
) - 16*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^4*
tan(b*c/d) + 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*d^2*x^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 8*b^2*c*d*
x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/
d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(1/2*a)^4*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 + 32*b^2*d^2*x^2*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 32*b^2*d^2*x^2*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) -
16*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)
^4*tan(b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/
d)*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 24*b^2*d^2*x^2*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b^2*d^2*x^2*
```

```
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d
)^2 + 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
1/2*a)^3*tan(b*c/d)^2 - 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 64*b^2*c*d*x*sin_integral(2*(b*d
*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*d^2*x^2*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*
c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*
c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(1/2*a)^4*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(1/2*a)^3 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 32*b^2*d^2*x^2*sin_integral(2*(b*d*
x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3 - 8*b^2*c*d*x*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 8*b^2*c*d*x*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 + 48*b^2*d^2*x^2*imag_part(cos
_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*d^2
*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(
b*c/d) + 96*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*
a)^2*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) - 8*b^2*d^2*x^2*imag_part(
cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*d^2*x^2*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) - 16*b^2*d^2*
x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d) - 8*b^2*c^2*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) +
8*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4
*tan(b*c/d) - 16*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2
*a)^4*tan(b*c/d) - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*d^2*x^2*sin_
integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 48*b^2*c*d
*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c
/d)^2 - 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*t
an(1/2*a)^2*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*d^2*x^2*sin_integral(2
*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*c^2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^
2 + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(
b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^
4*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
1/2*a)^4*tan(b*c/d)^2 + 24*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(1/2*a)^2 + 24*b^2*d^2*x^2*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 32*b^2*c*d*x*imag_part(cos_integral(
```

2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 + 32*b^2*c*d*x*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 64*b^2*c*d*x*sin_integral
(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3 - 4*b^2*d^2*x^2*real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 - 4*b^2*d^2*x^2*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 4*b^2*c^2*real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b^2*c^2*real_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 32*b^2*d^2*x^2*real_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*b^2*d^2*x^2*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) +
96*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)
^2*tan(b*c/d) - 96*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 192*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 32*b^2*d^2*x^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b^2*d^2*x^2*real_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b^2*c^2*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 32*b^
2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan
(b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^
4*tan(b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1
/2*a)^4*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a
)^4*tan(b*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(b*c/d)^2 - 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 32*b^2*c*d*x*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 64*b^2*c*d*x
sin_integral(2(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 24*b^
2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^
2 - 24*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*t
an(b*c/d)^2 - 24*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^
2*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b^2*c^2*real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + 32*b^2*c*d*x*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 32*b^2*c*d*x*imag_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 64*b^2*c*d*x*
sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + 16*b*d^2*x*tan(
b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 8*b*d^2*x*tan(b*x)*tan(1/2*a)^4
*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(1/2*a) - 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(1/2*a) + 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*t
an(b*x)^2*tan(1/2*a) + 48*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2*tan(1/2*a)^2 + 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(1/2*a)^3 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(1/2*a)^3 - 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*t

an(1/2*a)^3 - 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^
2*tan(1/2*a)^3 + 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b
*x)^2*tan(1/2*a)^3 - 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*
tan(1/2*a)^3 - 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2
*a)^4 - 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4
- 8*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c
/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(b*c/d) - 16*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*
c/d) - 64*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(1/2*a)*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 48*b^2*d^2*x^2*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*d^2*x^2*imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 96*b^2*d^2*x^2*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d) + 48*b^2*c^2*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*c^2*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)
 + 96*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^2*tan(b
*c/d) + 64*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*
tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2
*a)^3*tan(b*c/d) - 8*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1
/2*a)^4*tan(b*c/d) + 8*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(1/2*a)^4*tan(b*c/d) - 16*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*
a)^4*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*x)^2*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*d^2*x^2*sin_integral(2*(b*
d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2
*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 4
8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d
)^2 - 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*t
an(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a
)^3*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2
*a)^3*tan(b*c/d)^2 + 16*b*c*d*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 8*b*c*
d*tan(b*x)*tan(1/2*a)^4*tan(b*c/d)^2 + d^2*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/
d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 -
4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 32*b^2
*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a) - 32*
b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a) +
 64*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a) + 24*b^
2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2 + 24*b^2*d^
2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 + 24*b^2*c^2*r

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eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 + 24*b^2*c^
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 32*b^
2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 + 32*b^2*c*d*
x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 - 64*b^2*c*d*x*sin
_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3 + 16*b*d^2*x*tan(b*x)^2*tan(1/2*a
)^3 - 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 - 4*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 + 8*b*d^2*x*t
an(b*x)*tan(1/2*a)^4 - 16*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2*tan(b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(b*c/d) - 32*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(1/2*a)*tan(b*c/d) - 32*b^2*d^2*x^2*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(1/2*a)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 96*b^2*c*d*x*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 96*b^2*c*d
*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 192*
b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d) + 32*b^2*
c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) +
4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 4*b^2
*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 4*b^2*c^2
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 4*b^2*c
^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 32*b
^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 +
 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/
d)^2 - 64*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2
 - 16*b*d^2*x*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 24*b^2*c^2*real_part(cos
_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b^2*c^2*real_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 48*b*d^2*x*ta
n(b*x)*tan(1/2*a)^2*tan(b*c/d)^2 - 16*b*d^2*x*tan(1/2*a)^3*tan(b*c/d)^2 - 8
*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - 8*b^2*c*d*
x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 16*b^2*d^2*x^2*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a) - 16*b^2*d^2*x^2*imag_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 32*b^2*d^2*x^2*sin_integral(2
*(b*d*x + b*c)/d)*tan(1/2*a) + 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(1/2*a) - 16*b^2*c^2*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(b*x)^2*tan(1/2*a) + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d
)*tan(b*x)^2*tan(1/2*a) + 48*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(1/2*a)^2 + 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(1/2*a)^2 - 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*
a)^3 + 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 -
32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3 + 16*b*c*d*tan(b*x)
^2*tan(1/2*a)^3 + 8*b*c*d*tan(b*x)*tan(1/2*a)^4 + d^2*tan(b*x)^2*tan(1/2*a)
^4 - 8*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 8*
```

b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 16*b^2*d ^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) - 8*b^2*c^2*imag_part(cos _integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 8*b^2*c^2*imag_part(cos _integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 16*b^2*c^2*sin_integra l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_int egral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_ integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + 48*b^2*c^2*imag_part(co s_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*c^2*imag_part (cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 96*b^2*c^2*sin_i ntegral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_int egral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(2 *b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_integra l(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*c^2*sin_integral(2*(b *d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 - 16*b*c*d*tan(b*x)^2*tan(1/2*a)*tan (b*c/d)^2 - 48*b*c*d*tan(b*x)*tan(1/2*a)^2*tan(b*c/d)^2 - 14*d^2*tan(b*x)^2 *tan(1/2*a)^2*tan(b*c/d)^2 - 16*b*c*d*tan(1/2*a)^3*tan(b*c/d)^2 - 16*d^2*ta n(b*x)*tan(1/2*a)^3*tan(b*c/d)^2 - 3*d^2*tan(1/2*a)^4*tan(b*c/d)^2 - 4*b^2* d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)) - 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d)) - 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))* tan(b*x)^2 + 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2* a) - 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 64 *b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a) - 16*b*d^2*x*tan(b*x) ^2*tan(1/2*a) + 24*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2 *a)^2 + 24*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 - 48*b*d^2*x*tan(b*x)*tan(1/2*a)^2 - 16*b*d^2*x*tan(1/2*a)^3 - 16*b^2*c*d*x* imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 16*b^2*c*d*x*imag_par t(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2* (b*d*x + b*c)/d)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_integral(2*b*x + 2*b *c/d))*tan(1/2*a)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_integral(-2*b*x - 2 *b*c/d))*tan(1/2*a)*tan(b*c/d) + 4*b^2*c^2*real_part(cos_integral(2*b*x + 2 *b*c/d))*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d)) *tan(b*c/d)^2 + 8*b*d^2*x*tan(b*x)*tan(b*c/d)^2 + 16*b*d^2*x*tan(1/2*a)*tan (b*c/d)^2 - 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)) - 8*b^2*c* d*x*real_part(cos_integral(-2*b*x - 2*b*c/d)) + 16*b^2*c^2*imag_part(cos_in tegral(2*b*x + 2*b*c/d))*tan(1/2*a) - 16*b^2*c^2*imag_part(cos_integral(-2* b*x - 2*b*c/d))*tan(1/2*a) + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan (1/2*a) - 16*b*c*d*tan(b*x)^2*tan(1/2*a) - 48*b*c*d*tan(b*x)*tan(1/2*a)^2 - 14*d^2*tan(b*x)^2*tan(1/2*a)^2 - 16*b*c*d*tan(1/2*a)^3 - 16*d^2*tan(b*x)*t an(1/2*a)^3 - 3*d^2*tan(1/2*a)^4 - 8*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 8*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)) *tan(b*c/d) - 16*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 8*b*c *d*tan(b*x)*tan(b*c/d)^2 + d^2*tan(b*x)^2*tan(b*c/d)^2 + 16*b*c*d*tan(1/2*a)*tan(b*c/d)^2 + 16*d^2*tan(b*x)*tan(1/2*a)*tan(b*c/d)^2 + 10*d^2*tan(1/2*a

)^2*tan(b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d)) - 4*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d)) + 8*b*d^2*x*tan(b*x) + 16*
b*d^2*x*tan(1/2*a) + 8*b*c*d*tan(b*x) + d^2*tan(b*x)^2 + 16*b*c*d*tan(1/2*a
) + 16*d^2*tan(b*x)*tan(1/2*a) + 10*d^2*tan(1/2*a)^2 - 3*d^2*tan(b*c/d)^2 -
 3*d^2)/(d^5*x^2*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^
2*tan(1/2*a)^4*tan(b*c/d)^2 + d^5*x^2*tan(b*x)^2*tan(1/2*a)^4 + 2*d^5*x^2*t
an(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + d^5*x^2*tan(1/2*a)^4*tan(b*c/d)^2 + c
^2*d^3*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2*tan(1/2*
a)^4 + 4*c*d^4*x*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(1/2*a
)^4*tan(b*c/d)^2 + 2*d^5*x^2*tan(b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1/2*a)^4
 + c^2*d^3*tan(b*x)^2*tan(1/2*a)^4 + d^5*x^2*tan(b*x)^2*tan(b*c/d)^2 + 2*d^
5*x^2*tan(1/2*a)^2*tan(b*c/d)^2 + 2*c^2*d^3*tan(b*x)^2*tan(1/2*a)^2*tan(b*c
/d)^2 + c^2*d^3*tan(1/2*a)^4*tan(b*c/d)^2 + 4*c*d^4*x*tan(b*x)^2*tan(1/2*a)
^2 + 2*c*d^4*x*tan(1/2*a)^4 + 2*c*d^4*x*tan(b*x)^2*tan(b*c/d)^2 + 4*c*d^4*x
*tan(1/2*a)^2*tan(b*c/d)^2 + d^5*x^2*tan(b*x)^2 + 2*d^5*x^2*tan(1/2*a)^2 +
2*c^2*d^3*tan(b*x)^2*tan(1/2*a)^2 + c^2*d^3*tan(1/2*a)^4 + d^5*x^2*tan(b*c/
d)^2 + c^2*d^3*tan(b*x)^2*tan(b*c/d)^2 + 2*c^2*d^3*tan(1/2*a)^2*tan(b*c/d)^
2 + 2*c*d^4*x*tan(b*x)^2 + 4*c*d^4*x*tan(1/2*a)^2 + 2*c*d^4*x*tan(b*c/d)^2
+ d^5*x^2 + c^2*d^3*tan(b*x)^2 + 2*c^2*d^3*tan(1/2*a)^2 + c^2*d^3*tan(b*c/d
)^2 + 2*c*d^4*x + c^2*d^3)

3.375 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=205

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}+\frac{2b^2\cos^2(a+bx)}{d^3(c+dx)}$$

```
[Out] (-2*b^2)/(3*d^3*(c + d*x)) - Cos[a + b*x]^2/(d*(c + d*x)^3) + (2*b^2*Cos[a
+ b*x]^2)/(d^3*(c + d*x)) + (8*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a -
 (2*b*c)/d])/(3*d^4) + (4*b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^2)
+ Sin[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c + d*x
)) + (8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rubi [A] time = 0.379893, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4431, 3314, 32, 3313, 12, 3303, 3299, 3302}

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}+\frac{2b^2\cos^2(a+bx)}{d^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

```
[Out] (-2*b^2)/(3*d^3*(c + d*x)) - Cos[a + b*x]^2/(d*(c + d*x)^3) + (2*b^2*Cos[a
+ b*x]^2)/(d^3*(c + d*x)) + (8*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a -
 (2*b*c)/d])/(3*d^4) + (4*b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^2)
+ Sin[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c + d*x
)) + (8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^4} - \frac{\sin^2(a+bx)}{(c+dx)^4}\right) dx$$
$$= 3\int \frac{\cos^2(a+bx)}{(c+dx)^4}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx$$
$$= -\frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} + \frac{\sin^2(a+bx)}{3d(c+dx)^3} - \frac{b^2\int \frac{1}{(c+dx)^2}\,dx}{3d^2} + \frac{(2b^2}{}$$
$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} + \frac{\sin}{3d}$$
$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} + \frac{\sin}{3d}$$
$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} + \frac{\sin}{3d}$$
$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{8b^3\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^4}$$

Mathematica [A] time = 1.12737, size = 125, normalized size = 0.61

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+\frac{d\left(\cos(2(a+bx))\left(4b^2(c+dx)^2-2d^2\right)+d(2b(c+dx)\sin(2(a+bx))-d)\right)}{(c+dx)^3}+8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b}{}\right.}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

```
[Out] (8*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-2*d^2 +
4*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(-d + 2*b*(c + d*x)*Sin[2*(a + b*x)
])))/(c + d*x)^3 + 8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d
])/(3*d^4)
```

Maple [A] time = 0.039, size = 243, normalized size = 1.2

$$\frac{1}{3d(dx+c)^3} + 4\frac{1}{b}\left(1/4\,b^4\left(-2/3\,\frac{\cos(2bx+2a)}{((bx+a)d-ad+bc)^3 d} - 2/3\,\frac{1}{d}\left(-\frac{\sin(2bx+2a)}{((bx+a)d-ad+bc)^2 d} + \frac{1}{d}\left(-2\,\frac{\cos(2bx+2}{((bx+a)d-ad}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x)
```

```
[Out] 1/3/d/(d*x+c)^3+4/b*(1/4*b^4*(-2/3*cos(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^3/d-2
/3*(-sin(2*b*x+2*a)/((b*x+a)*d-a*d+b*c)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-a
*d+b*c)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*
x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/6*b^4/((b*x+a)*d-a*
d+b*c)^3/d)
```

Maxima [C] time = 1.51092, size = 190, normalized size = 0.93

$$-\frac{3\left(E_4\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_4\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(3i\,E_4\left(\frac{2i\,bdx+2i\,bc}{d}\right)-3i\,E_4\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+1}{3\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(exp_integral_e(4, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(4, -(2
*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (3*I*exp_integral_e(4, (2*I
*b*d*x + 2*I*b*c)/d) - 3*I*exp_integral_e(4, -(2*I*b*d*x + 2*I*b*c)/d))*sin
(-2*(b*c - a*d)/d) + 1)/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d)
```

Fricas [A] time = 0.556165, size = 748, normalized size = 3.65

$$-\frac{4\,b^2d^3x^2+8\,b^2cd^2x+4\,b^2c^2d-d^3-4\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2-4\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*b^2*d^3*x^2 + 8*b^2*c*d^2*x + 4*b^2*c^2*d - d^3 - 4*(2*b^2*d^3*x^2
+ 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 4*(b*d^3*x + b*c*d^2)
*cos(b*x + a)*sin(b*x + a) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 4*((b
```

```
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d
*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*co
s_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x
^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.376 $\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=255

$$-\frac{18d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{18d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{9id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{9id(c+$$

```
[Out] (-6*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (24*d^2*(c + d*x)*Cos[a + b*x
])/b^3 + (4*(c + d*x)^3*Cos[a + b*x])/b + ((9*I)*d*(c + d*x)^2*PolyLog[2, -
E^(I*(a + b*x))])/b^2 - ((9*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b
^2 - (18*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (18*d^2*(c + d*x
)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((18*I)*d^3*PolyLog[4, -E^(I*(a + b*x)
)])/b^4 + ((18*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^3*Sin[a + b*
x])/b^4 - (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.34706, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4431, 4408, 3296, 2637, 4183, 2531, 6609, 2282, 6589}

$$-\frac{18d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{18d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{9id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{9id(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```


```
[Out] (-6*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (24*d^2*(c + d*x)*Cos[a + b*x
])/b^3 + (4*(c + d*x)^3*Cos[a + b*x])/b + ((9*I)*d*(c + d*x)^2*PolyLog[2, -
E^(I*(a + b*x))])/b^2 - ((9*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b
^2 - (18*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (18*d^2*(c + d*x
)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((18*I)*d^3*PolyLog[4, -E^(I*(a + b*x)
)])/b^4 + ((18*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^3*Sin[a + b*
x])/b^4 - (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
```

```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\cos(a+bx)\cot(a+bx) - (c+dx)^3\sin(a+bx)\right)dx$$
$$= 3\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\,dx$$
$$= \frac{(c+dx)^3\cos(a+bx)}{b} + 3\int (c+dx)^3\csc(a+bx)\,dx - 3\int (c+dx)^3\sin(a+$$
$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)^3\cos(a+bx)}{b} - \frac{3d(c+dx)^2\sin(a+}{b^2}$$
$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+dx)^3\cos(a+}{b}$$
$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+dx)^3\cos(a}{b}$$
$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+dx)^3\cos(a}{b}$$
$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+dx)^3\cos(a}{b}$$

Mathematica [A] time = 1.57214, size = 459, normalized size = 1.8

$$\frac{3\left(3id\left(b^2(c+dx)^2\text{PolyLog}(2,-\cos(a+bx)-i\sin(a+bx)) + 2ibd(c+dx)\text{PolyLog}(3,-\cos(a+bx)-i\sin(a+bx)) - 2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^
2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d
```

```
*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a +
b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x]
+ I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a +
 b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]])))/b^4 + (4*Cos[b*
x]*(b^3*c^3*Cos[a] - 6*b*c*d^2*Cos[a] + 3*b^3*c^2*d*x*Cos[a] - 6*b*d^3*x*Co
s[a] + 3*b^3*c*d^2*x^2*Cos[a] + b^3*d^3*x^3*Cos[a] - 3*b^2*c^2*d*Sin[a] + 6
*d^3*Sin[a] - 6*b^2*c*d^2*x*Sin[a] - 3*b^2*d^3*x^2*Sin[a]))/b^4 - (4*(3*b^2
*c^2*d*Cos[a] - 6*d^3*Cos[a] + 6*b^2*c*d^2*x*Cos[a] + 3*b^2*d^3*x^2*Cos[a]
+ b^3*c^3*Sin[a] - 6*b*c*d^2*Sin[a] + 3*b^3*c^2*d*x*Sin[a] - 6*b*d^3*x*Sin[
a] + 3*b^3*c*d^2*x^2*Sin[a] + b^3*d^3*x^3*Sin[a])*Sin[b*x])/b^4
```

Maple [B] time = 0.196, size = 849, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] 2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^
3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-9/
b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+9/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+18*I*d^
3*polylog(4,exp(I*(b*x+a)))/b^4-9*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+9
*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-9*I/b^2*c^2*d*polylog(2,exp(I*(b*
x+a)))+6/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))-3/b^4*d^3*ln(exp(I*(b*x+a))+1)
*a^3+9/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))+1)+9*I/b^2*c^2*d*polylog(2,-exp(I*(b
*x+a)))+2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^
2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x
+a))+18/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))-18/b^3*c*d^2*a^2*arctanh(exp(I*
(b*x+a)))-9/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a-18*I/b^2*c*d^2*polylog(2,exp(I
*(b*x+a)))*x+18*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+3/b*d^3*ln(1-exp(I
*(b*x+a)))*x^3+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-3/b*d^3*ln(exp(I*(b*x+a))
+1)*x^3-9/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))-9/b*c^2*d*ln(exp(I*(b*x+a))+1)
*x+9/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+9/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-6/b*
c^3*arctanh(exp(I*(b*x+a)))-18*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+18/b^3*
d^3*polylog(3,exp(I*(b*x+a)))*x-18/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+18/
b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-18/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))
```

Maxima [B] time = 1.80947, size = 813, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)
*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2*(3
6*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I*a))
 + (6*I*b^3*d^3*x^3 + 18*I*b^3*c*d^2*x^2 + 18*I*b^3*c^2*d*x)*arctan2(sin(b*
x + a), cos(b*x + a) + 1) + (6*I*b^3*d^3*x^3 + 18*I*b^3*c*d^2*x^2 + 18*I*b^
3*c^2*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 8*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a) + (-18*I*
b^2*d^3*x^2 - 36*I*b^2*c*d^2*x - 18*I*b^2*c^2*d)*dilog(-e^(I*b*x + I*a)) +
(18*I*b^2*d^3*x^2 + 36*I*b^2*c*d^2*x + 18*I*b^2*c^2*d)*dilog(e^(I*b*x + I*a
)) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x)*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) +
36*(b*d^3*x + b*c*d^2)*polylog(3, -e^(I*b*x + I*a)) - 36*(b*d^3*x + b*c*d^2
)*polylog(3, e^(I*b*x + I*a)) + 24*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d
 - 2*d^3)*sin(b*x + a))/b^4
```

Fricas [C] time = 0.75591, size = 2314, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(18*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 18*I*d^3*polylog(
4, cos(b*x + a) - I*sin(b*x + a)) + 18*I*d^3*polylog(4, -cos(b*x + a) + I*s
in(b*x + a)) - 18*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*
x)*cos(b*x + a) + (-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9
*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-9*I*b^2*d^3*x^2 - 18
*I*b^2*c*d^2*x - 9*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (9*
I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*s
in(b*x + a)) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I
```

```
*sin(b*x + a) + 1/2) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) -
I*sin(b*x + a) + 1) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*si
n(b*x + a)) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x +
a)) - 18*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 1
8*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \csc(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

3.377 $\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=172

$$\frac{6id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{6d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}$$

```
[Out] (-6*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (8*d^2*Cos[a + b*x])/b^3 + (4
*(c + d*x)^2*Cos[a + b*x])/b + ((6*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x
))])/b^2 - ((6*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*Pol
yLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 -
(8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.228793, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4431, 4408, 3296, 2638, 4183, 2531, 2282, 6589}

$$\frac{6id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{6d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```


```
[Out] (-6*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (8*d^2*Cos[a + b*x])/b^3 + (4
*(c + d*x)^2*Cos[a + b*x])/b + ((6*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x
))])/b^2 - ((6*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*Pol
yLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 -
(8*d*(c + d*x)*Sin[a + b*x])/b^2
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
```

```
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\cos(a+bx)\cot(a+bx) - (c+dx)^2\sin(a+bx)\right)dx$$
$$= 3\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx - \int (c+dx)^2\sin(a+bx)\,dx$$
$$= \frac{(c+dx)^2\cos(a+bx)}{b} + 3\int (c+dx)^2\csc(a+bx)\,dx - 3\int (c+dx)^2\sin(a+$$
$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{2d(c+dx)\sin(a+}{b^2}$$
$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2\cos(a+bx)}{b} +$$
$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{8d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2\cos(a+bx)}{b} +$$
$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{8d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2\cos(a+bx)}{b} +$$

Mathematica [A] time = 1.15642, size = 223, normalized size = 1.3

$$\frac{6ibd(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right) - 6ibd(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right) - 6d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right) + 6d^2\text{PolyLog}\left(3,e^{i(a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*(c + d*x)^2*Log[1 + E^(
I*(a + b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*
d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))
] + 6*d^2*PolyLog[3, E^(I*(a + b*x))] + 4*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)
^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - 4*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2
+ b^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

Maple [B] time = 0.259, size = 481, normalized size = 2.8

$$2\,\frac{\left(d^2x^2b^2 + 2\,b^2cdx + b^2c^2 + 2\,ibd^2x - 2\,d^2 + 2\,ibcd\right)e^{i(bx+a)}}{b^3} + 2\,\frac{\left(d^2x^2b^2 + 2\,b^2cdx + b^2c^2 - 2\,ibd^2x - 2\,d^2 - 2\,ibcd\right)e^{-i(bx}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] 2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*(
b*x+a))+2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3
*exp(-I*(b*x+a))-6/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-6*d^2*polylog(3,-exp
(I*(b*x+a)))/b^3+6/b*c*d*ln(1-exp(I*(b*x+a)))*x+6/b^2*c*d*ln(1-exp(I*(b*x+a
)))*a-6/b*c*d*ln(exp(I*(b*x+a))+1)*x-6/b^2*c*d*ln(exp(I*(b*x+a))+1)*a+12/b^
2*c*d*a*arctanh(exp(I*(b*x+a)))-6*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I
/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+3/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^
3*d^2*ln(1-exp(I*(b*x+a)))*a^2+6*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-3/b*d
^2*ln(exp(I*(b*x+a))+1)*x^2+3/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2-6*I/b^2*c*d*
polylog(2,exp(I*(b*x+a)))+6*d^2*polylog(3,exp(I*(b*x+a)))/b^3-6/b*c^2*arcta
nh(exp(I*(b*x+a)))
```

Maxima [B] time = 1.56023, size = 552, normalized size = 3.21

$$\frac{c^2\left(8\cos(bx+a) - 3\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + 3\log\left(c\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)
*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2*(1
2*d^2*polylog(3, -e^(I*b*x + I*a)) - 12*d^2*polylog(3, e^(I*b*x + I*a)) + (
6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) +
 (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1
) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2)*cos(b*x + a) + (-12*I*b*d^2*x - 1
2*I*b*c*d)*dilog(-e^(I*b*x + I*a)) + (12*I*b*d^2*x + 12*I*b*c*d)*dilog(e^(I
*b*x + I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 + 2*cos(b*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a
)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 16*(b*d^2*x + b*c*d)*sin(b*x +
 a))/b^3
```

Fricas [C] time = 0.637698, size = 1480, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 6*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 6*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) + (-6*I*b*d^2*x - 6*I*b*c*d)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (6*I*b*d^2*x + 6*I*b*c*d)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) + (-6*I*b*d^2*x - 6*I*b*c*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (6*I*b*d^2*x + 6*I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
 + a)) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I
*sin(b*x + a) + 1) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*
cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2
*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1) - 16*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \csc (bx+a)^2 \sin (3\,bx+3\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

3.378 $\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=95

$$\frac{3idPolyLog\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3idPolyLog\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] (-6*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (4*(c + d*x)*Cos[a + b*x])/b +
((3*I)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*PolyLog[2, E^(I*(a +
b*x))])/b^2 - (4*d*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.108855, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4431, 4408, 3296, 2637, 4183, 2279, 2391}

$$\frac{3idPolyLog\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3idPolyLog\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```


```
[Out] (-6*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (4*(c + d*x)*Cos[a + b*x])/b +
((3*I)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*PolyLog[2, E^(I*(a +
b*x))])/b^2 - (4*d*Sin[a + b*x])/b^2
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \int (3(c+dx)\cos(a+bx)\cot(a+bx)-(c+dx)\sin(a+bx))\,dx$$

$$= 3\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx - \int (c+dx)\sin(a+bx)\,dx$$

$$= \frac{(c+dx)\cos(a+bx)}{b} + 3\int (c+dx)\csc(a+bx)\,dx - 3\int (c+dx)\sin(a+bx$$

$$= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)\cos(a+bx)}{b} - \frac{d\sin(a+bx)}{b^2} - \frac{(3d)}{}$$

$$= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)\cos(a+bx)}{b} - \frac{4d\sin(a+bx)}{b^2} + \frac{(3i}{}$$

$$= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)\cos(a+bx)}{b} + \frac{3id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2} -$$

Mathematica [A] time = 0.341125, size = 171, normalized size = 1.8

$$\frac{3d\left(i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{b^2} - \frac{4d\sin(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*c*Cos[a + b*x])/b + (4*d*x*Cos[a + b*x])/b - (3*c*Log[Cos[(a + b*x)/2]])
/b + (3*c*Log[Sin[(a + b*x)/2]])/b - (3*a*d*Log[Tan[(a + b*x)/2]])/b^2 + (3
*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(Po
lyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 - (4*d*Sin[
a + b*x])/b^2
```

Maple [B] time = 0.249, size = 205, normalized size = 2.2

$$2\,\frac{(dxb+bc+id)\,e^{i(bx+a)}}{b^2} + 2\,\frac{(dxb+bc-id)\,e^{-i(bx+a)}}{b^2} - 6\,\frac{c\text{Artanh}\left(e^{i(bx+a)}\right)}{b} + 3\,\frac{d\ln\left(1-e^{i(bx+a)}\right)x}{b} + 3\,\frac{d\ln\left(1-e^{i(bx+a}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] 2*(d*x*b+b*c+I*d)/b^2*exp(I*(b*x+a))+2*(d*x*b+b*c-I*d)/b^2*exp(-I*(b*x+a))-
6/b*c*arctanh(exp(I*(b*x+a)))+3/b*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*d*ln(1-exp
(I*(b*x+a)))*a-3*I*d*polylog(2,exp(I*(b*x+a)))/b^2-3/b*d*ln(exp(I*(b*x+a))+
1)*x-3/b^2*d*ln(exp(I*(b*x+a))+1)*a+3*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+6/
b^2*d*a*arctanh(exp(I*(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{c\left(8\,\cos\left(bx+a\right)-3\,\log\left(\cos\left(bx\right)^{2}+2\,\cos\left(bx\right)\cos\left(a\right)+\cos\left(a\right)^{2}+\sin\left(bx\right)^{2}-2\,\sin\left(bx\right)\sin\left(a\right)+\sin\left(a\right)^{2}\right)+3\,\log\left(\cos\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + s
in(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b + (4*b*x*c
os(b*x + a) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*cos(b*x + a) + 1), x) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1), x) - 4*sin(b*x + a))*d/b^2
```

Fricas [B] time = 0.58448, size = 814, normalized size = 8.57

$$\frac{8\,(bdx+bc)\cos\left(bx+a\right)-3i\,d\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+3i\,d\mathrm{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)-3i\,d\mathrm{Li}_2\left(-\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(8*(b*d*x + b*c)*cos(b*x + a) - 3*I*d*dilog(cos(b*x + a) + I*sin(b*x +
a)) + 3*I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*I*d*dilog(-cos(b*x + a
) + I*sin(b*x + a)) + 3*I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d*
x + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d*x + b*c)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I
*sin(b*x + a) + 1/2) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + 3*(b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(
b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 8*d*sin(b*x + a))/b^
```

2

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \csc (bx + a)^2 \sin (3\,bx + 3\,a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

3.379 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=71

$$3\text{Unintegrable}\left(\frac{\csc(a+bx)}{c+dx},x\right)-\frac{4\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] (-4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d - (4*Cos[a - (b*c)/d]*Si
nIntegral[(b*c)/d + b*x])/d + 3*Unintegrable[Csc[a + b*x]/(c + d*x), x]
```

Rubi [A] time = 0.2104, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (-4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d - (4*Cos[a - (b*c)/d]*Si
nIntegral[(b*c)/d + b*x])/d + 3*Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{c+dx}-\frac{\sin(a+bx)}{c+dx}\right)dx\\
&= 3\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)}{c+dx}\,dx\\
&= 3\int \frac{\csc(a+bx)}{c+dx}\,dx-3\int \frac{\sin(a+bx)}{c+dx}\,dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx-\sin\left(a\right.\\
&= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+3\int \frac{\csc(a+bx)}{c+dx}\,dx-\left(3\cos\right.\\
&= -\frac{4\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+3\int \frac{\csc(a+bx)}{c+dx}\,dx\end{aligned}$$

Mathematica [A] time = 6.23243, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

Maple [A] time = 0.349, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2\sin(3bx+3a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2iE_1\left(\frac{ibdx+ibc}{d}\right)-2iE_1\left(-\frac{ibdx+ibc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+3d\int\frac{\sin(bx+a)}{(dx+c)\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)}\,dx+3d\int\frac{}{(dx+c)\left(\cos(bx+\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((2*I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - 2*I*exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 3*d*integrate(sin(b*x + a)/((d*x + c
)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a
) + c), x) + 3*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x +
c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + 2*(exp_integr
al_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin
(-(b*c - a*d)/d))/d
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

3.380 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=91

$$3\text{Unintegrable}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)-\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}$$

```
[Out] (-4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 + (4*Sin[a + b*x])/(
d*(c + d*x)) + (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + 3*Un
integrable[Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.283036, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (-4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 + (4*Sin[a + b*x])/(
d*(c + d*x)) + (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + 3*De
fer[Int][Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^2} - \frac{\sin(a+bx)}{(c+dx)^2}\right)dx$$
$$= 3\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\sin(a+bx)}{d(c+dx)} + 3\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx - 3\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx - \frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}$$
$$= \frac{4\sin(a+bx)}{d(c+dx)} + 3\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx - \frac{(3b)\int \frac{\cos(a+bx)}{c+dx}\,dx}{d} - \frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+b\right.}{c+dx}}{d}$$
$$= -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{4\sin(a+bx)}{d(c+dx)} + \frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + 3\int \frac{\csc}{(c}$$
$$= -\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{4\sin(a+bx)}{d(c+dx)} + \frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + 3\int \frac{c}{}$$

Mathematica [A] time = 6.78663, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.7, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

3.381 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=114

$$3\text{Unintegrable}\left(\frac{\csc(a+bx)}{(c+dx)^3},x\right)+\frac{2b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{2b\cos(a+\ldots}{d^2(c+\ldots}$$

```
[Out] (2*b*Cos[a + b*x])/(d^2*(c + d*x)) + (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[
a - (b*c)/d])/d^3 + (2*Sin[a + b*x])/(d*(c + d*x)^2) + (2*b^2*Cos[a - (b*c)
/d]*SinIntegral[(b*c)/d + b*x])/d^3 + 3*Unintegrable[Csc[a + b*x]/(c + d*x)
^3, x]
```

Rubi [A] time = 0.328324, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (2*b*Cos[a + b*x])/(d^2*(c + d*x)) + (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[
a - (b*c)/d])/d^3 + (2*Sin[a + b*x])/(d*(c + d*x)^2) + (2*b^2*Cos[a - (b*c)
/d]*SinIntegral[(b*c)/d + b*x])/d^3 + 3*Defer[Int][Csc[a + b*x]/(c + d*x)^3
, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx)}{(c+dx)^3}\right)dx$$

$$= 3\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{\sin(a+bx)}{2d(c+dx)^2} + 3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx - 3\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx - \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d}$$

$$= \frac{b\cos(a+bx)}{2d^2(c+dx)} + \frac{2\sin(a+bx)}{d(c+dx)^2} + 3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx + \frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} - \frac{(3b)\int \frac{\cos(a}{(c+a}}{2d}$$

$$= \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{2\sin(a+bx)}{d(c+dx)^2} + 3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx + \frac{\left(3b^2\right)\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} + \frac{\left(b^2\cos\right.}{}$$

$$= \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3} + \frac{2\sin(a+bx)}{d(c+dx)^2} + \frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}\right.}{2d^3}$$

$$= \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{2b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} + \frac{2\sin(a+bx)}{d(c+dx)^2} + \frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\right.}{d^3}$$

Mathematica [A] time = 7.24875, size = 0, normalized size = 0.

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

Maple [A] time = 0.71, size = 0, normalized size = 0.

$$\int \frac{(\csc(bx+a))^2\sin(3bx+3a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\csc(bx+a)^2\sin(3\,bx+3\,a)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(bx+a)^2 \sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

3.382 $\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=299

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{3d^4}{}$$

```
[Out] (6*c*d^3*x)/b^3 + (3*d^4*x^2)/b^3 - (c + d*x)^4/b - ((I/5)*(c + d*x)^5)/d +
 ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*PolyLo
g[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(
a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^
4 - (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (6*d^3*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b
^2 + (3*d^4*Sin[a + b*x]^2)/b^5 - (6*d^2*(c + d*x)^2*Sin[a + b*x]^2)/b^3 +
(2*(c + d*x)^4*Sin[a + b*x]^2)/b
```

Rubi [A] time = 0.503125, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4431, 4404, 3311, 32, 3310, 4407, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{3d^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```



```
[Out] (6*c*d^3*x)/b^3 + (3*d^4*x^2)/b^3 - (c + d*x)^4/b - ((I/5)*(c + d*x)^5)/d +
 ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*PolyLo
g[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(
a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^
4 - (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (6*d^3*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b
^2 + (3*d^4*Sin[a + b*x]^2)/b^5 - (6*d^2*(c + d*x)^2*Sin[a + b*x]^2)/b^3 +
(2*(c + d*x)^4*Sin[a + b*x]^2)/b
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
```

```
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^4\cos(a+bx)\sin(a+bx) - (c+dx)^4\sin^2(a+bx)\tan(a+bx)\right) dx$$

$$= 3\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^4\sin^2(a+bx)\tan(a+bx$$

$$= \frac{3(c+dx)^4\sin^2(a+bx)}{2b} - \frac{(6d)\int (c+dx)^3\sin^2(a+bx)\,dx}{b} + \int (c+dx)^4\cos(a$$

$$= -\frac{i(c+dx)^5}{5d} + \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} - \frac{9d^2(c+dx)^2\sin^2(a+bx)}{2b^3}$$

$$= -\frac{3(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{9d^3(c+dx)\cos(a+}{2b^4}$$

$$= \frac{9cd^3x}{2b^3} + \frac{9d^4x^2}{4b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{2id(c}{}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{2id(c}{}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{2id(c}{}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{2id(c}{}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{2id(c}{}$$

Mathematica [B] time = 6.74213, size = 2482, normalized size = 8.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I
)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
- (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E
^(I*a)) + (I/2)*c*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*
a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*
PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*
x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] - (d^4
*((-4*I)*x^5 - (10*(1 + E^((2*I)*a))*x^4*Log[1 + E^((-2*I)*(a + b*x))])/b +
 (5*(1 + E^((2*I)*a))*((-4*I)*b^3*x^3*PolyLog[2, -E^((-2*I)*(a + b*x))] - 6
*b^2*x^2*PolyLog[3, -E^((-2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[4, -E^((-2*I
```

```
)*(a + b*x))] + 3*PolyLog[5, -E^((-2*I)*(a + b*x))]))/b^5)*Sec[a])/(20*E^(I
*a)) + (c^4*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin
[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (2*c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Co
t[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x
)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + P
i*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyL
og[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*
Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(40*b^5) -
 ((I/40)*Sin[2*a + 2*b*x])/b^5)*(-20*b^4*c^4*Cos[a] + (40*I)*b^3*c^3*d*Cos[
a] + 60*b^2*c^2*d^2*Cos[a] - (60*I)*b*c*d^3*Cos[a] - 30*d^4*Cos[a] - 80*b^4
*c^3*d*x*Cos[a] + (120*I)*b^3*c^2*d^2*x*Cos[a] + 120*b^2*c*d^3*x*Cos[a] - (
60*I)*b*d^4*x*Cos[a] - 120*b^4*c^2*d^2*x^2*Cos[a] + (120*I)*b^3*c*d^3*x^2*C
os[a] + 60*b^2*d^4*x^2*Cos[a] - 80*b^4*c*d^3*x^3*Cos[a] + (40*I)*b^3*d^4*x^
3*Cos[a] - 20*b^4*d^4*x^4*Cos[a] - (20*I)*b^5*c^4*x*Cos[a + 2*b*x] - (40*I)
*b^5*c^3*d*x^2*Cos[a + 2*b*x] - (40*I)*b^5*c^2*d^2*x^3*Cos[a + 2*b*x] - (20
*I)*b^5*c*d^3*x^4*Cos[a + 2*b*x] - (4*I)*b^5*d^4*x^5*Cos[a + 2*b*x] + (20*I
)*b^5*c^4*x*Cos[3*a + 2*b*x] + (40*I)*b^5*c^3*d*x^2*Cos[3*a + 2*b*x] + (40*
I)*b^5*c^2*d^2*x^3*Cos[3*a + 2*b*x] + (20*I)*b^5*c*d^3*x^4*Cos[3*a + 2*b*x]
 + (4*I)*b^5*d^4*x^5*Cos[3*a + 2*b*x] - 10*b^4*c^4*Cos[3*a + 4*b*x] - (20*I
)*b^3*c^3*d*Cos[3*a + 4*b*x] + 30*b^2*c^2*d^2*Cos[3*a + 4*b*x] + (30*I)*b*c
*d^3*Cos[3*a + 4*b*x] - 15*d^4*Cos[3*a + 4*b*x] - 40*b^4*c^3*d*x*Cos[3*a +
4*b*x] - (60*I)*b^3*c^2*d^2*x*Cos[3*a + 4*b*x] + 60*b^2*c*d^3*x*Cos[3*a + 4
*b*x] + (30*I)*b*d^4*x*Cos[3*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Cos[3*a + 4*b*
x] - (60*I)*b^3*c*d^3*x^2*Cos[3*a + 4*b*x] + 30*b^2*d^4*x^2*Cos[3*a + 4*b*x
] - 40*b^4*c*d^3*x^3*Cos[3*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Cos[3*a + 4*b*x]
 - 10*b^4*d^4*x^4*Cos[3*a + 4*b*x] - 10*b^4*c^4*Cos[5*a + 4*b*x] - (20*I)*b
^3*c^3*d*Cos[5*a + 4*b*x] + 30*b^2*c^2*d^2*Cos[5*a + 4*b*x] + (30*I)*b*c*d^
3*Cos[5*a + 4*b*x] - 15*d^4*Cos[5*a + 4*b*x] - 40*b^4*c^3*d*x*Cos[5*a + 4*b
*x] - (60*I)*b^3*c^2*d^2*x*Cos[5*a + 4*b*x] + 60*b^2*c*d^3*x*Cos[5*a + 4*b*
x] + (30*I)*b*d^4*x*Cos[5*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Cos[5*a + 4*b*x]
- (60*I)*b^3*c*d^3*x^2*Cos[5*a + 4*b*x] + 30*b^2*d^4*x^2*Cos[5*a + 4*b*x] -
 40*b^4*c*d^3*x^3*Cos[5*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Cos[5*a + 4*b*x] -
10*b^4*d^4*x^4*Cos[5*a + 4*b*x] + 20*b^5*c^4*x*Sin[a + 2*b*x] + 40*b^5*c^3*
d*x^2*Sin[a + 2*b*x] + 40*b^5*c^2*d^2*x^3*Sin[a + 2*b*x] + 20*b^5*c*d^3*x^4
*Sin[a + 2*b*x] + 4*b^5*d^4*x^5*Sin[a + 2*b*x] - 20*b^5*c^4*x*Sin[3*a + 2*b
*x] - 40*b^5*c^3*d*x^2*Sin[3*a + 2*b*x] - 40*b^5*c^2*d^2*x^3*Sin[3*a + 2*b*
x] - 20*b^5*c*d^3*x^4*Sin[3*a + 2*b*x] - 4*b^5*d^4*x^5*Sin[3*a + 2*b*x] - (
10*I)*b^4*c^4*Sin[3*a + 4*b*x] + 20*b^3*c^3*d*Sin[3*a + 4*b*x] + (30*I)*b^2
*c^2*d^2*Sin[3*a + 4*b*x] - 30*b*c*d^3*Sin[3*a + 4*b*x] - (15*I)*d^4*Sin[3*
a + 4*b*x] - (40*I)*b^4*c^3*d*x*Sin[3*a + 4*b*x] + 60*b^3*c^2*d^2*x*Sin[3*a
 + 4*b*x] + (60*I)*b^2*c*d^3*x*Sin[3*a + 4*b*x] - 30*b*d^4*x*Sin[3*a + 4*b*
x] - (60*I)*b^4*c^2*d^2*x^2*Sin[3*a + 4*b*x] + 60*b^3*c*d^3*x^2*Sin[3*a + 4
*b*x] + (30*I)*b^2*d^4*x^2*Sin[3*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3*Sin[3*a
+ 4*b*x] + 20*b^3*d^4*x^3*Sin[3*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Sin[3*a + 4
*b*x] - (10*I)*b^4*c^4*Sin[5*a + 4*b*x] + 20*b^3*c^3*d*Sin[5*a + 4*b*x] + (
```

```
30*I)*b^2*c^2*d^2*Sin[5*a + 4*b*x] - 30*b*c*d^3*Sin[5*a + 4*b*x] - (15*I)*d
^4*Sin[5*a + 4*b*x] - (40*I)*b^4*c^3*d*x*Sin[5*a + 4*b*x] + 60*b^3*c^2*d^2*
x*Sin[5*a + 4*b*x] + (60*I)*b^2*c*d^3*x*Sin[5*a + 4*b*x] - 30*b*d^4*x*Sin[5
*a + 4*b*x] - (60*I)*b^4*c^2*d^2*x^2*Sin[5*a + 4*b*x] + 60*b^3*c*d^3*x^2*Si
n[5*a + 4*b*x] + (30*I)*b^2*d^4*x^2*Sin[5*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3
*Sin[5*a + 4*b*x] + 20*b^3*d^4*x^3*Sin[5*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Si
n[5*a + 4*b*x])
```

Maple [B] time = 0.255, size = 956, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+6/b^3*c*d^3*polylog(3,-exp(2*I*(b
*x+a)))*x+1/b*d^4*ln(exp(2*I*(b*x+a))+1)*x^4+I*c^4*x-1/4*(2*d^4*x^4*b^4+4*I
*b^3*d^4*x^3+8*b^4*c*d^3*x^3+12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2+12*I*b^3
*c^2*d^2*x+8*b^4*c^3*d*x+4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2-6*I*b*d^4*x-
12*b^2*c*d^3*x-6*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp(2*I*(b*x+a))+6/b*c^
2*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+4/b*c^3*d*ln(exp(2*I*(b*x+a))+1)*x+8/5*I/b
^5*d^4*a^5-I*c*d^3*x^4-2*I*c^2*d^2*x^3-2*I*c^3*d*x^2+4/b*c*d^3*ln(exp(2*I*(
b*x+a))+1)*x^3-6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-6*I/b^2*c*d^3
*polylog(2,-exp(2*I*(b*x+a)))*x^2+3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a))
)-2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))-2*I/b^2*d^4*polylog(2,-exp(2*I
*(b*x+a)))*x^3+3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+3/b^3*c^2*d^2*pol
ylog(3,-exp(2*I*(b*x+a)))-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+1/b*c^4*ln(exp(2
*I*(b*x+a))+1)-1/5*I*d^4*x^5+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-12/b^3*c^2*d^
2*a^2*ln(exp(I*(b*x+a)))+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))+2*I/b^4*d^4*a^4
*x+8*I/b^3*c^2*d^2*a^3-4*I/b^2*c^3*d*a^2-6*I/b^4*c*d^3*a^4-8*I/b*c^3*d*a*x+
12*I/b^2*c^2*d^2*a^2*x-2/b*c^4*ln(exp(I*(b*x+a)))-1/4*(2*d^4*x^4*b^4-4*I*b^
3*d^4*x^3+8*b^4*c*d^3*x^3-12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2-12*I*b^3*c^
2*d^2*x+8*b^4*c^3*d*x-4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2+6*I*b*d^4*x-12*
b^2*c*d^3*x+6*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp(-2*I*(b*x+a))-8*I/b^3*
c*d^3*a^3*x+3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x
```

Maxima [B] time = 1.6788, size = 819, normalized size = 2.74

$$-\frac{c^4\left(2\cos\left(2\,bx+2\,a\right)-\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^2+\sin\left(2\,bx\right)^2-2\,\sin\left(2\,bx\right)\sin\left(2\,a\right)+\sin\left(2\,a\right.\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^4*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/30*(-
6*I*b^5*d^4*x^5 - 30*I*b^5*c*d^3*x^4 - 60*I*b^5*c^2*d^2*x^3 - 60*I*b^5*c^3*
d*x^2 - 90*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) + (60*I*b^4*d^4*x^4 + 160*I
*b^4*c*d^3*x^3 + 180*I*b^4*c^2*d^2*x^2 + 120*I*b^4*c^3*d*x)*arctan2(sin(2*b
*x + 2*a), cos(2*b*x + 2*a) + 1) - 15*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 6*
b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*
b^2*c*d^3)*x)*cos(2*b*x + 2*a) + (-120*I*b^3*d^4*x^3 - 240*I*b^3*c*d^3*x^2
- 180*I*b^3*c^2*d^2*x - 60*I*b^3*c^3*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 10*(3
*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 9*b^4*c^2*d^2*x^2 + 6*b^4*c^3*d*x)*log(cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (180*I*b*d
^4*x + 120*I*b*c*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 30*(6*b^2*d^4*x^2
+ 8*b^2*c*d^3*x + 3*b^2*c^2*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 30*(2*b
^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 -
 b*d^4)*x)*sin(2*b*x + 2*a))/b^5
```

Fricas [C] time = 1.00211, size = 3866, normalized size = 12.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 24*d^4*polylog(5, I*cos(b*x + a) + s
in(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*po
lylog(5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a
) - sin(b*x + a)) + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - 2*(2*b^4*d^4*x^4 + 8*
b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*
d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(2*b^3*d^4*x
^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*
x)*cos(b*x + a)*sin(b*x + a) + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x + (4*I*b^3*d
^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(I*c
os(b*x + a) + sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I
*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-4*
I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*di
log(-I*cos(b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2
 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)
```

```
) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) +
I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4
*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^
2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4
*d^4)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x
^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2
+ 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^4*d
^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*
d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(
b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*s
in(b*x + a) + I) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a)
 + sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a)
- sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a)
+ sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a)
 - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3
, I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^
2*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*
c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2
*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*
x + a)))/b^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^4 \sec(bx + a) \sin(3bx + 3a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

3.383 $\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=242

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\sin}{b^3}$$

```
[Out] (3*d^3*x)/(2*b^3) - (c + d*x)^3/b - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Lo
g[1 + E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2
*b^3) + (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^3*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])
/b^2 - (3*d^2*(c + d*x)*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^3*Sin[a + b*x]^2
)/b
```

Rubi [A] time = 0.445514, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {4431, 4404, 3311, 32, 2635, 8, 4407, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\sin}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```


```
[Out] (3*d^3*x)/(2*b^3) - (c + d*x)^3/b - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Lo
g[1 + E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2
*b^3) + (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^3*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])
/b^2 - (3*d^2*(c + d*x)*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^3*Sin[a + b*x]^2
)/b
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
```

[m, 0]

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\cos(a+bx)\sin(a+bx) - (c+dx)^3\sin^2(a+bx)\tan(a+bx)\right)$$
$$= 3\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^3\sin^2(a+bx)\tan(a+b$$
$$= \frac{3(c+dx)^3\sin^2(a+bx)}{2b} - \frac{(9d)\int (c+dx)^2\sin^2(a+bx)\,dx}{2b} + \int (c+dx)^3\cos($$
$$= -\frac{i(c+dx)^4}{4d} + \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{9d^2(c+dx)\sin^2(a+bx}{4b^3}$$
$$= -\frac{3(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{9d^3\cos(a+bx)\sin}{8b^4}$$
$$= \frac{9d^3x}{8b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{I}}{2}$$
$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{I}}{2}$$
$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{I}}{2}$$
$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{I}}{2}$$

Mathematica [B] time = 6.60769, size = 1719, normalized size = 7.1

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
(3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(
I*a)) + (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*
x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*Poly
Log[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))]
 - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + (c^3*Sec
[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a
]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a
]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - A
rcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]]
 - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*
(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2
```

```
*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(16*b^4) - ((I/16)*Sin[
2*a + 2*b*x])/b^4)*(-8*b^3*c^3*Cos[a] + (12*I)*b^2*c^2*d*Cos[a] + 12*b*c*d^
2*Cos[a] - (6*I)*d^3*Cos[a] - 24*b^3*c^2*d*x*Cos[a] + (24*I)*b^2*c*d^2*x*Co
s[a] + 12*b*d^3*x*Cos[a] - 24*b^3*c*d^2*x^2*Cos[a] + (12*I)*b^2*d^3*x^2*Cos
[a] - 8*b^3*d^3*x^3*Cos[a] - (8*I)*b^4*c^3*x*Cos[a + 2*b*x] - (12*I)*b^4*c^
2*d*x^2*Cos[a + 2*b*x] - (8*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] - (2*I)*b^4*d^3
*x^4*Cos[a + 2*b*x] + (8*I)*b^4*c^3*x*Cos[3*a + 2*b*x] + (12*I)*b^4*c^2*d*x
^2*Cos[3*a + 2*b*x] + (8*I)*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + (2*I)*b^4*d^3*
x^4*Cos[3*a + 2*b*x] - 4*b^3*c^3*Cos[3*a + 4*b*x] - (6*I)*b^2*c^2*d*Cos[3*a
 + 4*b*x] + 6*b*c*d^2*Cos[3*a + 4*b*x] + (3*I)*d^3*Cos[3*a + 4*b*x] - 12*b^
3*c^2*d*x*Cos[3*a + 4*b*x] - (12*I)*b^2*c*d^2*x*Cos[3*a + 4*b*x] + 6*b*d^3*
x*Cos[3*a + 4*b*x] - 12*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] - (6*I)*b^2*d^3*x^2*
Cos[3*a + 4*b*x] - 4*b^3*d^3*x^3*Cos[3*a + 4*b*x] - 4*b^3*c^3*Cos[5*a + 4*b
*x] - (6*I)*b^2*c^2*d*Cos[5*a + 4*b*x] + 6*b*c*d^2*Cos[5*a + 4*b*x] + (3*I)
*d^3*Cos[5*a + 4*b*x] - 12*b^3*c^2*d*x*Cos[5*a + 4*b*x] - (12*I)*b^2*c*d^2*
x*Cos[5*a + 4*b*x] + 6*b*d^3*x*Cos[5*a + 4*b*x] - 12*b^3*c*d^2*x^2*Cos[5*a
+ 4*b*x] - (6*I)*b^2*d^3*x^2*Cos[5*a + 4*b*x] - 4*b^3*d^3*x^3*Cos[5*a + 4*b
*x] + 8*b^4*c^3*x*Sin[a + 2*b*x] + 12*b^4*c^2*d*x^2*Sin[a + 2*b*x] + 8*b^4*
c*d^2*x^3*Sin[a + 2*b*x] + 2*b^4*d^3*x^4*Sin[a + 2*b*x] - 8*b^4*c^3*x*Sin[3
*a + 2*b*x] - 12*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] - 8*b^4*c*d^2*x^3*Sin[3*a +
 2*b*x] - 2*b^4*d^3*x^4*Sin[3*a + 2*b*x] - (4*I)*b^3*c^3*Sin[3*a + 4*b*x] +
 6*b^2*c^2*d*Sin[3*a + 4*b*x] + (6*I)*b*c*d^2*Sin[3*a + 4*b*x] - 3*d^3*Sin[
3*a + 4*b*x] - (12*I)*b^3*c^2*d*x*Sin[3*a + 4*b*x] + 12*b^2*c*d^2*x*Sin[3*a
 + 4*b*x] + (6*I)*b*d^3*x*Sin[3*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*Sin[3*a +
 4*b*x] + 6*b^2*d^3*x^2*Sin[3*a + 4*b*x] - (4*I)*b^3*d^3*x^3*Sin[3*a + 4*b*
x] - (4*I)*b^3*c^3*Sin[5*a + 4*b*x] + 6*b^2*c^2*d*Sin[5*a + 4*b*x] + (6*I)*
b*c*d^2*Sin[5*a + 4*b*x] - 3*d^3*Sin[5*a + 4*b*x] - (12*I)*b^3*c^2*d*x*Sin[
5*a + 4*b*x] + 12*b^2*c*d^2*x*Sin[5*a + 4*b*x] + (6*I)*b*d^3*x*Sin[5*a + 4*
b*x] - (12*I)*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] + 6*b^2*d^3*x^2*Sin[5*a + 4*b*
x] - (4*I)*b^3*d^3*x^3*Sin[5*a + 4*b*x])
```

Maple [B] time = 0.214, size = 639, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] 3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+1/b*c^3*ln(exp(2*I*(b*x+a))+1)+3
/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+3/2/b^3*d^3*polylog(3,-exp(2*I*(b
*x+a)))*x-3/2*I/b^4*d^3*a^4-3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-3/
```

```
2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+4*I/b^3*a^3*c*d^2-3*I/b^2*a^2*
c^2*d+3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x+3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x
^2+1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-1/8*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12
*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*
d^3*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*I*(b*x+a))-2*I/b^3*d^3*a^3*x-6/b^3*c*d^2
*a^2*ln(exp(I*(b*x+a)))+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+I*c^3*x-1/8*(4*d^3
*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6
*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b^4*exp(-2*I*(b*x+a))-I
*c*d^2*x^3-3/2*I*c^2*d*x^2+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))-3*I/b^2*polylog
(2,-exp(2*I*(b*x+a)))*c*d^2*x-6*I/b*a*c^2*d*x+6*I/b^2*a^2*c*d^2*x-2/b*c^3*l
n(exp(I*(b*x+a)))-1/4*I*d^3*x^4
```

Maxima [B] time = 1.50017, size = 597, normalized size = 2.47

$$-\frac{c^3\left(2\cos(2bx+2a)-\log\left(\cos(2bx)^2+2\cos(2bx)\cos(2a)+\cos(2a)^2+\sin(2bx)^2-2\sin(2bx)\sin(2a)+\sin(2\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^3*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/12*(-
3*I*b^4*d^3*x^4 - 12*I*b^4*c*d^2*x^3 - 18*I*b^4*c^2*d*x^2 + 12*I*d^3*polylo
g(4, -e^(2*I*b*x + 2*I*a)) + (16*I*b^3*d^3*x^3 + 36*I*b^3*c*d^2*x^2 + 36*I*
b^3*c^2*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(2*b^3*d^3
*x^3 + 6*b^3*c*d^2*x^2 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(2*b*x +
 2*a) + (-24*I*b^2*d^3*x^2 - 36*I*b^2*c*d^2*x - 18*I*b^2*c^2*d)*dilog(-e^(2
*I*b*x + 2*I*a)) + 2*(4*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(4*b*
d^3*x + 3*b*c*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 9*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a))/b^4
```

Fricas [C] time = 0.865917, size = 2703, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 6*I*d^3*polylog(4, I*cos(b*x + a) +
sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3
*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x
 + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*
b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*c^2*d
 - b*d^3)*x + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(I*c
os(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2
*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*
c*d^2*x - 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d
^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x +
 a)) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a)
 + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a
) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) +
1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*
(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x
+ b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^
2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/b^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \sec (bx+a) \sin (3bx+3a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

3.384 $\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=173

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\sin(a+bx)\cos(a+bx)}{b^2}-\frac{d^2\sin^2(a+bx)}{b^3}+$$

```
[Out] (-2*c*d*x)/b - (d^2*x^2)/b - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E
^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b
^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (2*d*(c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/b^2 - (d^2*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^2*Sin[a
+ b*x]^2)/b
```

Rubi [A] time = 0.328273, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {4431, 4404, 3310, 4407, 3719, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\sin(a+bx)\cos(a+bx)}{b^2}-\frac{d^2\sin^2(a+bx)}{b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```


```
[Out] (-2*c*d*x)/b - (d^2*x^2)/b - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E
^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b
^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (2*d*(c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/b^2 - (d^2*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^2*Sin[a
+ b*x]^2)/b
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
```

```
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\cos(a+bx)\sin(a+bx)-(c+dx)^2\sin^2(a+bx)\tan(a+bx)\right) d\cdots$$

$$= 3\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^2\sin^2(a+bx)\tan(a+b\cdots$$

$$= \frac{3(c+dx)^2\sin^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)\sin^2(a+bx)\,dx}{b} + \int (c+dx)^2\cos(a\cdots$$

$$= -\frac{i(c+dx)^3}{3d} + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} - \frac{3d^2\sin^2(a+bx)}{4b^3} + \frac{2(c+\cdots}{\cdots}$$

$$= -\frac{3cdx}{2b} - \frac{3d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2d(c+dx)\cos(a\cdots}{\cdots}$$

$$= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\text{Li}_2\left(-e^{\cdots}\right.}{b^2}$$

$$= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\text{Li}_2\left(-e^{\cdots}\right.}{b^2}$$

$$= -\frac{2cdx}{b} - \frac{d^2x^2}{b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\text{Li}_2\left(-e^{\cdots}\right.}{b^2}$$

Mathematica [B] time = 6.54109, size = 516, normalized size = 2.98

$$\frac{cd\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\text{PolyLog}\left(2,e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}\cdots\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\cdots\right.}{b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(
a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (
3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I
*a)) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin
[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]
]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] -
 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Lo
g[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2
, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt
[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a] - d^2*C
os[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*Sin[2*a]
- 2*b*d^2*x*Sin[2*a]))/(2*b^3) + ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Cos[2*a] +
2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d^2*x^2*Si
n[2*a])*Sin[2*b*x])/(2*b^3) - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

Maple [B] time = 0.243, size = 377, normalized size = 2.2

$$-icdx^2 - \frac{4\,iacdx}{b} + ic^2x - \frac{\left(2\,d^2x^2b^2 + 2\,ibd^2x + 4\,b^2cdx + 2\,ibcd + 2\,b^2c^2 - d^2\right)\mathrm{e}^{2\,i(bx+a)}}{4\,b^3} - \frac{\left(2\,d^2x^2b^2 - 2\,ibd^2x + 4\,b^2cdx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -I*c*d*x^2-4*I/b*a*c*d*x+I*c^2*x-1/4*(2*d^2*x^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x
+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*I*(b*x+a))-1/4*(2*d^2*x^2*b^2-2*I*b*d^2
*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(-2*I*(b*x+a))+1/b*c^2*ln(ex
p(2*I*(b*x+a))+1)-2/b*c^2*ln(exp(I*(b*x+a)))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)
))+2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x-I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*
x-1/3*I*d^2*x^3+1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-I/b^2*c*d*polylog(2,-exp
(2*I*(b*x+a)))+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+4/b^2*c*d*a*ln(exp(
I*(b*x+a)))-2*I/b^2*a^2*c*d+2*I/b^2*a^2*d^2*x+4/3*I/b^3*a^3*d^2
```

Maxima [A] time = 1.39856, size = 406, normalized size = 2.35

$$-\frac{c^2\left(2\,\cos\left(2\,bx+2\,a\right) - \log\left(\cos\left(2\,bx\right)^2 + 2\,\cos\left(2\,bx\right)\cos\left(2\,a\right) + \cos\left(2\,a\right)^2 + \sin\left(2\,bx\right)^2 - 2\,\sin\left(2\,bx\right)\sin\left(2\,a\right) + \sin\left(2\right.\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/6*(-2
*I*b^3*d^2*x^3 - 6*I*b^3*c*d*x^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) +
 (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2
*a) + 1) - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - d^2)*cos(2*b*x + 2*a) + (-6*I*b
*d^2*x - 6*I*b*c*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*
d*x)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
+ 6*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))/b^3
```

Fricas [C] time = 0.718819, size = 1728, normalized size = 9.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - 2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c
^2 - d^2)*cos(b*x + a)^2 + 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
+ 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)
) + 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*
d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(
I*cos(b*x + a) - sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-I*cos(b*
x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) -
sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x
+ a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x
 + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^
2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*
a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I)
+ (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/
b^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^2 \sec (bx+a) \sin (3bx+3a) \ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

3.385 $\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=107

$$-\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)\sin^2(a+bx)}{b}-\frac{dx}{b}-\frac{i(c$$

```
[Out] -((d*x)/b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))
])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d*Cos[a + b*x]*Sin
[a + b*x])/b^2 + (2*(c + d*x)*Sin[a + b*x]^2)/b
```

Rubi [A] time = 0.180171, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4431, 4404, 2635, 8, 4407, 3719, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)\sin^2(a+bx)}{b}-\frac{dx}{b}-\frac{i(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -((d*x)/b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))
])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d*Cos[a + b*x]*Sin
[a + b*x])/b^2 + (2*(c + d*x)*Sin[a + b*x]^2)/b
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)\cos(a+bx)\sin(a+bx)-(c+dx)\sin^2(a+bx)\tan(a+bx)\right)\,dx$$
$$= 3\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)\sin^2(a+bx)\tan(a+bx)\,d$$
$$= \frac{3(c+dx)\sin^2(a+bx)}{2b} - \frac{(3d)\int \sin^2(a+bx)\,dx}{2b} + \int (c+dx)\cos(a+bx)\sin(a$$
$$= -\frac{i(c+dx)^2}{2d} + \frac{3d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{2(c+dx)\sin^2(a+bx)}{b} + 2i\int \frac{e^{2i(a}}{1}$$
$$= -\frac{3dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\cos(a+bx)\sin(a+bx)}{b^2} +$$
$$= -\frac{dx}{b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\cos(a+bx)\sin(a+bx)}{b^2} + \frac{2}{}$$
$$= -\frac{dx}{b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} + \frac{d\cos(a+}{}$$

Mathematica [B] time = 5.82378, size = 254, normalized size = 2.37

$$\frac{d\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\text{PolyLog}\left(2,e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}(c\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{2b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[
Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^
((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[S
in[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))])
)/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)])
- (d*Cos[2*b*x]*(2*b*x*Cos[2*a] - Sin[2*a]))/(2*b^2) + (d*(Cos[2*a] + 2*b*x
*Sin[2*a])*Sin[2*b*x])/(2*b^2) + (c*(Log[Cos[a + b*x]] + 2*Sin[a + b*x]^2))
/b - (d*x^2*Tan[a])/2
```

Maple [A] time = 0.324, size = 177, normalized size = 1.7

$$-\frac{i}{2}dx^2 + icx - \frac{(2\,dxb + id + 2\,bc)\,\mathrm{e}^{2\,i(bx+a)}}{4\,b^2} - \frac{(2\,dxb - id + 2\,bc)\,\mathrm{e}^{-2\,i(bx+a)}}{4\,b^2} - 2\,\frac{c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b} + \frac{c\ln\left(\mathrm{e}^{2\,i(bx+a)}+1\right)}{b} - \frac{2\,ida}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -1/2*I*d*x^2+I*c*x-1/4*(2*d*x*b+I*d+2*b*c)/b^2*exp(2*I*(b*x+a))-1/4*(2*d*x*
b-I*d+2*b*c)/b^2*exp(-2*I*(b*x+a))-2/b*c*ln(exp(I*(b*x+a)))+1/b*c*ln(exp(2*
I*(b*x+a))+1)-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(exp(2*I*(b*x+a))+1)*x-1/2*I*
d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{c\left(2\,\cos\left(2\,bx+2\,a\right) - \log\left(\cos\left(2\,bx\right)^2 + 2\,\cos\left(2\,bx\right)\cos\left(2\,a\right) + \cos\left(2\,a\right)^2 + \sin\left(2\,bx\right)^2 - 2\,\sin\left(2\,bx\right)\sin\left(2\,a\right) + \sin\left(2\right.\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos
(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b - 1/2*(2*b*
x*cos(2*b*x + 2*a) + 4*b^2*integrate(x*sin(2*b*x + 2*a)/(cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - sin(2*b*x + 2*a))*d/b
^2
```

Fricas [B] time = 0.621951, size = 937, normalized size = 8.76

$$\frac{2\,bdx - 4\,(bdx + bc)\cos\left(bx+a\right)^2 + 2\,d\cos\left(bx+a\right)\sin\left(bx+a\right) + i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right) + \sin\left(bx+a\right)\right) - i\,d\mathrm{Li}_2\left(i\,\cos\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - 4*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) -
```

```
sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I*co
s(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
+ I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a*d)*l
og(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(I*cos(b*x + a) -
sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) +
 (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-c
os(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-cos(b*x + a) - I*sin(b
*x + a) + I))/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \sec (bx + a) \sin (3 bx + 3 a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

3.386 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=79

$$-\text{Unintegrable}\left(\frac{\tan(a+bx)}{c+dx},x\right)+\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}$$

```
[Out] (2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (2*Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d - Unintegrable[Tan[a + b*x]/(c +
 d*x), x]
```

Rubi [A] time = 0.299071, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (2*Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d - Defer[Int][Tan[a + b*x]/(c + d
*x), x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{c+dx} - \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\right) dx \\
&= 3\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx - \int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx \\
&= 3\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx + \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx - \int \frac{\tan(a+bx)}{c+dx}\,dx \\
&= \frac{3}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx + \int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx - \int \frac{\tan(a+bx)}{c+dx}\,dx \\
&= \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx + \frac{1}{2}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx + \frac{1}{2}\left(3\sin\left(2a-\right.\right. \\
&= \frac{3\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} + \frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \\
&= \frac{2\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d} + \frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d} - \int \frac{\tan(a+bx)}{c+dx}\,dx
\end{aligned}
$$

Mathematica [A] time = 3.15739, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x), x]
```

Maple [A] time = 0.371, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(iE_1\left(\frac{2i\,bdx+2i\,bc}{d}\right)-iE_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+2\,d\int\frac{\sin(2\,bx+2\,a)}{(dx+c)\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)}\,dx+\left(E_1\left(2\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(1, -(2*I
*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*d*integrate(sin(2*b*x + 2*a
)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d
*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, (2*I*b*d*x + 2*I*b*c
)/d) + exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))/
d
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(bx+a\right)\sin\left(3\,bx+3\,a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

3.387 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=102

$$-\text{Unintegrable}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)+\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{2\,\text{s}}{}$$

```
[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - (2*Sin[2*a
+ 2*b*x])/(d*(c + d*x)) - (4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d +
 2*b*x])/d^2 - Unintegrable[Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi [A] time = 0.341935, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - (2*Sin[2*a
+ 2*b*x])/(d*(c + d*x)) - (4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d +
 2*b*x])/d^2 - Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{(c+dx)^2} - \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\right)dx \\ &= 3\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx \\ &= 3\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx + \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx \\ &= \frac{3}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx + \int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx \\ &= -\frac{3\sin(2a+2bx)}{2d(c+dx)} + \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx + \frac{(3b)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d} - \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx \\ &= -\frac{2\sin(2a+2bx)}{d(c+dx)} + \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d} + \frac{\left(3b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} - \frac{(3b\sin \\ &= \frac{3b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{2\sin(2a+2bx)}{d(c+dx)} - \frac{3b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2} \\ &= \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{2\sin(2a+2bx)}{d(c+dx)} - \frac{4b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}\end{aligned}$$

Mathematica [A] time = 4.25604, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.663, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(i E_2\left(\frac{2 i b d x+2 i b c}{d}\right)-i E_2\left(-\frac{2 i b d x+2 i b c}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+2\left(d^2 x+cd\right)\int\frac{\sin(2bx+2a)}{(dx+c)^2\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)}\,d}{d^2 x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((I*exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(2, -(2*I
*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin
(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*
a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*
c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, (2*I*b*d*x + 2*I*b*
c)/d) + exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))
/(d^2*x + c*d)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)\sin(3bx+3a)}{d^2x^2+2cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx + a)\sin(3\,bx + 3\,a)}{(dx + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

3.388 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=128

$$-\text{Unintegrable}\left(\frac{\tan(a+bx)}{(c+dx)^3},x\right)-\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-$$

```
[Out] (-2*b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) - (4*b^2*CosIntegral[(2*b*c)/d + 2*
b*x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(d*(c + d*x)^2) - (4*b^2*
Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3 - Unintegrable[Tan
[a + b*x]/(c + d*x)^3, x]
```

Rubi [A] time = 0.397779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-2*b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) - (4*b^2*CosIntegral[(2*b*c)/d + 2*
b*x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(d*(c + d*x)^2) - (4*b^2*
Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3 - Defer[Int][Tan[a
 + b*x]/(c + d*x)^3, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\right)dx$$

$$= 3\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\,dx$$

$$= 3\int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx + \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{3}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx + \int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3\sin(2a+2bx)}{4d(c+dx)^2} + \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx + \frac{(3b)\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{2d} - \int \frac{\tan(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{\left(3b^2\right)\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{2d} - \int$$

$$= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{b^2\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2} - \frac{\left(3b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin(}{}}{d^2}$$

$$= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{3b^2\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{3b^2\cos\left(2a\right.}{}$$

$$= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{4b^2\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{4b^2\cos\left(2a\right.}{}$$

Mathematica [A] time = 5.89447, size = 0, normalized size = 0.

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

Maple [A] time = 0.504, size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(bx+a)\sin(3\,bx+3\,a)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
 c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (bx + a) \sin (3\,bx + 3\,a)}{(dx + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

3.389 $\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx) \, dx$

Optimal. Leaf size=230

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}\left(3\right.}{b^4}$$

```
[Out] ((-6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 + (24*d^2*(c + d*x)*Cos[
a + b*x])/b^3 - (4*(c + d*x)^3*Cos[a + b*x])/b + ((6*I)*d^2*(c + d*x)*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 - (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 + (6*d^3*Pol
yLog[3, I*E^(I*(a + b*x))])/b^4 - ((c + d*x)^3*Sec[a + b*x])/b - (24*d^3*Si
n[a + b*x])/b^4 + (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.329964, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4431, 3296, 2637, 4407, 4409, 4181, 2531, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}\left(3\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```



```
[Out] ((-6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 + (24*d^2*(c + d*x)*Cos[
a + b*x])/b^3 - (4*(c + d*x)^3*Cos[a + b*x])/b + ((6*I)*d^2*(c + d*x)*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 - (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 + (6*d^3*Pol
yLog[3, I*E^(I*(a + b*x))])/b^4 - ((c + d*x)^3*Sec[a + b*x])/b - (24*d^3*Si
n[a + b*x])/b^4 + (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\sin(a+bx) - (c+dx)^3\sin(a+bx)\tan^2(a+bx)\right)dx$$
$$= 3\int (c+dx)^3\sin(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\tan^2(a+bx)\,dx$$
$$= -\frac{3(c+dx)^3\cos(a+bx)}{b} + \frac{(9d)\int (c+dx)^2\cos(a+bx)\,dx}{b} + \int (c+dx)^3 \sin$$
$$= -\frac{4(c+dx)^3\cos(a+bx)}{b} - \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{9d(c+dx)^2\sin(a+bx)}{b^2}$$
$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{18d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx)^3\cos(}{b}$$
$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx)^3\cos(}{b}$$
$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx)^3\cos(}{b}$$
$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx)^3\cos(}{b}$$

Mathematica [B] time = 2.33488, size = 547, normalized size = 2.38

$$-\frac{\sec(a+bx)\left(6ibcd^2\cos(a+bx)\text{PolyLog}(2,-\sin(a+bx)+i\cos(a+bx)) - 6ibcd^2\cos(a+bx)\text{PolyLog}(2,\sin(a+bx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] -((Sec[a + b*x]*(3*b^3*c^3 - 12*b*c*d^2 + 9*b^3*c^2*d*x - 12*b*d^3*x + 9*b^
3*c*d^2*x^2 + 3*b^3*d^3*x^3 - (6*I)*b^2*d^3*x^2*ArcTan[Cos[a + b*x] - I*Sin
[a + b*x]]*Cos[a + b*x] + (6*I)*b^2*c^2*d*ArcTan[Cos[a + b*x] + I*Sin[a + b
*x]]*Cos[a + b*x] + (12*I)*b^2*c*d^2*x*ArcTan[Cos[a + b*x] + I*Sin[a + b*x]
```

```
]*Cos[a + b*x] + 2*b^3*c^3*Cos[2*(a + b*x)] - 12*b*c*d^2*Cos[2*(a + b*x)] +
 6*b^3*c^2*d*x*Cos[2*(a + b*x)] - 12*b*d^3*x*Cos[2*(a + b*x)] + 6*b^3*c*d^2
*x^2*Cos[2*(a + b*x)] + 2*b^3*d^3*x^3*Cos[2*(a + b*x)] - (6*I)*b*d^3*x*Cos[
a + b*x]*PolyLog[2, (-I)*Cos[a + b*x] - Sin[a + b*x]] + (6*I)*b*c*d^2*Cos[a
 + b*x]*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - (6*I)*b*c*d^2*Cos[a + b
*x]*PolyLog[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] + (6*I)*b*d^3*x*Cos[a + b*
x]*PolyLog[2, I*Cos[a + b*x] + Sin[a + b*x]] - 6*d^3*Cos[a + b*x]*PolyLog[3
, (-I)*Cos[a + b*x] - Sin[a + b*x]] + 6*d^3*Cos[a + b*x]*PolyLog[3, I*Cos[a
 + b*x] + Sin[a + b*x]] - 6*b^2*c^2*d*Sin[2*(a + b*x)] + 12*d^3*Sin[2*(a +
b*x)] - 12*b^2*c*d^2*x*Sin[2*(a + b*x)] - 6*b^2*d^3*x^2*Sin[2*(a + b*x)]))/
b^4)
```

Maple [B] time = 0.302, size = 677, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d
^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-2*
(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*
x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-2*(d
^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))+1)+12*
I*d^2/b^3*c*a*arctan(exp(I*(b*x+a)))-6*I*d/b^2*c^2*arctan(exp(I*(b*x+a)))-6
*I*d^3/b^4*a^2*arctan(exp(I*(b*x+a)))-6*d^2/b^3*c*ln(1+I*exp(I*(b*x+a)))*a-
6*d^2/b^2*c*ln(1+I*exp(I*(b*x+a)))*x-3*d^3/b^2*ln(1+I*exp(I*(b*x+a)))*x^2-6
*I*d^3/b^3*polylog(2,I*exp(I*(b*x+a)))*x-3*d^3/b^4*a^2*ln(1-I*exp(I*(b*x+a)
))-6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*I*d^2/b^3*c*polylog(2,I*exp(I*(
b*x+a)))+6*d^2/b^3*c*ln(1-I*exp(I*(b*x+a)))*a+6*d^2/b^2*c*ln(1-I*exp(I*(b*x
+a)))*x+3*d^3/b^2*ln(1-I*exp(I*(b*x+a)))*x^2+6*d^3*polylog(3,I*exp(I*(b*x+a
)))/b^4+6*I*d^3/b^3*polylog(2,-I*exp(I*(b*x+a)))*x+6*I*d^2/b^3*c*polylog(2,
-I*exp(I*(b*x+a)))+3*d^3/b^4*a^2*ln(1+I*exp(I*(b*x+a)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c^3/(b*cos(3*b*x + 3*a)^2
+ 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)
^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 3/2*(4*(cos(a)
^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) +
 b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(
b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))
*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x
 + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*s
in(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x +
 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a)
)*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*
x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*
x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) +
(b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b
*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*
(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + s
in(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x
+ a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*
x + a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x +
3*a)^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(
b*x + 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x +
 a)*sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*
b*x*cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b
*x + 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^
2*cos(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2
 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x
+ 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*
x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*
sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 +
 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a
) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a
) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^
2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*
sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*
sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^
2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)
^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*
```

```
x + a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 +
 sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a
)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + si
n(a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) +
cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos
(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*c
os(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*c
os(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*c
os(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b
*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*
sin(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x
 + a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(co
s(b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) +
b*x*sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) -
cos(a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3
*a) - cos(a))*sin(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(a))*sin(b*x + a)^2 + 2
*(b*x*cos(b*x + a)*sin(2*b*x + 3*a) + (b*x*sin(a) - cos(a))*cos(b*x + a) -
cos(2*b*x + 3*a)*cos(b*x + a))*cos(3*b*x + 3*a) - (cos(b*x + a)^2 + sin(b*x
 + a)^2)*cos(2*b*x + 3*a) + 2*(b*x*sin(2*b*x + 3*a)*sin(b*x + a) + (b*x*sin
(a) - cos(a))*sin(b*x + a) - cos(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a
) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x +
 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(a)*sin(b*x + a) + 6*b*x*sin(b*x + 2*a)*
sin(b*x + a)*sin(a) + 2*(3*b*x*cos(b*x + 2*a)*sin(b*x + a) - cos(a))*cos(2*
b*x + 3*a) - cos(2*b*x + 3*a)^2 - cos(a)^2 + 2*(3*b*x*sin(b*x + 2*a)*sin(b*
x + a) - sin(a))*sin(2*b*x + 3*a) - sin(2*b*x + 3*a)^2 - sin(a)^2)*sin(3*b*
x + 3*a) + 4*(2*b*x*cos(b*x + a)*sin(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b
*x + a)^2)*sin(b*x + 2*a) - 2*sin(b*x + a)*sin(a))*sin(2*b*x + 3*a) + 12*(b
*x*cos(b*x + a)^2*sin(a) + b*x*sin(b*x + a)^2*sin(a))*sin(b*x + 2*a) - 4*(c
os(a)^2 + sin(a)^2)*sin(b*x + a))*c^2*d/((cos(a)^2 + sin(a)^2)*b^2*cos(b*x
+ a)^2 + (cos(a)^2 + sin(a)^2)*b^2*sin(b*x + a)^2 + (b^2*cos(2*b*x + 3*a)^2
 + 2*b^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*x
 + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*cos(3*b*x + 3*a)^2 + (b^2*cos(b
*x + a)^2 + b^2*sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (b^2*cos(2*b*x + 3*a)^
2 + 2*b^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*
x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*sin(3*b*x + 3*a)^2 + (b^2*cos(
b*x + a)^2 + b^2*sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + 2*(b^2*cos(2*b*x + 3*
a)^2*cos(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + b^2*cos(b*
x + a)*sin(2*b*x + 3*a)^2 + 2*b^2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (c
os(a)^2 + sin(a)^2)*b^2*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(b^2*cos(b*x + a
)^2*cos(a) + b^2*cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^2*cos(2*b*x
 + 3*a)^2*sin(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + b^2*s
in(2*b*x + 3*a)^2*sin(b*x + a) + 2*b^2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a)
 + (cos(a)^2 + sin(a)^2)*b^2*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(b^2*cos(b*
```

x + a)^2*sin(a) + b^2*sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a)) - (6*((b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(2*b*x + 3*a)*cos(b*x
+ 2*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*sin(2*b*x
+ 3*a)*sin(b*x + 2*a) + (b^3*d^3*x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*
d^3*x*cos(a) - 4*b*c*d^2*cos(a))*cos(b*x + 2*a) + (b^3*d^3*x^3*sin(a) + 3*b
^3*c*d^2*x^2*sin(a) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*sin(b*x + 2*a))*
cos(3*b*x + 3*a)^2 + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*
d^2)*cos(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*sin(b*x + a))*c
os(2*b*x + 3*a)^2 + 6*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d
^2)*cos(2*b*x + 3*a)*cos(b*x + 2*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*
d^3*x - 4*b*c*d^2)*sin(2*b*x + 3*a)*sin(b*x + 2*a) + (b^3*d^3*x^3*cos(a) +
3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*cos(b*x + 2*a
) + (b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*sin(a) - 4*b*d^3*x*sin(a) - 4*b*c
*d^2*sin(a))*sin(b*x + 2*a))*sin(3*b*x + 3*a)^2 + 2*((b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d
^2*x - 2*d^3)*sin(b*x + a))*sin(2*b*x + 3*a)^2 + 2*((b^3*d^3*x^3*cos(a) - 6
*b*c*d^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2
+ 6*(b^2*c*d^2*sin(a) - b*d^3*cos(a))*x + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 -
6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x -
2*d^3)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a)^2 + (b^3*d^3*x^3*cos(a) - 6*b*c*d
^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2 + 6*(b
^2*c*d^2*sin(a) - b*d^3*cos(a))*x)*cos(b*x + a)^2 + (b^3*d^3*x^3*cos(a) - 6
*b*c*d^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2
+ 6*(b^2*c*d^2*sin(a) - b*d^3*cos(a))*x + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 -
6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x -
2*d^3)*sin(2*b*x + 3*a))*sin(3*b*x + 3*a)^2 + (b^3*d^3*x^3*cos(a) - 6*b*c*d
^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2 + 6*(b
^2*c*d^2*sin(a) - b*d^3*cos(a))*x)*sin(b*x + a)^2 + 2*((b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a)*cos(b*x + a) + 3*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(b*x + a)*sin(2*b*x + 3*a) + (b^3*d^3*x
^3*cos(a) - 6*b*c*d^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3
*sin(a))*x^2 + 6*(b^2*c*d^2*sin(a) - b*d^3*cos(a))*x)*cos(b*x + a))*cos(3*b
*x + 3*a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*
x + a)^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x
+ a)^2)*cos(2*b*x + 3*a) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x -
6*b*c*d^2)*cos(2*b*x + 3*a)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x -
2*d^3)*sin(2*b*x + 3*a)*sin(b*x + a) + (b^3*d^3*x^3*cos(a) - 6*b*c*d^2*cos
(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2 + 6*(b^2*c*d
^2*sin(a) - b*d^3*cos(a))*x)*sin(b*x + a))*sin(3*b*x + 3*a) + 3*((b^2*d^3*x
^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(b*x + a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x -
2*d^3)*sin(b*x + a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 2*((cos(a)^2 +
sin(a)^2)*b^3*d^3*x^3 + 3*(cos(a)^2 + sin(a)^2)*b^3*c*d^2*x^2 - 6*(cos(a)^
2 + sin(a)^2)*b*d^3*x - 6*(cos(a)^2 + sin(a)^2)*b*c*d^2 + (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a)^2 + 6*(b^3*d^3*x^3*
cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*cos(

b*x + 2*a)*cos(b*x + a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*
c*d^2)*sin(2*b*x + 3*a)^2 + 6*(b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*sin(a)
- 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*cos(b*x + a)*sin(b*x + 2*a) + 2*(b^3
*d^3*x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 6*b*d^3*x*cos(a) - 6*b*c*d^2*cos
(a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + 2
*a)*cos(b*x + a))*cos(2*b*x + 3*a) + 2*(b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^
2*sin(a) - 6*b*d^3*x*sin(a) - 6*b*c*d^2*sin(a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + a)*sin(b*x + 2*a))*sin(2*b*x + 3*
a))*cos(3*b*x + 3*a) + 2*(3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4
*b*c*d^2)*cos(b*x + a)^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b
*c*d^2)*sin(b*x + a)^2)*cos(b*x + 2*a) + 2*(b^3*d^3*x^3*cos(a) + 3*b^3*c*d^
2*x^2*cos(a) - 6*b*d^3*x*cos(a) - 6*b*c*d^2*cos(a))*cos(b*x + a) - 6*(b^2*d
^3*x^2*cos(a) + 2*b^2*c*d^2*x*cos(a) - 2*d^3*cos(a))*sin(b*x + a))*cos(2*b*
x + 3*a) + 6*((b^3*d^3*x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(
a) - 4*b*c*d^2*cos(a))*cos(b*x + a)^2 + (b^3*d^3*x^3*cos(a) + 3*b^3*c*d^2*x
^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*sin(b*x + a)^2)*cos(b*x +
2*a) + 2*((cos(a)^2 + sin(a)^2)*b^3*d^3*x^3 + 3*(cos(a)^2 + sin(a)^2)*b^3*c
*d^2*x^2 - 6*(cos(a)^2 + sin(a)^2)*b*d^3*x - 6*(cos(a)^2 + sin(a)^2)*b*c*d^
2)*cos(b*x + a) - ((cos(a)^2 + sin(a)^2)*b^4*cos(b*x + a)^2 + (cos(a)^2 + s
in(a)^2)*b^4*sin(b*x + a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x + 3
*a)*cos(a) + b^4*sin(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos(
a)^2 + sin(a)^2)*b^4)*cos(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b*
x + a)^2)*cos(2*b*x + 3*a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x +
3*a)*cos(a) + b^4*sin(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos
(a)^2 + sin(a)^2)*b^4)*sin(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b
*x + a)^2)*sin(2*b*x + 3*a)^2 + 2*(b^4*cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*
b^4*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + b^4*cos(b*x + a)*sin(2*b*x + 3*a
)^2 + 2*b^4*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^
4*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*cos(a) + b^4*cos(a
)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^4*cos(2*b*x + 3*a)^2*sin(b*x + a)
 + 2*b^4*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + b^4*sin(2*b*x + 3*a)^2*sin(
b*x + a) + 2*b^4*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^
2)*b^4*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*sin(a) + b^4*
sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*integrate(6*((d^3*x^2 + 2*c*d^2*x)
*cos(2*b*x + 2*a)*cos(b*x + a) + (d^3*x^2 + 2*c*d^2*x)*sin(2*b*x + 2*a)*sin
(b*x + a) + (d^3*x^2 + 2*c*d^2*x)*cos(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*s
in(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*a) + b), x) + 2*((b^3*d^3*x^3*sin(a)
- 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x
^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 - 2*d^3)*cos(2*b*x + 3*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6
*b*c*d^2)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a)^2 + (b^3*d^3*x^3*sin(a) - 6*b*
c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x^2 - 6
*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*cos(b*x + a)^2 + (b^3*d^3*x^3*sin(a)
- 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x
^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x

- 2*d^3)*cos(2*b*x + 3*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6
*b*c*d^2)*sin(2*b*x + 3*a))*sin(3*b*x + 3*a)^2 + (b^3*d^3*x^3*sin(a) - 6*b*
c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x^2 - 6
*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*sin(b*x + a)^2 - 2*(3*(b^2*d^3*x^2 +
2*b^2*c*d^2*x - 2*d^3)*cos(2*b*x + 3*a)*cos(b*x + a) - (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)*sin(2*b*x + 3*a) - (b^3*d^
3*x^3*sin(a) - 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*
d^3*cos(a))*x^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*cos(b*x + a))*cos(
3*b*x + 3*a) - 3*((b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(b*x + a)^2 + (b
^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*sin(b*x + a)^2)*cos(2*b*x + 3*a) - 2*(3
*(b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(2*b*x + 3*a)*sin(b*x + a) - (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*sin(2*b*x + 3*a)*sin(b*
x + a) - (b^3*d^3*x^3*sin(a) - 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d
^2*sin(a) - b^2*d^3*cos(a))*x^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*si
n(b*x + a))*sin(3*b*x + 3*a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x
- 6*b*c*d^2)*cos(b*x + a)^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x -
6*b*c*d^2)*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x + 4*a) - 6*((cos(a)^
2 + sin(a)^2)*b^2*d^3*x^2 + 2*(cos(a)^2 + sin(a)^2)*b^2*c*d^2*x - 2*(cos(a)
^2 + sin(a)^2)*d^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(2*b*x + 3*a)
^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*sin(2*b*x + 3*a)^2 - 2*(b^3*d^3*
x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*
cos(b*x + 2*a)*sin(b*x + a) - 2*(b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*sin(a
) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*sin(b*x + 2*a)*sin(b*x + a) + 2*(b
^2*d^3*x^2*cos(a) + 2*b^2*c*d^2*x*cos(a) - 2*d^3*cos(a) - (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + 2*a)*sin(b*x + a))*cos(2*b
*x + 3*a) + 2*(b^2*d^3*x^2*sin(a) + 2*b^2*c*d^2*x*sin(a) - 2*d^3*sin(a) - (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*sin(b*x + 2*a)*sin(b
*x + a))*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + 2*(2*(b^3*d^3*x^3*sin(a) + 3*
b^3*c*d^2*x^2*sin(a) - 6*b*d^3*x*sin(a) - 6*b*c*d^2*sin(a))*cos(b*x + a) +
3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + a)^2 +
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*sin(b*x + a)^2)*si
n(b*x + 2*a) - 6*(b^2*d^3*x^2*sin(a) + 2*b^2*c*d^2*x*sin(a) - 2*d^3*sin(a))
*sin(b*x + a))*sin(2*b*x + 3*a) + 6*((b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*
sin(a) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*cos(b*x + a)^2 + (b^3*d^3*x^3
*sin(a) + 3*b^3*c*d^2*x^2*sin(a) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*sin
(b*x + a)^2)*sin(b*x + 2*a) - 6*((cos(a)^2 + sin(a)^2)*b^2*d^3*x^2 + 2*(cos
(a)^2 + sin(a)^2)*b^2*c*d^2*x - 2*(cos(a)^2 + sin(a)^2)*d^3)*sin(b*x + a))/
((cos(a)^2 + sin(a)^2)*b^4*cos(b*x + a)^2 + (cos(a)^2 + sin(a)^2)*b^4*sin(b
*x + a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x + 3*a)*cos(a) + b^4*s
in(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b
^4)*cos(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b*x + a)^2)*cos(2*b*
x + 3*a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x + 3*a)*cos(a) + b^4*
sin(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*
b^4)*sin(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b*x + a)^2)*sin(2*b
*x + 3*a)^2 + 2*(b^4*cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*b^4*cos(2*b*x + 3*

```
a)*cos(b*x + a)*cos(a) + b^4*cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*b^4*cos(b*
x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^4*cos(b*x + a))*co
s(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*cos(a) + b^4*cos(a)*sin(b*x + a)^2)*
cos(2*b*x + 3*a) + 2*(b^4*cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*b^4*cos(2*b*x
 + 3*a)*cos(a)*sin(b*x + a) + b^4*sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*b^4*s
in(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^4*sin(b*x + a
))*sin(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*sin(a) + b^4*sin(b*x + a)^2*sin
(a))*sin(2*b*x + 3*a))
```

Fricas [C] time = 0.755471, size = 2240, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*
polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -
I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
 a) - sin(b*x + a)) + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^
2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 - (-6*I*b*d^3*x - 6*I*b*c*d^2
)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - (-6*I*b*d^3*x - 6*I*b
*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (6*I*b*d^3*x +
6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (6*I*b*d^
3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*
x + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2
 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^
3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x +
 a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^
2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^
2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a)
 - sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*l
og(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 24*(b^2*d^3*x^2
+ 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*
x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx+c)^3 \sec (bx+a)^2 \sin (3\,bx+3\,a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

3.390 $\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=147

$$\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{8d(c+dx)\sin(a+bx)}{b^2}-\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{8d^2\cos}{}$$

```
[Out] ((-4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + (8*d^2*Cos[a + b*x])/b^3
 - (4*(c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*
x))])/b^3 - ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((c + d*x)^2*Se
c[a + b*x])/b + (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.212481, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4431, 3296, 2638, 4407, 4409, 4181, 2279, 2391}

$$\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{8d(c+dx)\sin(a+bx)}{b^2}-\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{8d^2\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] ((-4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + (8*d^2*Cos[a + b*x])/b^3
 - (4*(c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*
x))])/b^3 - ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((c + d*x)^2*Se
c[a + b*x])/b + (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int (c+dx)^2\sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\sin(a+bx)-(c+dx)^2\sin(a+bx)\tan^2(a+bx)\right)dx$$
$$= 3\int (c+dx)^2\sin(a+bx)\,dx - \int (c+dx)^2\sin(a+bx)\tan^2(a+bx)\,dx$$
$$= -\frac{3(c+dx)^2\cos(a+bx)}{b} + \frac{(6d)\int (c+dx)\cos(a+bx)\,dx}{b} + \int (c+dx)^2\sin(a$$
$$= -\frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{6d(c+dx)\sin(a+bx)}{b^2} +$$
$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{(}{}$$
$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{(}{}$$
$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(a+bx)}{b} + \frac{2}{}$$

Mathematica [B] time = 3.7371, size = 364, normalized size = 2.48

$$2d^2\left(2\tan^{-1}(\cot(a))\tanh^{-1}\left(\cos(a)\tan\left(\frac{bx}{2}\right)+\sin(a)\right)-\frac{\csc(a)\left(i\text{PolyLog}\left(2,-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-i\text{PolyLog}\left(2,e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+\left(bx-\text{ta}\right.\right.}{\sqrt{\csc^2(a)}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + 2*d^2*(2*ArcTan[Cot[a]]*Ar
cTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - (Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[
1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]]))])
+ I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x - Ar
cTan[Cot[a]]))]))/Sqrt[Csc[a]^2]) - b^2*(c + d*x)^2*Sec[a] - 4*Cos[b*x]*((-
2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) + 4*(2*b*d*(c + d
*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] - (b^2*(c + d*x)^2
*Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])
) + (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2]
 + Sin[(a + b*x)/2])))/b^3
```

Maple [B] time = 0.308, size = 345, normalized size = 2.4

$$-2\frac{\left(d^{2}x^{2}b^{2}+2\,b^{2}cdx+b^{2}c^{2}+2\,ibd^{2}x-2\,d^{2}+2\,ibcd\right)\mathrm{e}^{i(bx+a)}}{b^{3}}-2\frac{\left(d^{2}x^{2}b^{2}+2\,b^{2}cdx+b^{2}c^{2}-2\,ibd^{2}x-2\,d^{2}-2\,ibcd\right)\mathrm{e}^{-i}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*
(b*x+a))-2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^
3*exp(-I*(b*x+a))-2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a)
)+1)-4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-2*d^2/b^2*ln(1+I*exp(I*(b*x+a)))*x-
2*d^2/b^3*ln(1+I*exp(I*(b*x+a)))*a+2*d^2/b^2*ln(1-I*exp(I*(b*x+a)))*x+2*d^2
/b^3*ln(1-I*exp(I*(b*x+a)))*a+2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))-2*I*d^2
/b^3*dilog(1-I*exp(I*(b*x+a)))+4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.638272, size = 1320, normalized size = 8.98

$$-\frac{b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}+i\,d^{2}\cos\left(bx+a\right)\mathrm{Li}_{2}\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+i\,d^{2}\cos\left(bx+a\right)\mathrm{Li}_{2}\left(i\,\cos\left(bx+a\right)-\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
+ a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x +
```

```
a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(
b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(cos
(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x
+ a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a)
 - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - s
in(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b
*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x +
 a) + I) - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \sec (bx + a)^2 \sin (3\,bx + 3\,a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

3.391 $\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=57

$$\frac{4d\sin(a+bx)}{b^2}+\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}$$

```
[Out] (d*ArcTanh[Sin[a + b*x]])/b^2 - (4*(c + d*x)*Cos[a + b*x])/b - ((c + d*x)*S
ec[a + b*x])/b + (4*d*Sin[a + b*x])/b^2
```

Rubi [A] time = 0.0914535, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4431, 3296, 2637, 4407, 4409, 3770}

$$\frac{4d\sin(a+bx)}{b^2}+\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*ArcTanh[Sin[a + b*x]])/b^2 - (4*(c + d*x)*Cos[a + b*x])/b - ((c + d*x)*S
ec[a + b*x])/b + (4*d*Sin[a + b*x])/b^2
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)\sin(a+bx)-(c+dx)\sin(a+bx)\tan^2(a+bx)\right)\,dx\\ &= 3\int (c+dx)\sin(a+bx)\,dx-\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx\\ &= -\frac{3(c+dx)\cos(a+bx)}{b}+\frac{(3d)\int \cos(a+bx)\,dx}{b}+\int (c+dx)\sin(a+bx)\,dx -\\ &= -\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}+\frac{3d\sin(a+bx)}{b^2}+\frac{d\int \cos(a+\\ &= \frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}+\frac{4d\sin\end{aligned}$$

Mathematica [A] time = 0.453908, size = 105, normalized size = 1.84

$$-\frac{\sec(a+bx)\left(2b(c+dx)\cos(2(a+bx))-2d\sin(2(a+bx))+d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)-\log\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] -((Sec[a + b*x]*(3*b*c + 3*b*d*x + 2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Cos[a
 + b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] +
Sin[(a + b*x)/2]]) - 2*d*Sin[2*(a + b*x)]))/b^2)
```

Maple [A] time = 0.046, size = 87, normalized size = 1.5

$$-4\,{\frac{c\cos \left(bx+a \right) }{b}}-{\frac{c}{b\cos \left(bx+a \right) }}-4\,{\frac{d\cos \left(bx+a \right) x}{b}}+4\,{\frac{d\sin \left(bx+a \right) }{{b}^{2}}}-{\frac{dx}{b\cos \left(bx+a \right) }}+{\frac{d\ln \left(\sec \left(bx+a \right) +\tan \left(bx \right.}{{b}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -4*c/b*cos(b*x+a)-c/b/cos(b*x+a)-4*d/b*cos(b*x+a)*x+4*d*sin(b*x+a)/b^2-d/b/
cos(b*x+a)*x+d/b^2*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 1.85269, size = 4496, normalized size = 78.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c/(b*cos(3*b*x + 3*a)^2 +
2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)^2
 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 1/2*(4*(cos(a)^2
 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b
*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*
x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*c
os(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x +
 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*sin
(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 3
*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*
cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*x
+ 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*x*
cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) + (b
```

```
*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b*x
 + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b
*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + sin
(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x +
a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*x
+ a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x + 3*
a)^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(b*
x + 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x + a
)*sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*b*
x*cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b*x
 + 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^2*
cos(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2 +
 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x +
3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)
^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*x
+ 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*si
n(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2
*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a)
+ cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a)
+ (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2*
cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*si
n(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*si
n(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)
*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)^2
*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x
+ a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + s
in(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^
2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(
a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + co
s(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b
*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a
)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos
(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos
(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos
(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x
 + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*si
n(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x +
 a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(
b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) + b*
x*sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) - co
s(a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3*a
) - cos(a))*sin(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(a))*sin(b*x + a)^2 + 2*(
b*x*cos(b*x + a)*sin(2*b*x + 3*a) + (b*x*sin(a) - cos(a))*cos(b*x + a) - co
```

```
s(2*b*x + 3*a)*cos(b*x + a))*cos(3*b*x + 3*a) - (cos(b*x + a)^2 + sin(b*x +
 a)^2)*cos(2*b*x + 3*a) + 2*(b*x*sin(2*b*x + 3*a)*sin(b*x + a) + (b*x*sin(a
) - cos(a))*sin(b*x + a) - cos(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a)
+ (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x + 4
*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(a)*sin(b*x + a) + 6*b*x*sin(b*x + 2*a)*si
n(b*x + a)*sin(a) + 2*(3*b*x*cos(b*x + 2*a)*sin(b*x + a) - cos(a))*cos(2*b*
x + 3*a) - cos(2*b*x + 3*a)^2 - cos(a)^2 + 2*(3*b*x*sin(b*x + 2*a)*sin(b*x
+ a) - sin(a))*sin(2*b*x + 3*a) - sin(2*b*x + 3*a)^2 - sin(a)^2)*sin(3*b*x
+ 3*a) + 4*(2*b*x*cos(b*x + a)*sin(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x
 + a)^2)*sin(b*x + 2*a) - 2*sin(b*x + a)*sin(a))*sin(2*b*x + 3*a) + 12*(b*x
*cos(b*x + a)^2*sin(a) + b*x*sin(b*x + a)^2*sin(a))*sin(b*x + 2*a) - 4*(cos
(a)^2 + sin(a)^2)*sin(b*x + a))*d/((cos(a)^2 + sin(a)^2)*b^2*cos(b*x + a)^2
 + (cos(a)^2 + sin(a)^2)*b^2*sin(b*x + a)^2 + (b^2*cos(2*b*x + 3*a)^2 + 2*b
^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*x + 3*a
)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*cos(3*b*x + 3*a)^2 + (b^2*cos(b*x + a
)^2 + b^2*sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (b^2*cos(2*b*x + 3*a)^2 + 2*
b^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*x + 3*
a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*sin(3*b*x + 3*a)^2 + (b^2*cos(b*x +
a)^2 + b^2*sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + 2*(b^2*cos(2*b*x + 3*a)^2*c
os(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + b^2*cos(b*x + a)
*sin(2*b*x + 3*a)^2 + 2*b^2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^
2 + sin(a)^2)*b^2*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(b^2*cos(b*x + a)^2*co
s(a) + b^2*cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^2*cos(2*b*x + 3*a
)^2*sin(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + b^2*sin(2*b
*x + 3*a)^2*sin(b*x + a) + 2*b^2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (co
s(a)^2 + sin(a)^2)*b^2*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(b^2*cos(b*x + a)
^2*sin(a) + b^2*sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))
```

Fricas [A] time = 0.513506, size = 252, normalized size = 4.42

$$-\frac{2\,bdx + 8\,(bdx + bc)\cos{(bx + a)}^2 - d\cos{(bx + a)}\log{(\sin{(bx + a)} + 1)} + d\cos{(bx + a)}\log{(-\sin{(bx + a)} + 1)} - 8\,d\,c}{2\,b^2\cos{(bx + a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + 8*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
 + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 8*d*cos(b*x + a)*sin(b
*x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.2886, size = 3883, normalized size = 68.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] -1/2*(10*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b*c*tan(1/2*b*x)^4*tan(1/2*
a)^4 + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*
b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(
1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 +
2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1
/2*b*x)^4*tan(1/2*a)^4 - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2
*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1
/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*
x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(
1/2*a) + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 - 12*b*d*x*tan(1/2*b*x)^4*tan(1/2*
a)^2 - 64*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3 - 12*b*d*x*tan(1/2*b*x)^2*tan(1
/2*a)^4 - 12*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 64*b*c*tan(1/2*b*x)^3*tan(1/
2*a)^3 - 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(
1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*
tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^
2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*t
an(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*t
an(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 +
 tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(
1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) +
2*tan(1/2*a) + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 16*d*tan(1/2*b*x)^4*tan(1/
2*a)^3 - 12*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 16*d*tan(1/2*b*x)^3*tan(1/2*a
)^4 + 10*b*d*x*tan(1/2*b*x)^4 + 64*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) + 168*b*
d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 64*b*d*x*tan(1/2*b*x)*tan(1/2*a)^3 + 10*b
```

*d*x*tan(1/2*a)^4 + 10*b*c*tan(1/2*b*x)^4 - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*b*x)^4 + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)^4 + 64*b*c*tan(1/2*b*x)^3*tan(1/2*a) - 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)^3*tan(1/2*a) - 16*d*tan(1/2*b*x)^4*tan(1/2*a) + 168*b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 96*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 64*b*c*tan(1/2*b*x)*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a)^3 + 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a)^3 - 96*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + 10*b*c*tan(1/2*a)^4 - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*a)^4 + d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*a)^4 - 16*d*tan(1/2*b*x)*tan(1/2*a)^4 - 12*b*d*x*tan(1/2*b*x)^2 - 64*b*d*x*tan(1/2*b*x)*tan(1/2*a) - 12*b*d*x*tan(1/2*a)^2 - 12*b*c*tan(1/2*b*x)^2 + 16*d*tan(1/2*b*x)^3 - 64*b*c*tan(1/2*b*x)*tan(1/2*a) - 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1

```
/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2
*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*
a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1))*tan(1/2*b*x)*tan(1/2*a) + 96*d*t
an(1/2*b*x)^2*tan(1/2*a) - 12*b*c*tan(1/2*a)^2 + 96*d*tan(1/2*b*x)*tan(1/2*
a)^2 + 16*d*tan(1/2*a)^3 + 10*b*d*x + 10*b*c + d*log(2*(tan(1/2*a)^2 + 1)/(
tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^
3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2
*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2
*tan(1/2*b*x) - 2*tan(1/2*a) + 1)) - d*log(2*(tan(1/2*a)^2 + 1)/(tan(1/2*b*
x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*
a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 -
2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b
*x) + 2*tan(1/2*a) + 1)) - 16*d*tan(1/2*b*x) - 16*d*tan(1/2*a))/(b^2*tan(1/
2*b*x)^4*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - b^2*tan(1/2*b*x
)^4 - 4*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*b^2*tan(1/2*b*x)*tan(1/2*a)^3 - b
^2*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2)
```

3.392 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=77

$$-\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec(a+bx)}{c+dx},x\right)+\frac{4\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x] + (4*CosIntegral
[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (4*Cos[a - (b*c)/d]*SinIntegral[(b*c)
/d + b*x])/d
```

Rubi [A] time = 0.266208, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (4*Cos[a - (b*c)/d]*Sin
Integral[(b*c)/d + b*x])/d - Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*
x), x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \left(\frac{3\sin(a+bx)}{c+dx} - \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\right)dx$$
$$= 3\int \frac{\sin(a+bx)}{c+dx}\,dx - \int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$
$$= \left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx + \left(3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx + \int \frac{\sin(}{c}$$
$$= \frac{3\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} + \frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} + \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+b\right.}{c+dx}$$
$$= \frac{4\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} + \frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} - \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}$$

Mathematica [A] time = 13.5578, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

Maple [A] time = 0.388, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx+a))^2\sin(3bx+3a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

3.393 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=97

$$-\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)+\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}$$

```
[Out] -CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x] + (4*b*Cos[a -
 (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - (4*Sin[a + b*x])/(d*(c + d*x))
- (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2
```

Rubi [A] time = 0.344218, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - (4*Sin[a + b*x])/(d
*(c + d*x)) - (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 - Defer
[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \left(\frac{3\sin(a+bx)}{(c+dx)^2} - \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\right)dx$$
$$= 3\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$
$$= -\frac{3\sin(a+bx)}{d(c+dx)} + \frac{(3b)\int \frac{\cos(a+bx)}{c+dx}\,dx}{d} + \int \frac{\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sec(a+bx)\tan(a+bx}{(c+dx)^2}$$
$$= -\frac{4\sin(a+bx)}{d(c+dx)} + \frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d} + \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} - \frac{\left(3b\sin\left(a-\frac{b}{a}\right.\right.}{}$$
$$= \frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{4\sin(a+bx)}{d(c+dx)} - \frac{3b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{\left(b\cos\left(\right.\right.}{}$$
$$= \frac{4b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{4\sin(a+bx)}{d(c+dx)} - \frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} - \int \frac{\sec(}{}$$

Mathematica [A] time = 16.5538, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

Maple [A] time = 0.5, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx+a))^2\sin(3bx+3a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

3.394 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=120

$$-\text{CannotIntegrate}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^3},x\right)-\frac{2b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^3}-\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+\right.}{d^3}$$

```
[Out] -CannotIntegrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^3, x] - (2*b*Cos[a +
 b*x])/(d^2*(c + d*x)) - (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^3 - (2*Sin[a + b*x])/(d*(c + d*x)^2) - (2*b^2*Cos[a - (b*c)/d]*SinInteg
ral[(b*c)/d + b*x])/d^3
```

Rubi [A] time = 0.436226, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-2*b*Cos[a + b*x])/(d^2*(c + d*x)) - (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin
[a - (b*c)/d])/d^3 - (2*Sin[a + b*x])/(d*(c + d*x)^2) - (2*b^2*Cos[a - (b*c
)/d]*SinIntegral[(b*c)/d + b*x])/d^3 - Defer[Int][(Sec[a + b*x]*Tan[a + b*x
])/(c + d*x)^3, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\sin(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\right)dx$$

$$= 3\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3\sin(a+bx)}{2d(c+dx)^2} + \frac{(3b)\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d} + \int \frac{\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sec(a+bx)\tan(a+bx}{(c+dx)^3}$$

$$= -\frac{3b\cos(a+bx)}{2d^2(c+dx)} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{\left(3b^2\right)\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d} - \int \frac{\sec(a}{}$$

$$= -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} - \frac{\left(3b^2\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}}{2d^2}$$

$$= -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{3b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}}{2d^3}$$

$$= -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{2b^2\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}}{d^3}$$

Mathematica [A] time = 18.9277, size = 0, normalized size = 0.

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

Maple [A] time = 0.996, size = 0, normalized size = 0.

$$\int \frac{(\sec(bx+a))^2\sin(3bx+3a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(bx+a)^2 \sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

3.395 $\int x\cos(2x)\sec(x)\,dx$

Optimal. Leaf size=57

$$-i\text{PolyLog}\left(2,-ie^{ix}\right)+i\text{PolyLog}\left(2,ie^{ix}\right)+2x\sin(x)+2\cos(x)+2ix\tan^{-1}\left(e^{ix}\right)$$

```
[Out] (2*I)*x*ArcTan[E^(I*x)] + 2*Cos[x] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLog
[2, I*E^(I*x)] + 2*x*Sin[x]
```

Rubi [A] time = 0.0674738, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4431, 3296, 2638, 4407, 4181, 2279, 2391}

$$-i\text{PolyLog}\left(2,-ie^{ix}\right)+i\text{PolyLog}\left(2,ie^{ix}\right)+2x\sin(x)+2\cos(x)+2ix\tan^{-1}\left(e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x],x]
```

```
[Out] (2*I)*x*ArcTan[E^(I*x)] + 2*Cos[x] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLog
[2, I*E^(I*x)] + 2*x*Sin[x]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int x\cos(2x)\sec(x)\,dx &= \int (x\cos(x)-x\sin(x)\tan(x))\,dx\\ &= \int x\cos(x)\,dx-\int x\sin(x)\tan(x)\,dx\\ &= x\sin(x)+\int x\cos(x)\,dx-\int x\sec(x)\,dx-\int \sin(x)\,dx\\ &= 2ix\tan^{-1}\left(e^{ix}\right)+\cos(x)+2x\sin(x)+\int \log\left(1-ie^{ix}\right)\,dx-\int \log\left(1+ie^{ix}\right)\,dx-\int \sin(x)\,dx\\ &= 2ix\tan^{-1}\left(e^{ix}\right)+2\cos(x)+2x\sin(x)-i\,\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)+i\,\text{Subst}\left(\int \frac{\log(1+i}{x}\right.\\ &= 2ix\tan^{-1}\left(e^{ix}\right)+2\cos(x)-i\text{Li}_2\left(-ie^{ix}\right)+i\text{Li}_2\left(ie^{ix}\right)+2x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.031047, size = 77, normalized size = 1.35

$$-i\left(\text{PolyLog}\left(2,-ie^{ix}\right)-\text{PolyLog}\left(2,ie^{ix}\right)\right)-x\left(\log\left(1-ie^{ix}\right)-\log\left(1+ie^{ix}\right)\right)+2x\sin(x)+2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x],x]
```

```
[Out] 2*Cos[x] - x*(Log[1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) - I*(PolyLog[2, (-I)
*E^(I*x)] - PolyLog[2, I*E^(I*x)]) + 2*x*Sin[x]
```

Maple [A] time = 0.192, size = 81, normalized size = 1.4

$$-i(x+i)\,\mathrm{e}^{ix}+i(x-i)\,\mathrm{e}^{-ix}+x\ln\left(1+i\mathrm{e}^{ix}\right)-x\ln\left(1-i\mathrm{e}^{ix}\right)-i\mathrm{dilog}\left(1+i\mathrm{e}^{ix}\right)+i\mathrm{dilog}\left(1-i\mathrm{e}^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x),x)
```

```
[Out] -I*(x+I)*exp(I*x)+I*(x-I)*exp(-I*x)+x*ln(1+I*exp(I*x))-x*ln(1-I*exp(I*x))-I
*dilog(1+I*exp(I*x))+I*dilog(1-I*exp(I*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$2\,x\sin(x)+2\,\cos(x)-2\int\frac{x\cos(2\,x)\cos(x)+x\sin(2\,x)\sin(x)+x\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2+2\,\cos(2\,x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x, algorithm="maxima")
```

```
[Out] 2*x*sin(x) + 2*cos(x) - 2*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x)
+ x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x)
```

Fricas [B] time = 0.529158, size = 405, normalized size = 7.11

$$-\frac{1}{2}\,x\log(i\,\cos(x)+\sin(x)+1)+\frac{1}{2}\,x\log(i\,\cos(x)-\sin(x)+1)-\frac{1}{2}\,x\log(-i\,\cos(x)+\sin(x)+1)+\frac{1}{2}\,x\log(-i\,\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x, algorithm="fricas")
```

```
[Out] -1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - sin(x) + 1) - 1/2*
x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(x) + 1) + 2*x*sin
(x) + 2*cos(x) + 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) - si
n(x)) - 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(2x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x)
```

```
[Out] Integral(x*cos(2*x)*sec(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(2x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x, algorithm="giac")
```

```
[Out] integrate(x*cos(2*x)*sec(x), x)
```

3.396 $\int x\cos(2x)\sec^2(x)\,dx$

Optimal. Leaf size=14

$$x^2 - x\tan(x) - \log(\cos(x))$$

```
[Out] x^2 - Log[Cos[x]] - x*Tan[x]
```

Rubi [A] time = 0.0331439, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4431, 3720, 3475, 30}

$$x^2 - x\tan(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x]^2,x]
```

```
[Out] x^2 - Log[Cos[x]] - x*Tan[x]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\cos(2x)\sec^2(x)\,dx &= \int \left(x - x\tan^2(x)\right)\,dx\\ &= \frac{x^2}{2} - \int x\tan^2(x)\,dx\\ &= \frac{x^2}{2} - x\tan(x) + \int x\,dx + \int \tan(x)\,dx\\ &= x^2 - \log(\cos(x)) - x\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0232502, size = 14, normalized size = 1.

$$x^2 - x\tan(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x]^2,x]
```

```
[Out] x^2 - Log[Cos[x]] - x*Tan[x]
```

Maple [A] time = 0.042, size = 15, normalized size = 1.1

$$x^2 - \ln(\cos(x)) - x\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x)^2,x)
```

```
[Out] x^2-ln(cos(x))-x*tan(x)
```

Maxima [B] time = 1.56808, size = 150, normalized size = 10.71

$$\frac{2\,x^2\cos(2\,x)^2 + 2\,x^2\sin(2\,x)^2 + 4\,x^2\cos(2\,x) + 2\,x^2 - \left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1\right)\log\left(\cos(2\,x)^2 + \sin(2\,x\right.}{2\left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*cos(2*x)^2 + 2*x^2*sin(2*x)^2 + 4*x^2*cos(2*x) + 2*x^2 - (cos(2*
x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x
) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 0.486144, size = 73, normalized size = 5.21

$$\frac{x^2\cos(x) - \cos(x)\log(-\cos(x)) - x\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="fricas")
```

```
[Out] (x^2*cos(x) - cos(x)*log(-cos(x)) - x*sin(x))/cos(x)
```

Sympy [B] time = 53.0657, size = 144, normalized size = 10.29

$$x^2 + \frac{2x\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)-1} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)**2,x)
```

```
[Out] x**2 + 2*x*tan(x/2)/(tan(x/2)**2 - 1) - log(tan(x/2) - 1)*tan(x/2)**2/(tan(
x/2)**2 - 1) + log(tan(x/2) - 1)/(tan(x/2)**2 - 1) - log(tan(x/2) + 1)*tan(
x/2)**2/(tan(x/2)**2 - 1) + log(tan(x/2) + 1)/(tan(x/2)**2 - 1) + log(tan(x
/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**2 - 1) - log(tan(x/2)**2 + 1)/(tan(x/2)*
*2 - 1)
```

Giac [B] time = 1.16055, size = 159, normalized size = 11.36

$$\frac{2\,x^2\tan\left(\frac{1}{2}\,x\right)^2-\log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4-2\,\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2-2\,x^2+4\,x\tan\left(\frac{1}{2}\,x\right)+\log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4-2\,\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)}{2\left(\tan\left(\frac{1}{2}\,x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*x^2*tan(1/2*x)^2 - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*x^2 + 4*x*tan(1/2*x) + log(4
*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/
(tan(1/2*x)^2 - 1)
```

3.397 $\int x\cos(2x)\sec^3(x)\,dx$

Optimal. Leaf size=67

$$\frac{3}{2}i\text{PolyLog}\left(2,-ie^{ix}\right)-\frac{3}{2}i\text{PolyLog}\left(2,ie^{ix}\right)-3ix\tan^{-1}\left(e^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\tan(x)\sec(x)$$

```
[Out] (-3*I)*x*ArcTan[E^(I*x)] + ((3*I)/2)*PolyLog[2, (-I)*E^(I*x)] - ((3*I)/2)*P
olyLog[2, I*E^(I*x)] + Sec[x]/2 - (x*Sec[x]*Tan[x])/2
```

Rubi [A] time = 0.137247, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {4431, 4181, 2279, 2391, 4413, 4185}

$$\frac{3}{2}i\text{PolyLog}\left(2,-ie^{ix}\right)-\frac{3}{2}i\text{PolyLog}\left(2,ie^{ix}\right)-3ix\tan^{-1}\left(e^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x]^3,x]
```


```
[Out] (-3*I)*x*ArcTan[E^(I*x)] + ((3*I)/2)*PolyLog[2, (-I)*E^(I*x)] - ((3*I)/2)*P
olyLog[2, I*E^(I*x)] + Sec[x]/2 - (x*Sec[x]*Tan[x])/2
```


Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int x\cos(2x)\sec^3(x)\,dx &= \int \left(x\sec(x)-x\sec(x)\tan^2(x)\right)\,dx\\
&= \int x\sec(x)\,dx-\int x\sec(x)\tan^2(x)\,dx\\
&= -2ix\tan^{-1}\left(e^{ix}\right)-\int \log\left(1-ie^{ix}\right)\,dx+\int \log\left(1+ie^{ix}\right)\,dx+\int x\sec(x)\,dx-\int x\sec^3(x)\,dx\\
&= -4ix\tan^{-1}\left(e^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)+i\,\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)-i\,\mathrm{Subst}\left(\int\right.\\
&= -3ix\tan^{-1}\left(e^{ix}\right)+i\mathrm{Li}_2\left(-ie^{ix}\right)-i\mathrm{Li}_2\left(ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)+i\,\mathrm{Subst}\left(\int \frac{\log(1-i}{x}\right.\\
&= -3ix\tan^{-1}\left(e^{ix}\right)+2i\mathrm{Li}_2\left(-ie^{ix}\right)-2i\mathrm{Li}_2\left(ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)-\frac{1}{2}i\,\mathrm{Subst}\left(\int \frac{\log(1}{}\right.\\
&= -3ix\tan^{-1}\left(e^{ix}\right)+\frac{3}{2}i\mathrm{Li}_2\left(-ie^{ix}\right)-\frac{3}{2}i\mathrm{Li}_2\left(ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)
\end{aligned}$$

Mathematica [B] time = 0.27406, size = 146, normalized size = 2.18

$$\frac{1}{4}\left(6i\text{PolyLog}\left(2,-ie^{ix}\right)-6i\text{PolyLog}\left(2,ie^{ix}\right)+6x\log\left(1-ie^{ix}\right)-6x\log\left(1+ie^{ix}\right)+\frac{x}{\sin(x)-1}+\frac{x}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x]^3,x]
```

```
[Out] (6*x*Log[1 - I*E^(I*x)] - 6*x*Log[1 + I*E^(I*x)] + (6*I)*PolyLog[2, (-I)*E^
(I*x)] - (6*I)*PolyLog[2, I*E^(I*x)] + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2]) +
 x/(Cos[x/2] + Sin[x/2])^2 - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + x/(-1 + S
in[x]))/4
```

Maple [B] time = 0.223, size = 102, normalized size = 1.5

$$\frac{i\left(x e^{3ix}-x e^{ix}-i e^{3ix}-i e^{ix}\right)}{\left(e^{2ix}+1\right)^2}-\frac{3x\ln\left(1+ie^{ix}\right)}{2}+\frac{3x\ln\left(1-ie^{ix}\right)}{2}+\frac{3i}{2}\text{dilog}\left(1+ie^{ix}\right)-\frac{3i}{2}\text{dilog}\left(1-ie^{ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x)^3,x)
```

```
[Out] I/(exp(2*I*x)+1)^2*(x*exp(3*I*x)-x*exp(I*x)-I*exp(3*I*x)-I*exp(I*x))-3/2*x*
ln(1+I*exp(I*x))+3/2*x*ln(1-I*exp(I*x))+3/2*I*dilog(1+I*exp(I*x))-3/2*I*dil
og(1-I*exp(I*x))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 0.538735, size = 500, normalized size = 7.46

$$\frac{3\,x\cos(x)^2\log(i\,\cos(x)+\sin(x)+1)-3\,x\cos(x)^2\log(i\,\cos(x)-\sin(x)+1)+3\,x\cos(x)^2\log(-i\,\cos(x)+\sin(x)+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(I*cos(x) -
sin(x) + 1) + 3*x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(-
I*cos(x) - sin(x) + 1) - 3*I*cos(x)^2*dilog(I*cos(x) + sin(x)) - 3*I*cos(x)
^2*dilog(I*cos(x) - sin(x)) + 3*I*cos(x)^2*dilog(-I*cos(x) + sin(x)) + 3*I*
cos(x)^2*dilog(-I*cos(x) - sin(x)) - 2*x*sin(x) + 2*cos(x))/cos(x)^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\cos(2\,x)\sec(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(2*x)*sec(x)^3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
   as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```